Nonperturbative Heavy-Quark Interactions in the QGP

Ralf Rapp[a], Felix Riek[a], Hendrik van Hees[b], Vincenzo Greco[c], Massimo Mannarelli[d]

[a] *Texas A&M University, Cyclotron Institute and Physics Department, College Station, TX, 77843-3666, USA*
[b] *Justus-Liebig-Universität Giessen, Heinrich-Buff-Ring 16, D-35392 Giessen, Germany*
[c] *INFN-LNS, Laboratori Nazionali del Sud, and Dipartimento di Fisica e Astronomia, Universitá di Catania, Italy*
[d] *IEEC/CSIC, Universitat Autònoma de Barcelona, Torre C5, E-08193 Bellaterra (Barcelona), Spain*

Abstract

We adopt a T-matrix approach to study quarkonium properties and heavy-quark transport in a Quark-Gluon Plasma. The T-matrix approach is well suited to implement potential scattering and thus provides a common framework for low-momentum transfer interactions in heavy-heavy and heavy-light quark systems. We assume that the underlying potentials can be estimated from the heavy-quark free energy computed in lattice QCD. We discuss constraints from vacuum spectroscopy, uncertainties arising from different choices of the potential, and the role of elastic and inelastic widths which are naturally accounted for in the T-matrix formalism.

---

1. Introduction

Interactions of heavy quarks at low momentum transfer, $|q^2| \le m_Q^2$, are parametrically dominated by elastic scattering; Bremsstrahlung is suppressed by the large mass, m_Q, of the quark. Furthermore, 3-momentum transfer dominates over energy transfer, $q_0 \sim \vec{q}^2/2m_Q \ll |\vec{q}|$, which corresponds to the static limit or potential-type interactions. Both heavy-quark (HQ) transport and quarkonium binding are governed by low-momentum interactions. It is therefore suggestive to (i) address both problems in a common framework, and (ii) invoke a potential-based description. A finite-temperature T-matrix approach is well suited for these purposes; it provides a consistent framework to evaluate both bound-state and scattering solutions, based on a two-body static potential (see, e.g., Ref. [1] for a recent review). An extra benefit arises if the latter can be defined as a parameter-free input, e.g., from finite-temperature lattice QCD (lQCD), or at least be constrained by lQCD “data”. In the regime of moderate or even strong coupling, resummations of large diagrams are necessary, which in the T-matrix equation is realized via the standard ladder sum. In this paper, we set up a scattering equation with heavy quarks, check constraints in the zero-temperature and small-coupling (perturbative) limits and discuss the question of input potentials at finite temperature (Sec. 2), followed by an application to HQ diffusion and nonphotonic electron spectra (Sec. 3) and conclusions (Sec. 4).

2. *T*-Matrix Approach with Heavy Quarks

Starting from a relativistic Bethe-Salpeter equation, a 3-dimensional (3D) Lippmann-Schwinger equation can be derived by reducing the energy-transfer variable. A partial-wave expansion then yields a 1D integral equation for the T-matrix,

$$T_\alpha(E;q',q) = V_\alpha(q',q) + \int \frac{2dk\,k^2}{\pi} V_\alpha(q',k)\,G_2(E;k)\,T_\alpha(E;k,q)\,[1 - f_F(\omega_k^Q) - f_F(\omega_k^q)]\,, \quad (1)$$

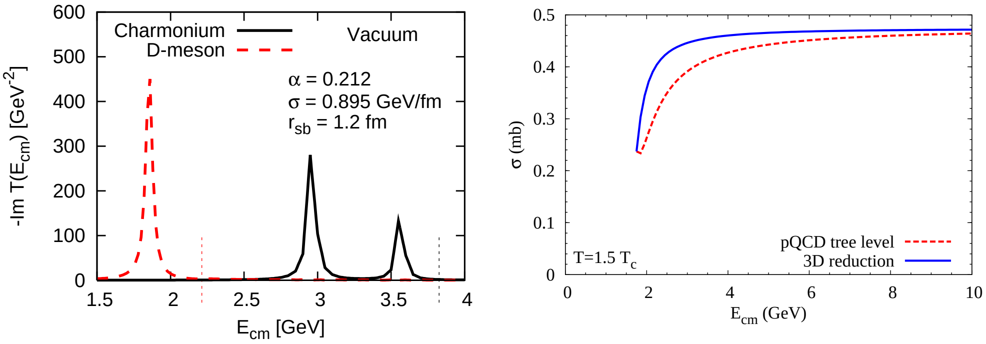

Figure 1: Left: imaginary part of the T-matrix in vacuum for S-wave $c\bar{q}$ (D, D^* mesons) and $c\bar{c}$ (η_c, J/ψ mesons) channels using a T=0 HQ free (=internal) energy with string breaking at $r_{\rm sb}$=1.2 fm [3]. Right: comparison of the heavy-light quark cross section in LO pQCD with the Born term of a T-matrix calculation with screenend Yukawa potential.

in a given quantum-number channel α. The concrete form of the intermediate 2-particle propagator, G_2, depends on the reduction scheme; applications to the the nuclear many-body problem or to quark scattering in the QGP [2] yield rather robust results.

In the HQ sector in vacuum, potential models have been established as the proper realization of low-energy QCD. The underlying potential is identified with the HQ free energy computed in lQCD (and agrees well with the phenomenological Cornell potential). As a first application, we inject this potential [3] (subtracted at a typical string-breaking scale of $r_{\rm sb}$=1.2 fm and Fourier-transformed) into Eq. (1) to compute the vacuum spectrum in the S-wave Q-$\bar{Q}$ and Q-$\bar{q}$ channels. With bare HQ masses of $m^0_{c,b}$=1.4, 4.75 GeV and a light-quark mass of m_q=0.35 GeV, the D, B, J/ψ and ψ' as well as Υ, Υ' and Υ'' states are reproduced within an accuracy of ∼0.1 GeV (spin-spin interactions, of order $1/m_Q$, are neglected here), cf. left panel of Fig. 1.

For applications to HQ spectra in heavy-ion collisions it is important to check the high-energy and perturbative limit of the T-matrix. To this end we compare in the right panel of Fig. 1 two calculations for the c-$\bar{q}$ cross section: (i) a leading-order (LO) perturbative QCD (pQCD) calculation with a screened one-gluon exchange propagator $1/(q^2-\mu_D^2)$; (ii) the Born approximation to the T-matrix using a Yukawa potential (including a Breit correction to account for color-magnetic interactions [5]) with identical Debye-mass (μ_D=gT) and strong coupling as in (i). The agreement is within ∼20% and shows that the Born term of the T-matrix is consistent with pQCD at high energies ($T(E_{\rm cm})\to V(E_{\rm cm})$ for large $E_{\rm cm}$, see Ref. [6]).

In Fig. 2 we summarize our results for the in-medium T-matrices in the open and hidden charm sector. If the potential is identified with the singlet free energy computed in thermal lQCD, the effects are rather weak: at 1.2 T_c the charmonium ground-state is about to become unbound while in the D-meson sector only a moderate threshold enhancement is visible; at 1.8 T_c little strength is left even at threshold. However, if the corresponding internal energy is employed, the charmonium bound state is still (slightly) bound at 1.2(1.8) T_c, while the D-meson channel exhibits an appreciable (moderate) “Feshbach resonance” at threshold [7].

3. Heavy-Quark Diffusion in QGP

The coupling of heavy quarks to the expanding medium in heavy-ion collisions is a direct means to extract transport properties of the QGP. E.g., the degree of c-quark thermalization ap-

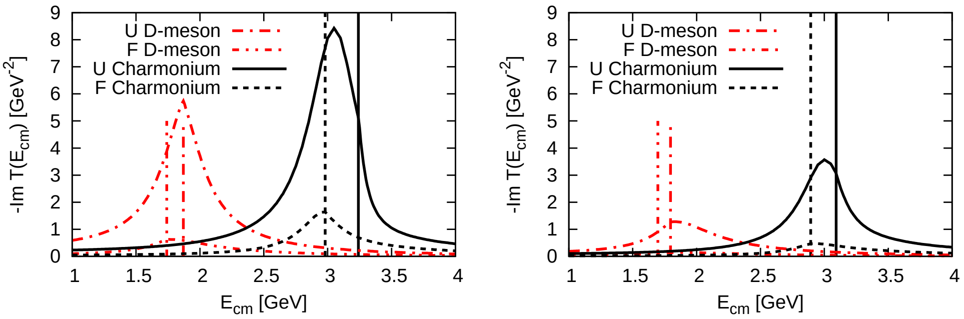

Figure 2: Imaginary parts of the c-$\bar{q}$ and c-$\bar{c}$ S-wave T-matrices in the QGP at T=1.2 T_c (left) and 1.8 T_c (right) using potentials corresponding to the lQCD free (dashed lines) or internal (solid lines) energy [4]; vertical lines indicate the quark-antiquark thresholds, $2m_Q$ and m_Q+m_q; a single-quark width of 0.2 GeV is used in the 2-particle propagator, G_2.

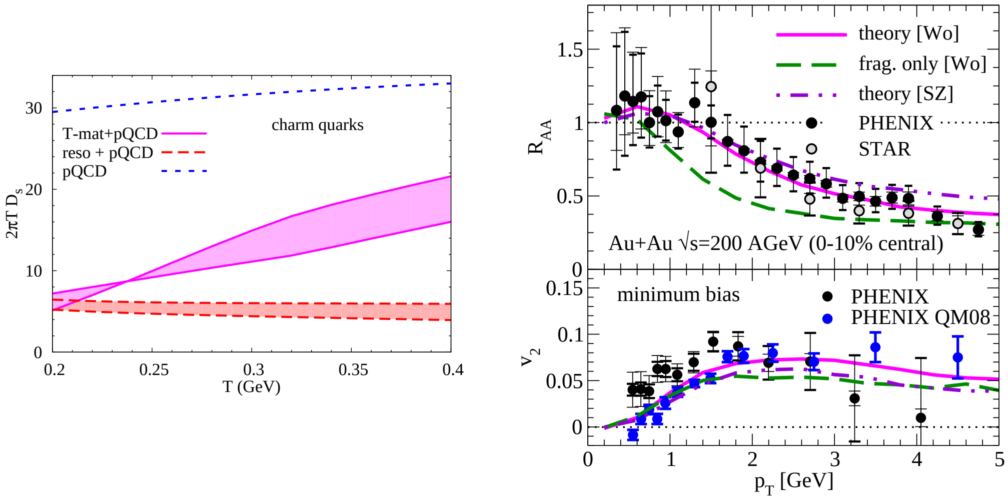

Figure 3: Left: spatial c-quark diffusion constant in the T-matrix approach [7], effective resonance model [8] and LO pQCD (α_s=0.4) [9]. Right: comparison of semiletponic $e^{\pm}$ RHIC data [10, 11] to relativistic Langevin simulations for c and b quarks using T-matrix+pQCD interactions in an expanding QGP (including hadronization into D-/B-mesons and subsequent 3-body decays); solid and dash-dotted lines are obtained with different lQCD internal energies; the dashed line is computed without coalescence contributions.

pears to be high but not complete [10, 11], thus enabling a quantitative determination of the thermal relaxation time, τ_c (i.e., the latter is comparable to the QGP lifetime). The large HQ mass enables to compute transport coefficients within a Fokker-Planck equation. These are readily calculated using in-medium heavy-light quark T-matrices [7]. The resulting spatial diffusion coefficient (related to the thermal relaxation time by D_s=$\tau_Q T/m_Q$), is compared to other approaches in the left panel of Fig. 3. Close to T_c, the value of D_s in the T-matrix approach (with internal energy as potential) is about a factor of 4-5 smaller than a LO pQCD calculation (with α_s=0.4), but comparable to an effective D-meson resonance model [8]. The increase of D_s with temperature within the T-matrix calculations is due to a decreasing interaction strength caused by color-screening in the potential. It is a direct reflection of the dissolving D-resonance structures (recall Fig. 2); toward high temperatures the pQCD results are approached. We note that the use of free energy as potential leads to a diffusion constant similar to the LO pQCD calculations.

The temperature- and momentum-dependent diffusion coefficients have been implemented into Langevin simulations of c and b quarks in Au-Au collisions at RHIC [12, 7]. The elliptically expanding medium has been modeled by a thermal fireball in close resemblance of hydrodynamic simulations [13]. A QGP evolution with initial temperature $T_0 \simeq 340$ MeV is followed by a QGP-hadron-gas mixed phase with a constant total entropy to reproduce the observed number of charged hadrons. The final HQ spectra are hadronized in a combined coalescence/fragmentation scheme and decayed into electrons to compare to experiment, cf. right panel of Fig. 3. Note the role of quark coalescence processes at T_c in increasing both the R_{AA} and v_2. Coalescence may be viewed as a manifestation of the T-matrix interaction in the hadronization process.

4. Conclusions

A T-matrix approach for elastic HQ scattering in the QGP has been used to study open and hidden heavy flavor in a common framework. Assuming interaction potentials given by the HQ internal energy extracted from thermal lQCD, HQ thermalization is substantially accelerated compared to pQCD estimates. This is caused by resonant correlations in heavy-light quark scattering at temperatures up to $\sim 1.5\, T_c$. Charmonia remain bound up to $\sim 2\, T_c$. Much weaker effects emerge when the HQ free energy is employed.

Acknowledgments

This work has been supported by a U.S. National Science Foundation CAREER award under grant no. PHY-0449489 and by the A.-v.-Humboldt foundation through a Bessel award (RR).

References

[1] R. Rapp and H. van Hees, to be published in *QGP-4* (R.C. Hwa and X.-N. Wang, eds.); arXiv:0903.1096 [hep-ph]

[2] M. Mannarelli and R. Rapp, Phys. Rev. C **72** (2005) 064905.

[3] O. Kaczmarek and F. Zantow, Phys. Rev. D **71** (2005) 114510.

[4] P. Petreczky and K. Petrov, Phys. Rev. D **70** (2004) 054503.

[5] G.E. Brown, C.H. Lee, M. Rho and E. Shuryak, Nucl. Phys. A **740** (2004) 171.

[6] D. Cabrera and R. Rapp, Phys. Rev. D **76** (2007) 114506.

[7] H. van Hees, M. Mannarelli, V. Greco and R. Rapp, Phys. Rev. Lett. **100** (2008) 192301.

[8] H. van Hees and R. Rapp, Phys. Rev. C **71** (2005) 034907.

[9] B. Svetitsky, Phys. Rev. D **37** (1988) 2484.

[10] A. Adare et al. [PHENIX Collaboration], Phys. Rev. Lett. **98** (2007) 172301; T.C. Awes [PHENIX Collaboration], J. Phys. G **35** (2008) 104007.

[11] B.I. Abelev et al. (STAR Collaboration), Phys. Rev. Lett. **98** (2007) 192301.

[12] H. van Hees, V. Greco and R. Rapp, Phys. Rev. C **73** (2006) 034913.

[13] P.F. Kolb and U.W. Heinz (2003), published in R.C. Hwa and X.-N. Wang (editors), *Quark-gluon plasma* vol. 3 (World Scientific, 2004) p. 634, arXiv:nucl-th/0305084.