Nonequilibrium quark-pair and photon production

Hendrik van Hees

Frank Michler, Dennis Dietrich, Stefan Leupold, and Carsten Greiner

Goethe-Universität Frankfurt

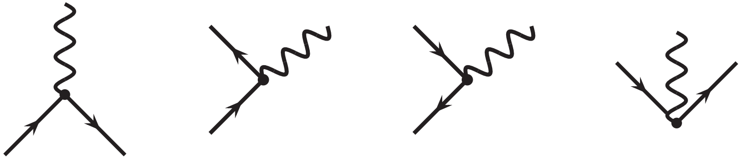
Outline

Motivation: Chiral symmetry and quark-mass changes

- approximate chiral symmetry of QCD (light-quark sector)
  - spontaneously broken in the vacuum due to formation of $\langle \bar{q}q \rangle \neq 0$
  - pions (+kaons): pseudo-Nambu-Goldstone bosons (massless in χ limit)
- at high temperatures and/or densities chiral-symmetry restoration
  - chiral-partner hadrons should become mass degenerate
  - expect large in-medium modifications of spectral properties
- electromagnetic probe in heavy-ion collisions
  - photons and dileptons nearly unaffected by final-state interactions
  - provide undisturbed signal from hot and dense fireball
  - $M_{\ell^+\ell^-}$ spectra $\Leftrightarrow$ medium modifications of light vector mesons
  - p_T spectra of real and virtual photons $\Leftrightarrow$ collective flow/temperature
- time-dependent problem: nonequilibrium production of em. probes

Earlier Work

- Wang, Boyanovsky [WB01]: naive definition of transient photon numbers
- first-order processes (kinematically forbidden in equilibrium!)
  - allowed because of violation of energy conservation
  - spontaneous pair annihilation, particle/antiparticle bremsstrahlung, pair+γ production



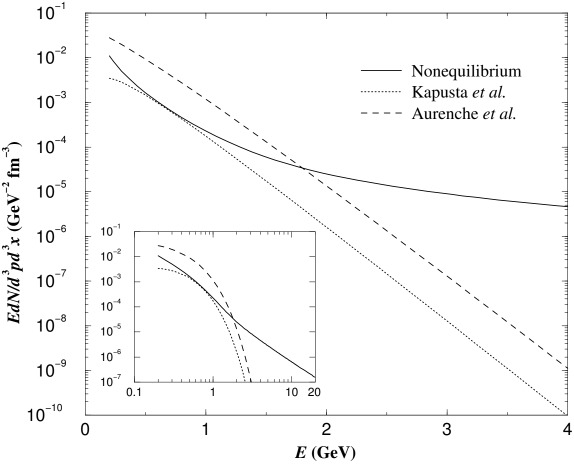
- Boyanovsky, Vega [BV03]
  - vacuum contribution to γ self-energy divergent; renormalization?!?
  - other contributions $\cong_{k\to\infty} 1/k^3$: γ number UV divergent
- Fraga, Gelis, Schiff [FGS05]
  - vacuum contribution unphysical; renormalization prescription criticized
  - no alternative ansatz
  - counter arguments by Boyanovsky and Vega [BV05]

Earlier Work

- Wang, Boyanovsky [WB01]: comparison to LO equilibrium processes
  - contribution from vacuum polarization divergent
  - attempt to renormalization
  - remaining "finite" contributions to the spectrum $\sim 1/k^3$
  - total photon number and energy divergent
- Fraga et al [FGS05]
  - vacuum contribution claimed to be unphysical
  - renormalization procedure claimed to be ad hoc
  - no suggestion of alternative ansatz
- Boyanovsky et al [BV05]
  - devaluated criticism of Fraga et al

Earlier Work

- Wang, Boyanovsky [WB01]: comparison to LO equilibrium processes



- at large k non-equilibrium γ's outshining thermal contributions
- spectra $\cong_{k\to\infty} 1/k^3$ vs. $\propto \exp(-k/T)$ for thermal contrib.
- but total photon number divergent

QED with external Yukawa field

- address toy model
  - start with Dirac (quark) field coupled to a Yukawa-background field
  - couple photon field minimally to quarks
  - keep basic principles: current conservation, em. gauge symmetry
- "matter" Lagrangian

$$\mathscr{L}^{(0)} = \overline{\psi}(\mathrm{i}\not{\partial} - m - g\phi)\psi$$

- gauge phase symmetry $\Leftrightarrow \mathrm{U}(1)_{\text{em}}$
  - add minimal gauge coupling $\partial_\mu \to \partial_\mu + \mathrm{i}q\mathrm{A}_\mu$
  - kinetic term for photons $\mathscr{L}_\gamma^{(0)} = -\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}/4$

$$\mathscr{L} = \mathscr{L}^{(0)} + \mathscr{L}_\gamma^{(0)} \underbrace{-\overline{\psi}q\not{\mathrm{A}}\psi}_{\mathscr{L}_{\text{int}}}$$

- make classical Yukawa field time dependent: $\phi = \phi(t)$
- invariant under $\mathrm{U}(1)_{\text{em}}$ gauge symmetry

$$\psi(x) \to \exp[\mathrm{i}q\chi(x)]\psi(x), \quad \overline{\psi}(x) \to \exp[-\mathrm{i}q\chi(x)]\overline{\psi}(x),$$
$$\mathrm{A}_\mu(x) \to \mathrm{A}_\mu(x) - \partial_\mu\chi, \quad \phi(t) \to \phi(t)$$

Mode decomposition

- start with
$$\mathscr{L}^{(0)} = \overline{\psi}(x)[\mathrm{i}\not{\partial} - m - g\phi(t)]\psi(x)$$
- equation of motion for field operators: free Dirac eq. with t-dep. mass
$$[\mathrm{i}\not{\partial} - M(t)]\psi(t) = 0, \quad M(t) = m + g\phi(t)$$
- spatial translation invariance ⇒ find momentum-eigenmodes:
$$\psi(x) = \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^{3/2}} \sum_{\sigma=\pm 1/2} \exp(\mathrm{i}\vec{p}\cdot\vec{x})[\mathrm{b}(\vec{p},\sigma)u_{\vec{p},\sigma}^{(+)}(t) + \mathrm{d}^{\dagger}(-\vec{p},\sigma)u_{\vec{p},\sigma}^{(-)}(t)]$$
- mode functions
$$u_{\vec{p},\sigma}^{(\lambda)}(t) = [\mathrm{i}\gamma^0\partial_t - \vec{\gamma}\cdot\vec{p} + M(t)]\chi_\sigma^{(\lambda)}\varphi_{\vec{p}}^{(\lambda)}(t),$$
$$\gamma^0\chi_\sigma^{(\lambda)} = \lambda\chi_\sigma^{(\lambda)}, \quad \Sigma^3\chi_\sigma^{(\lambda)} = \sigma\chi_\sigma^{(\lambda)}, \quad \Sigma^3 = \frac{\mathrm{i}}{4}[\gamma^1,\gamma^2],$$
$$[-\partial_t^2 - \vec{p}^2 - M^2(t) + \mathrm{i}\lambda\dot{M}(t)]\varphi_{\vec{p}}^{\lambda} = 0.$$

Particle Interpretation?

- investigate time-dependences of mass with

$$M(t) \underset{t \to \pm\infty}{\cong} M_{\pm} = \text{const}$$

- define modes such that they allow particle interpretation for $t \to \pm\infty$

$$\varphi_{\text{in},\vec{p}}^{(\lambda=1)}(t) \underset{t \to -\infty}{\cong} N_{\text{in},\vec{p}} \exp[-\mathrm{i}\omega_{-}(\vec{p})t], \quad \omega_{-}(\vec{p}) = +\sqrt{M_{-}^2 + \vec{p}^2}$$

$$\varphi_{\text{out},\vec{p}}^{(\lambda=1)}(t) \underset{t \to +\infty}{\cong} N_{\text{out},\vec{p}} \exp[-\mathrm{i}\omega_{+}(\vec{p})t], \quad \omega_{+}(\vec{p}) = +\sqrt{M_{+}^2 + \vec{p}^2}$$

$$\varphi_{\text{in/out}}^{(\lambda=-1)}(t) := \varphi_{\text{in/out},\vec{p}}^{(\lambda=+1)*}(t), \quad |\Omega_{\text{in}}\rangle \neq |\Omega_{\text{out}}\rangle!$$

- normalization of mode functions from equal-time anticommutators
- for $M(t) \neq \text{const} \Rightarrow \varphi_{\text{in},\vec{p}}^{(\lambda)} \neq \varphi_{\text{out},\vec{p}}^{(\lambda)} \Rightarrow |\text{vac}_{\text{in}}\rangle \neq |\text{vac}_{\text{out}}\rangle$
- particle interpretation uniquely defined only for asymptotic states!
- in the following: $\mathrm{b}(\vec{p},\sigma) = \mathrm{b}_{\text{in}}(\vec{p},\sigma)$, $\mathrm{d}(\vec{p},\sigma) = \mathrm{d}_{\text{in}}(\vec{p},\sigma)$

Particle Interpretation at finite times?

- diagonalize time-dependent Hamiltonian
- Bogoliubov transformation to new creation and annihilation operators

$$\begin{pmatrix} \tilde{\mathrm{b}}(t,\vec{p},\sigma) \\ \tilde{\mathrm{d}}^{\dagger}(t,\vec{p},\sigma) \end{pmatrix} = \begin{pmatrix} \xi_{\vec{p},\sigma}(t) & \eta_{\vec{p},\sigma}(t) \\ -\eta^{*}_{\vec{p},\sigma}(t) & \xi^{*}_{\vec{p},\sigma}(t) \end{pmatrix} \begin{pmatrix} \mathrm{b}(\vec{p},\sigma) \\ \mathrm{d}^{\dagger}(\vec{p},\sigma) \end{pmatrix},$$

$$|\xi_{\vec{p},\sigma}(t)|^2 + |\eta_{\vec{p},\sigma}(t)|^2 = 1$$

- after normal ordering wrt. instantaneous vacuum

$$\mathrm{H}(t) = \sum_{\sigma} \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} \omega_{\vec{p}}(t) \left[\tilde{\mathrm{b}}^{\dagger}(t,\vec{p},s)\tilde{\mathrm{b}}(t,\vec{p},s) + \tilde{\mathrm{d}}^{\dagger}(t,\vec{p},s)\tilde{\mathrm{d}}(t,\vec{p},s)\right],$$

$$\omega_{\vec{p}}(t) = \sqrt{M^2(t) + \vec{p}^2}$$

- instantaneous particle/antiparticle number operators

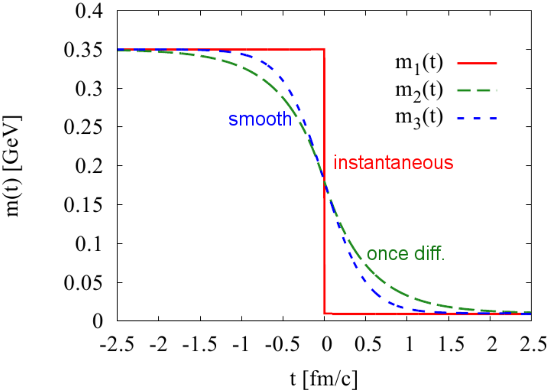
$$\mathrm{N}_{\vec{p},\sigma}(t) = \tilde{\mathrm{b}}^{\dagger}(t,\vec{p},\sigma)\tilde{\mathrm{b}}(t,\vec{p},\sigma), \quad \overline{\mathrm{N}}_{\vec{p},\sigma}(t) = \tilde{\mathrm{d}}^{\dagger}(t,\vec{p},\sigma)\tilde{\mathrm{d}}(t,\vec{p},\sigma)$$

Well-defined initial-value problem!

- give initial state R_0 at $t \to -\infty$; here vacuum: $R_0 = |\Omega_{\text{in}}\rangle \langle \Omega_{\text{in}}|$
- calculate spectra for particles as

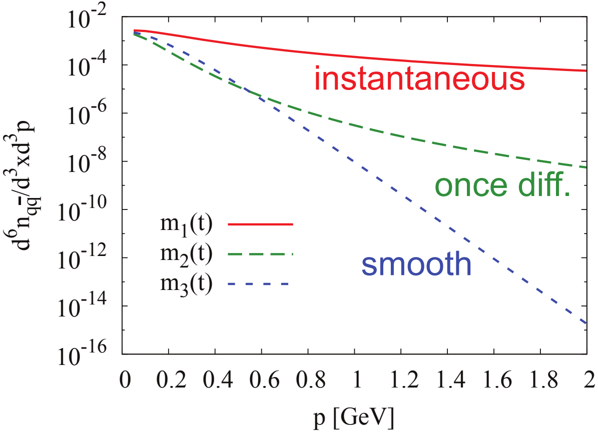
$$\frac{dN_{\vec{p}}}{d^3\vec{x}d^3\vec{p}} = \sum_{\sigma} \left\langle \Omega_{\text{in}} \left| N_{\vec{p},\sigma}(t) \right| \Omega_{\text{in}} \right\rangle = \frac{d\overline{N}_{\vec{p}}}{d^3\vec{x}d^3\vec{p}} = \sum_{\sigma} \left| \eta_{\vec{p},\sigma}(t) \right|^2$$

- Mass-switching functions



Asymptotic pair spectra

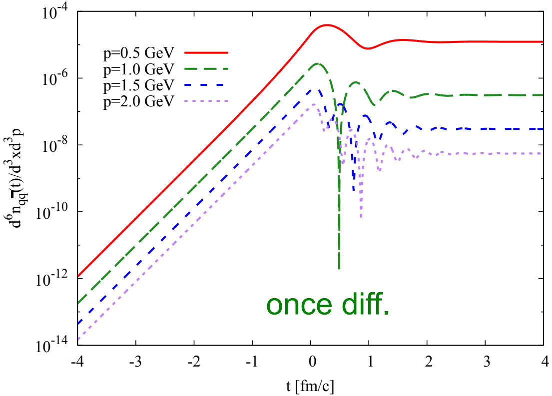
- asymptotic spectra ($t \to \infty$)



- behavior for $p \to \infty$:

  $\propto (m_c - m_b)^2/|\vec{p}|^2$, $\propto 1/|\vec{p}|^6$, $\propto \exp(-p/T_{\text{eff}})$

Time dependence of spectra

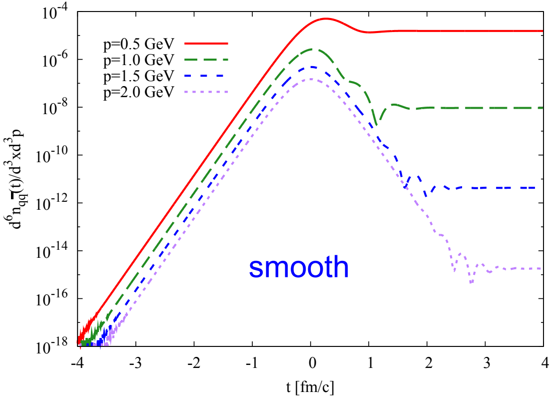


- large overshoot at times with large variations in $M(t)$
- reason:

$$\eta_{\vec{p},\sigma}(t) \underset{|\vec{p}| \gg m(t)}{\cong} \frac{\exp(-\mathrm{i}|\vec{p}|t)}{2|\vec{p}|^2} \int_{-\infty}^{t} \mathrm{d}t' \dot{M}(t') \exp(2\mathrm{i}|\vec{p}|t')$$

- well-behaved UV limit only for asymptotic spectra

Time dependence of spectra



- for finite t: spectra $\cong_{|\vec{p}|\to\infty} 1/|\vec{p}|^4$
- asymptotic spectrum exponential for large $|\vec{p}|$
- total energy density divergent for finite t!
- artifact of Yukawa background field independent of $\vec{x}$?

Adiabatic particle definitions

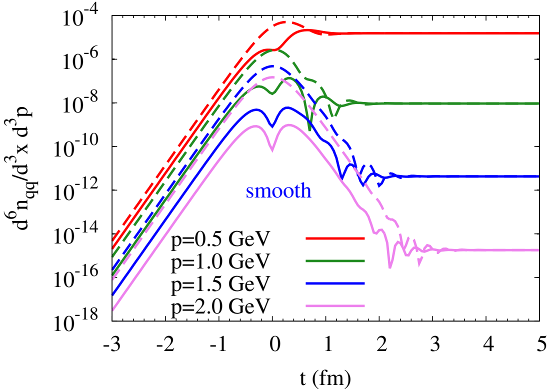
- problems with definition of transient particle numbers
  - instantaneous single-particle energy-eigenmodes: no clear definition of "positive frequency solutions"
  - particle interpretation of those modes ambiguous
- possible well-defined "particle models" @ $t \neq \pm\infty$:
  - adiabatic solution to the mode function (1st order WKB)

$$\tilde{\varphi}_{\vec{p}}^{(+)}(t) = \frac{1}{\sqrt{2\omega(t)[\omega(t)+M(t)]}} \exp\left[-\mathrm{i}\int_{t_0}^{t} \mathrm{d}t' \omega(t')\right]$$

  - positive definite $\omega(t) \Rightarrow \tilde{\varphi}_{\vec{p}}^{(+)} =$ "positive frequency solutions"
  - usually well behaved finite particle densities [Ful79, Ful89]
  - admit proper particle definition (but not unique!)
  - asymptotic particle interpretation unique!
- if WKB (semiclassical approximations) are good
  - transient particle definitions approximately equivalent
  - transient particle spectra/yields good under "transport conditions"
  - fulfilled for "slow" external fields
  - fulfilled for times $\gg$ "formation times" $\tau_{\mathrm{f}} \sim 1/|\vec{p}|$

Adiabatic particle definition

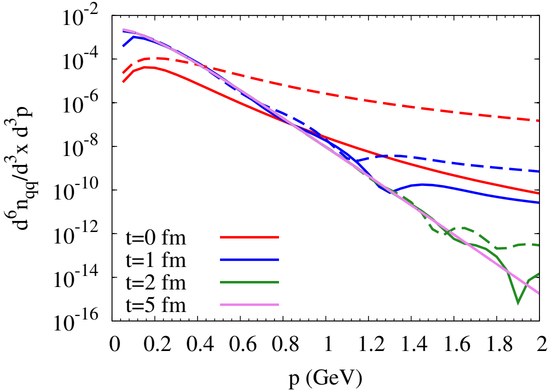
- for smooth mass-switching function
  - solid lines: adiabatic particle definition (1^{st} order WKB)
  - dashed lines: instantenous energy-mode particle definition



- in transient region: still large overshooting of asymptotic value
- more moderate than for instantaneous energy-eigenmodes

Transient particle spectra

- for smooth mass-switching function
  - solid lines: adiabatic particle definition (1st order WKB)
  - dashed lines: instantenous energy-mode particle definition



- in transient region: adiabatic spectra softer
- converge faster towards asymptotic spectra
- only asymptotic spectra unique and well-defined!

Perturbation theory for em. interaction

- treat photon production perturbatively
  - use quark fields from $\mathscr{L}^{(0)}$
  - J-interaction picture includes exact dynamics from $\phi(t)$ (all orders in g)
  - photon field operators: free-field evolution in J-interaction picture
- to get correct asymptotic limit
  - adiabatic switching a la Gell-Mann-Low theorem crucial
  - $\mathrm{H}_{\text{int}} \to \exp(-\varepsilon |t|) \mathrm{H}_{\text{int}}$ $[\mathrm{H}_{\text{int}} = q \int \mathrm{d}^3\vec{x} \overline{\psi}(x) \not{A}(x) \psi(x)]$
  - $\mathrm{U}_{\varepsilon}(t_1, t_2)$: interaction-picture time evolution for states

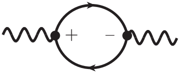
$$|\Omega_{\text{out}}\rangle = \lim_{\varepsilon \to 0} \lim_{t \to \infty} \frac{\mathrm{U}_{\varepsilon}(-\infty, t) |\Omega_{\text{in}}\rangle}{\langle \Omega_{\text{in}} | \mathrm{U}_{\varepsilon}(-\infty, t) | \Omega_{\text{in}} \rangle}$$

  - order of limits crucial: first $t \to \infty$ and then $\varepsilon \to 0$
  - projects to vacuum state
  - damping out transient contributions from higher states
- only applicable for asymptotic observables (S-matrix prescription)
  - advantage: perturbation theory applicable in NEqQFT
  - disadvantage: no transient photon numbers but only asymptotic!
  - definition for γ numbers for interacting γ fields???

Photon production in leading order in α_e

- asymptotic photon-production yield
- photon polarization in terms of Schwinger-Keldysh real-time diagrams

$$\mathrm{i}\Pi^{<}_{\mu\nu}(\vec{k},t_1,t_2) =$$

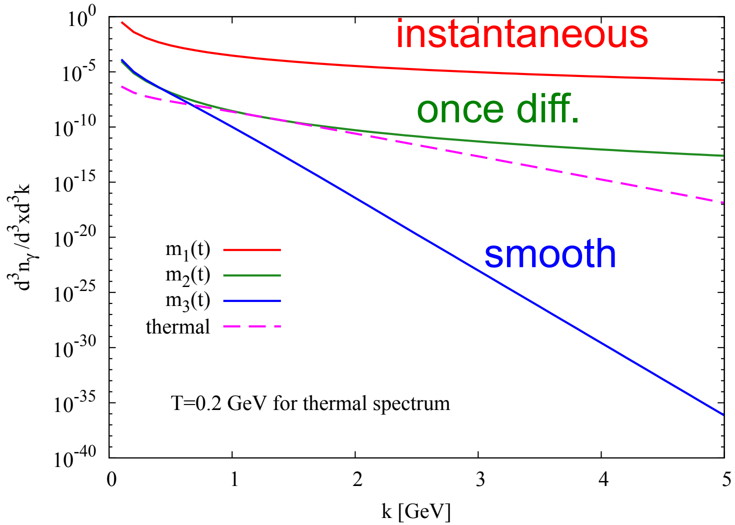


- $\Pi^{<}_{\mu\nu} = \Pi^{<}_{\text{vac},\mu\nu} + \Delta\Pi^{<}_{\mu\nu}$
- photon yield turns out to be absolute square

$$(2\pi)^3|\vec{k}|\frac{\mathrm{d}n_\gamma}{\mathrm{d}^3\vec{x}\mathrm{d}^3\vec{k}} = -\operatorname{Im}\left\{\int_{-\infty}^{\infty}\overline{\mathrm{d}t_1}\int_{-\infty}^{\infty}\overline{\mathrm{d}t_2}\,\Pi^{<}_{\perp}(\vec{k},t_1,t_2) \times \exp\left[\mathrm{i}|\vec{k}|(t_1-t_2)\right]\right\},$$

- $\overline{\mathrm{d}t} := \mathrm{d}t\exp(-\varepsilon|t|)$ with limit $\varepsilon \to 0^+$ after all time integrations!
- quark propagators: full time evolution from external Yukawa field!
- in correct asymptotic limit: vacuum contribution vanishes!

Asymptotic photon spectra

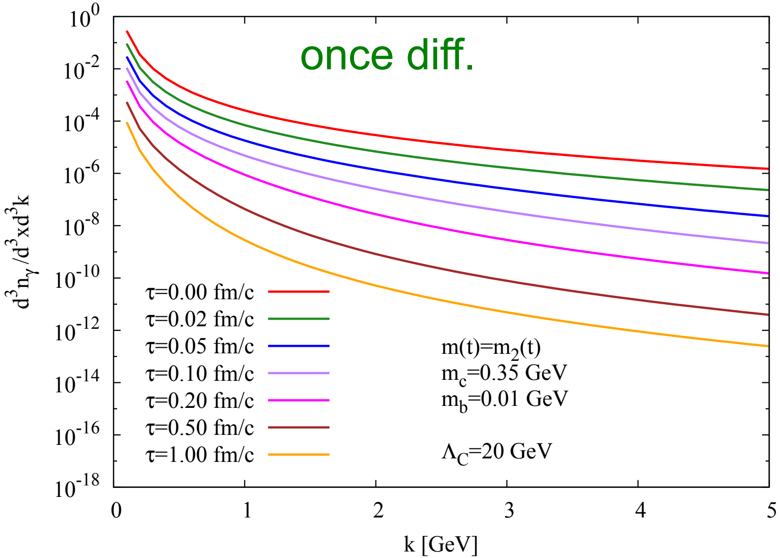


Asymptotic photon spectra

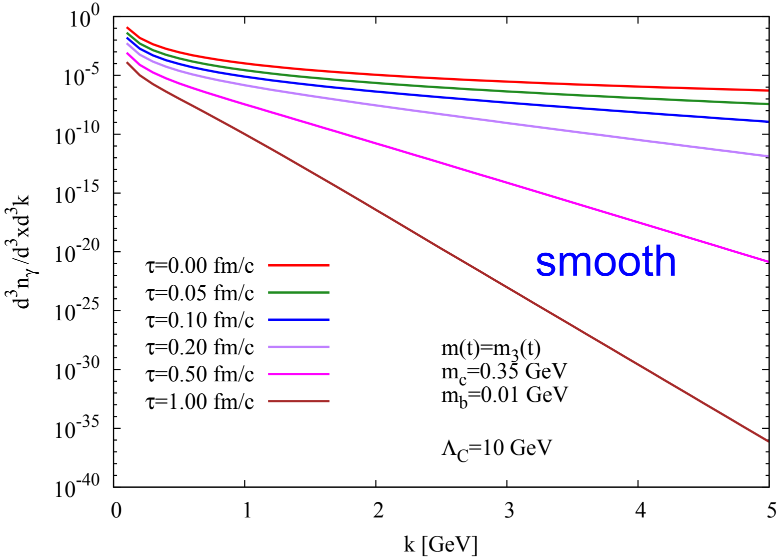
- spectra=modulus squared $\Rightarrow$ positive photon number
- all pure vacuum contributions vanish for asymptotic spectra
- one-loop $\Pi^{<}$ convergent (except for instantaneous switching!)
- instantaneous switching
  - loop integral divergent (here cut-off regulated with $\Lambda = 100\,\text{GeV}$)
  - asymptotic photon spectrum $\cong_{|\vec{k}| \to \infty} 1/|\vec{k}|^3$
  - total photon number and energy UV divergent
- once-differentiable switching function
  - loop integral convergent (here: extrapolated spectrum for $\Lambda \to \infty$)
  - asymptotic photon spectrum $\cong_{|\vec{k}| \to \infty} 1/|\vec{k}|^6$
  - total photon number and energy UV convergent
- Smooth switching function
  - loop integral convergent (here: extrapolated spectrum for $\Lambda \to \infty$)
  - asymptotic photon spectrum $\cong_{|\vec{k}| \to \infty} \exp(-|\vec{k}|/T_{\text{eff}})$
  - total photon number and energy UV convergent

Asymptotic photon spectra

- dependence on switching duration

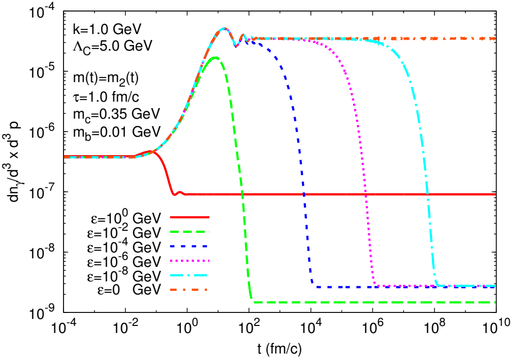


- dependence on switching duration



Transient photon spectra?

- naive attempt: keep time-integration limit finite
- keep only non-vacuum contributions (ad-hoc renormalization)



once differentiable

- spectra at finite t many orders of magnitude above asymptotic limit
- ill-defined transient photon numbers
- no sensible physical interpretation! of naive rates
- interchanging orders of limits $t \to \infty$ and $\varepsilon \to 0 \Rightarrow$ forbidden!

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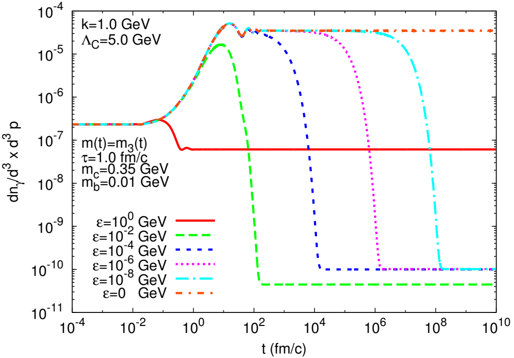
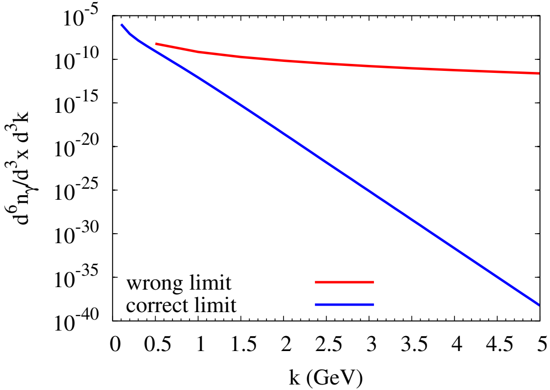


smooth

- spectra at finite t many orders of magnitude above asymptotic limit
- ill-defined transient photon numbers
- no sensible physical interpretation! of naive rates
- interchanging orders of limits $t \to \infty$ and $\varepsilon \to 0 \Rightarrow$ forbidden!

Photon spectrum

- make $\varepsilon \to 0$ before $t_f \to \infty$ in Fourier integrals
- switching em. interactions adiabatically on but not off again
- subtract pure vacuum piece (divergent!)
- vacuum piece vanishes only in correct limit with full time integral
- only then asymptotic limit of vacuum part "on-shell"
- "wrong" limit: spectra $\cong_{k \to \infty} 1/k^{3.8}$



smooth

Conclusions

- spontaneous $q\bar{q}$-pair creation in time-dep. Yukawa background
  - simplified model for transient effect of chiral phase transition $m_{\text{const}} \rightarrow m_{\text{curr}}$
  - definition of transient particle numbers non-trivial
  - naive interpretation wrt. instant energy eigenmodes unphysical
  - probably cured by using particle interpretations wrt. adiabatic modes
  - asymptotic particle yields well defined
- Nonequilibrium photon radiation
  - well-defined perturbation theory for asymptotic photon numbers
  - contributions to $\mathscr{O}\alpha_{\text{em}}$ (kinematically forbidden in equil.)
  - LO neq. asymptotic photon yield convergent (except for instantaneous mass shift)
  - naively defined transient photon spectra ill-defined

Outlook

- straight-forward extension: asymptotic non-eq. dilepton spectra
- Challenges
  - what about IR divergences (soft photons, LPM effect)?
  - physically sensible definition of transient photon numbers?
  - relation to quantum-kinetic/transport picture/equations?
  - self-consistent electromagnetic mean field (“back-reaction problem”)?
  - Noneq. photon (dilepton) asymptotic spectra in realistic HIC fireball?

BACKUP SLIDES

Normalization of fermion-mode functions

- equal-time anticommutators

$$\left\{\psi_\alpha(t,\vec{x}),\psi_\beta^\dagger(t,\vec{y})\right\} = \delta_{\alpha\beta}\delta^{(3)}(\vec{x}-\vec{y})$$

- fulfilled if $\mathrm{b}(\vec{p},\sigma)$, $\mathrm{d}(\vec{p},\sigma)$ like usual fermionic annihilation operators

$$u_{\vec{p},\sigma}^{(\lambda)\dagger} u_{\vec{p},\sigma'}^{(\lambda')} = \delta_{\sigma\sigma'}\delta_{\lambda\lambda'}$$

- with ansatz

$$u_{\vec{p},\sigma}^{(\lambda)}(t) = [\mathrm{i}\gamma^0\partial_t - \vec{\gamma}\cdot\vec{p} + M(t)]\varphi_{\vec{p}}^{(\lambda)}(t)\chi_\sigma^{(\lambda)}$$

fulfilled with $\varphi_{\vec{p}}^{(\lambda=1)} = \varphi_{\vec{p}}^{(\lambda=1)*} =: \varphi_{\vec{p}}(t)$ if

$$|\dot{\varphi}_{\vec{p}}|^2 + \mathrm{i}m\varphi_{\vec{p}}^* \overleftrightarrow{\partial}_t \varphi_{\vec{p}} + \omega_{\vec{p}}^2(t)|\varphi_{\vec{p}}|^2 \equiv 1, \quad \omega_{\vec{p}}^2(t) = M^2(t) + \vec{p}^2$$

- lhs time-independent due to EoM for φ (charge conservation!)
- can express normalization factors in terms of asymptotic frequencies:

$$N_{\text{in/out},\vec{p}}^{(\lambda)} = \frac{1}{\sqrt{2\omega_\pm(\omega_\pm + \lambda M_\pm)}}$$

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