# T-Matrix Approach to Heavy-Quark Diffusion and Quarkonia

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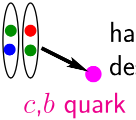
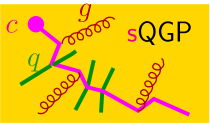
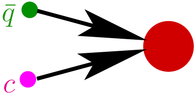
# Outline



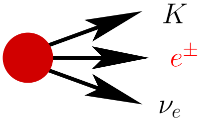

hard production of HQs
described by PDF's + pQCD (PYTHIA)




HQ rescattering in QGP: Langevin simulation
drag and diffusion coefficients from
microscopic model for HQ interactions in the sQGP




Hadronization to $D$,$B$ mesons via
quark coalescence + fragmentation
V. Greco, C. M. Ko, R. Rapp, PLB **595**, 202 (2004)




semileptonic decay ⇒
"non-photonic" electron observables
$R_{AA}^{e^+e^-}(p_T)$, $v_2^{e^+e^-}(p_T)$

# Relativistic Langevin process

- Langevin process: friction force + Gaussian random force
- in the (local) rest frame of the heat bath

$$\begin{aligned} \mathrm{d}\vec{x} &= \frac{\vec{p}}{E_p}\mathrm{d}t, \\ \mathrm{d}\vec{p} &= -A\,\vec{p}\,\mathrm{d}t + \sqrt{2\mathrm{d}t}[\sqrt{B_0}P_\perp + \sqrt{B_1}P_\parallel]\vec{w} \end{aligned}$$

- $\vec{w}$: normal-distributed random variable
- $A$: friction (drag) coefficient
- $B_{0,1}$: diffusion coefficients
- dependent on realization of stochastic process

# Local-Equilibrium Limit

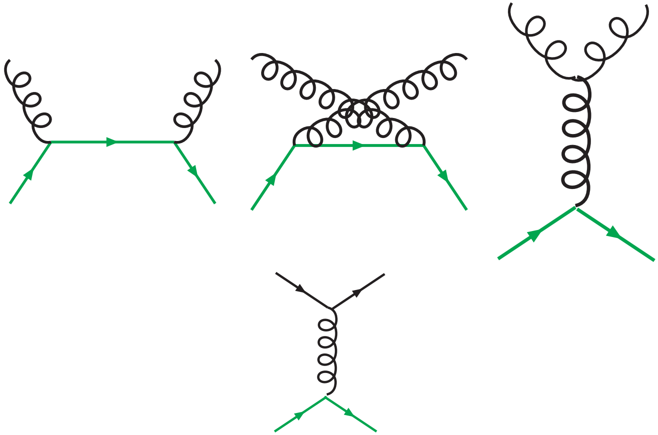
- to guarantee correct (local) equilibrium limit:
  - use Hänggi-Klimontovich calculus, i.e., use $B_{0/1}(t, \vec{p} + \mathrm{d}\vec{p})$
  - Einstein dissipation-fluctuation relation $B_0 = B_1 = E_p T A$.
- to implement flow of the medium
  - use Lorentz boost to change into local "heat-bath frame"
  - use update rule in heat-bath frame
  - boost back into "lab frame"
- Realizes Milekhin-freeze-out distribution for $t \to \infty$

$$\frac{\mathrm{d}N}{\mathrm{d}^3x \mathrm{d}^3p} = \frac{g}{(2\pi)^3} \frac{p \cdot u(x)}{E} \exp\left[-\frac{p \cdot u(x)}{T(x)}\right].$$

  - $E = p^0 = \sqrt{m^2 + \vec{p}^2}$
  - $u(x)$: four-velocity-flow field of the medium
  - $T(x)$: temperature field of the medium
  - thermal fireball adjusted such as to lead to correct light-meson and baryon observables
  - using the same coalescence/fragmentation model for hadronization
  - bulk $v_2$ sensitive to freeze-out description (see next talk by P. Gossiaux)

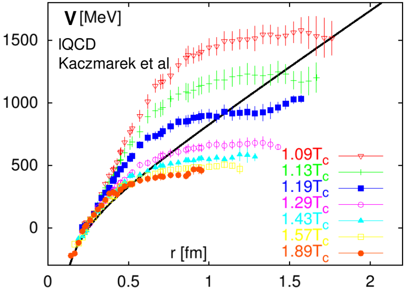
# Elastic pQCD processes

- Lowest-order matrix elements [Combridge 79]



- Debye-screening mass for $t$-channel gluon exch. $\mu_g = gT$, $\alpha_s = 0.4$
- not sufficient to understand RHIC data on "non-photonic" electrons

# Microscopic model: Static potentials from lattice QCD




- color-singlet free energy from lattice
- use internal energy

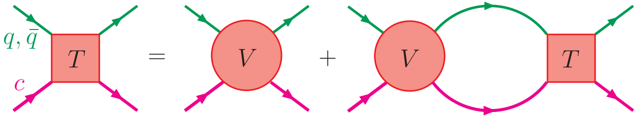
$$U_1(r,T) = F_1(r,T) - T\frac{\partial F_1(r,T)}{\partial T},$$
$$V_1(r,T) = U_1(r,T) - U_1(r \to \infty, T)$$

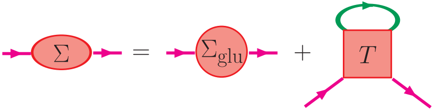
- Casimir scaling for other color channels [Nakamura et al 05; Döring et al 07]

$$V_{\bar{3}} = \frac{1}{2}V_1, \quad V_6 = -\frac{1}{4}V_1, \quad V_8 = -\frac{1}{8}V_1$$

# T-matrix

- Brueckner many-body approach for elastic $Qq$, $Q\bar{q}$ scattering

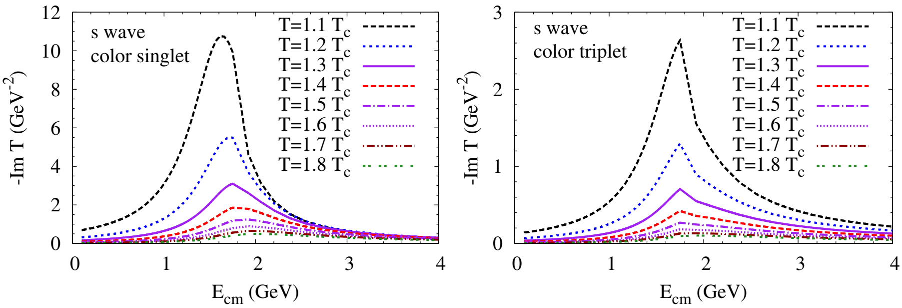



- reduction scheme: 4D Bethe-Salpeter → 3D Lippmann-Schwinger
- $S$- and $P$ waves
- same scheme for light quarks (self consistent!)
- Relation to invariant matrix elements

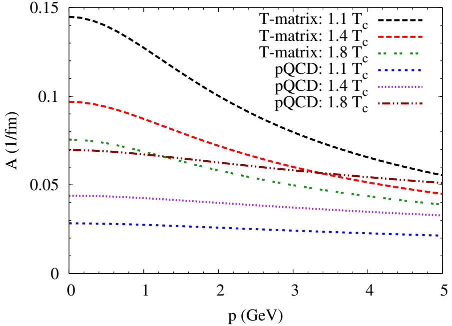
$$\sum |\mathcal{M}(s)|^2 \propto \sum_q d_a \left( |T_{a,l=0}(s)|^2 + 3|T_{a,l=1}(s)|^2 \cos\theta_{\text{cm}} \right)$$

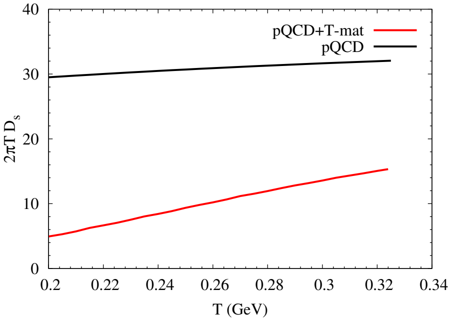
# Resonance formation: T-matrix calculation




- use static heavy-quark potentials from lQCD
- resonance formation at lower temperatures $T \simeq T_c$
- melting of resonances at higher $T$! $\Rightarrow$ sQGP
- model-independent assessment of elastic $Qq$, $Q\bar{q}$ scattering
- problems: uncertainties in extracting potential from lQCD
  in-medium potential $V$ vs. $F$?

# Transport coefficients




- from non-pert. interactions reach $A_{\text{non-pert}} \simeq 1/(7\ \text{fm}/c) \simeq 4 A_{\text{pQCD}}$
- A decreases with higher temperature
- higher density (over)compensated by melting of resonances!
- spatial diffusion coefficient

$$D_s = \frac{T}{mA}$$

increases with temperature

# Time evolution of the fire ball

- Elliptic fire-ball parameterization
  fitted to hydrodynamical flow pattern [Kolb '00]

$$V(t) = \pi(z_0 + v_z t) a(t) b(t), \quad a, b: \text{ semi-axes of ellipse},$$
$$v_{a,b} = v_\infty [1 - \exp(-\alpha t)] \mp \Delta v [1 - \exp(-\beta t)]$$

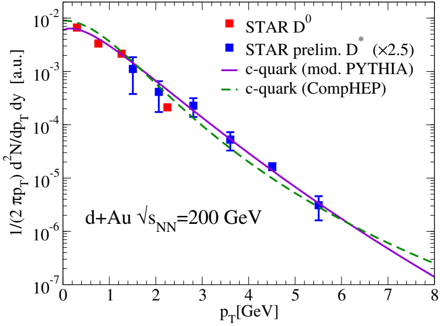
- Isentropic expansion: $S = \mathrm{const}$ (fixed from $N_{\mathrm{ch}}$)
- QGP Equation of state:

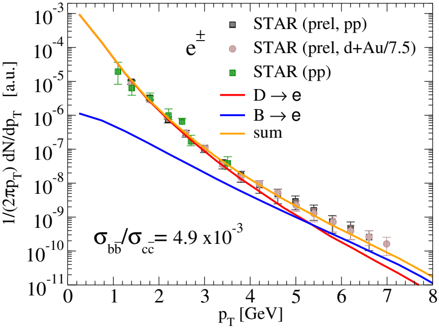
$$s = \frac{S}{V(t)} = \frac{4\pi^2}{90} T^3 (16 + 10.5 n_f^*), \quad n_f^* = 2.5$$

- obtain $T(t) \Rightarrow A(t,p)$, $B_0(t,p)$ and $B_1 = TEA$
- for semicentral collisions ($b = 7 \text{ fm}$): $T_0 = 340 \text{ MeV}$,
  QGP lifetime $\simeq 5 \text{ fm}/c$.
- simulate FP equation as relativistic Langevin process

# Initial conditions

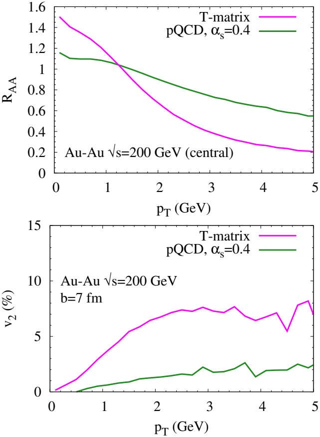
- need initial $p_T$-spectra of charm and bottom quarks
  - (modified) PYTHIA to describe exp. D meson spectra, assuming $\delta$-function fragmentation
  - exp. non-photonic single-$e^{\pm}$ spectra: Fix bottom/charm ratio

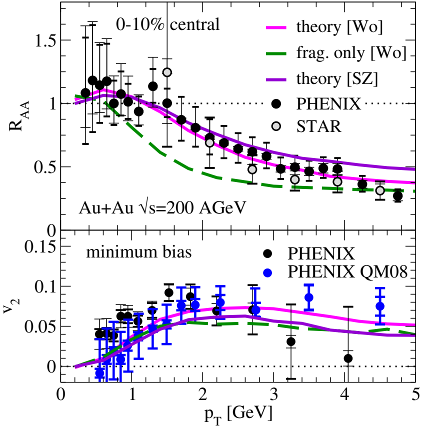


# Non-photonic electrons at RHIC

- same model for bottom
- quark coalescence+fragmentation $\rightarrow D/B \rightarrow e + X$

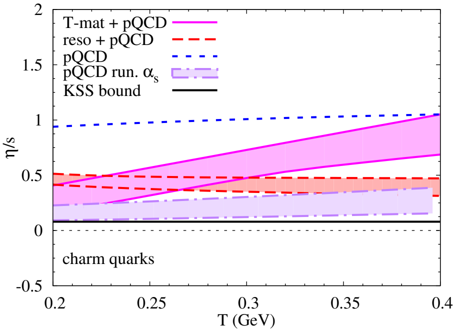


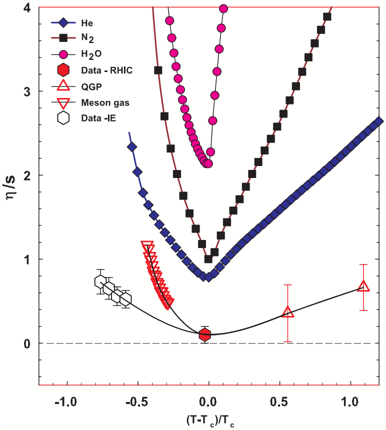




- coalescence crucial for description of data
- increases both, $R_{AA}$ and $v_2 \Leftrightarrow$ "momentum kick" from light quarks!
- "resonance formation" towards $T_c \Rightarrow$ coalescence natural [Ravagli, Rapp 07]

- spatial diffusion coefficient: Fokker-Planck $\Rightarrow D_s = \frac{T}{mA} = \frac{T^2}{D}$
- measure for coupling strength in plasma: $\eta/s$

$$\frac{\eta}{s} \simeq \frac{1}{2} T D_s \quad \text{(AdS/CFT)}, \qquad \frac{\eta}{s} \simeq \frac{1}{5} T D_s \quad \text{(wQGP)}$$

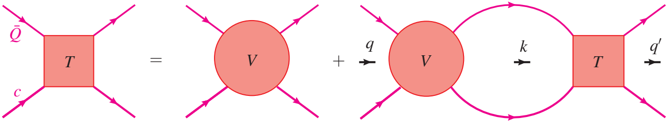


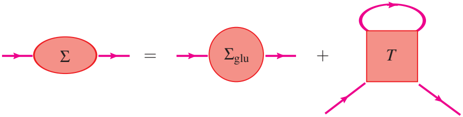




[Lacey, Taranenko (2006)]

# T-matrix approach for quarkonium-bound-state problem

- T-matrix Brückner approach for heavy quarkonia as for HQ diffusion
- consistency between HQ diffusion and $\bar{Q}Q$ suppression!



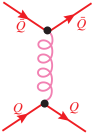




- 4D Bethe-Salpeter equation → 3D Lippmann-Schwinger equation
- relativistic interaction → static heavy-quark potential (lQCD)

$$T_\alpha(E;q',q) = V_\alpha(q',q) + \frac{2}{\pi}\int_0^\infty \mathrm{d}k\, k^2 V_\alpha(q',k) G_{Q\bar{Q}}(E;k) T_\alpha(E;k,q)$$
$$\times \{1 - n_F[\omega_1(\vec{k})] - n_F[\omega_2(k)]\}$$

- $q$, $q'$, $k$ relative 3-momentum of initial, final, intermediate $\bar{Q}Q$ state
  [F. Riek, R. Rapp, PRC **82**, 035201 (2010)]

# The potential




- non-perturbative static gluon propagator

$$D_{00}(\vec{k}) = 1/(\vec{k}^2 + \mu_D^2) + m_G^2/(\vec{k}^2 + \tilde{m}_D^2)^2$$

- finite-T HQ color-singlet-free energy from Polyakov loops

$$\exp[-F_1(r,T)/T] = \left\langle \mathrm{Tr}[\Omega(x)\Omega^\dagger(y)]/N_c \right\rangle$$
$$= \exp\left[\frac{g^2}{2N_c T^2} \left\langle A_{0,\alpha}(x) A_{0,\alpha}(y) - A_{0,\alpha}^2(x) \right\rangle\right] + \mathcal{O}(g^6)$$

- identify $\langle A_{0,\alpha}(x) A_{0,\alpha}(y) \rangle = D_{00}(x-y)$
- color-singlet free energy

$$F_1(r,T) = -\frac{4}{3}\alpha_s \left\{ \frac{\exp(-m_D r)}{r} + \frac{m_G^2}{2\tilde{m}_D}[\exp(-\tilde{m}_D r) - 1] + m_D \right\}$$

- in vacuo $m_D, \tilde{m}_D \to 0$

$$F_1(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r, \quad \sigma = \frac{2\alpha_s m_G^2}{3}$$

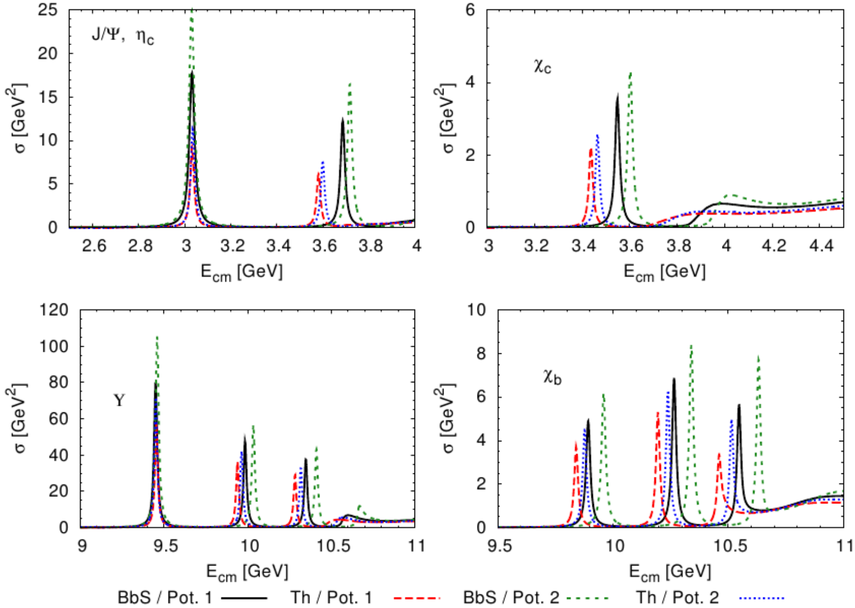
[F. Riek, R. Rapp, arXiv:1005.0769 [hep-ph]]

# Heavy quarkonia

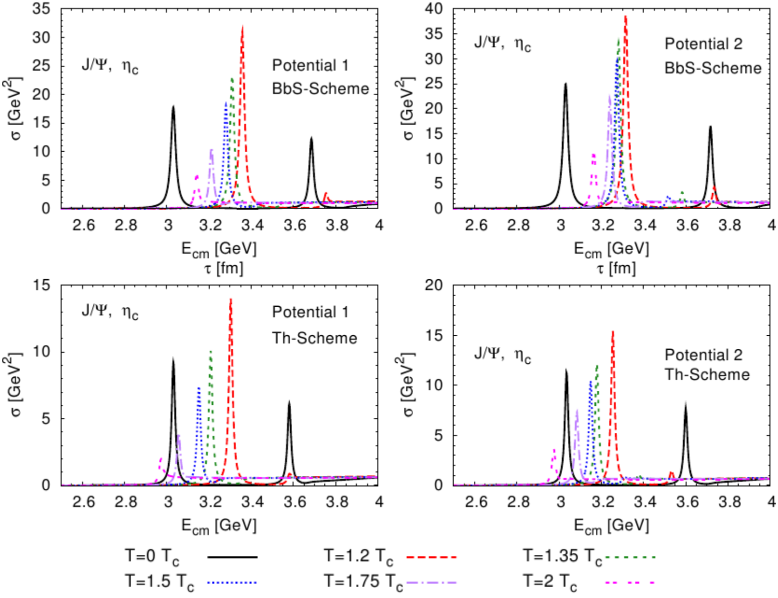
- fit parameters, $\alpha_s(T)$, $m_D(T)$, $\tilde{m}_D(T)$, $\tilde{m}_G(T)$ to lQCD
- calculate internal energy $U(r,T) = F(r,T) - T\frac{\partial}{\partial T}F(r,T)$
- solve Lippmann-Schwinger equation $\Rightarrow$ adjust $m_Q$ to get $s$-wave charmonia/bottomonia masses in vacuum
- in the following
  - potential 1: $N_f = 2+1$ [O. Kaczmarek]
  - potential 2: $N_f = 3$ [P. Petreczky]
  - BbS: Blancenblecler-Sugar reduction scheme
  - Th: Thompson reduction scheme
- vacuum-mass splittings
  - uncertainty for charmonia $50\text{-}100\ \mathrm{MeV}$
  - uncertainty for bottomonia $30\text{-}70\ \mathrm{MeV}$
  - overall uncertainty $\simeq 10\%$
- melting temperatures with $U$ and $F$
  - $s$-wave ($\eta_c$, $J/\psi$): $2\text{-}2.5T_c$, $\gtrsim 1.3T_c$,
    $\Upsilon$: $> 2T_c, \gtrsim 1.7T_c, 1T_c$, $\gtrsim 2T_c,\ 1T_c,\ 1T_c$
  - $p$-wave ($\chi_c$): $\gtrsim 1.2T_c$, $\gtrsim 1T_c$, $\chi_b$: $\gtrsim 1.7T_c, 1.2T_c$, all $\gtrsim 1T_c$

[F. Riek, R. Rapp, arXiv:1005.0769 [hep-ph]]

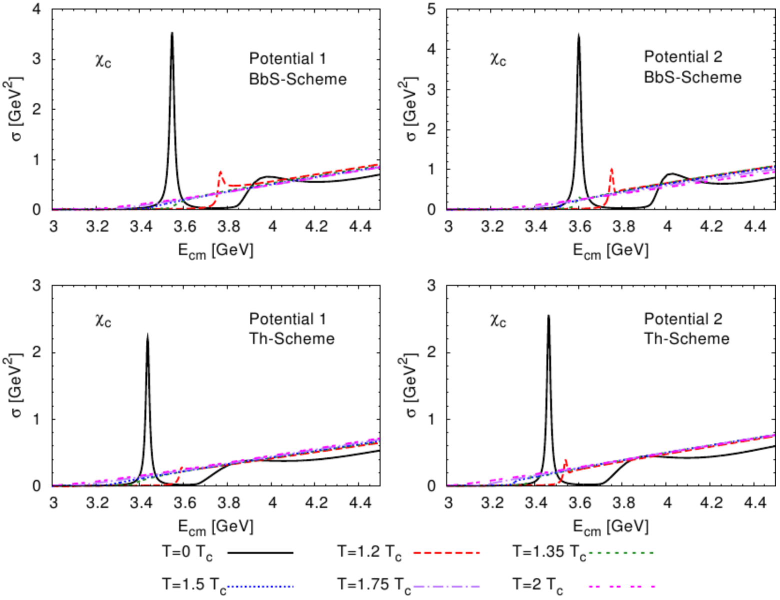
# Quarkonium-spectral functions in the vacuum



# In-medium charmonium-spectral functions (s states)



# In-medium charmonium-spectral functions (p states)



# Summary and Outlook

- Summary
  - Heavy quarks in the sQGP
  - non-perturbative interactions
    - mechanism for strong coupling: resonance formation at $T \gtrsim T_c$
    - lQCD potentials parameter free
    - res. melt at higher temperatures ⇔ consistency betw. $R_{AA}$ and $v_2$!
    - same model also used for quarkonia in medium
  - also provides "natural" mechanism for quark coalescence
  - resonance-recombination model [L. Ravagli, HvH, R. Rapp, Phys. Rev. C **79**, 064902 (2009)]
  - problems
    - potential approach at finite $T$: $F$, $V$ or combination?
- Outlook
  - use more realistic bulk-medium description (→ following talks by P. Gossiaux and M. He)
  - include inelastic heavy-quark processes (gluo-radiative processes)
  - take into account D/B-meson rescattering in the hadronic phase
  - other heavy-quark observables like charmonium suppression/regeneration (→ talk by X. Zhao on Thursday)