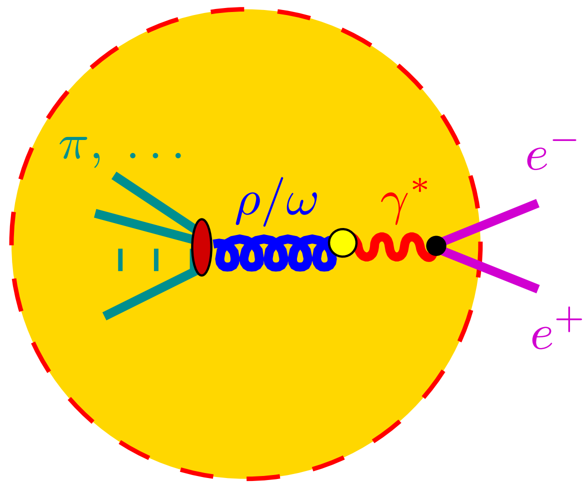
Symmetries and Self-consistency

Vector mesons in the fireball



Hendrik van Hees

Fakultät für Physik

Universität Bielefeld

Content

- Concepts
  - Real time formalism
  - 2PI formalism
  - Symmetries and trouble with 2PI formalism

Content

- Concepts
  - Real time formalism
  - 2PI formalism
  - Symmetries and trouble with 2PI formalism
- Application to the $\pi\rho$-system
  - Kroll-Lee-Zumino (KLZ) model
  - Perturbative results
  - 2PI: Trouble in paradise (due to violation of symmetries)
  - First way out: Projection formalism
  - First toy calculations:
- Outlook

Real time formalism

- Initial statistical operator $\boldsymbol{\rho}_i$ at $t = t_i$
- Time evolution

$$\langle O(t)\rangle = \mathrm{Tr}\left[\boldsymbol{\rho}(t_i)\,\underbrace{\mathcal{T}_a\left\{\exp\left[+\mathsf{i}\int_{t_i}^{t}\mathrm{d}t'\,\mathbf{H}_I(t')\right]\right\}}_{\text{anti time–orderd}}\right.$$

$$\mathbf{O}_I(t)$$

$$\left.\underbrace{\mathcal{T}_c\left\{\exp\left[-\mathsf{i}\int_{t_i}^{t}\mathrm{d}t'\,\mathbf{H}_I(t')\right]\right\}}_{\text{time–ordered}}\right].$$

Real time formalism

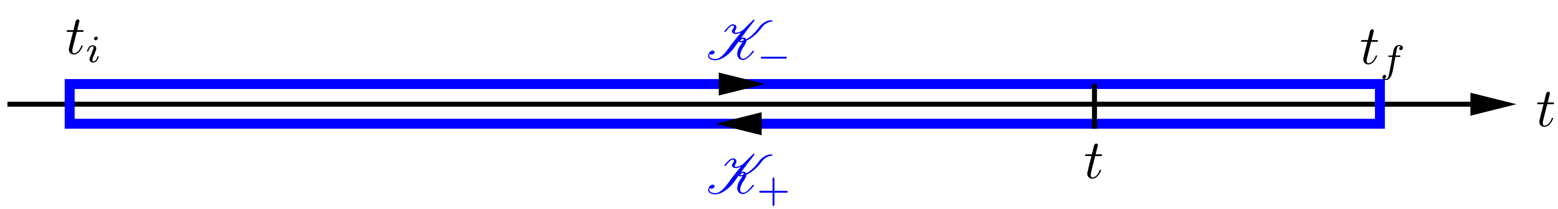
- Initial statistical operator $\boldsymbol{\rho}_i$ at $t = t_i$
- Time evolution

$$\langle O(t)\rangle = \mathrm{Tr}\Bigg[\boldsymbol{\rho}(t_i)\,\underbrace{\mathcal{T}_a\left\{\exp\left[+\mathrm{i}\int_{t_i}^{t}\mathrm{d}t'\,\mathbf{H}_I(t')\right]\right\}}_{\text{anti time–orderd}}$$

$$\mathbf{O}_I(t)$$

$$\underbrace{\mathcal{T}_c\left\{\exp\left[-\mathrm{i}\int_{t_i}^{t}\mathrm{d}t'\,\mathbf{H}_I(t')\right]\right\}}_{\text{time–ordered}}\Bigg].$$

- Contour ordered Green's functions



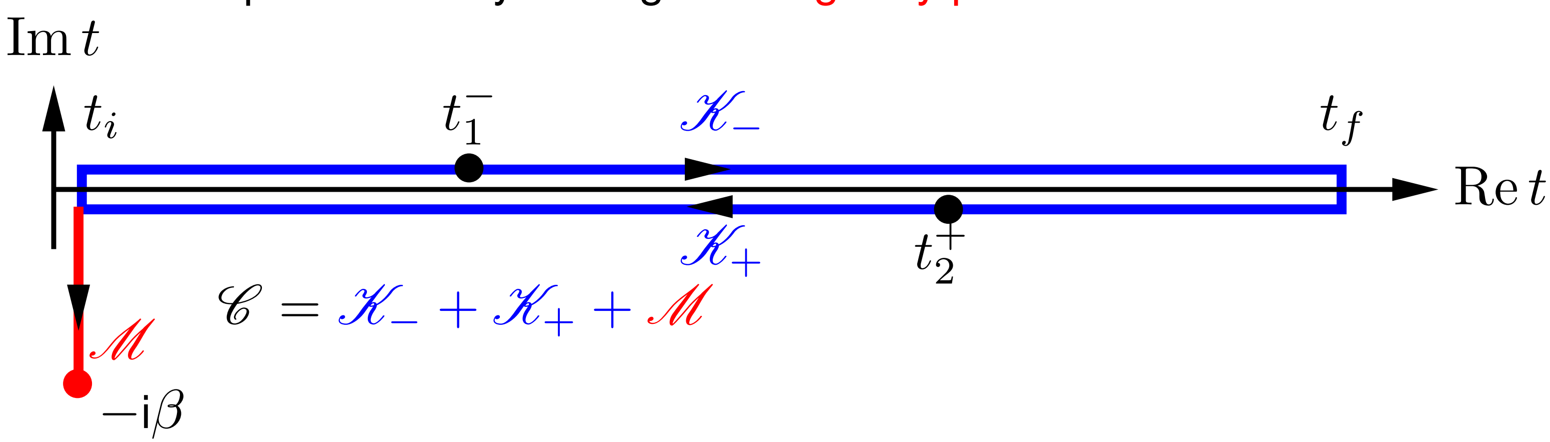
$$\mathscr{C} = \mathscr{K}_- + \mathscr{K}_+$$

Real-time formalism: Equilibrium

- In equilibrium

$$\boldsymbol{\rho} = \exp(-\beta \mathbf{H})/Z \text{ with } Z = \mathrm{Tr}\exp(-\beta \mathbf{H}), \quad \beta = 1/T$$

- Can be implemented by adding an imaginary part to the contour



- Correlation functions with real times: $\mathrm{i}G_{\mathscr{C}}(x_1^-, x_2^+)$
- Fields periodic (bosons) or anti-periodic (fermions) in imaginary time
- Feynman rules $\Rightarrow$ time integrals $\rightarrow$ contour integrals

2PI-Formalism: The Φ-functional

- Introduce local and bilocal sources

$$Z[J,K] = N \int \mathrm{D}\phi \exp\left[\mathrm{i}S[\phi] + \mathrm{i}\,\{J_1\phi_1\}_1 + \left\{\frac{\mathrm{i}}{2}K_{12}\phi_1\phi_2\right\}_{12}\right]$$

- Generating functional for connected diagrams

$$Z[J,K] = \exp(\mathrm{i}W[J,K])$$

- The mean field and the connected Green's function

$$\underbrace{\varphi_1 = \frac{\delta W}{\delta J_1},\ G_{12} = -\frac{\delta^2 W}{\delta J_1 \delta J_2}}_{\text{standard quantum field theory}} \Rightarrow \frac{\delta W}{\delta K_{12}} = \frac{1}{2}[\varphi_1\varphi_2 + \mathrm{i}G_{12}]$$

- Legendre transformation for φ and G:

$$\Gamma[\varphi, G] = W[J,K] - \{\varphi_1 J_1\}_1 - \frac{1}{2}\{(\varphi_1\varphi_2 + \mathrm{i}G_{12})K_{12}\}_{12}$$

2PI-formalism: The Φ-functional

- Exact closed form:

$$\mathbb{\Gamma}[\varphi, G] = S_0[\varphi] + \frac{\mathrm{i}}{2}\,\mathrm{Tr}\ln(-\mathrm{i}G^{-1}) + \frac{\mathrm{i}}{2}\left\{D_{12}^{-1}(G_{12} - D_{12})\right\}_{12}$$

$$+ \Phi[\varphi, G] \Leftarrow \text{all closed 2PI interaction diagrams}, \quad D_{12} = \left(-\Box - m^2\right)^{-1}$$

- Equations of motion

$$\frac{\delta \mathbb{\Gamma}}{\delta \varphi_1} = -J_1 - \{K_{12}\varphi_2\}_2 \stackrel{!}{=} 0, \quad \frac{\delta \mathbb{\Gamma}}{\delta G_{12}} = -\frac{\mathrm{i}}{2} K_{12} \stackrel{!}{=} 0,$$

- Equation of motion for the mean field φ and the “full” propagator G

$$-\Box\varphi - m^2\varphi := j = -\frac{\delta \Phi}{\delta \varphi}, \quad -\mathrm{i}(D_{12}^{-1} - {G_{12}}^{-1}) := -\mathrm{i}\Sigma = 2\frac{\delta \Phi}{\delta G_{21}}$$

- Integral form of Dyson’s equation:

$$G_{12} = D_{12} + \{D_{11'}\Sigma_{1'2'}G_{2'2}\}_{1'2'}$$

- Closed set of equations of for φ and G

2PI-formalism: Features

- Truncation of the Series of diagrams for Φ
- Expectation values for currents are conserved
  $\Rightarrow$ "Conserving Approximations"
- In equilibrium i$\Gamma[\varphi, G] = \ln Z(\beta)$
  (thermodynamical potential)
- consistent treatment of Dynamical quantities (real time formalism) and thermodynamical bulk properties (imaginary time formalism) like energy, pressure, entropy
- Real- and Imaginary-Time quantities "glued" together by Analytic properties from (anti-)periodicity conditions of the fields (KMS-condition)
- Self-consistent set of equations for self-energies and mean fields

Symmetries

- Problem with Φ–Functional: Most approximations break symmetry!
- Reason: Only conserving for Expectation values for currents, not for correlation functions
- Dyson's equation as functional of φ:

$$\left. \frac{\delta \mathbb{\Gamma}[\varphi, G]}{\delta G} \right|_{G = G_{\text{eff}}[\varphi]} \equiv 0$$

- Define new effective action functional

$$\Gamma_{\text{eff}}[\varphi] = \mathbb{\Gamma}[\varphi, G_{\text{eff}}[\varphi]]$$

- Symmetry analysis $\Rightarrow \Gamma_{\text{eff}}[\varphi]$ symmetric functional in φ
- Stationary point

$$\left. \frac{\delta \Gamma_{\text{eff}}}{\delta \phi} \right|_{\varphi = \varphi_0} = 0$$

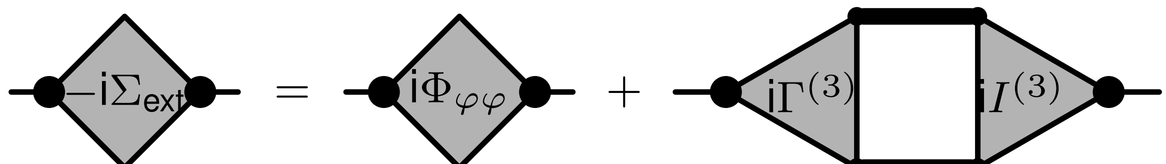
- φ_0 and $G = G_{\text{eff}}[\varphi_0]$: self–consistent Φ–Functional solutions!

Symmetries

- Γ_{eff} generates external vertex functions fulfilling Ward–Takahashi identities of symmetries
- External Propagator

$$(G_{\text{ext}}^{-1})_{12} = \left. \frac{\delta^2 \Gamma_{\text{eff}}[\varphi]}{\delta\varphi_1 \delta\varphi_2} \right|_{\varphi=\varphi_0}$$

- G_{ext} generally not identical with Dyson resummed propagator
- Problem: Calculation of Σ_{ext} needs resummation of Bethe-Salpeter ladders



Gauge theories

- Abelian massive gauge boson: Do not need Higgs! (Stueckelberg formalism)
- Gauge invariant classical Lagrangian:

$$\mathscr{L} = -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} m^2 V_\mu V^\mu + \frac{1}{2} (\partial^\mu \varphi)(\partial_\mu \varphi) + m \varphi \partial_\mu V^\mu$$

- Gauge invariance:

$$\delta V_\mu(x) = \partial_\mu \chi(x), \; \delta\varphi = m\chi(x)$$

- Quantisation: Gauge fixing and ghosts

$$\begin{aligned} \mathscr{L}_V = & -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{m}{2} V_\mu V^\mu - \frac{1}{2\xi} (\partial_\mu V^\mu)^2 + \\ & + \frac{1}{2} (\partial_\mu \varphi)(\partial^\mu \varphi) - \frac{\xi m^2}{2} \varphi^2 + \\ & + (\partial_\mu \eta^*)(\partial_\mu \eta) - \xi m^2 \eta^* \eta . \end{aligned}$$

Gauge theories

- Free vacuum propagators

$$\Delta_V^{\mu\nu}(p) = -\frac{g^{\mu\nu}}{p^2 - m^2 + \mathrm{i}\eta} + \frac{(1-\xi)p^\mu p^\nu}{(p^2 - m^2 + \mathrm{i}\eta)(p^2 - \xi m^2 + \mathrm{i}\eta)}$$

$$\Delta_\varphi(p) = \frac{1}{p^2 - \xi m^2 + \mathrm{i}\eta}$$

$$\Delta_\eta(p) = \frac{1}{p^2 - \xi m^2 + \mathrm{i}\eta}.$$

- Usual power counting $\Rightarrow$ renormalisable
- Partition sum: Three bosonic degrees of freedom!

The π-ρ-system: KLZ-action

- Adding $\pi^{\pm}$ and γ
- Gauge–covariant derivative

$$\mathrm{D}_\mu \pi = \partial_\mu \pi + \mathrm{i} g V_\mu \pi + \mathrm{i} e A_\mu$$

- Quantisation of free photon as usual
- Minimal coupling and KLZ-interaction:

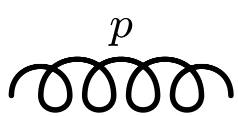
$$\mathscr{L}_{\pi V} = \mathscr{L}_V + (\mathrm{D}_\mu \pi)^* (\mathrm{D}^\mu \pi) - m_\pi^2 \pi^* \pi - \frac{\lambda}{8} (\pi^* \pi)^2 - \frac{e}{2 g_{\rho\gamma}} A_{\mu\nu} V^{\mu\nu}$$

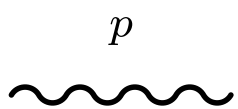
- Eqs. of motion: Vector meson dominance (Kroll, Lee, Zumino) Photons couple to pions only over “mixing” with (neutral) ρ-mesons!
- Adding Leptons like in usual QED:

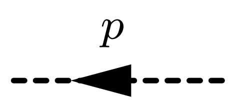
$$\mathscr{L}_{e\gamma} = \bar{\psi}(\mathrm{i}\not{\mathrm{D}} - m_e)\psi \quad \text{with} \quad \mathrm{D}_\mu \psi = \partial_\mu \psi - \mathrm{i} e A_\mu \psi$$

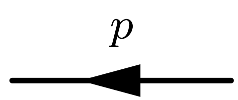
The Feynman rules

- Propagators

ρ-meson: μ  ν $= -\frac{\mathrm{i}g^{\mu\nu}}{p^2 - m_\rho^2 + \mathrm{i}\eta} + \frac{\mathrm{i}(1-\xi_\rho)p^\mu p^\nu}{(p^2 - m_\rho^2 + \mathrm{i}\eta)(p^2 - \xi m_\rho^2 + \mathrm{i}\eta)}$

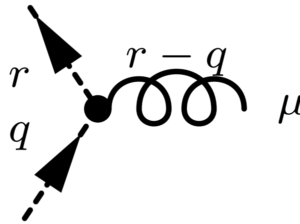
Photon: μ  ν $= -\frac{\mathrm{i}g^{\mu\nu}}{p^2 + \mathrm{i}\eta} + \frac{\mathrm{i}(1-\xi_\gamma)p^\mu p^\nu}{(p^2 + \mathrm{i}\eta)^2}$

Pion:  $= \frac{\mathrm{i}}{p^2 - m_\pi^2 + \mathrm{i}\eta}$

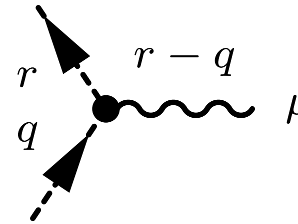
Elektron:  $= \frac{\mathrm{i}(\not{p} + m)}{p^2 - m_l^2 + \mathrm{i}\eta}$

The Feynman rules

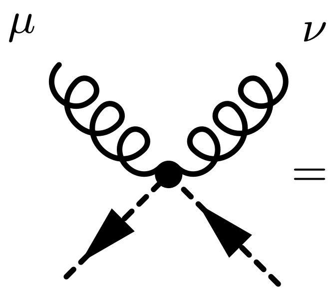
- The vertices



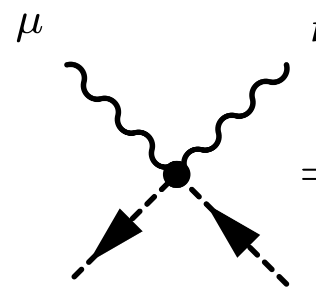
$$= -\mathrm{i}g_{\rho\pi}(q^{\mu} + r^{\mu})$$



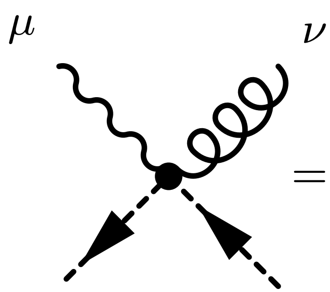
$$= -\mathrm{i}e(q^{\mu} + r^{\mu})$$



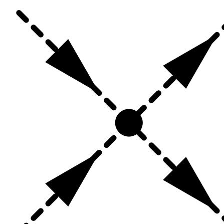
$$= \mathrm{i}g_{\rho\pi}^{2} g^{\mu\nu}$$



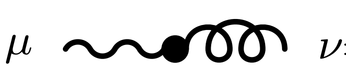
$$= \mathrm{i}e^{2} g^{\mu\nu}$$



$$= 2\mathrm{i}e g_{\rho\pi} g^{\mu\nu}$$



$$= -\frac{\mathrm{i}\lambda}{8}$$

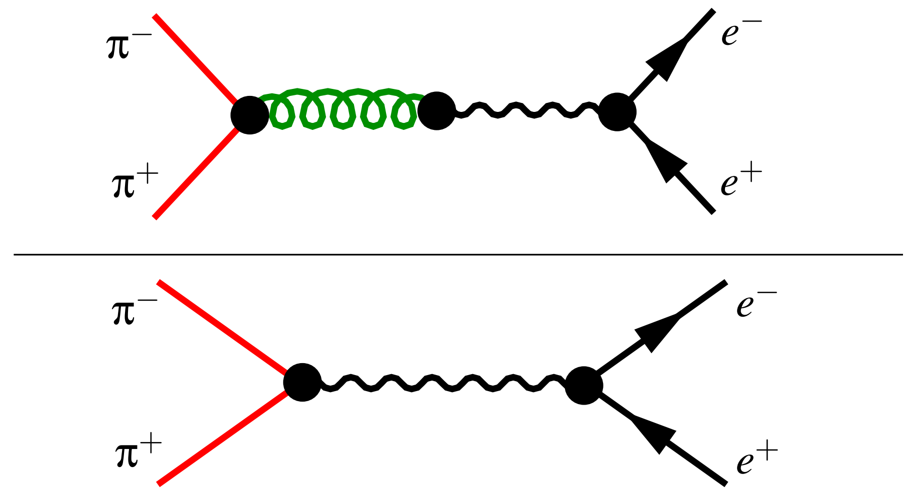


$$= \mathrm{i}\frac{e}{g_{\rho\gamma}} p^{2} \Theta^{\mu\nu}(p)$$

Pion form factor

- Definition

$F(k^2) =$

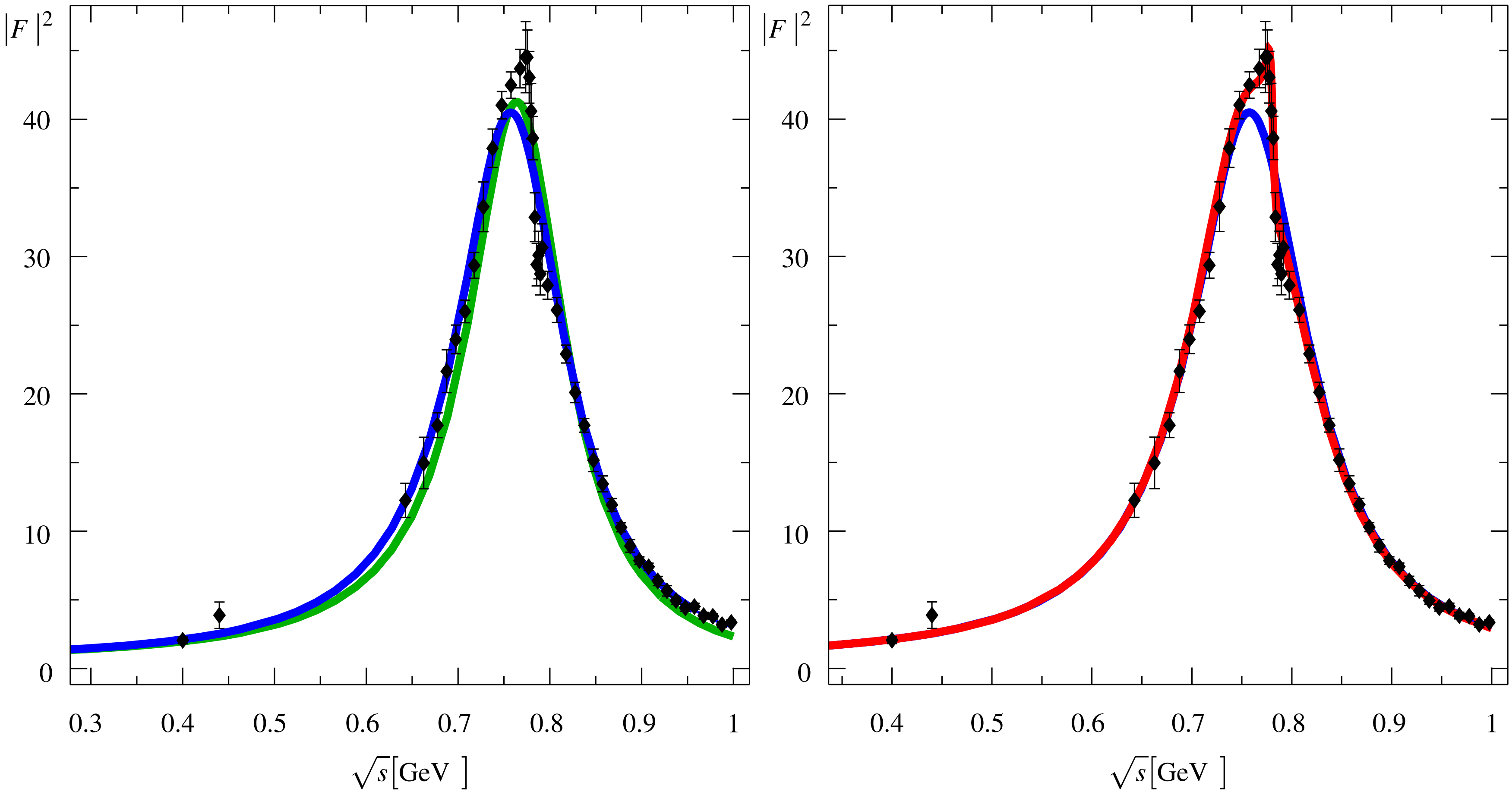


- Measurable quantity: Pion form factor

$$|F(s)|^2 = \frac{m_\rho^4}{|s - m_\rho^2 - \Pi_\rho(s)|^2}$$

- Π_ρ: ρ-self-energy!

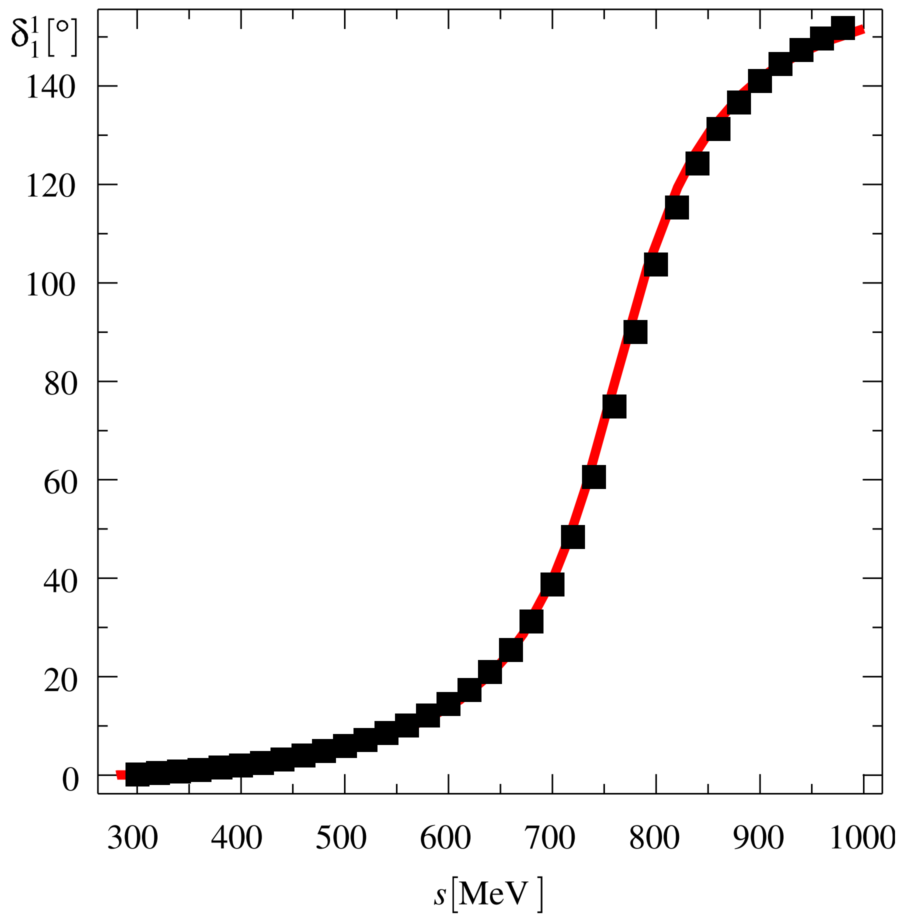
Pion form factor



- "Strict" vector-dominance, i.e., $g_{\rho\gamma} = g$
- Including corrections from ρ-ω mixing
- Fit with $g_{\rho\gamma} \neq g$
- data: L. M. Barkov, et al., Electromagnetic Pion Form Factor in the Timelike Region, Nucl. Phys. B256 (1985) 365

Pion phase-shift δ_1^1

- Phase shift in the $s = 1$, $t = 1$-channel of pion scattering



- Plot with data from form-factor fit (without ω mixing)
- data: C. D. Frogatt, J. L. Petersen. Phase-Shift Analysis of $\pi^+\pi^-$ Scattering between 1.0 and 1.8 GeV Based on Fixed Transfer Analyticity (II). Nuclear Physics B129 (1977) 89

Big trouble in paradise!

- Kroll–Lee–Zumino interaction: Coupling of massive vector bosons to conserved currents $\Rightarrow$ gauge theory
- Symmetry breaking at correlator level
- Internal propagators contain spurious degrees of freedom
- Negative norm states
- Numerically instable due to light cone singularities

Digression: Classical transport picture

- Classical picture (Fokker–Planck–equation):

$$\Pi^{\mu\nu}(\tau, \vec{p} = 0) \propto \langle v^\mu(\tau) v^\nu(0) \rangle$$

- „One–loop" approximation in the classical limit

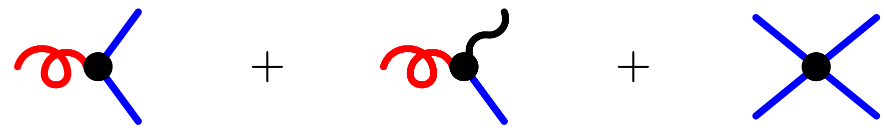
$$\Pi^{\mu\nu}(\tau, \vec{p} = 0) \propto \exp(-\Gamma\tau)$$

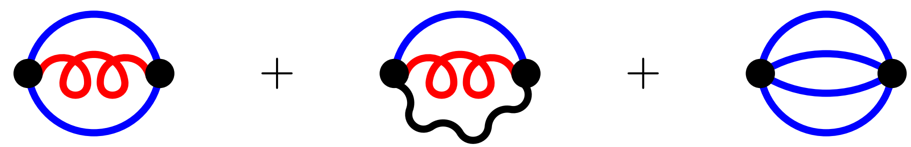
- $1/\Gamma$: Relaxation time scale due to scattering
- Exact behaviour due to Conservation law:

$$\Pi^{00}(\tau, \vec{p} = 0) \propto \langle 1 \cdot 1 \rangle = \text{const}, \quad \Pi^{jk}(\tau, \vec{p} = 0) \propto \left\langle v^j v^k \right\rangle \propto \exp(-\Gamma_x \tau)$$

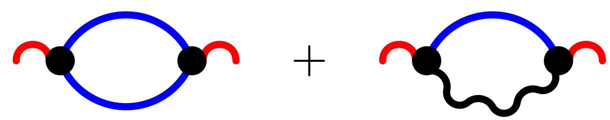
- For Π^{jk}: If $\Gamma \approx \Gamma_x \Rightarrow$ 1–loop approximation justified
- Classical limit also shows: Π^{jk} only slightly modified by ladder resummation
- For self–consistent approximations: Use only $p_j p_k \Pi^{jk}$ and $g_{jk} \Pi^{jk}$
- Construct Π_T and Π_L

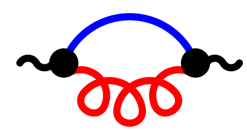
First toy calculations

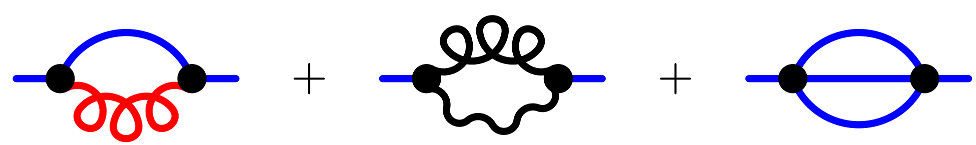
Lagrangian: $\mathscr{L}_{\text{int}} =$ 

$\Phi =$ 

Self–energies:

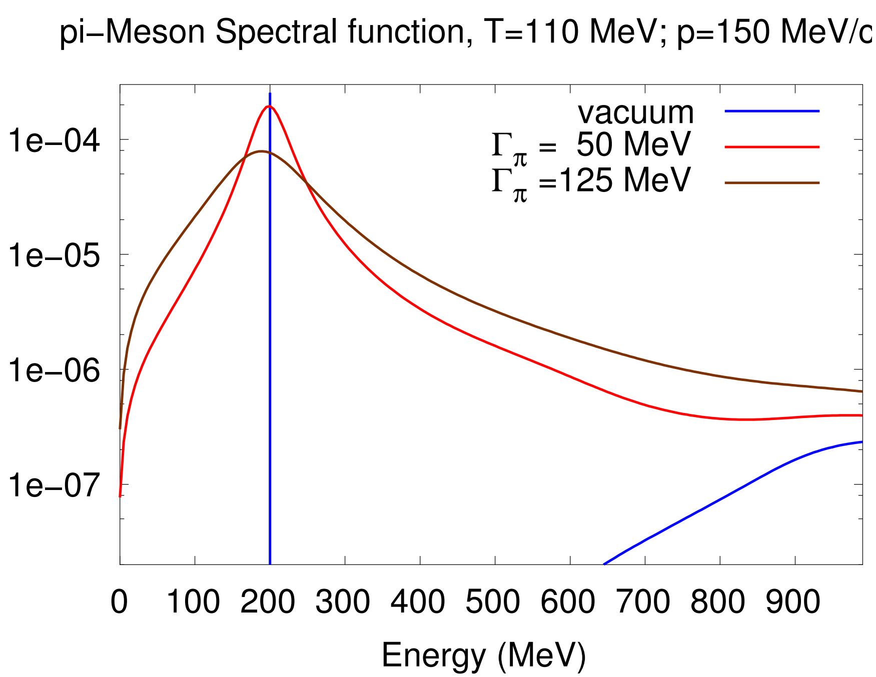
$\Pi_\rho =$ 

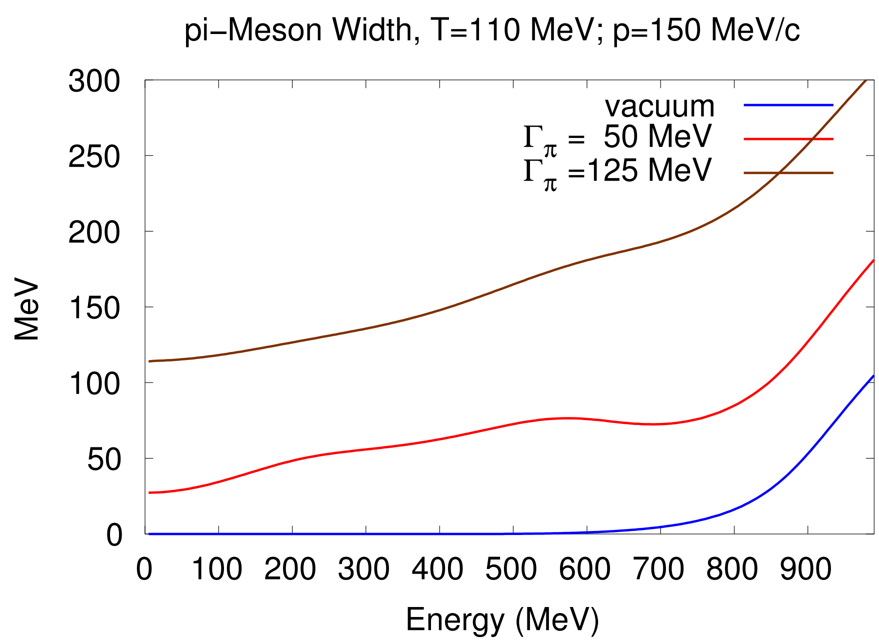
$\Pi_{a_1} =$ 

$\Sigma_\pi =$ 

Results: Broad pions in the medium

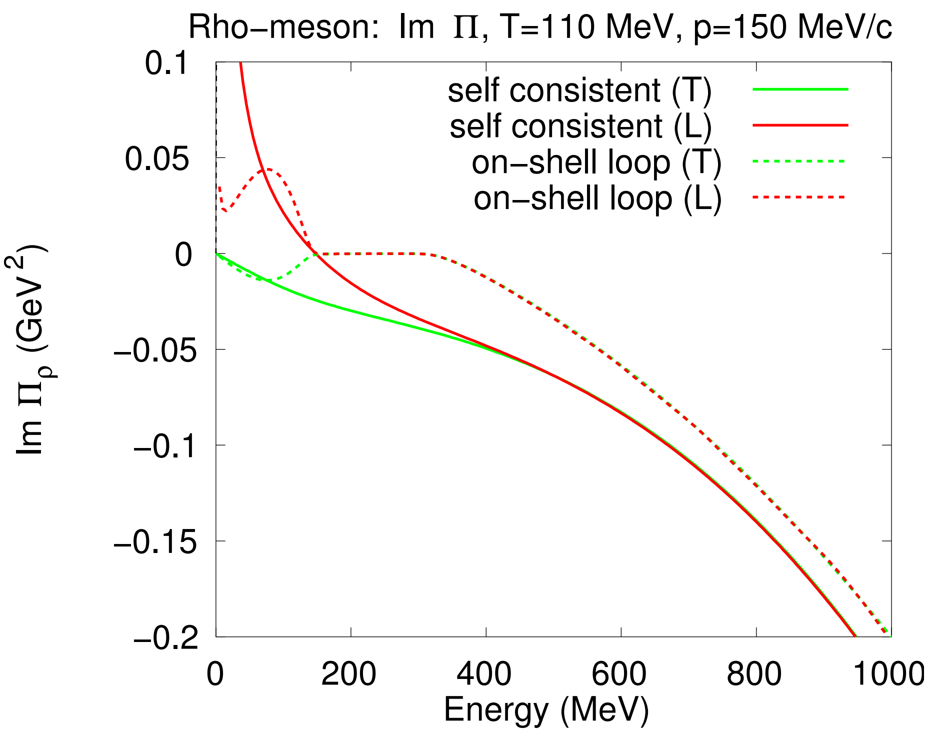
- Avoided renormalization problem: Took only imaginary parts of the self-energies!
- Tuned "π^4-interaction such that pions get an arbitrarily chosen width
- In "reality": pion-width due to interactions with baryons (not included in the toy model yet!)

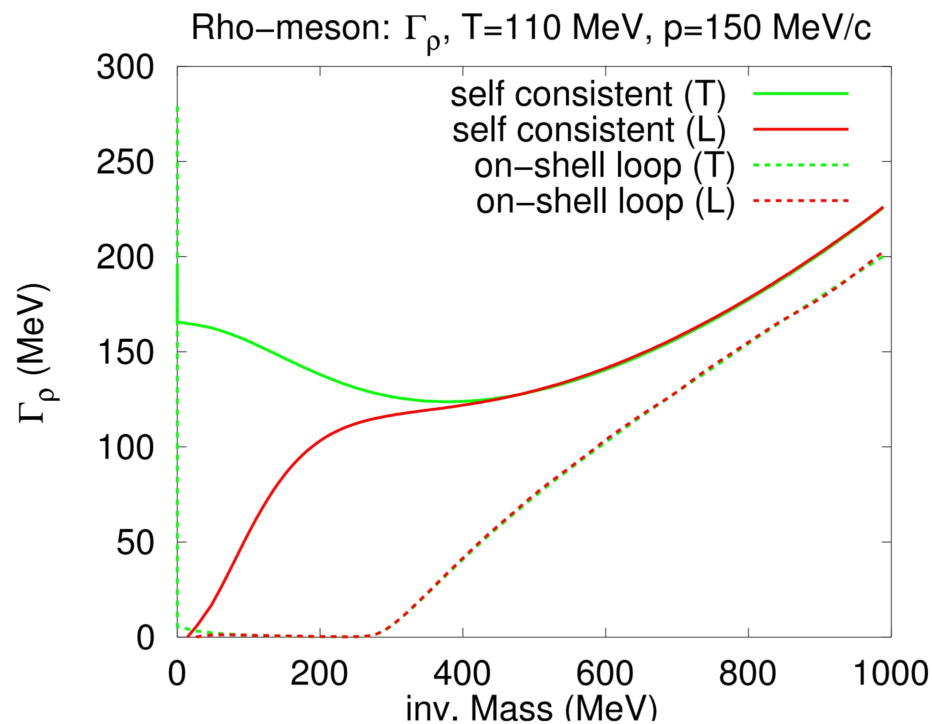




Results: ρ-meson spectral functions

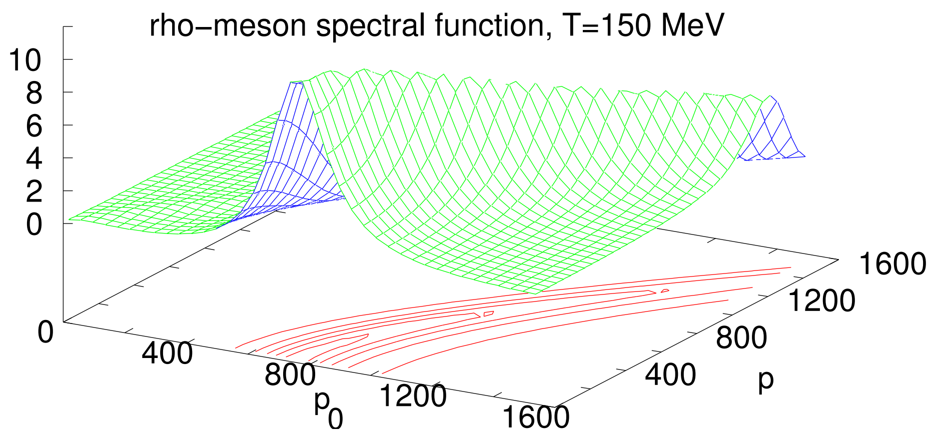
- 3D transverse and longitudinal quantities: Π_T, Π_L etc.

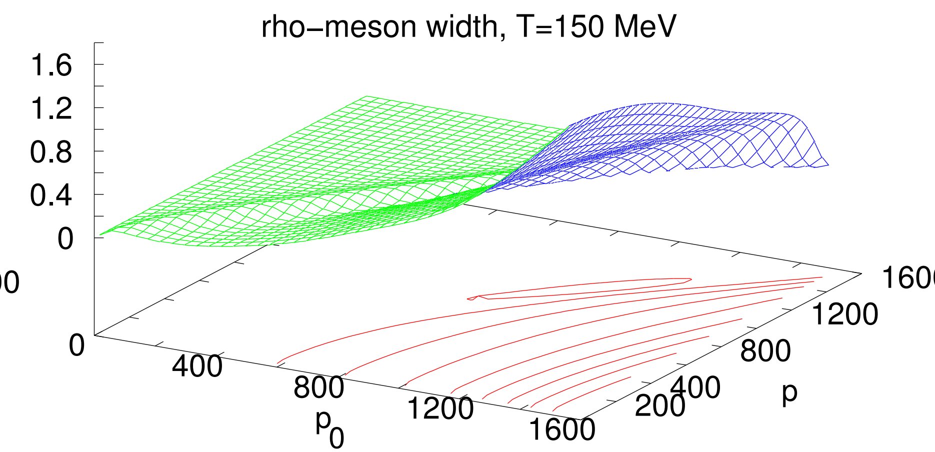




Results: ρ-meson properties

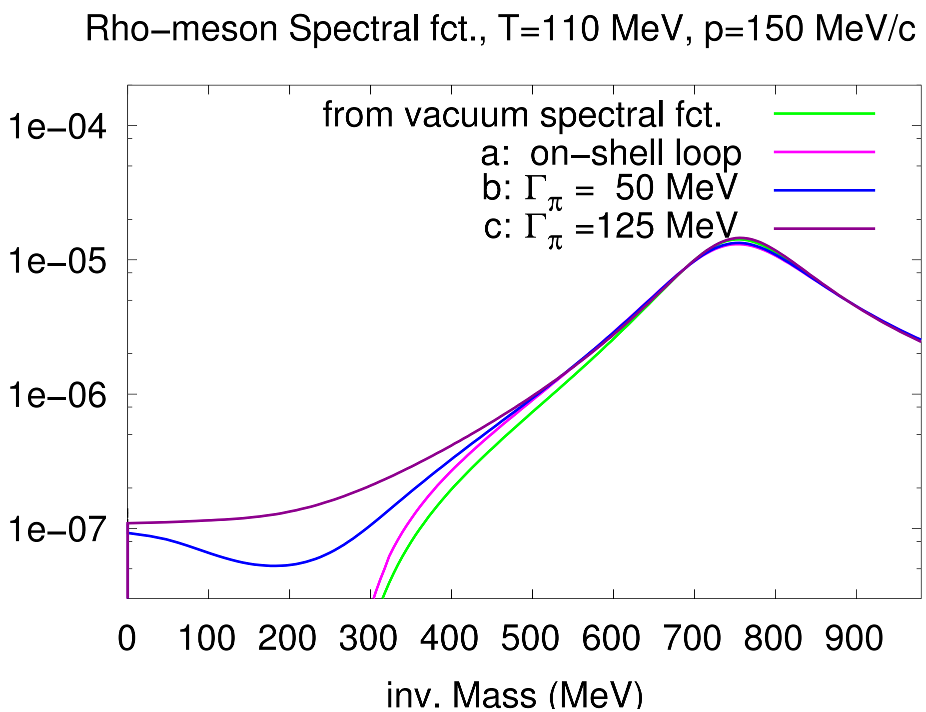
- Averaged quantities: $(2\Pi_T + \Pi_L)/3$

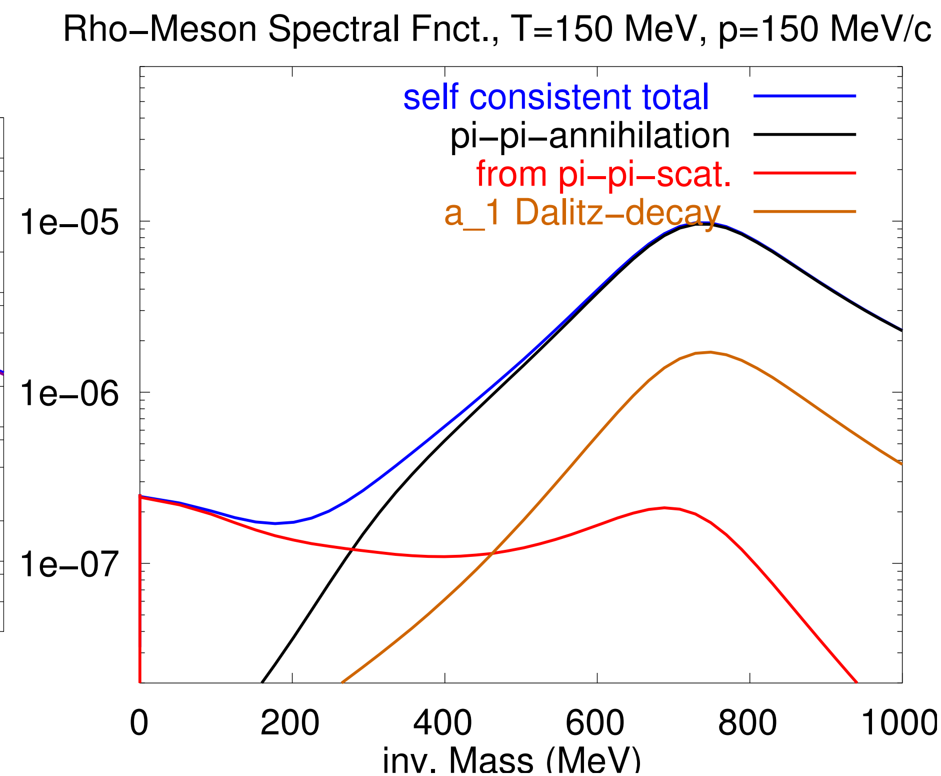




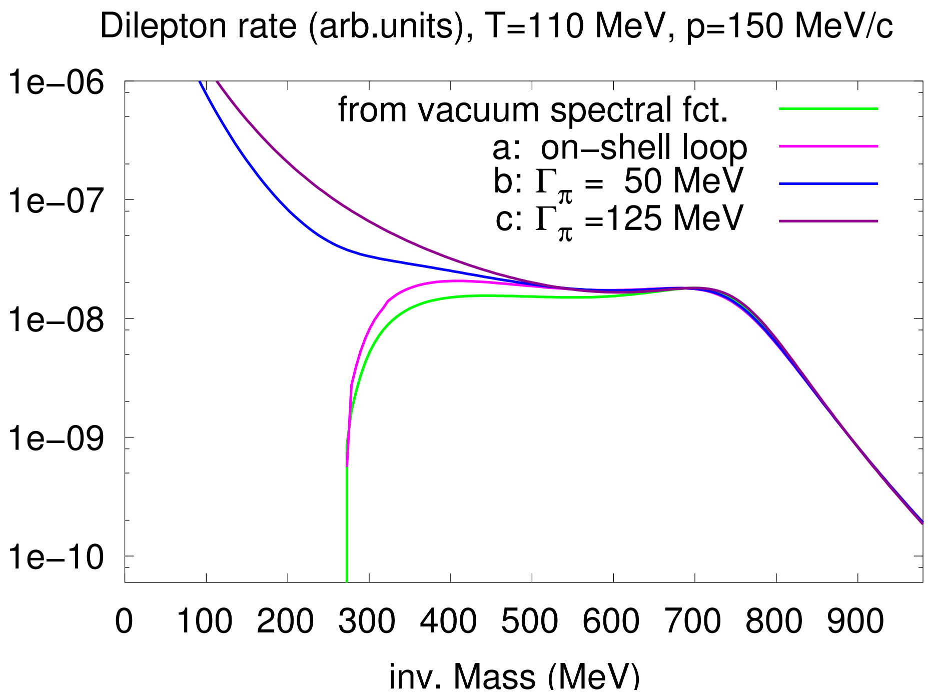
Results: ρ-meson properties

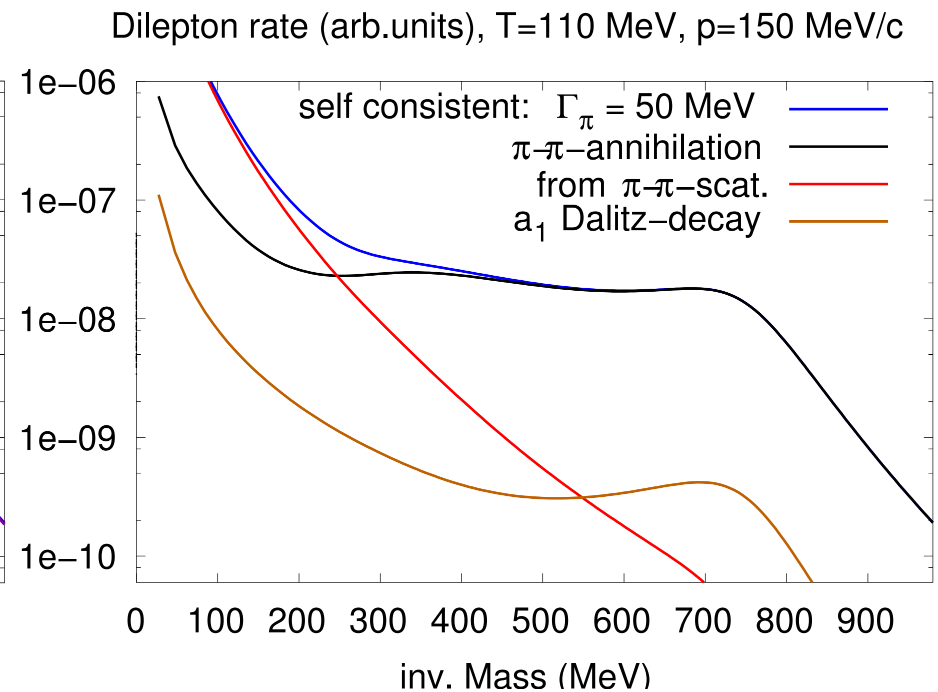
- Averaged quantities: $(2\Pi_T + \Pi_L)/3$





Results: Dilepton rates





Summary and Outlook

- Self–consistent Φ–derivable schemes
- Renormalization
- Symmetry analysis
- Scheme for vector particles
- Numerical treatment
- “Toolbox” for application to realistic models
- Perspectives for self–consistent treatment of gauge theories
- QCD e.g. beyond HTL?
- Transport equations for particles with finite width

References

[Hee97] H. van Hees, Quantenfeldtheoretische Beschreibung des $\pi\rho$-Systems, Master's thesis, Technische Hochschule Darmstadt (1997), URL http://theory.gsi.de/~vanhees/index.html

[Hee00] H. van Hees, Renormierung selbstkonsistenter Näherungen in der Quantenfeldtheorie bei endlichen Temperaturen, Ph.D. thesis, TU Darmstadt (2000), URL http://elib.tu-darmstadt.de/diss/000082/

[HK01] H. van Hees, J. Knoll, Finite Width Effects And Dilepton Spectra, Nucl. Phys. **A683** (2001) 369, URL http://arXiv.org/abs/hep-ph/0007070

[HK02a] H. van Hees, J. Knoll, Renormalization of Self-consistent Approximations II: applications: Applications to the sunset diagram, Phys. Rev. D **65** (2002) 105005, URL http://arxiv.org/abs/hep-ph/0111193

[HK02b] H. van Hees, J. Knoll, Renormalization of Self-consistent Approximations III: Symmetries, Phys. Rev. D (2002), accepted

[HK02c] H. van Hees, J. Knoll, Renormalization of self-consistent approximations: theoretical concepts, Phys. Rev. D **65** (2002) 025010, URL http://arXiv.org/abs/hep-ph/0107200

[KV96] J. Knoll, D. Voskresensky, Classical and Quantum Many-Body Description of Bremsstrahlung in Dense Matter (Landau-Pomeranchuk-Migdal Effect), Ann. Phys. (NY) **249** (1996) 532