

# *Symmetries and Self-consistency: Vector mesons at finite Temperature*

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## *Motivation*

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- Thermodynamics of strongly interacting systems
- Conservation laws, detailed balance, thermodynamical consistency
- Finite width effects (resonance, damping,  $\dots$ )

## *Concepts*

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- Real time quantum field theory
- The  $\Phi$ -derivable scheme (example  $O(N)$ )
- Renormalization
- Restoration of symmetries
- Gauge Symmetries and Vector Mesons

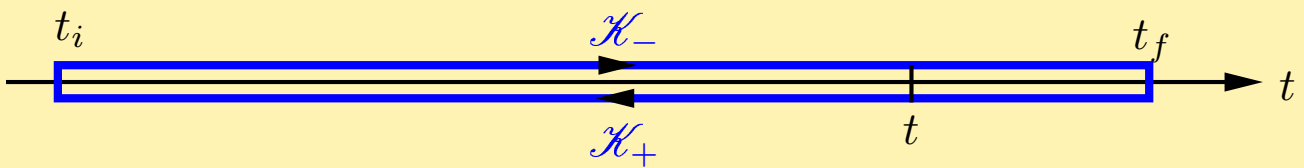
# Schwinger-Keldysh Formalism

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- Initial statistical operator  $\rho_i$  at  $t = t_i$
- Time evolution

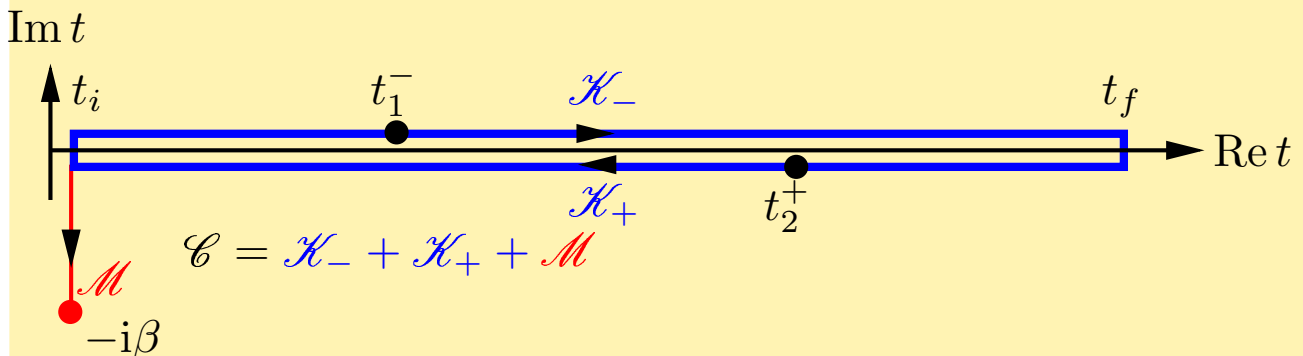
$$\langle O(t) \rangle = \text{Tr} \left[ \rho(t_i) \underbrace{\mathcal{T}_a \left\{ \exp \left[ +i \int_{t_i}^t dt' \mathbf{H}_I(t') \right] \right\}}_{\text{anti time-ordered}} \mathbf{O}_I(t) \underbrace{\mathcal{T}_c \left\{ \exp \left[ -i \int_{t_i}^t dt' \mathbf{H}_I(t') \right] \right\}}_{\text{time-ordered}} \right].$$

- Difference to vacuum: Contour-ordered Green's functions



$$\mathcal{C} = \mathcal{K}_- + \mathcal{K}_+$$

- In equilibrium:  $\rho = \exp(-\beta\mathbf{H})/Z$  with  $Z = \text{Tr} \exp(-\beta\mathbf{H})$
- Imaginary part of the time contour



- Correlation functions with real times:  $iG_{\mathcal{C}}(x_1^-, x_2^+)$
- Fields periodic (bosons) or anti-periodic (fermions)
- Feynman rules  $\Rightarrow$  time integrals  $\rightarrow$  contour integrals

# The $\Phi$ -Functional

#3

- Introduce **local** and **bilocal** auxiliary sources
- Generating functional

$$Z[J, K] = N \int D\phi \exp \left[ iS[\phi] + i \{J_1 \phi_1\}_1 + \left\{ \frac{i}{2} K_{12} \phi_1 \phi_2 \right\}_{12} \right]$$

- Generating functional for **connected diagrams**

$$Z[J, K] = \exp(iW[J, K])$$

- The **mean field** and the **connected Green's** function

$$\underbrace{\varphi_1 = \frac{\delta W}{\delta J_1}, G_{12} = -\frac{\delta^2 W}{\delta J_1 \delta J_2}}_{\text{standard quantum field theory}} \Rightarrow \frac{\delta W}{\delta K_{12}} = \frac{1}{2} [\varphi_1 \varphi_2 + iG_{12}]$$

- Legendre transformation for  $\varphi$  and  $G$ :

$$\mathbb{F}[\varphi, G] = W[J, K] - \{\varphi_1 J_1\}_1 - \frac{1}{2} \{(\varphi_1 \varphi_2 + iG_{12}) K_{12}\}_{12}$$

- Exact closed form:

$$\begin{aligned} \mathbb{F}[\varphi, G] = & S_0[\varphi] + \frac{i}{2} \text{Tr} \ln(-iG^{-1}) + \frac{i}{2} \{D_{12}^{-1}(G_{12} - D_{12})\}_{12} \\ & + \Phi[\varphi, G] \Leftarrow \text{all closed 2PI interaction diagrams} \\ D_{12} = & (-\square - m^2)^{-1} \end{aligned}$$

# Equations of Motion

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- Physical solution defined by vanishing **auxiliary sources**:

$$\frac{\delta\mathbb{I}}{\delta\varphi_1} = -J_1 - \{K_{12}\varphi_2\}_2 \stackrel{!}{=} 0$$

$$\frac{\delta\mathbb{I}}{\delta G_{12}} = -\frac{i}{2}K_{12} \stackrel{!}{=} 0$$

- Equation of motion for the **mean field**  $\varphi$

$$-\square\varphi - m^2\varphi := j = -\frac{\delta\Phi}{\delta\varphi}$$

- for the “**full**” **propagator**  $G \Rightarrow$  Dyson’s equation:

$$-i(D_{12}^{-1} - G_{12}^{-1}) := -i\Sigma = 2\frac{\delta\Phi}{\delta G_{21}}$$

- Integral form of Dyson’s equation:

$$G_{12} = D_{12} + \{D_{11'}\Sigma_{1'2'}G_{2'2}\}_{1'2'}$$

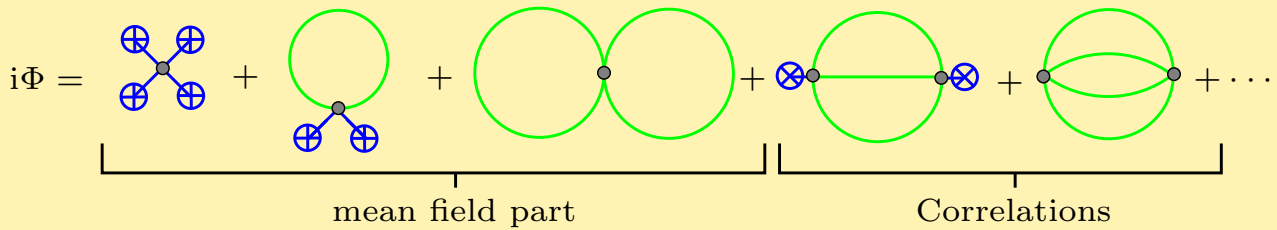
- Closed set** of equations of for  $\varphi$  and  $G$

# “Diagrammar”

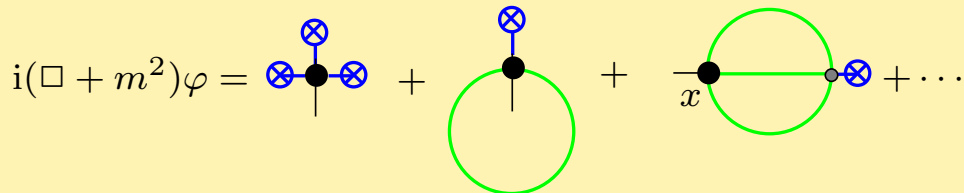
- $O(N)$ -theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \vec{\phi}) - \frac{m^2}{2}\vec{\phi}^2 - \frac{\lambda}{4!}(\vec{\phi}^2)^2$$

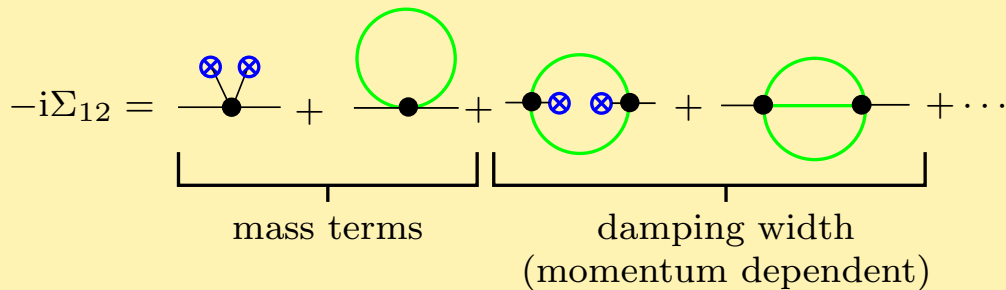
- 2PI Generating Functional



- Mean field equation of motion



- Self-energy



# Properties of the $\Phi$ -derivable Approximations

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## Why using the $\Phi$ -functional?

- Truncation of the Series of diagrams for  $\Phi$
- ☞ Expectation values for currents are conserved  
⇒ “Conserving Approximations”
- In equilibrium  $i\Gamma[\varphi, G] = \ln Z(\beta)$   
(thermodynamical potential)
- consistent treatment of **Dynamical quantities** (real time formalism) and **thermodynamical bulk properties** (imaginary time formalism) like **energy, pressure, entropy**
- Real- and Imaginary-Time quantities “glued” together by **Analytic properties** from (anti-)periodicity conditions of the fields (**KMS-condition**)
- Self-consistent set of equations for self-energies and mean fields

# Problem of Renormalization

#7

## Why renormalization?

- ☞ Diagrams UV-divergent
- ☞ Control the physical parameters in vacuum
- ☞ Temperature dependence from theory alone

## How to renormalize self-consistent diagrams?

- ☞ In terms of perturbation theory: Resummation of all self-energy insertions in propagators
- ☞ Self-consistent diagrams with explicit nested and overlapping sub-divergences
- ☞ “Hidden” sub-divergences from self-consistency

## How to manage it numerically?

- ☞ Power counting (Weinberg) valid for self-consistent diagrams
- ☞ At finite temperatures:  
Self-consistent scheme rendered finite with local counterterms independent of temperature
- ☞ Analytical properties  $\Rightarrow$  subtracted dispersion relations
- ☞ BPHZ-renormalization  $\Rightarrow$  Subtracting the integrands
- ☞ Advantage: Clear scheme how to subtract temperature independent sub-divergences
- ☞  $\Phi$ -functional  $\Rightarrow$  consistency of counterterms

# Self-Consistent Renormalisation

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## Example: Tadpole approximation

$$\Phi = \text{[Diagram: two green circles connected at a central grey dot]} \quad -i\Sigma = \text{[Diagram: a green circle with a grey dot on its bottom edge, connected to a horizontal line below it]}$$

- Here: Only time-ordered propagator needed
- The renormalized tadpole  $d = 2\omega = 2(2 - \epsilon)$ :

$$-i\Sigma = -\frac{i\lambda}{2} \int \frac{d^{2\omega}p}{(2\pi)^{2\omega}} \mu^{2\epsilon} iG(p) + \text{CT}$$

- Self-energy constant in  $p$
- ☞ temperature dependent effective mass
- Dyson's equation can be resummed:

$$iG(p) = \frac{i}{p^2 - M^2 + i\eta} + 2\pi n(p_0) \delta(p^2 - M^2)$$

$$\text{with } M^2 = m^2 + \Sigma, \quad n(p_0) = \frac{1}{\exp(\beta|p_0|) - 1}$$

- Use standard formulae for dimensional regularized Feynman integrals:

$$\Sigma_{\text{inf}} = -\frac{\lambda}{32\pi^2} M^2 \left[ \frac{1}{\epsilon} - \gamma + 1 + \ln \left( \frac{4\pi\mu^2}{M^2} \right) \right]$$

- Does one need temperature dependent counter terms ( $\propto \lambda M^2/\epsilon$ )?



# Self-Consistent Renormalisation

How to determine the counterterms?

- Use **BPHZ-Renormalization**: Subtract the **integrand**s

$$-i\Sigma_{\text{ren}} = \text{red diamond} = \frac{\lambda}{2} G(l) - \frac{\lambda}{2} G_v^2(l) \Sigma_{\text{ren}} - \frac{\lambda}{2} G_v(l)$$

👉 vertex counterterm

$$\text{blue loop with crossed lines} = \frac{\lambda^2}{2} G_v^2(l)$$

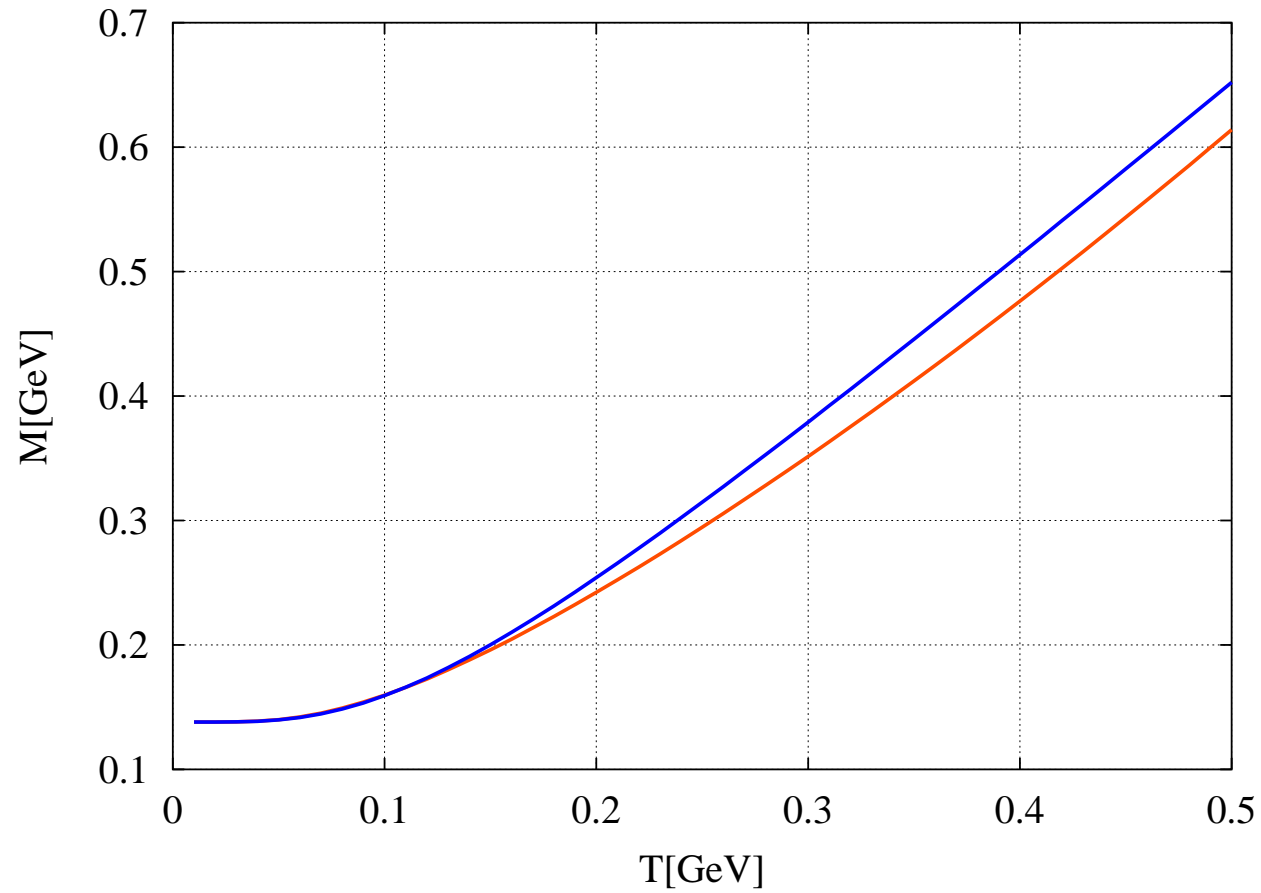
Finite Self-consistent equation (gap equation)

$$M^2 = m^2 + \Sigma_{\text{ren}} = m^2 + \frac{\lambda}{32\pi^2} \left( M^2 \ln \frac{M^2}{m^2} - \Sigma_{\text{ren}} \right) + \underbrace{\frac{\lambda}{2} \int \frac{d^4p}{(2\pi)^4} 2\pi \delta(p^2 - M^2) n(p_0)}_{\rightarrow 0 \text{ for } T \rightarrow 0}$$

👉 Renormalisation conditions:  $\Sigma_{\text{vac}} = 0$ ,  $\Gamma_{\text{vac}}^{(4)}(p = 0) = \lambda$

# Numerical Results

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Numerical solution of the **self-consistent tadpole equation** compared to the **perturbative result** for  $m = 140\text{MeV}$  and  $\lambda = 50$

# The Sunset Diagram

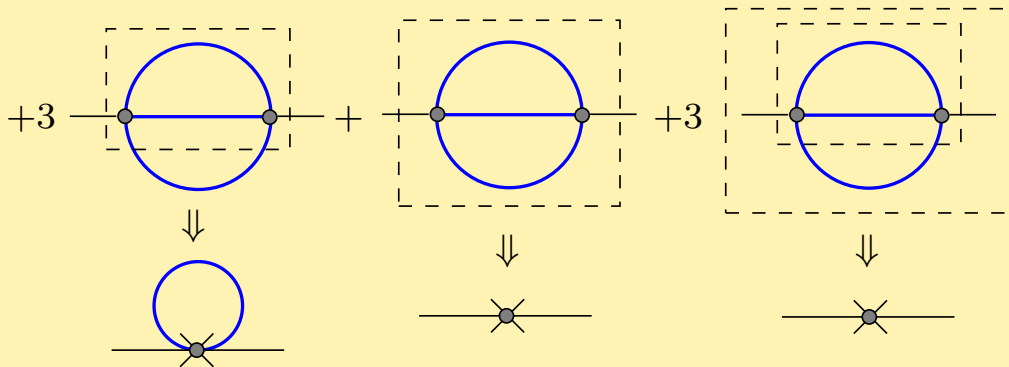
## The vacuum part

$$i\Phi = \text{Sunset Diagram} \quad \Leftrightarrow \quad -i\Sigma = \text{Sunset Diagram}$$

$\frac{1}{2 \cdot 4!}$ 
 $\frac{1}{3!}$

- Overall and sub-divergences to all orders perturbation theory

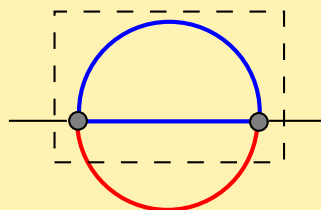
☞ Subtracted dispersion relations for vacuum divergences



## At finite temperature

- Split Propagator:  $G = G_v + G_T$
- $\delta(G_T) < -4$

☞ Only one sub-divergence:



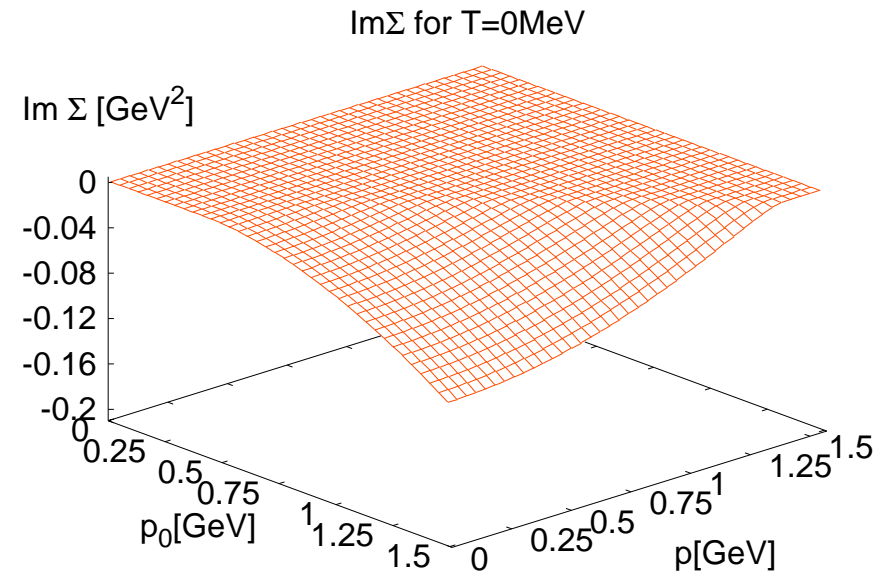
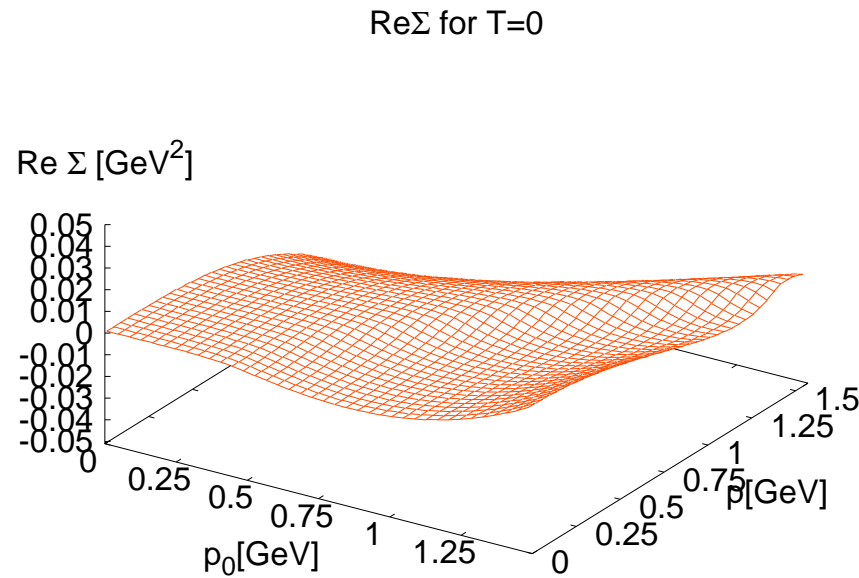
☞ Adding also tadpole  $\Rightarrow \delta(G_T) = -4$

☞ Additional complications treatable in the same way!

☞ Coupled Eqs. for  $\Sigma_{\text{ren}}^{\text{tad}}$  and  $\Sigma_{\text{ren}}^{\text{sunset}}$

# Results for the Vacuum Sunset Diagram

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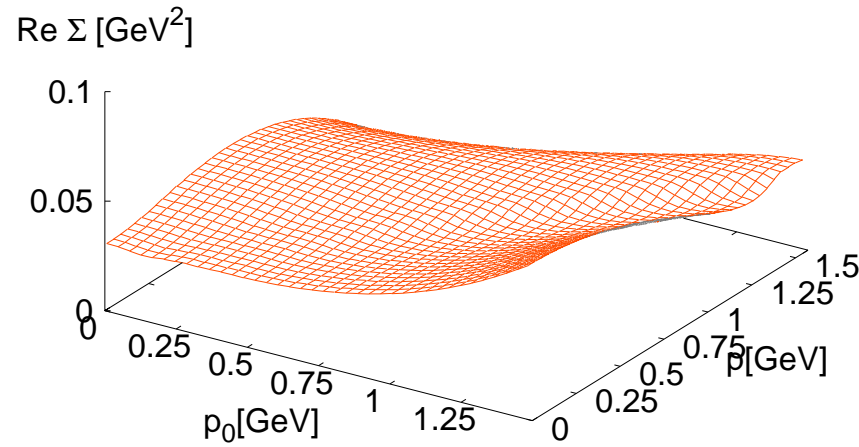


Vacuum:  $m = 140\text{MeV}$ ,  $\lambda = 65$

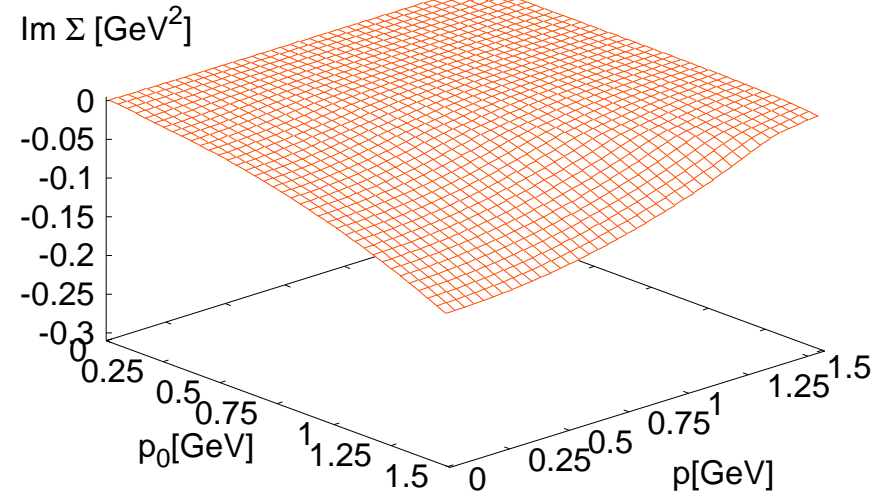
# Perturbative Result for “Sunset + Tadpole” at $T > 0$

#13

Re $\Sigma$  for T=100MeV



Im $\Sigma$  for T=100MeV

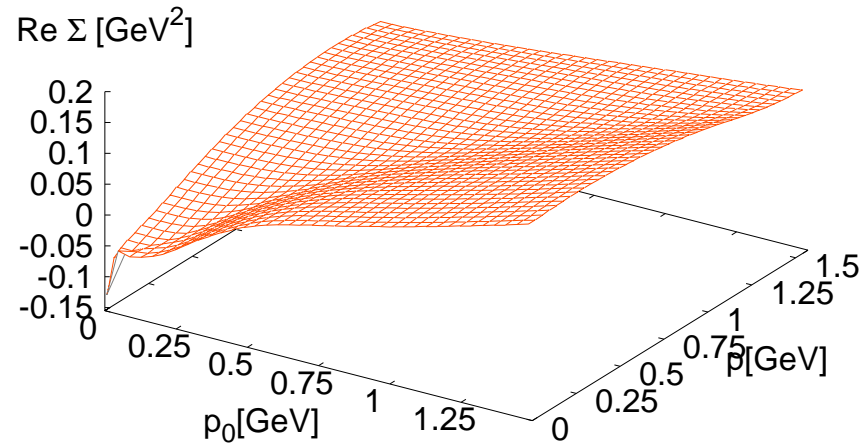


$T = 100$  MeV

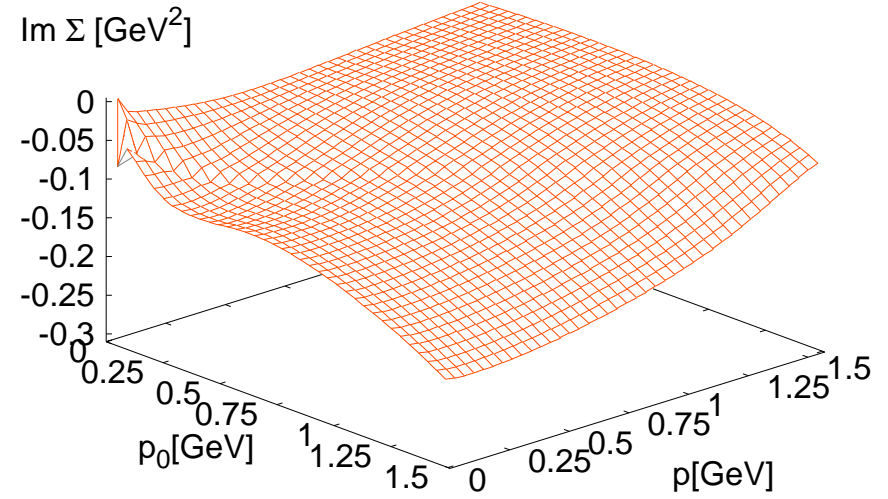
# Self-consistent Result for “Sunset + Tadpole” at $T > 0$

#14

Re $\Sigma$  for T=100MeV



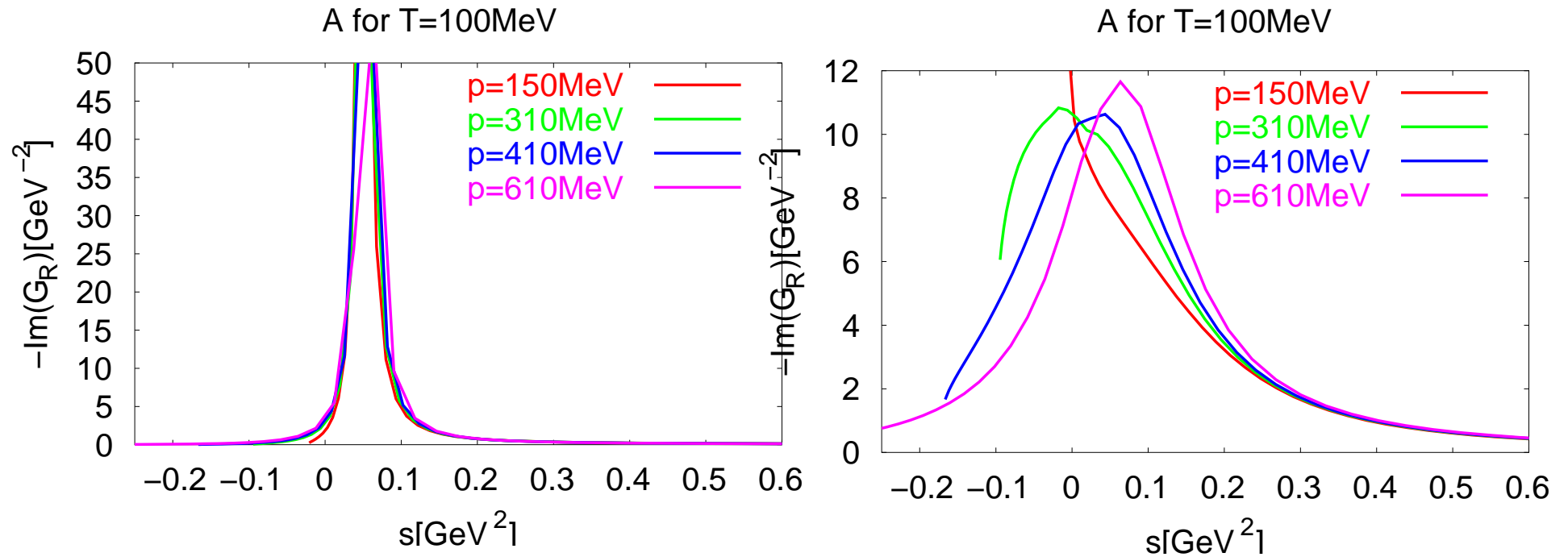
Im $\Sigma$  for T=100MeV



$T = 100$  MeV

# Spectral function of the “Meson”

#15



$T = 100\text{MeV}$ : Perturbative (left) and self-consistent (right) calculation

# The Analytic Green's Function

## The imaginary part of the contour

- So far **real-time formalism**
- **Entropy, pressure, mean energy, ...**
- **Analytic propagator**
- Branch of analytic continuation of  $G_{\text{Matsubara}}$

$$G_C(p_0, \vec{p}) = \int \frac{dz'}{2\pi} \frac{A(z', \vec{p})}{z' - p_0} \text{ with}$$
$$\forall z \in \mathbb{R} : \mathbb{R} \ni A(z, \vec{p}) = -A(-z, \vec{p}) = -2 \text{Im} G_R(z, \vec{p})$$

- Causality structure of  $G_R$  and  $G_A$

$$G_C(p_0 \pm i0) = -G_{R/A}(p) \text{ for } p_0 \in \mathbb{R}$$

- **Matsubara-propagator**

$$G_M(i\omega_n, \vec{p}) = G_C(i\omega_n, \vec{p}) \text{ with } \omega_n = \frac{2\pi i}{\beta} n = 2\pi i n T, n \in \mathbb{Z}$$



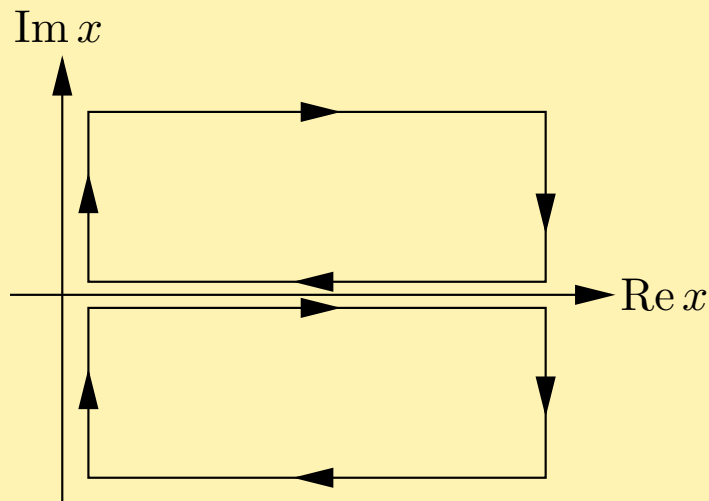
# Matsubara Sums

## Summing over Matsubara frequencies

- $F(z)$ : analytic in an open strip around imaginary axis

$$\frac{1}{\beta} \sum_{n \in \mathbb{Z}} F(i\omega_n) = \frac{1}{2\pi i} \int_{-i\infty+\epsilon}^{i\infty+\epsilon} dx [F(x) + F(-x)] \left[ \frac{1}{2} + \frac{1}{\exp(\beta x) - 1} \right]$$

- $F(z)$ : also analytic **away from the real axis**



$$\begin{aligned} \frac{1}{\beta} \sum_{n \in \mathbb{Z}} F(i\omega_n) &= \frac{1}{2\pi i} \int_{-\infty}^{\infty} dx \frac{1}{2} [F(x - i\epsilon\sigma(x)) - F(x + i\epsilon\sigma(x))] + \\ &+ \frac{1}{2\pi i} \int_{-\infty}^{\infty} dx n(x) [F(x - i\epsilon\sigma(x)) - F(x + i\epsilon\sigma(x))] \end{aligned}$$

# The Entropy

## Thermodynamical properties

- Expectation values from thermal quantum field theory:

$$Z(\beta, V) = \text{Tr} \exp(-\beta \mathbf{H})$$

$$\Rightarrow \varepsilon = \frac{1}{V} \langle \mathbf{H} \rangle = -\frac{1}{V} \partial_\beta \ln Z, \quad \frac{1}{V} d(\beta \ln Z) = -\varepsilon d\beta$$

- Define thermodynamical quantities:

$$P = \frac{\ln Z}{\beta V}, \quad s = \beta(P + \varepsilon) \Rightarrow dP = s dT$$

- Solution of the real time  $\Phi$ -derived self-consistent equations

$$\ln Z = i\Gamma \Rightarrow P = i\Gamma \Rightarrow s = i\partial_T \Gamma$$

- Stationarity with respect to  $G_R$ : Need to derive only with respect to explicit temperature dependency

$$s = -2 \int_{p_0 > 0} \frac{d^4 p}{(2\pi)^4} \partial_T n(p_0) \{ \text{Im} \ln[-G_R^{-1}(p)] + \text{Im}(\Sigma_R G_R) \} +$$

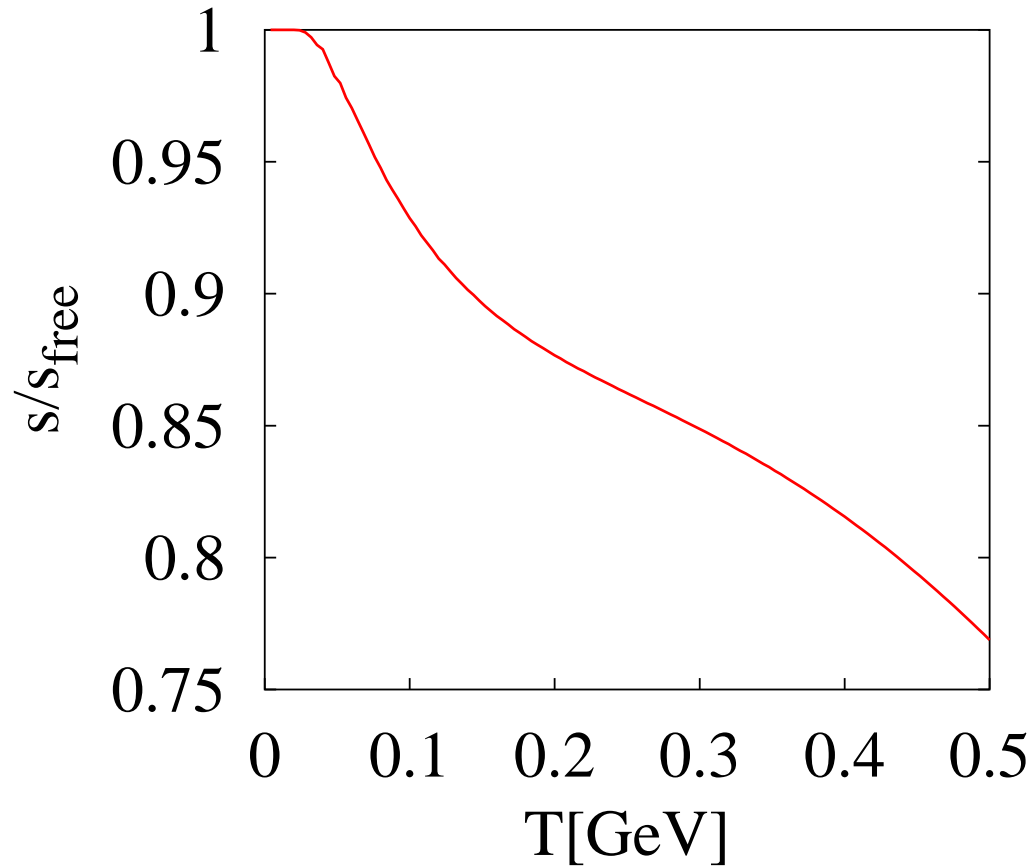
$$+ i \left\{ \frac{\delta \Phi[\varphi, G]}{\delta n} \right\} \Big|_{G_R, \varphi \text{ fixed}} \partial_T n$$

- Especially for 2-point  $\Phi$ -functionals

$$s = -2 \int_{p_0 > 0} \frac{d^4 p}{(2\pi)^4} \partial_T n(p_0) \{ \text{Im} \ln[-G_R^{-1}(p)] + (\text{Im} \Sigma_R)(\text{Re} G_R) \}$$

# Result for Tadpole Resummation

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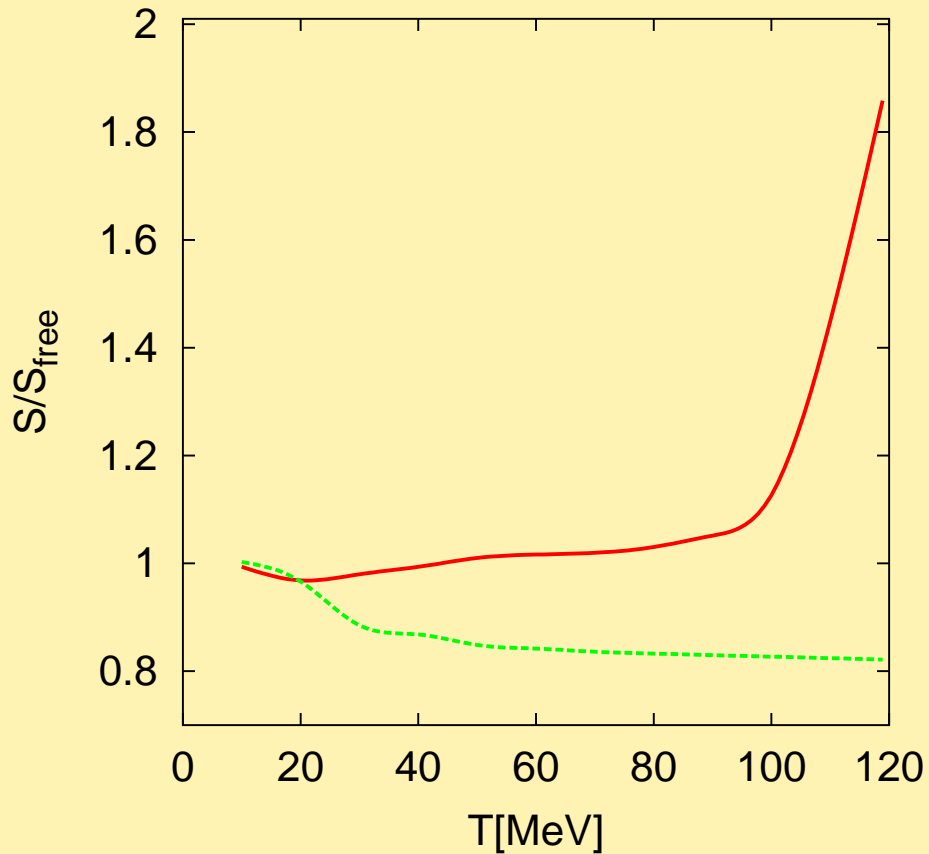


The entropy density of the **selfconsistent tadpole approximation** per entropy of the free gas (for  $\lambda = 50$ ).

# Result for the sunset approximation

#20

The entropy



Entropy per ideal gas entropy for the perturbative (green) and self-consistent calculation (red)

# Symmetries at the correlator level

#21

## Symmetry restoration

- Problem with  $\Phi$ -Functional: **Most approximations break symmetry!**
- Reason: Only conserving for **Expectation values for currents, not for correlation functions**
- Dyson's equation as functional of  $\varphi$ :

$$\left. \frac{\delta \mathbb{I}[\varphi, G]}{\delta G} \right|_{G=G_{\text{eff}}[\varphi]} \equiv 0$$

- Define new effective action functional

$$\Gamma_{\text{eff}}[\varphi] = \mathbb{I}[\varphi, G_{\text{eff}}[\varphi]]$$

- Symmetry analysis  $\Rightarrow \Gamma_{\text{eff}}[\varphi]$  symmetric functional in  $\varphi$
- Stationary point

$$\left. \frac{\delta \Gamma_{\text{eff}}}{\delta \phi} \right|_{\varphi=\varphi_0} = 0$$

☞  $\varphi_0$  and  $G = G_{\text{eff}}[\varphi_0]$ : self-consistent  $\Phi$ -Functional solutions!

☞  $\Gamma_{\text{eff}}$  generates **external** vertex functions fulfilling **Ward-Takahashi identities** of symmetries

☞ External Propagator

$$(G_{\text{ext}}^{-1})_{12} = \left. \frac{\delta^2 \Gamma_{\text{eff}}[\varphi]}{\delta \varphi_1 \delta \varphi_2} \right|_{\varphi=\varphi_0}$$

☞  $G_{\text{ext}}$  generally **not** identical with Dyson resummed propagator

# Example: Hartree approximation

#22

## External self-energy

- Hartree approximation:

$$i\Phi = \text{[Hartree diagram]} + \text{[Hartree diagram]} + \text{[Hartree diagram]}$$

- External self-energy defined on top of Hartree approximation

$$-i\Sigma_{\text{ext}} = \underbrace{\text{[Hartree diagram]} + \text{[Hartree diagram]} + \text{[Hartree diagram]} + \text{[Hartree diagram]} + \dots}_{\Sigma_{\text{int}}}$$

👉 RPA-Resummation restores symmetry

# Diagrammar for external vertices I

1st step: define  $\Phi$  and internal propagator

$$\begin{aligned}
 i\Phi &= \text{tree} + \text{self-energy} + \text{tadpole} + \frac{1}{2} \text{bubble} + \frac{1}{2} \text{fish} \\
 i(\square - \tilde{m}^2)\varphi &= \text{tree} + \text{self-energy} + \text{bubble} \\
 -i\Sigma &= \text{tadpole} + \text{self-energy} + \text{fish} + \text{fish}
 \end{aligned}$$

☞ Defines mean field and Dyson resummed **internal** propagator

2nd step: Derivatives

$$\frac{\delta G_{\text{eff}}}{\delta \varphi} = i\Gamma^{(3)} = \text{triangle} = \text{triangle} + \text{triangle} \text{ with } iK \text{ box}$$

External self-energy

$$\text{diamond} = \text{diamond} + \text{triangle} \text{ with } iK \text{ box}$$

# Diagrammar for external vertices II

#24

## Definition of Bethe–Salpeter equation elements

$$i\Phi_{\varphi,\varphi} = \text{diagram 1} + \text{diagram 2} + \text{diagram 3}$$

The equation shows three diagrams representing the Bethe–Salpeter equation for  $i\Phi_{\varphi,\varphi}$ . The first diagram is a vertex with two external lines and two internal lines meeting at a central point. The second diagram is a loop with two external lines. The third diagram is a loop with two external lines and two internal lines.

$$iI^{(3)} = i\Phi_{iG,\varphi} = \text{diagram 4} + \text{diagram 5}$$

The equation shows two diagrams representing the Bethe–Salpeter equation for  $iI^{(3)}$ . The first diagram is a vertex with one external line and two internal lines. The second diagram is a loop with two external lines and two internal lines.

$$iK = i\Phi_{iG,iG} = \text{diagram 6} + \text{diagram 7} + \text{diagram 8}$$

The equation shows three diagrams representing the Bethe–Salpeter equation for  $iK$ . The first diagram is a vertex with two external lines and two internal lines. The second diagram is a loop with two external lines and two internal lines. The third diagram is a loop with two external lines and two internal lines.

- Here: Green's function lines and mean fields: fixed from self-consistent  $\Phi$ -Functional solution



# Application to the $\pi$ - $\rho$ -System

#25

## The free vector meson

- Gauge invariant classical Lagrangian:

$$\mathcal{L} = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{1}{2}m^2V_\mu V^\mu + \frac{1}{2}(\partial^\mu\varphi)(\partial_\mu\varphi) + m\varphi\partial_\mu V^\mu$$

- Gauge invariance:

$$\delta V_\mu(x) = \partial_\mu\chi(x), \quad \delta\varphi = m\chi(x)$$

- Quantisation: Gauge fixing and ghosts

$$\begin{aligned}\mathcal{L}_V = & -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{m}{2}V_\mu V^\mu - \frac{1}{2\xi}(\partial_\mu V^\mu)^2 + \\ & + \frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) - \frac{\xi m^2}{2}\varphi^2 + \\ & + (\partial_\mu\eta^*)(\partial_\mu\eta) - \xi m^2\eta^*\eta.\end{aligned}$$

- Free vacuum propagators

$$\Delta_V^{\mu\nu}(p) = -\frac{g^{\mu\nu}}{p^2 - m^2 + i\eta} + \frac{(1 - \xi)p^\mu p^\nu}{(p^2 - m^2 + i\eta)(p^2 - \xi m^2 + i\eta)}$$

$$\Delta_\varphi(p) = \frac{1}{p^2 - \xi m^2 + i\eta}$$

$$\Delta_\eta(p) = \frac{1}{p^2 - \xi m^2 + i\eta}.$$

☞ Usual power counting  $\Rightarrow$  renormalisable

☞ Partition sum: Three bosonic degrees of freedom!

# Application to the $\pi$ - $\rho$ -System

#26

## Adding $\pi^\pm$ and $\gamma$

- Gauge-covariant derivative

$$D_\mu \pi = \partial_\mu \pi + igV_\mu \pi + ieA_\mu$$

☞ Quantisation of free photon as usual

- Minimal coupling:

$$\mathcal{L}_{\pi V} = \mathcal{L}_V + (D_\mu \pi)^* (D^\mu \pi) - m_\pi^2 \pi^* \pi - \frac{\lambda}{8} (\pi^* \pi)^2 - \frac{e}{2g_{\rho\gamma}} A_{\mu\nu} V^{\mu\nu}$$

☞ Eqs. of motion: Vector meson dominance (Kroll, Lee, Zumino)

- Adding Leptons like in QED:

$$\mathcal{L}_{e\gamma} = \bar{\psi}(i\not{D} - m_e)\psi$$

with

$$D_\mu \psi = \partial_\mu \psi + ie\psi \tag{1}$$

# Application to the $\pi$ - $\rho$ -System

#27

## The Propagators

$$\mu \text{ --- } \overset{p}{\text{oooo}} \text{ --- } \nu = -\frac{ig^{\mu\nu}}{p^2 - m_\rho^2 + i\eta} + \frac{i(1 - \xi_\rho)p^\mu p^\nu}{(p^2 - m_\rho^2 + i\eta)(p^2 - \xi m_\rho^2 + i\eta)}$$

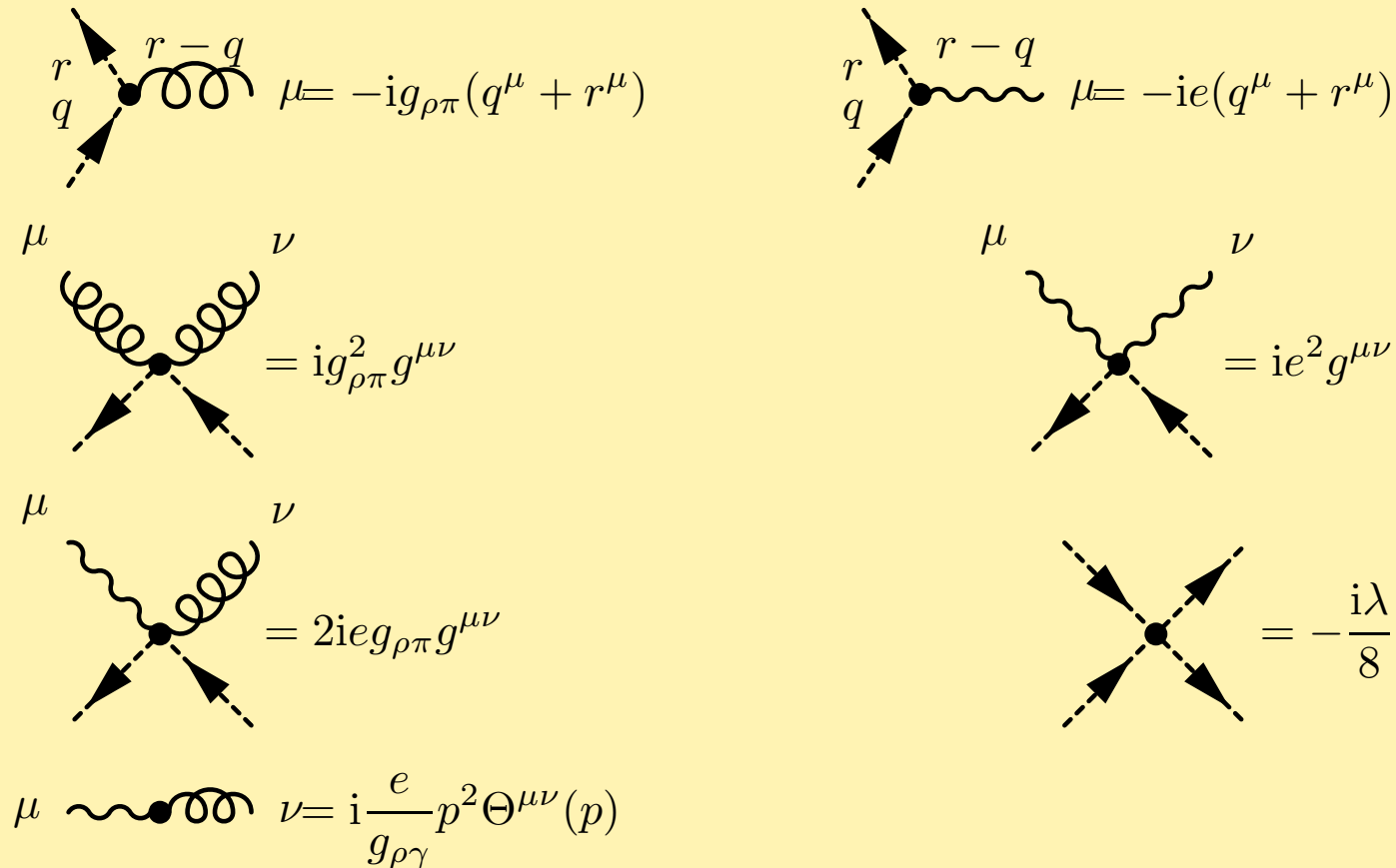
$$\mu \text{ --- } \overset{p}{\text{~~~~~}} \text{ --- } \nu = -\frac{ig^{\mu\nu}}{p^2 + i\eta} + \frac{i(1 - \xi_\gamma)p^\mu p^\nu}{(p^2 + i\eta)^2}$$

$$\overset{p}{\text{---}} \blacktriangleleft = \frac{i}{p^2 - m_\pi^2 + i\eta}$$

$$\overset{p}{\text{---}} \blacktriangleleft = \frac{i(\not{p} + m)}{p^2 - m_l^2 + i\eta}$$

# Application to the $\pi$ - $\rho$ -System

## The Vertices



# Application to Vector bosons

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- Kroll–Lee–Zumino interaction: Coupling of massive vector bosons to conserved currents  $\Rightarrow$  **gauge theory**
- Symmetry breaking at correlator level

Problems:

- ☞ Internal propagators contain spurious degrees of freedom
- ☞ Negative norm states
- ☞ Numerically instable due to light cone singularities

- Classical picture (Fokker–Planck–equation):

$$\Pi^{\mu\nu}(\tau, \vec{p} = 0) \propto \langle v^\mu(\tau) v^\nu(0) \rangle$$

- „One–loop” approximation in the classical limit

$$\Pi^{\mu\nu}(\tau, \vec{p} = 0) \propto \exp(-\Gamma\tau)$$

- ☞  $1/\Gamma$ : Relaxation time scale due to scattering

- Exact behaviour:

$$\Pi^{00}(\tau, \vec{p} = 0) \propto \langle 1 \cdot 1 \rangle = \text{const}$$

$$\Pi^{jk}(\tau, \vec{p} = 0) \propto \langle v^j v^k \rangle \propto \exp(-\Gamma_x \tau)$$

- ☞ For  $\Pi^{jk}$ : If  $\Gamma \approx \Gamma_x \Rightarrow$  1–loop approximation justified

- ☞ Classical limit also shows:

$\Pi^{jk}$  only slightly modified by ladder resummation

- In self–consistent approximations:

Use only  $p_j p_k \Pi^{jk}$  and  $g_{jk} \Pi^{jk}$

- ☞ Construct  $\Pi_T$  and  $\Pi_L$

# The interacting $\pi$ - $\rho$ - $a_1$ system

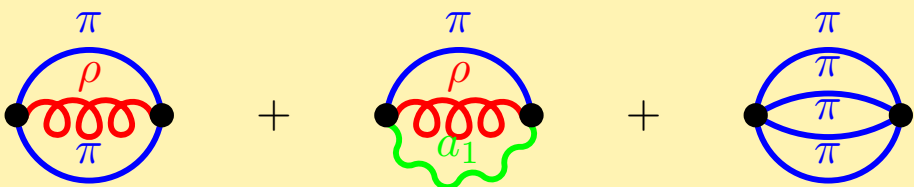
#30

## The self-consistent approximation

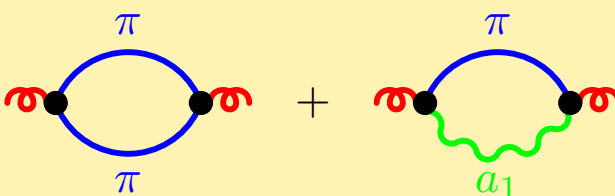
Lagrangian:

$$\mathcal{L}_{\text{int}} =$$

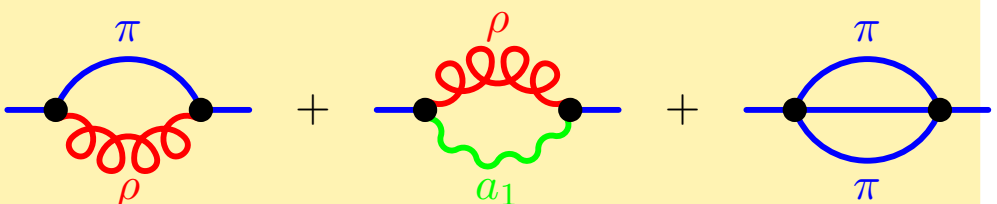

$\Phi$ -Funktional:

$$\Phi =$$


Self-energies:

$$\Pi_\rho =$$


$$\Pi_{a_1} =$$

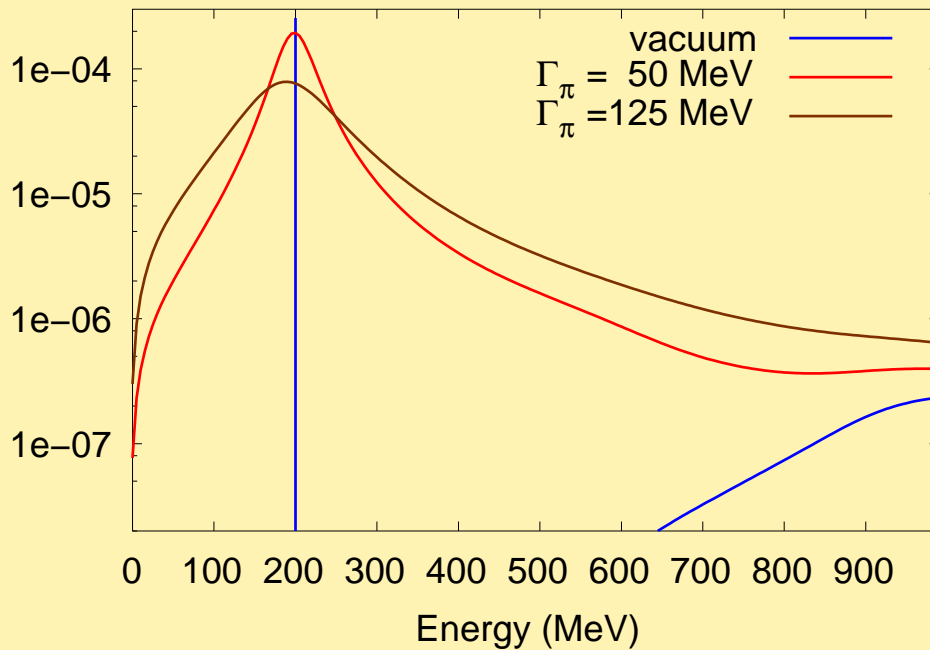

$$\Sigma_\pi =$$


# Results for the $\pi\rho a_1$ -System

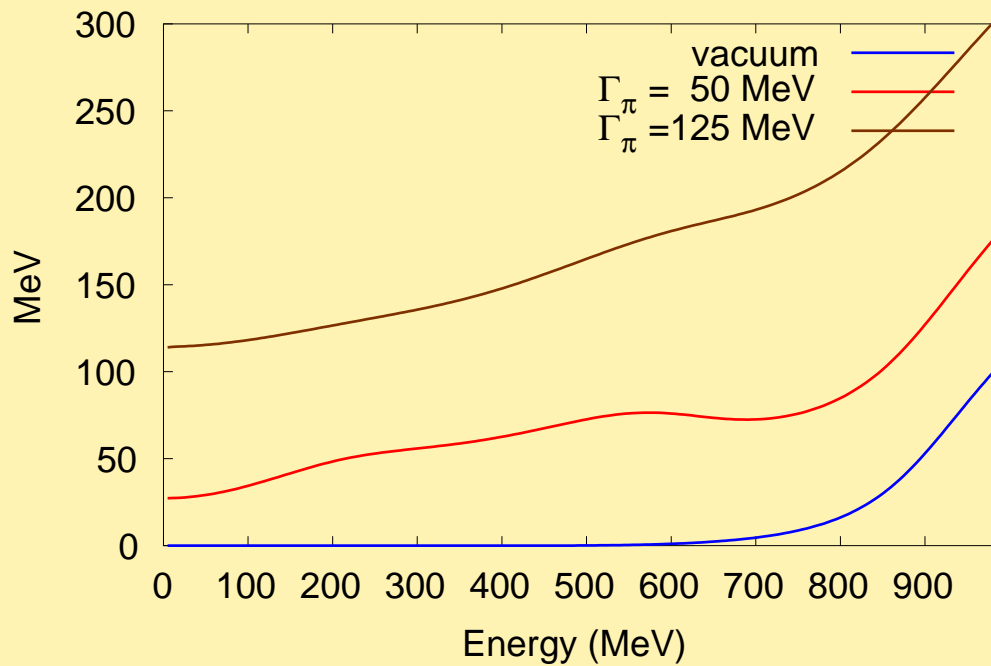
#31

## Broad pions in the medium

$\pi$ -Meson Spectral function,  $T=110$  MeV;  $p=150$  MeV/c



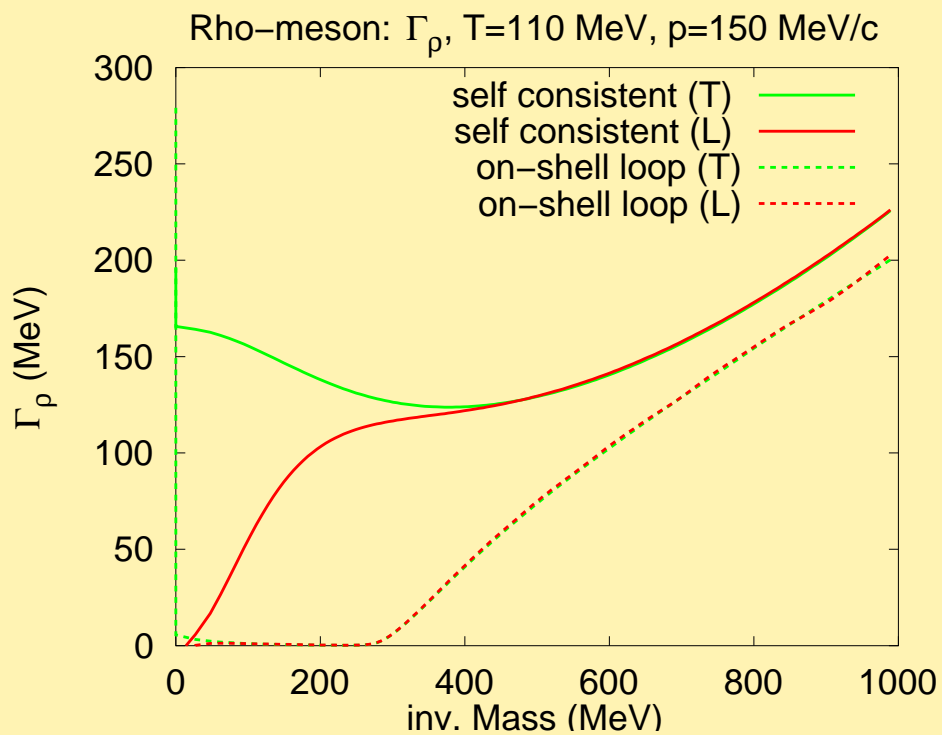
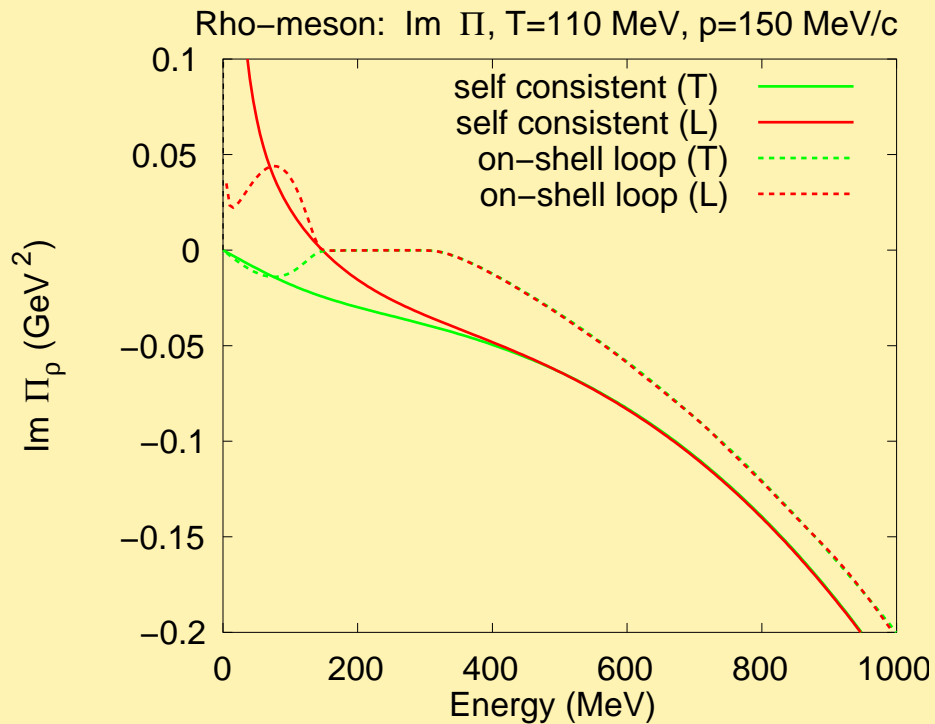
$\pi$ -Meson Width,  $T=110$  MeV;  $p=150$  MeV/c



# Results for the $\pi\rho a_1$ -System

#32

## $\rho$ -meson Polarisation tensor

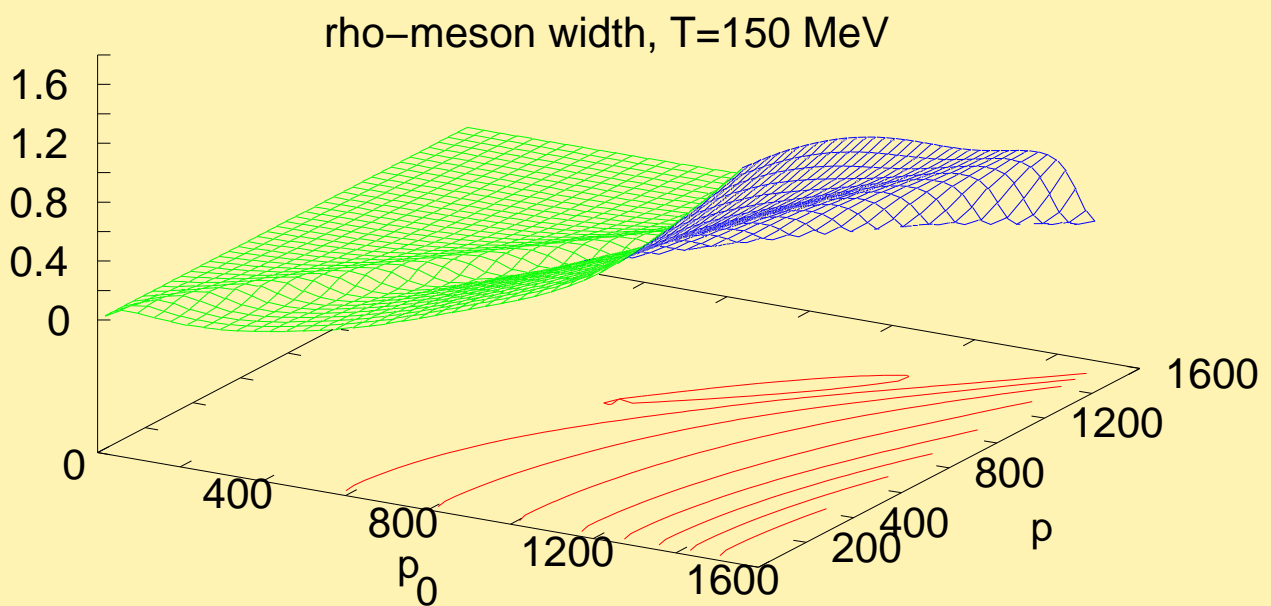
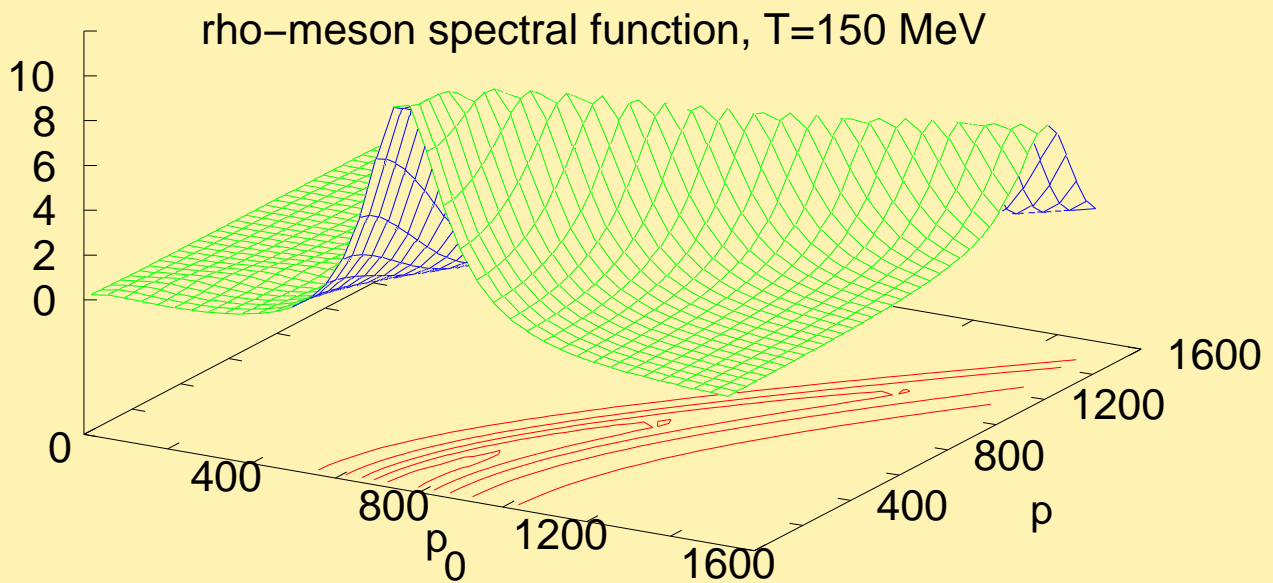




# Results for the $\pi\rho a_1$ -System

#33

## $\rho$ -meson properties I

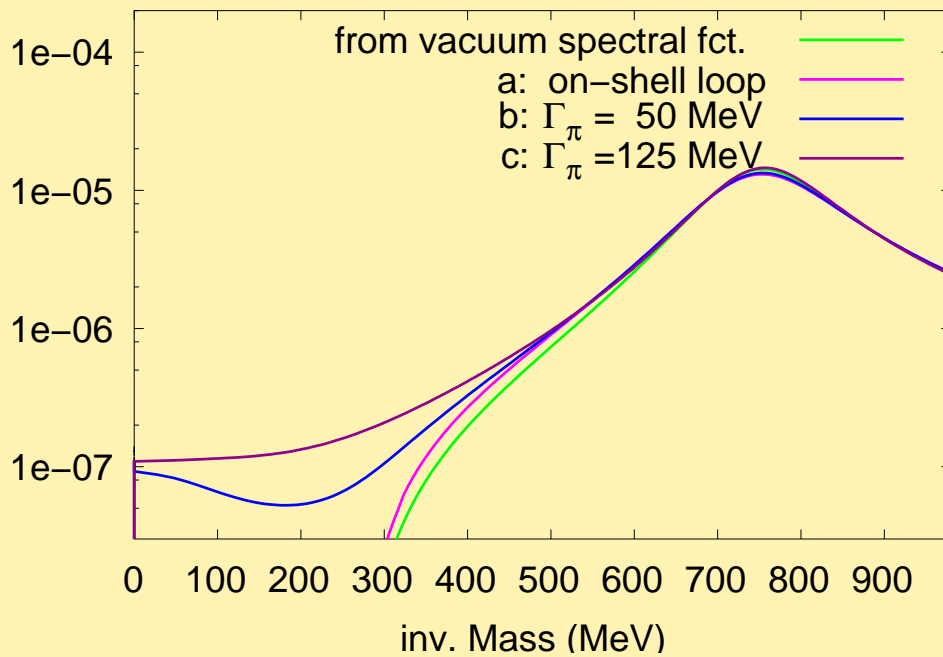


# Results for the $\pi\rho a_1$ -System

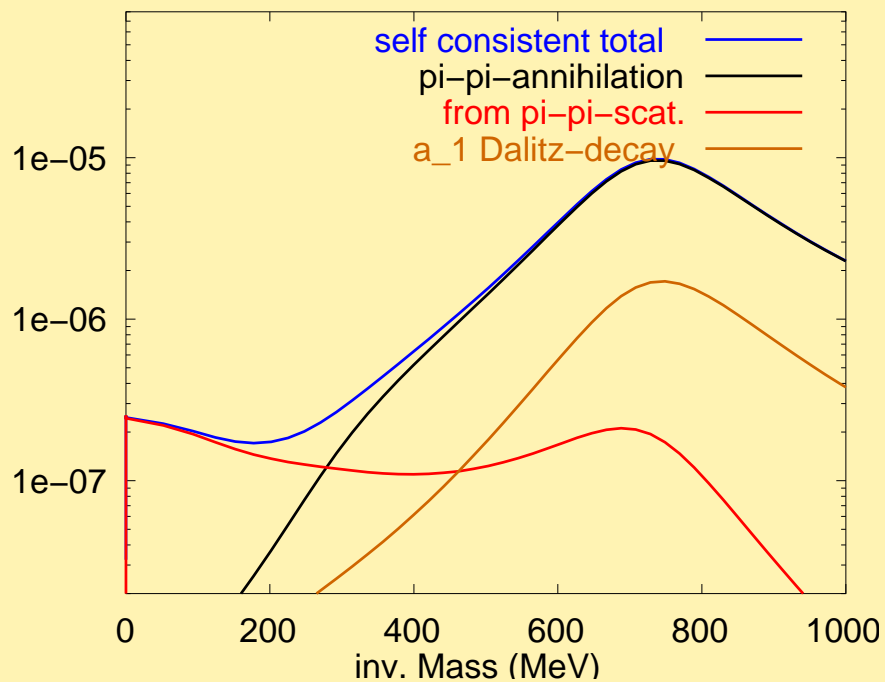
#34

## $\rho$ -meson properties II

Rho-meson Spectral fct.,  $T=110$  MeV,  $p=150$  MeV/c



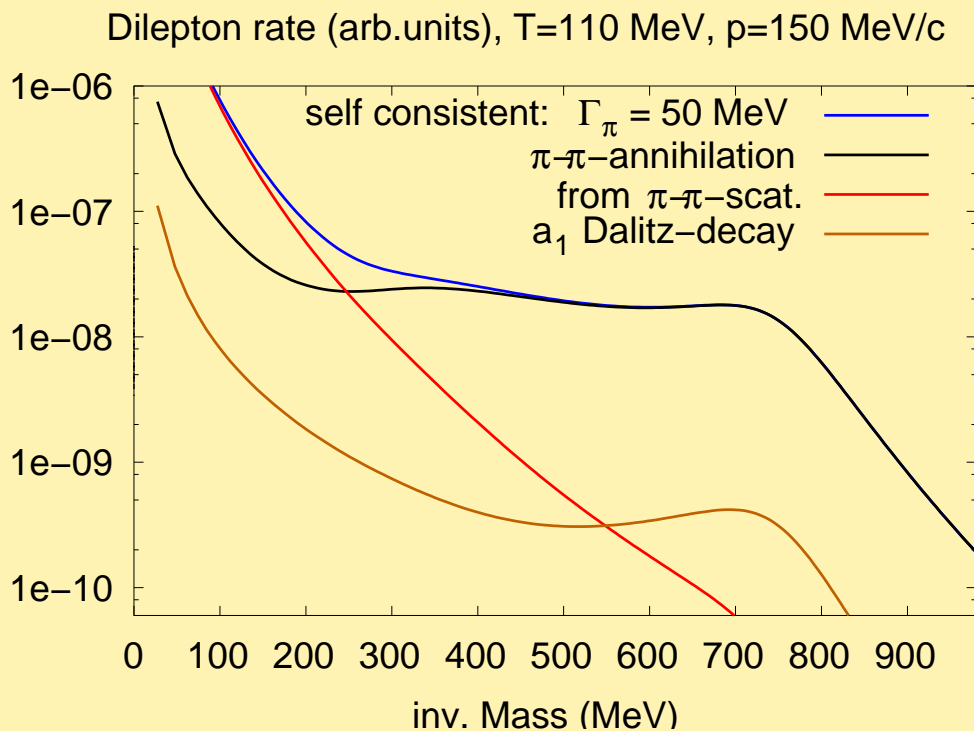
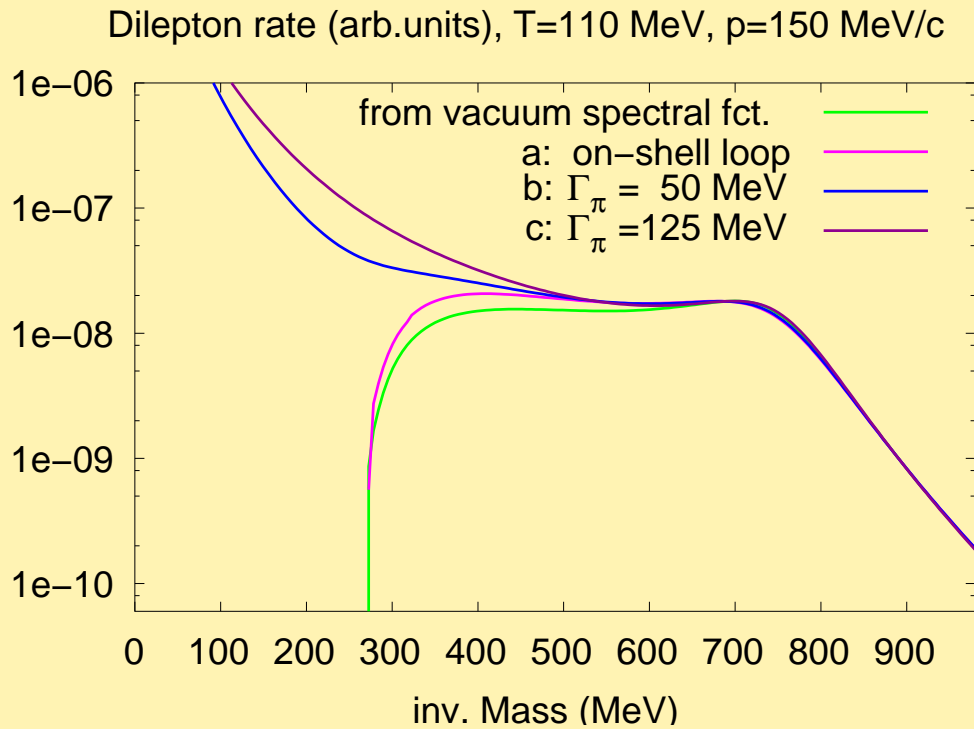
Rho-Meson Spectral Fnc.,  $T=150$  MeV,  $p=150$  MeV/c



# Results for the $\pi\rho a_1$ -System

#35

## Dilepton rate



# Conclusions

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## *Summary*

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- Self-consistent  $\Phi$ -derivable schemes
- Renormalization
- Symmetry analysis
- Scheme for vector particles
- Numerical treatment

## *Outlook*

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- “Toolbox” for application to realistic models
- Perspectives for self-consistent treatment of gauge theories
- QCD e.g. beyond HTL?
- Transport equations for particles with finite width