Einführung in die theoretische Kern- und Teilchenphysik 2
Vorlesung 1: Heavy-Ion Overview

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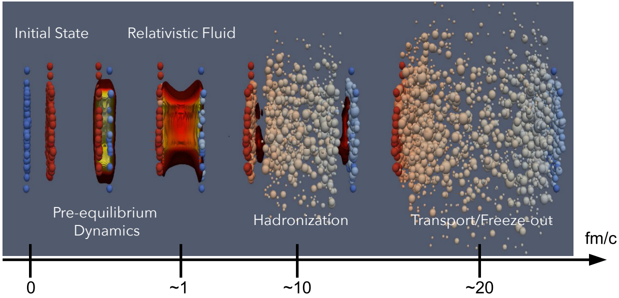
Outline

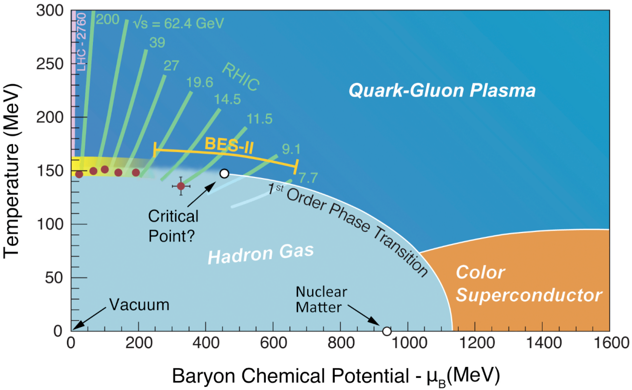
Ultrarelativistic Heavy-Ion Collisions

- ultra-relativistic collisions of heavy nuclei
- creates hot and dense fireball behaving like a strongly coupled medium
- early thermalization, starting in QGP phase
- rapidly expanding and cooling
- (cross-over) transition to hadron-resonance gas ($T_{\text{pc}} \simeq 150\text{-}160\,\text{MeV}$)



from [EM23]

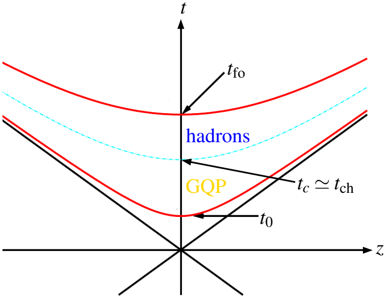
QCD Phase Diagram

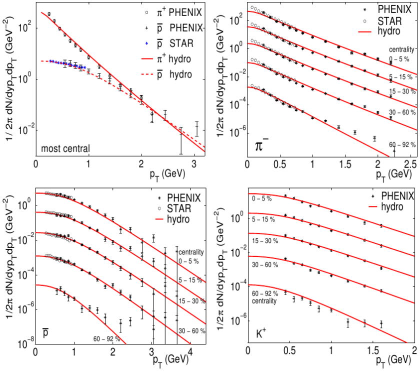


[Fig. from A. Aprahamian et al. Reaching for the horizon]

Collective flow of the fireball (Hydrodynamics)

- hydrodynamical model for ultra-relativistic heavy-ion collisions
  - after short formation time ($t_0 \lesssim 1\,\text{fm}/c$)
  - QGP in local thermal equilibrium → hadronization at $T_{\text{pc}} \simeq 150\text{-}160\,\text{MeV}$
  - chemical freeze-out: (inelastic collisions cease) $T_{\text{ch}} \simeq 150\text{-}160\,\text{MeV}$
  - thermal freeze-out: (also elastic scatterings cease) $T \sim 100\,\text{MeV}$

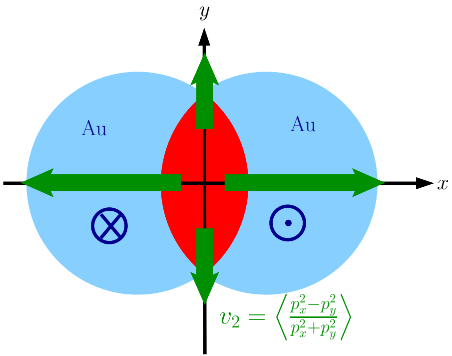


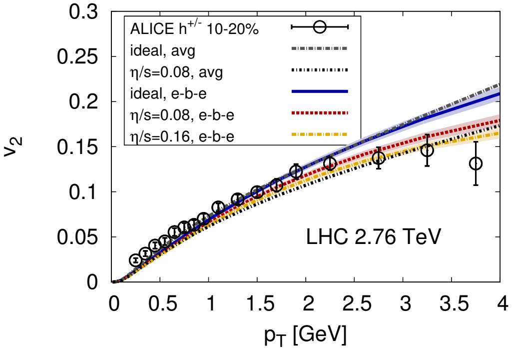


[KH03]

Hydrodynamical Behavior

- particle spectra compatible with collective flow (hydrodynamical expansion)
- elliptic flow as signature of pressure
- (nearly) ideal hydrodynamics $\eta/s \simeq 1\text{-}2 \times 1/4\pi$

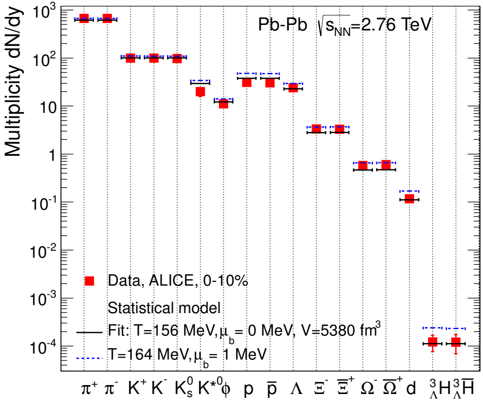




[SJG11]

Chemical freeze-out: Statistical hadronization model

- hadron abundancies: can be described by (grand-)canonical hadron-resonance-gas model ($T_{\text{ch}} \simeq T_{\text{pc}}$, $\mu_B = 0$)
- even light (anti-hyper-)nuclei follow the systematics!



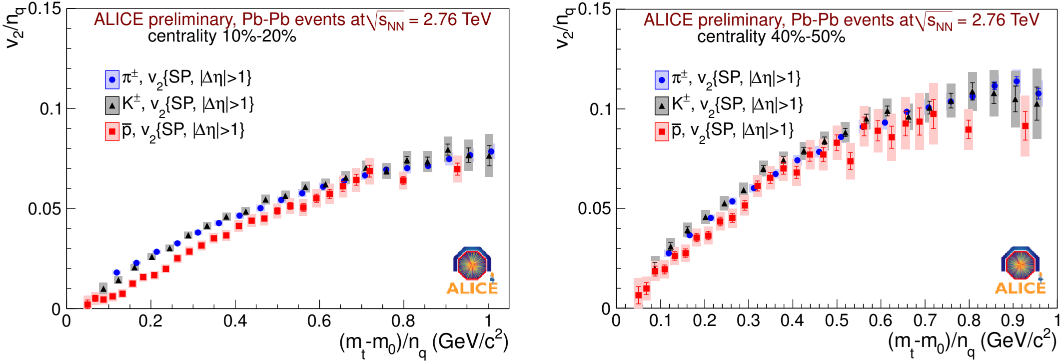
thermal hadronization model: J. Stachel et al [SABMR14]

Constituent-quark-number scaling of v_2

- v_2 scales with number of constituent quarks

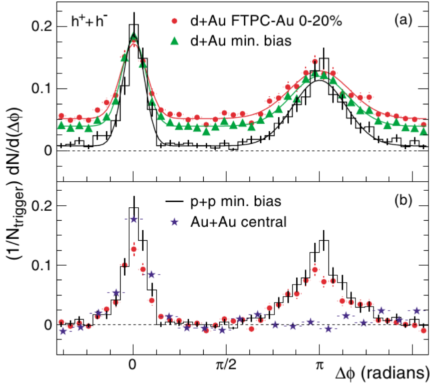
$$v_2^{(\text{had})}(p_T^{(\text{had})}) = n_q\, v_2^{(q)}(p_T^{(\text{had})}/n_q)$$

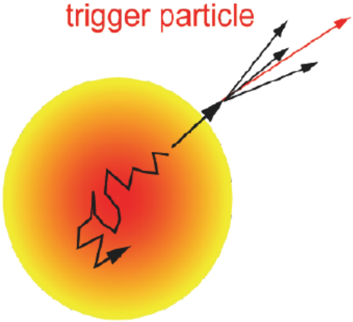
- indicates recombination of quarks in medium around T_{pc}
- "coalescence" of partonic degrees of freedom!



[Krz11]

Jet Quenching





- high p_T: jets going through medium suppressed
- high-density medium $\Rightarrow \rho > \rho_{\text{krit}}$
- energy loss due to elastic scattering and gluon bremsstrahlung
- more on heavy-ion phenomenology: [FHK+11]

QCD

$$\mathscr{L}_{\text{QCD}} = \mathscr{L}_{\psi,A_\mu} + \mathscr{L}_G := \sum_{i\in\{u,d,s,c,b,t\}} \bar{\psi}_{i,j} \left(\mathrm{i}\gamma^\mu (D_\mu)^j{}_k - m_i \delta^j_k\right) \psi^k_i - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a, \quad \hat{D}_\mu = \partial_\mu + \mathrm{i} g \hat{T}^a A^a_\mu(x)$$

$$\mathscr{L}_{\psi,A_\mu} = \sum_{i\in\{u,d\}} \left[\bar{\psi}_{i,R} \left(i\gamma^\mu D_\mu\right) \psi_{i,R} + \bar{\psi}_{i,L} \left(i\gamma^\mu D_\mu\right) \psi_{i,L}\right] - \sum_{i\in\{u,d\}} m_i \left[\bar{\psi}_{i,R} \psi_{i,L} + \bar{\psi}_{i,L} \psi_{i,R}\right]$$

- asymptotic freedom: “running coupling” small at high energy scales
- non-perturbative at low energy scales
- confinement: only color-neutral objects observable (hadrons: mesons, baryons,...)
- lattice-QCD: Euclidean QCD, equilibrium many-body properties
- to describe dynamics: effective models based on “accidental” symmetries of QCD
- light-quark sector (u+d quarks): approximate chiral symmetry $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$

Quark-Meson linear-σ Model

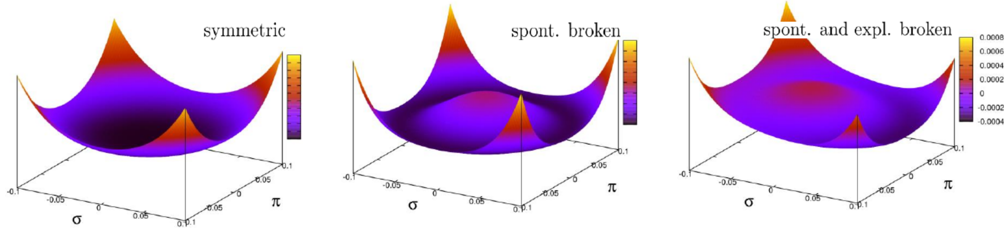
- from Meistrenko (PhD Thesis): [MHG21]
- $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$ linear-σ model
- mesons: σ, $\vec{\pi}$, quarks: $\psi = (u, d)$

$$\mathscr{L} = \bar{\psi}\left[\mathrm{i}\not{\partial} - g\left(\sigma + i\gamma_5 \vec{\pi} \cdot \vec{\tau}\right)\right]\psi + \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi}\right) - \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 + f_\pi m_\pi^2 \sigma + U_0$$

parameter	value	description
λ	20	coupling constant for σ and $\vec{\pi}$
g	$2-5$	coupling constant between $\sigma, \vec{\pi}$ and ψ
f_π	$93\,\mathrm{MeV}$	pion decay constant
m_π	$138\,\mathrm{MeV}$	pion mass
ν^2	$f_\pi^2 - m_\pi^2/\lambda$	field shift term
U_0	$m_\pi^4/(4\lambda) - f_\pi^2 m_\pi^2$	ground state

Quark-Meson linear-σ Model: meson potential

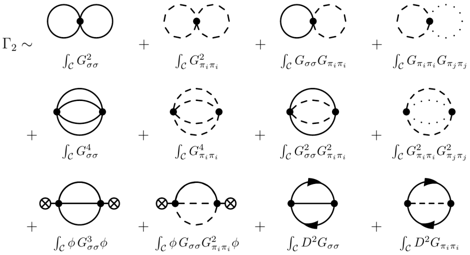
$$\mathscr{L} = \bar{\psi}\left[\mathrm{i}\not{\partial} - g\left(\sigma + i\gamma_5 \vec{\pi}\cdot\vec{\tau}\right)\right]\psi + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - \frac{\lambda}{4}\left(\sigma^2+\vec{\pi}^2-\nu^2\right)^2 + f_\pi m_\pi^2\sigma + U_0$$



Quark-Meson linear-σ Model: 2PI action

$$\mathscr{L} = \bar{\psi}\left[\mathrm{i}\partial\!\!\!/ - g\left(\sigma + i\gamma_5 \vec{\pi}\cdot\vec{\tau}\right)\right]\psi + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 + f_\pi m_\pi^2 \sigma$$

$$\Gamma[\sigma,\vec{\pi},G,D] = S[\sigma,\vec{\pi}] + \frac{\mathrm{i}}{2}\mathrm{Tr}\ln G^{-1} + \frac{\mathrm{i}}{2} G_0^{-1} G - \mathrm{i}\,\mathrm{Tr}\ln D^{-1} - \mathrm{i}\,\mathrm{Tr}\, D_0^{-1} D + \Gamma_2[\sigma,\vec{\pi},G,D]$$



Equations of motion: Kadanoff-Baym equations for Green's functions + mean-field equations

$$\frac{\delta\Gamma}{\delta\sigma} = \frac{\delta\Gamma}{\delta\vec{\pi}} = \frac{\delta\Gamma}{\delta G} = \frac{\delta\Gamma}{\delta D} = 0.$$

Quark-Meson linear-σ Model: off-equilibrium equations

- real-time Keldysh contour $\Rightarrow$ 2PI/Kadanoff Baym $\Rightarrow$ transport equation (spatially homogeneous)
- Mean-field equation

$$\partial_t^2 \phi + D(t) + J(t) = 0, \; J(t) := \lambda \left(\phi^2 - \nu^2 + 3 G^{11}_{\sigma\sigma} + \sum_i G^{11}_{\pi_i \pi_i} \right) \phi - f_\pi m_\pi^2 + g \langle \bar{\psi} \psi \rangle$$

- transport equations for meson- and quark-phase-space-distribution functions

$$\partial_t f^{\sigma}(t, \vec{p}_1) = \mathscr{C}^{b.}_{\sigma\sigma\leftrightarrow\sigma\sigma} + \sum_i \mathscr{C}^{b.}_{\sigma\pi_i\leftrightarrow\sigma\pi_i} + \sum_i \mathscr{C}^{b.}_{\sigma\sigma\leftrightarrow\pi_i\pi_i} + \mathscr{C}^{b.s.}_{\sigma\phi\leftrightarrow\sigma\sigma} + \sum_i \mathscr{C}^{b.s.}_{\sigma\phi\leftrightarrow\pi_i\pi_i} + \mathscr{C}^{f}_{\sigma}$$

$$\partial_t f^{\pi_i}(t, \vec{p}_1) = \mathscr{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_i\pi_i} + \sum_{j\neq i} \mathscr{C}^{b.}_{\pi_i\pi_j\leftrightarrow\pi_i\pi_j} + \sum_{j\neq i} \mathscr{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_j\pi_j} + \mathscr{C}^{b.}_{\pi_i\sigma\leftrightarrow\pi_i\sigma} + \mathscr{C}^{b.}_{\pi_i\pi_i\leftrightarrow\sigma\sigma}$$
$$+ \mathscr{C}^{b.s.}_{\pi_i\phi\leftrightarrow\pi_i\sigma} + \mathscr{C}^{f.s.}_{\pi_i\leftrightarrow\psi\bar{\psi}}$$

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$$\partial_t f^{\bar{\psi}}(t, \vec{p}_1) = \mathscr{C}^{f.s.}_{\bar{\psi}\psi\leftrightarrow\sigma} + \sum_i \mathscr{C}^{f.s.}_{\bar{\psi}\psi\leftrightarrow\pi_i},$$

Hydrodynamics

- ideal hydrodynamics: local thermal equilibrium

$$f^{(0)}(x,p) = g \exp[-\beta(x) u(x) \cdot p + \beta(x)\mu(x)]$$

- $u^{\mu}(x)$ with $u_{\mu} u^{\mu} \equiv 1$: fluid four-velocity, $\beta(x)$: inverse temperature, $\mu(x)$: chemical potential, $p^0 = \sqrt{m^2 + \vec{p}^2}$
- Boltzmann equation (collision term vanishes) $\Rightarrow$ conservation of energy, momentum, and conserved charges

$$T^{\mu\nu}(x) = \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 p}{(2\pi)^3 p^0} p^{\mu} p^{\nu} f(x,p) = u^{\mu} u^{\nu} [\epsilon(x) + P(x)] - \eta^{\mu\nu} P(x),$$

$$N^{\mu} = \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 p}{(2\pi)^3 p^0} p^{\mu} f(x,p) = n(x) u^{\mu}(x),$$

$$\partial_{\mu} T^{\mu\nu} = 0, \quad \partial_{\mu} N^{\mu} = 0.$$

- to close system: equation of state $p = p(\epsilon, n)$
- extended to dissipative hydrodynamics: systematic expansion via moments of f more on hydro: [DR21]

Cumulants of net-baryon number fluctuations

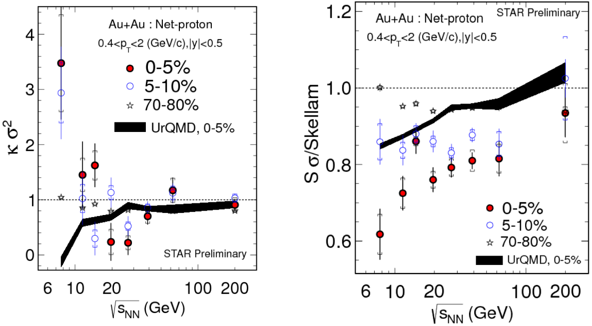
Quark-number susceptibilities

$$c_1 = \frac{N_{q,\text{net}}}{V T^3}, \quad c_2 = \frac{1}{V T^3} \left\langle \left(N_{q,\text{net}} - \langle N_{q,\text{net}} \rangle \right)^2 \right\rangle \equiv \frac{1}{V T^3} \sigma_{q,\text{net}}^2$$

$$c_3 = \frac{1}{V T^3} \left\langle \left(N_{q,\text{net}} - \langle N_{q,\text{net}} \rangle \right)^3 \right\rangle, \quad c_4 = \frac{1}{V T^3} \left[\left\langle \left(N_{q,\text{net}} - \langle N_{q,\text{net}} \rangle \right)^4 \right\rangle - 3\sigma_{q,\text{net}}^4 \right],$$

$$\kappa\sigma^2 = c_4/c_2$$

$$S\sigma = c_3/c_2$$



[Luo16]

Quark-Meson linear-σ Model: 2PI action (A. Meistrenko et al)

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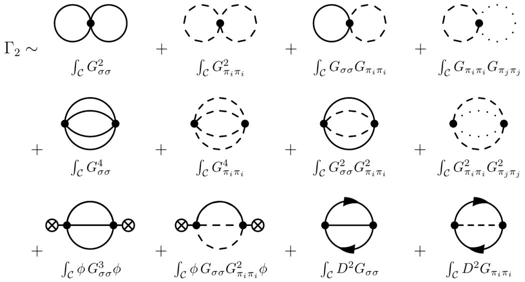
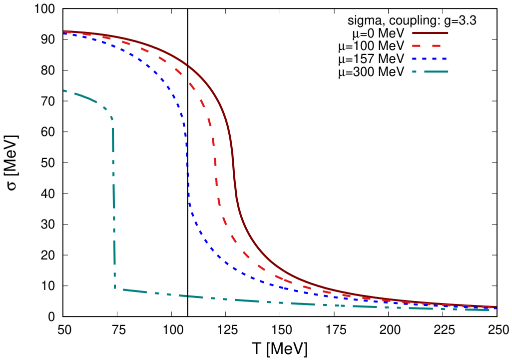


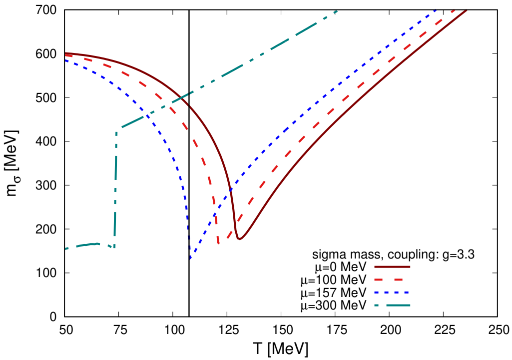
Equations of motion:

$$\frac{\delta\Gamma}{\delta\sigma} = \frac{\delta\Gamma}{\delta\vec{\pi}} = \frac{\delta\Gamma}{\delta G} = \frac{\delta\Gamma}{\delta D} = 0.$$

Quark-Meson linear-σ Model: equilibrium order parameter

$$\Omega_{\text{eff}}[\sigma,\vec{\pi},G,D] = -\frac{1}{\beta V}\mathrm{i}\Gamma[\sigma,\vec{\pi},G,D], \qquad \frac{\partial \Omega_{\text{eff}}}{\partial \sigma} \stackrel{!}{=} 0, \quad M_\sigma^2 = \frac{\partial^2 \Omega_{\text{eff}}}{\partial \sigma^2}, \quad M_\pi^2 = \frac{\partial^2 \Omega_{\text{eff}}}{\partial \pi_i^2}$$





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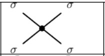
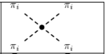
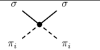
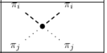
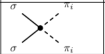
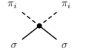
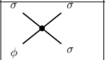
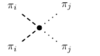
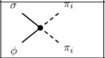
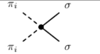
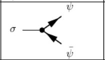
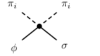
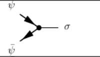
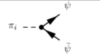
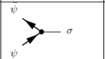
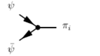
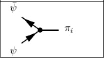
$$\partial_t f^{\pi_i}(t, \vec{p}_1) = \mathscr{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_i\pi_i} + \sum_{j \neq i} \mathscr{C}^{b.}_{\pi_i\pi_j\leftrightarrow\pi_i\pi_j} + \sum_{j \neq i} \mathscr{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_j\pi_j} + \mathscr{C}^{b.}_{\pi_i\sigma\leftrightarrow\pi_i\sigma} + \mathscr{C}^{b.}_{\pi_i\pi_i\leftrightarrow\sigma\sigma}$$
$$+ \mathscr{C}^{b.s.}_{\pi_i\phi\leftrightarrow\pi_i\sigma} + \mathscr{C}^{f.s.}_{\pi_i\leftrightarrow\psi\bar{\psi}}$$

$$\partial_t f^{\psi}(t, \vec{p}_1) = \mathscr{C}^{f.s.}_{\psi\bar{\psi}\leftrightarrow\sigma} + \sum_i \mathscr{C}^{f.s.}_{\psi\bar{\psi}\leftrightarrow\pi_i}$$

$$\partial_t f^{\bar{\psi}}(t, \vec{p}_1) = \mathscr{C}^{f.s.}_{\bar{\psi}\psi\leftrightarrow\sigma} + \sum_i \mathscr{C}^{f.s.}_{\bar{\psi}\psi\leftrightarrow\pi_i},$$

Quark-Meson linear-σ Model: collision terms

collision integral	diagram	collision integral	diagram
$C^{b.}_{\sigma\sigma\leftrightarrow\sigma\sigma}$		$C^{b.}_{\pi_i\pi_i\leftrightarrow\pi_i\pi_i}$	
$C^{b.}_{\sigma\pi_i\leftrightarrow\sigma\pi_i}$		$C^{b.}_{\pi_i\pi_j\leftrightarrow\pi_i\pi_j}$	
$C^{b.}_{\sigma\sigma\leftrightarrow\pi_i\pi_i}$		$C^{b.}_{\pi_i\sigma\leftrightarrow\pi_i\sigma}$	
$C^{b.s.}_{\sigma\phi\leftrightarrow\sigma\sigma}$		$C^{b.}_{\pi_i\pi_i\leftrightarrow\pi_j\pi_j}$	
$C^{b.s.}_{\sigma\phi\leftrightarrow\pi_i\pi_i}$		$C^{b.}_{\pi_i\pi_i\leftrightarrow\sigma\sigma}$	
$C^{f.s.}_{\sigma\leftrightarrow\psi\bar{\psi}}$		$C^{b.s.}_{\pi_i\phi\leftrightarrow\pi_i\sigma}$	
$C^{f.s.}_{\psi\bar{\psi}\leftrightarrow\sigma}$		$C^{f.s.}_{\pi_i\leftrightarrow\psi\bar{\psi}}$	
$C^{f.s.}_{\bar{\psi}\psi\leftrightarrow\sigma}$		$C^{f.s.}_{\psi\bar{\psi}\leftrightarrow\pi_i}$	
		$C^{f.s.}_{\bar{\psi}\psi\leftrightarrow\pi_i}$	

Quark-Meson linear-σ Model: expanding-fireball geometry

- Friedmann-Lemaître-Robertson-Walker metric (spatially flat)

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)(\mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2), \quad H = \dot{a}/a$$

- expanding fireball with radius $R(t) = R_0 + v_e t$, $\dot{a}/a = \dot{R}/R$
- mean-field equation

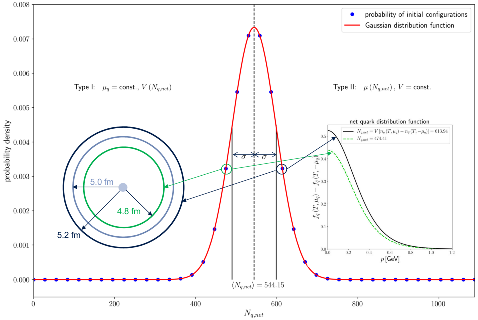
$$\partial_t^2 \phi + 3H \partial_t \phi + D(t) + J(t) = 0$$

- Boltzmann equation

$$\left(\frac{\partial}{\partial t} - H p \frac{\partial}{\partial p}\right) f = \mathscr{I}$$

Initialization of net-quark numbers

- goal: time-evolution of net-quark number fluctuations
- ensembles with fluctuating initial conditions



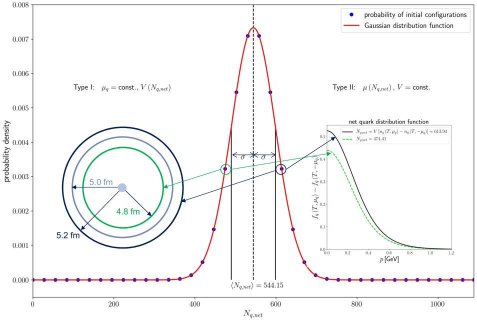
- mean net-quark number

$$\langle N_{\text{q,net}} \rangle = \frac{4\pi}{3} R_0^2 \int \frac{\mathrm{d}^3 p}{(2\pi)^3} [f_q(T,\mu_q) - f_q(T, -\mu_q$$

- standard deviation: $\sigma_{\text{q,net}} = \langle N_{\text{q,net}} \rangle / 10$
- choose $M = 200$-1000 values for $N_{\text{q,net}}$
- initialize type I or type II for each $N_{\text{q,net}}$

Observables

- goal: time-evolution of net-quark number fluctuations
- ensembles with fluctuating initial conditions

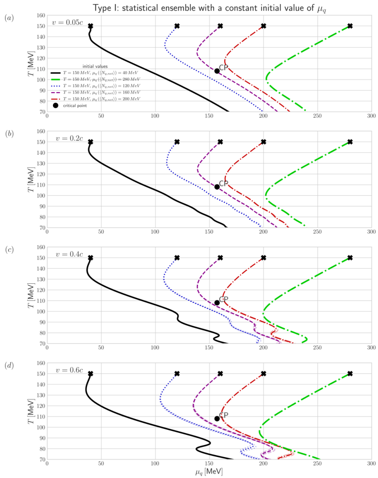


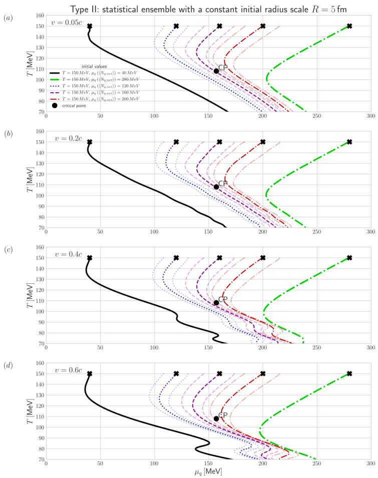
- ensembles with fluctuating initial conditions

$$\langle O\rangle = \frac{p_0 O_0 + p_M O_M}{2} + \sum_{k=1}^{M-1} p_k O_k$$

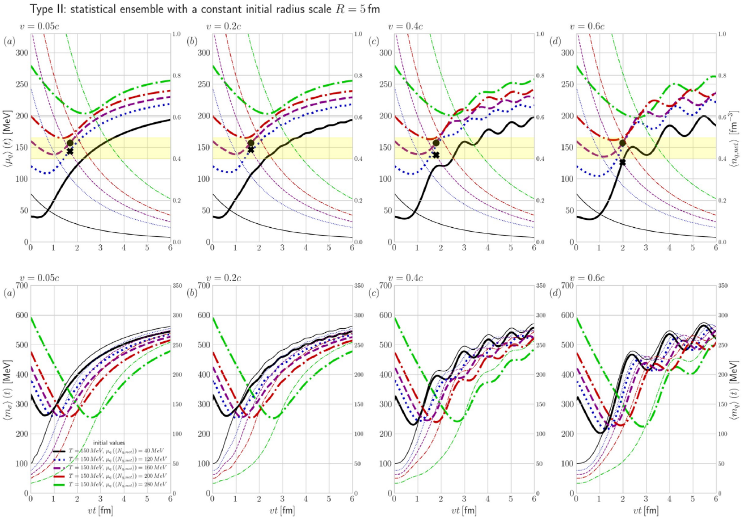
- cumulant ratios $R_{3,1} = c_3/c_1$,
  $R_{4,2} = c_1/c_2 = \kappa\sigma^2$,
  $c_1 = \langle m\rangle$,
  $c_2 = \tilde{m}_2 = \sigma^2$,
  $c_3 = \tilde{m}_3$,
  $c_4 = \tilde{m}_4 - 3\tilde{m}_2^2$

“Trajectories” in phase diagram





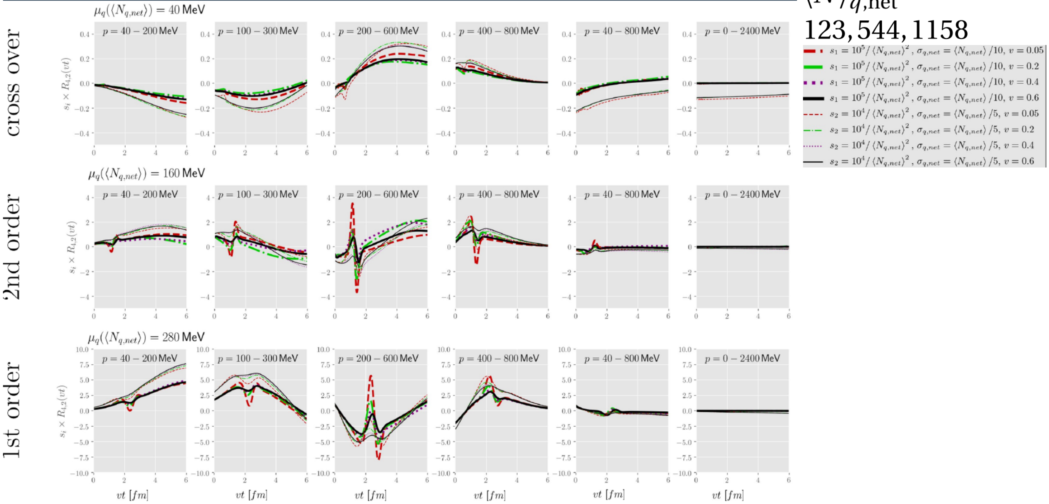
“Trajectories” in phase diagram



“Trajectories” in phase diagram

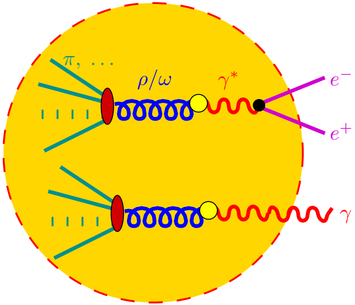
$T_{\text{ini}} = 160$ MeV

$\langle N \rangle_{q,\text{net}} = 123, 544, 1158$



Electromagnetic probes in heavy-ion collisions

- $\gamma, \ell^{\pm}$: no strong interactions
- reflect whole “history” of collision:
  - from pre-equilibrium phase
  - from thermalized medium QGP and hot hadron gas
  - from VM decays after thermal freezeout



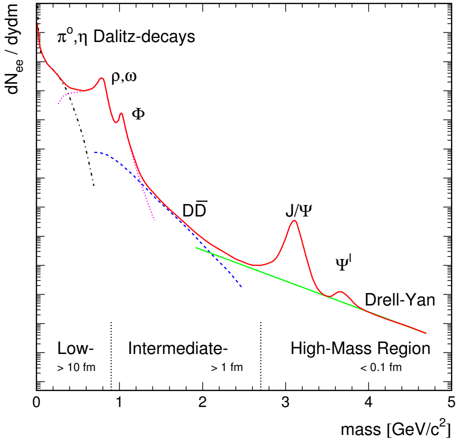


Fig. by A. Drees

Electromagnetic probes from thermal source

- retarded electromagnetic-current-correlation function

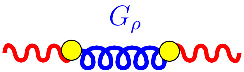
$$\Pi^{\mu\nu}_{\text{em},i} = \mathrm{i}\int \mathrm{d}^4x \; \exp(\mathrm{i}q\,x)\Theta(x^0)\left\langle\left[j^{\mu}_{\text{em},i}(x), j^{\nu}_{\text{em},i}(0)\right]\right\rangle$$

- McLerran-Toimela formula [MT85, GK91]

$$q_0 \frac{\mathrm{d}N_\gamma}{\mathrm{d}^4x\,\mathrm{d}^3\vec{q}} = -\frac{\alpha_{\text{em}}}{2\pi^2} g^{\mu\nu} \,\mathrm{Im}\, \Pi^{(\text{ret})}_{\mu\nu}(q,u)\Big|_{q_0=|\vec{q}|} f_B(q\cdot u)$$

$$\frac{\mathrm{d}N_{e^+e^-}}{\mathrm{d}^4x\,\mathrm{d}^4q} = -g^{\mu\nu}\frac{\alpha^2}{3q^2\pi^3}\,\mathrm{Im}\,\Pi^{(\text{ret})}_{\mu\nu}(q,u)\Big|_{q^2=M^2_{e^+e^-}} f_B(q\cdot u)$$

- Lorentz covariant (dependent on four-velocity of fluid cell, u)
- $q\cdot u = E_{\text{cm}}$: Doppler blue shift of q_T spectra!
- to lowest order in α: $4\pi\alpha\Pi_{\mu\nu} \simeq \Sigma^{(\gamma)}_{\mu\nu}$
- vector-meson dominance model:

$\Sigma^{\gamma}_{\mu\nu} =$ 

- $\ell^+\ell^-$-inv.-mass spectra $\Rightarrow$ in-med. spectral functions of vector mesons (ρ, ω, ϕ)!

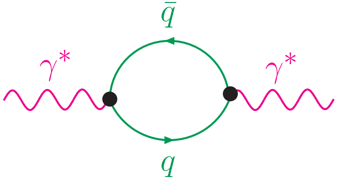
Radiation from thermal QGP: $q\bar{q}$ annihilation

- General: McLerran-Toimela formula

$$\frac{\mathrm{d}N^{(\mathrm{MT})}_{l^+l^-}}{\mathrm{d}^4 x \mathrm{d}^4 q} = -\frac{\alpha^2}{3\pi^3} \frac{L(M^2)}{M^2} g_{\mu\nu} \mathrm{Im} \sum_i \Pi^{\mu\nu}_{\mathrm{em},i}(M,\vec{q}) f_B(q \cdot u)$$

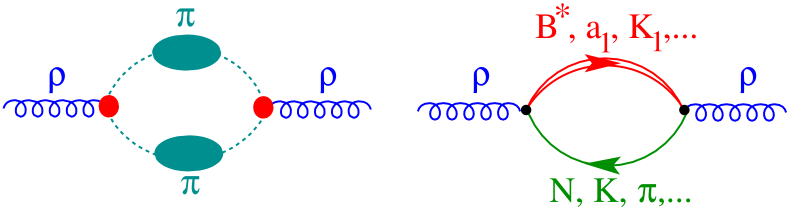
- in QGP phase: $q\bar{q}$ annihilation
- hard-thermal-loop improved em. current-current correlator

$$-\mathrm{i}\Pi_{\mathrm{em,QGP}} =$$



Hadronic many-body theory

- hadronic many-body theory (HMBT) for vector mesons
  [Ko et al, Chanfray et al, Herrmann et al, Rapp et al, …]
- $\pi\pi$ interactions and baryonic excitations
- effective hadronic models, implementing symmetries
- parameters fixed from phenomenology (photon absorption at nucleons and nuclei, $\pi N \rightarrow \rho N$)
- evaluated at finite temperature and density
- self-energies ⇒ mass shift and broadening in the medium



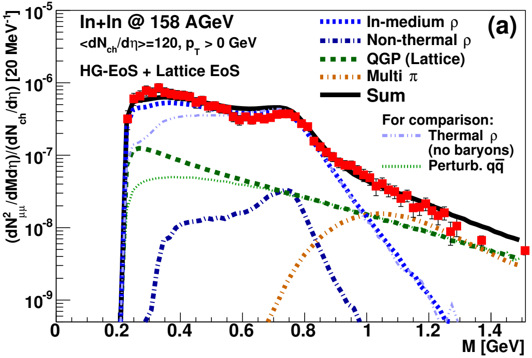
- Baryons important, even at low **net** baryon density $n_B - n_{\bar{B}}$
- reason: $n_B + n_{\bar{B}}$ relevant (CP inv. of strong interactions)

Bulk evolution with transport and coarse graining

- established transport models for bulk evolution
  - e.g., UrQMD, GiBUU, BAMPS, (p)HSD,...
  - solve Boltzmann equation for hadrons and/or partons
- dilemma: need medium-modified dilepton/photon emission rates
- usually available only in equilibrium QFT calculations
- ways out:
  - (ideal) hydrodynamics ⇒ local thermal equilibrium ⇒ use equilibrium rates
  - transport-hydro hybrid model: treat early stage with transport, then coarse grain ⇒ switch to hydro ⇒ switch back to transport (Cooper-Frye "particlization")
- here: UrQMD transport for entire bulk evolution
  ⇒ use coarse graining in space-time cells ⇒ extract $T, \mu_B, \mu_\pi, \ldots$ ⇒ use equilibrium rates locally

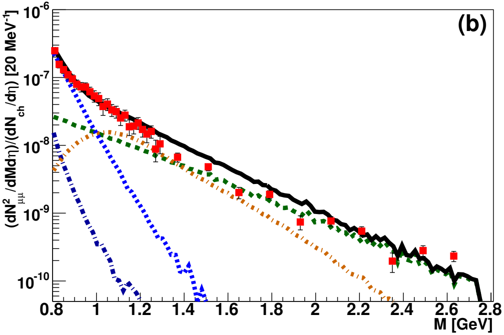
CGUrQMD: In+In (158 AGeV) (SPS/NA60)

- dimuon spectra from $\mathrm{In}+\mathrm{In}(158\,A\mathrm{GeV}) \rightarrow \mu^+\mu^-$ (NA60) [EHWB15]
- min-bias data ($\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}y = 120$)



CGUrQMD: In+In (158 *A*GeV) (SPS/NA60)

- dimuon spectra from $\mathrm{In}+\mathrm{In}(158\,A\mathrm{GeV}) \rightarrow \mu^+\mu^-$ (NA60) [EHWB15]
- min-bias data $(\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}y = 120)$
- higher IMR: provides averaged true temperature
  $\langle T \rangle_{1.5\,\mathrm{GeV} \lesssim M \lesssim 2.4\,\mathrm{GeV}} = 205\text{-}230\,\mathrm{MeV}$
- clearly above $T_c \simeq 150\text{-}160\,\mathrm{MeV}$ (no blueshifts in the invariant-mass spectra!)

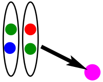


- more on electromagnetic probes: [RW00, RWH10]

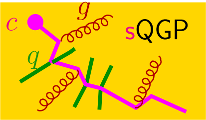
Heavy-Quark Observables

- heavy quarks (charm, bottom) produced in early hard collisions
- suffer whole history of fireball evolution
- open charm/bottom flow (via non-photonic single electrons@RHIC)
  - drag of heavy quarks with thermalized QGP (light quarks + gluons)
  - extract transport properties of QGP!?
  - theoretical challenges: describe motion of heavy quarks in QGP + hadronization to open-charm/bottom mesons
- Heavy quarkonia (e.g., J/ψ, Υ, ...)
  - "J/ψ" suppression (beyond cold-nuclear matter effects): "classical" prediction as QGP signal [T. Matsui, H. Satz, PLB **178**, 416 (1986)]
  - probes in-medium properties of strong force (deconfinement $\Rightarrow$ less binding!)
  - "observation" in lQCD: heavy quarkonia may "survive" above T_c
  - dissociation/melting vs. regeneration of heavy quarkonia in QGP
  - theoretical challenges: in-medium bound-state problem (potential @ $T, \mu > 0$?)
  - describe dissociation/melting + regeneration processes
  - evaluate (take out) "cold-nuclear matter effects" (shadowing, Cronin effect,...)

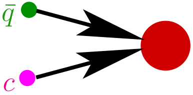
Heavy Quarks in Heavy-Ion collisions



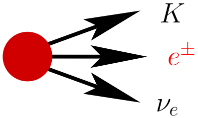
hard production of HQs
described by PDF's + pQCD (PYTHIA)



HQ rescattering in QGP: Langevin simulation
drag and diffusion coefficients from
microscopic model for HQ interactions in the sQGP



Hadronization to D,B mesons via
quark coalescence + fragmentation



semileptonic decay $\Rightarrow$
"non-photonic" electron observables
$R_{AA}^{e^+e^-}(p_T)$, $v_2^{e^+e^-}(p_T)$

Relativistic Langevin process

- Langevin process: friction force + Gaussian random force
- in the (local) rest frame of the heat bath

$$\mathrm{d}\vec{x} = \frac{\vec{p}}{E_p}\mathrm{d}t,$$

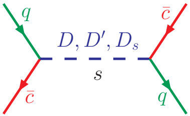
$$\mathrm{d}\vec{p} = -A\,\vec{p}\,\mathrm{d}t + \sqrt{2\mathrm{d}t}\left[\sqrt{B_0}P_\perp + \sqrt{B_1}P_\parallel\right]\vec{w}$$

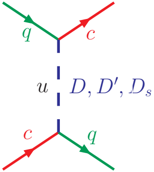
- $\vec{w}$: normal-distributed random variable
- A: friction (drag) coefficient
- $B_{0,1}$: diffusion coefficients
- Einstein dissipation-fluctuation relation $B_1 = E_p T A$.
- flow via Lorentz boosts between "heat-bath frame" and "lab frame"
- A and B_0 from microscopic models for qQ, gQ scattering
- background medium: UrQMD → hydro → UrQMD

[R. Rapp, HvH, R. C. Hwa and X. N. Wang (eds.), Quark-Gluon Plasma Vol. IV, World Sientific (2010), arXiv: 0903.1096 [hep-ph]; M. He, HvH, P. B. Gossiaux, R. J. Fries, R. Rapp, Phys. Rev. E **88**, 032138 (2013)]

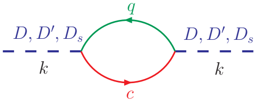
Non-perturbative interactions: Resonance Scattering

- General idea: Survival of D- and B-meson like resonances above T_c
- model based on chiral symmetry (light quarks) HQ-effective theory
- elastic heavy-light-(anti-)quark scattering





- D- and B-meson like resonances in sQGP

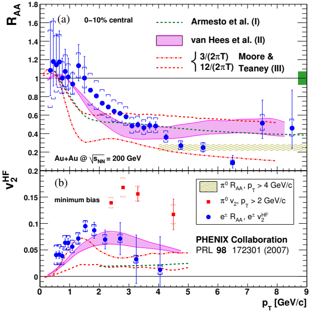


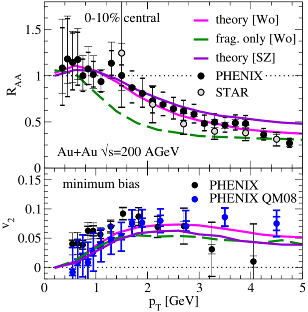
- parameters
  - $m_D = 2\,\text{GeV}, \Gamma_D = 0.4 \ldots 0.75\,\text{GeV}$
  - $m_B = 5\,\text{GeV}, \Gamma_B = 0.4 \ldots 0.75\,\text{GeV}$

[HvH, R. Rapp, Phys. Rev. C **71**, 034907 (2005); HvH, V. Greco, R. Rapp, Phys. Rev. C 73, 034913 (2006)]

Open-Charm/Bottom Observables

- "Non-photonic single electron" spectra at RHIC
- come from decay of D and B mesons ($\bar{q}Q$- and $\bar{Q}q$-bound states)
- p_T spectra ($R_{AA}(p_T)$): energy loss/degree of thermalization
- $v_2(p_T)$: participation of heavy quarks in (anisotropic) flow

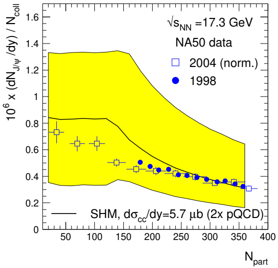


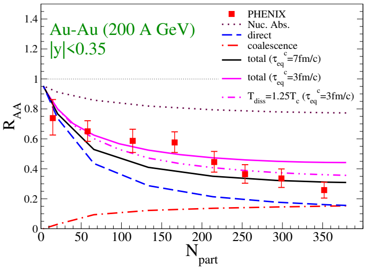


- surprisingly large suppression and $v_2 \Rightarrow$ strongly interacting QGP (sQGP)
  - microscopic energy-loss mechanism?
  - pQCD vs. non-perturbative interactions
  - elastic vs. (gluo-)radiative energy loss

Heavy quarkonia in hot and dense matter

- J/ψ yields in AA compared to pp and pA collisions
- already suppression in pA (initial- and final-state effects)
- understanding of pA crucial to determine QGP effects

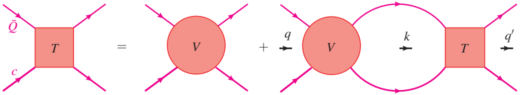


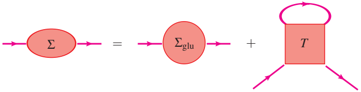


- J/ψ suppression the same at SPS and RHIC
  - in-medium color screening (Mott-like transition)?
  - microscopic dissociation processes?
  - J/ψ survive phase transition $\Rightarrow$ regeneration of J/ψ in QGP
- connections between results from heavy quarkonia and HQ diffusion?

T-matrix approach for quarkonium-bound-state problem

- T-matrix Brückner approach for heavy quarkonia as for HQ diffusion
- consistency between HQ diffusion and $\bar{Q}Q$ suppression!





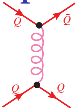
- 4D Bethe-Salpeter equation ⟶ 3D Lippmann-Schwinger equation
- relativistic interaction ⟶ static heavy-quark potential (lQCD)

$$T_\alpha(E;q',q) = V_\alpha(q',q) + \frac{2}{\pi}\int_0^\infty \mathrm{d}k\, k^2 V_\alpha(q',k) G_{Q\bar{Q}}(E;k) T_\alpha(E;k,q) \times \{1 - n_F[\omega_1(\vec{k})] - n_F[\omega_2(k)]\}$$

- q, q', k relative 3-momentum of initial, final, intermediate $\bar{Q}Q$ state

[F. Riek, R. Rapp, arXiv:1005.0769 [hep-ph]]

The potential fit to lattice data



- non-perturbative static gluon propagator

$$D_{00}(\vec{k}) = 1/(\vec{k}^2 + \mu_D^2) + m_G^2/(\vec{k}^2 + \tilde{m}_D^2)^2$$

- finite-T HQ color-singlet-free energy from Polyakov loops

$$\exp[-F_1(r,T)/T] = \left\langle \mathrm{Tr}[\Omega(x)\Omega^\dagger(y)]/N_c \right\rangle$$

$$= \exp\left[\frac{g^2}{2N_c T^2}\left\langle A_{0,\alpha}(x)A_{0,\alpha}(y) - A_{0,\alpha}^2(x)\right\rangle\right] + \mathcal{O}(g^6)$$

- identify $\left\langle A_{0,\alpha}(x)A_{0,\alpha}(y)\right\rangle = D_{00}(x-y)$
- color-singlet free energy

$$F_1(r,T) = -\frac{4}{3}\alpha_s\left\{\frac{\exp(-m_D r)}{r} + \frac{m_G^2}{2\tilde{m}_D}[\exp(-\tilde{m}_D r) - 1] + m_D\right\}$$

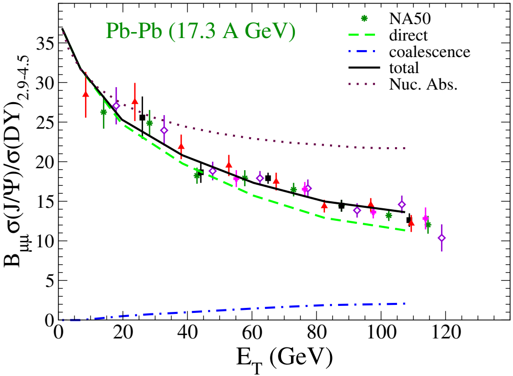
- in vacuo $m_D, \tilde{m}_D \to 0$

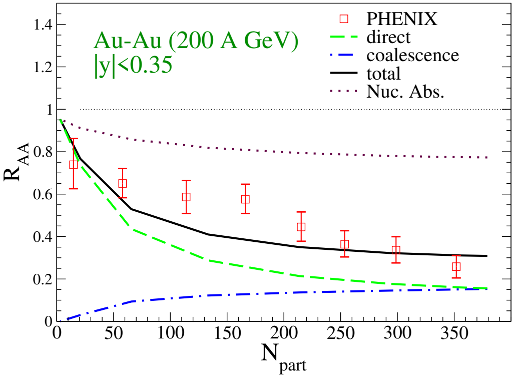
$$F_1(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r, \quad \sigma = \frac{2\alpha_s m_G^2}{3}$$

[F. Riek, R. Rapp, arXiv:1005.0769 [hep-ph]]

Centrality dependence of J/ψ in AA collisions

- mid rapidity

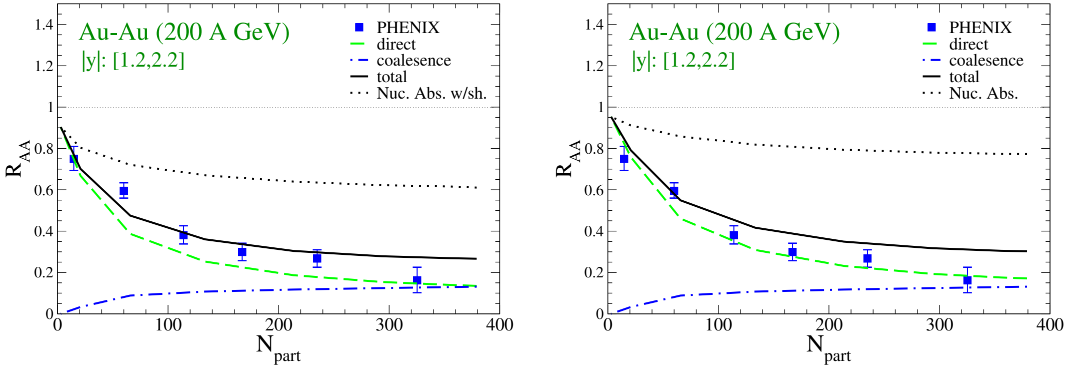




[X. Zhao, R. Rapp, EPC **62**, 109 (2009)]

Centrality dependence of J/ψ in AA collisions

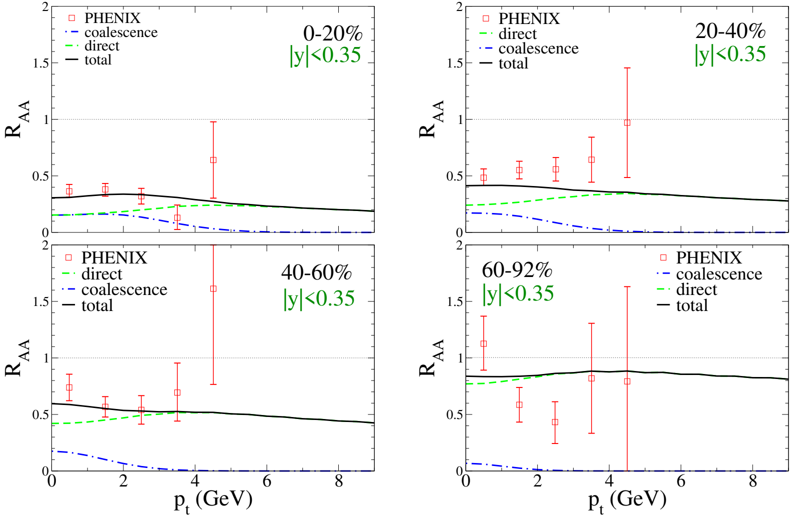
- forward rapidity
- with and without shadowing



[X. Zhao, R. Rapp, EPC **62**, 109 (2009)]

p_T dependence of J/ψ R_{AA}

- mid rapidity



[X. Zhao, R. Rapp, EPC **62**, 109 (2009)]

Conclusion

- QCD medium created in heavy-ion collisions ⇒ can be described as collectively moving fluid
  - p_T spectra, anisotropic flow, v_2
  - high-density medium: jet quenching
  - at highest beam energies: particle/light (anti-) nuclei ⟷ chemical freeze-out close to T_{pc}
  - electromagnetic probes: medium modifications of hadrons
  - heavy quarks: interaction strength ⟷ transport coefficients; quarkonia: screening, dissociation vs. regeneration
- Theory toolbox
  - fundamental level: QCD ⟷ effective (hadronic) QFT models (chiral symmetry,...)
  - many-body QFT: equilibrium ⇒ "imaginary time"/Matsubara formalism/lQCD/hadronic many-body calculations; non-equilibrium ⇒ "real-time"/Schwinger-Keldysh/2PI/Kadanoff-Baym equations
  - coarse graining I: gradient ($\hbar$) expansion ⇒ transport models (on- and off-shell)

Conclusion

  - coarse graning II: expansion around local thermal equilibrium/method of moments ⇒ derivation of transport coefficients (shear+bulk viscosity, electric conductivity, diffusion constants,...)
- Further “applications”
  - nuclear astro physics: neutron stars, neutron-star mergers/kilonovae

Outlook

- many open questions
  - phase diagram: is there a confinement-deconfinement 1st-order phasetransition line with critical endpoint? ⇒ kinetics of "grand-canonical fluctuations" of conserved charges?
  - equation of state? ⇒ neutron stars/kilonovae?
  - do we understand hadronization? ⇒ kinetic theory vs. "naive coalescence"?
  - "spin transport/hydro"? ⇔ polarization measurements (Λ, ϕ mesons at RHIC?)
  - initial state? early "off-equilibrium" phase of "fireball evolution"?

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