Electromagnetic Probes in Heavy-Ion Collisions I
Foundations

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Outline

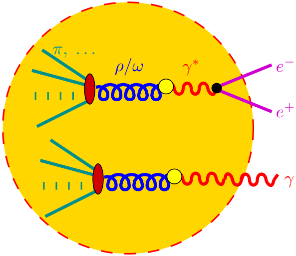
1. Plan of the lectures
2. Electromagnetic Probes: Phenomenology
3. QCD and its (“accidental”) symmetries
4. Strongly interacting matter: QCD/hadronic models at finite T, μ
5. References

Plan of the Lectures

- Lecture I: Fundamentals
  - symmetries and conservation laws in (quantum) field theory
  - QCD, chiral symmetry, and the relation with electromagnetic probes
  - radiation from a transparent thermal source (McLerran-Toimela formula)
- Lecure II: Phenomenology from SIS to LHC energies
  - transport and hydrodynamics
  - collective flow
  - effective hadronic models for vector mesons
  - dileptons at SIS (HADES), SPS (NA60), RHIC (STAR, PHENIX), FAIR, LHC
  - direct photons at RHIC (STAR, PHENIX) and LHC (ALICE)

Why Electromagnetic Probes?

- $\gamma, \ell^{\pm}$: only e. m. interactions
- reflect whole "history" of collision
- chance to see chiral symm. rest. directly?



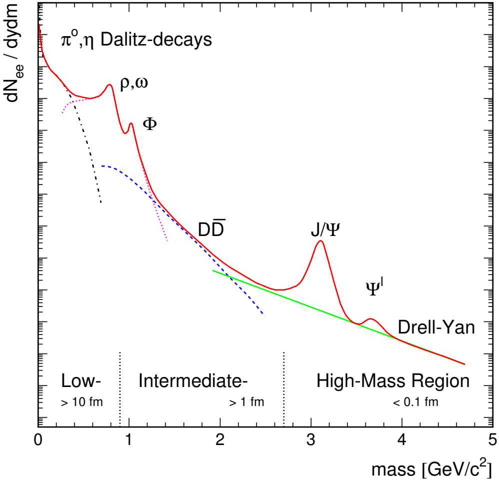
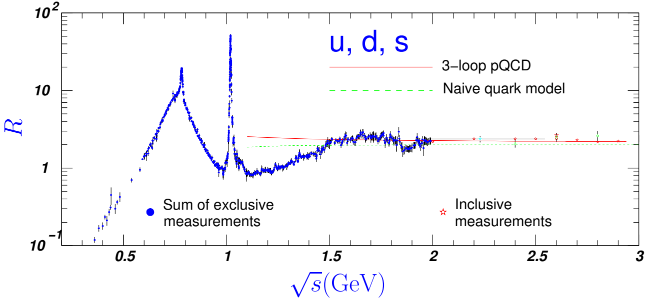


Fig. by A. Drees (from [RW00])

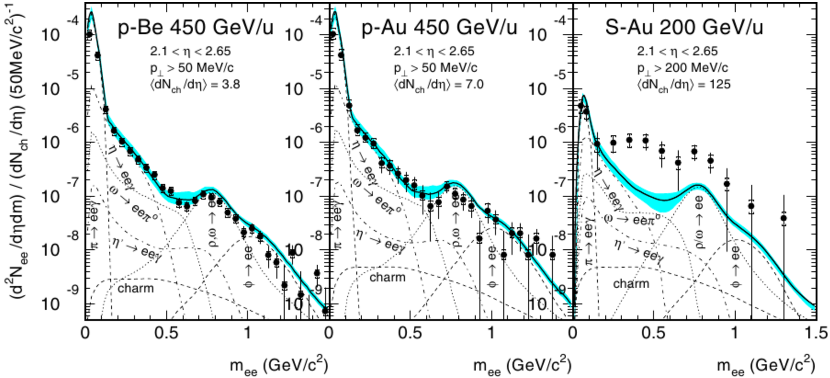
Vacuum Baseline: $e^+e^- \to$ hadrons



$$R := \frac{\sigma_{\mathrm{e^+e^-}\to \mathrm{hadrons}}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$$

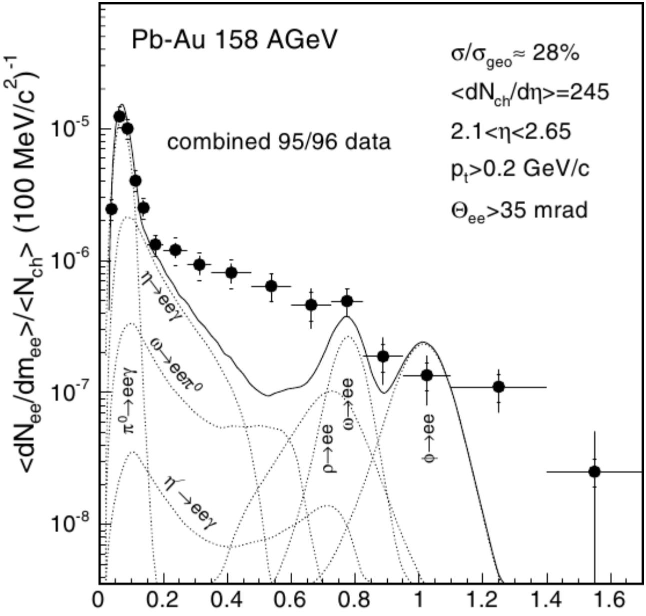
- probes all hadrons with quantum numbers of γ^*
- $R_{\mathrm{QM}} = N_c \sum_{f=u,d,s} Q_f^2 = 3 \times [(2/3)^2 + (-1/3)^2 + (-1/3)^2] = 2$
- Our aim pp $\to \ell^+\ell^-$, pA $\to \ell^+\ell^-$, AA $\to \ell^+\ell^-$ ($\ell = \mathrm{e}, \mu$)

The CERES findings: Dilepton enhancement

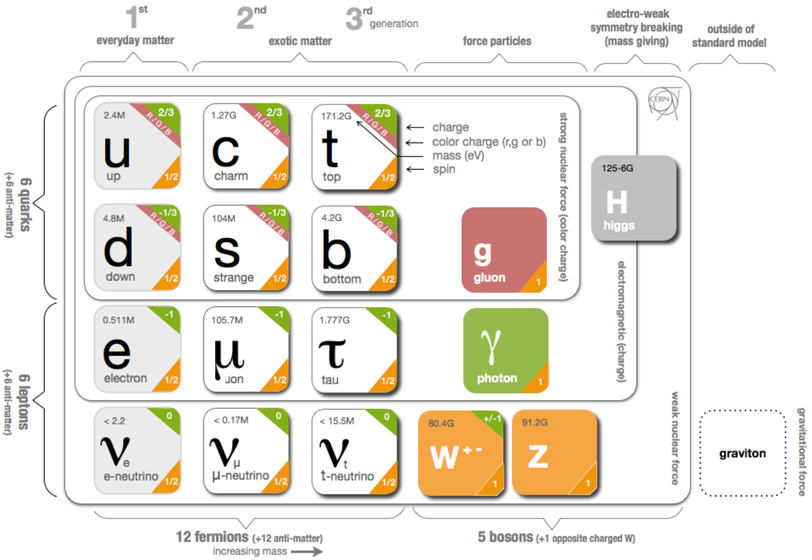


- pp (pBe): "elementary reactions"; baseline (mandatory to understand first!)
- pA: "cold nuclear matter effects"; next step (important as baseline for other observables like "J/ψ suppression")
- AA: "medium effects"; hope to learn something about in-medium properties of vector mesons, fundamental QCD properties

The CERES findings: Dilepton enhancement



The standard model in a nutshell: particles and forces



[graphics from http://www.isgtw.org/spotlight/go-particle-quest-first-cern-hackfest]

Quantum Chromodynamics: QCD

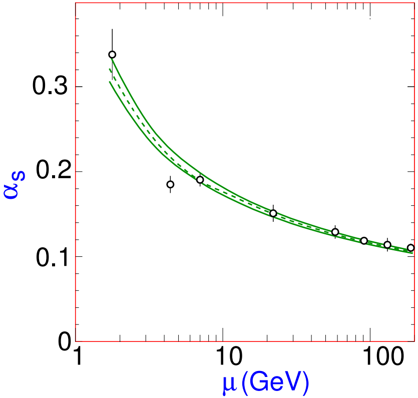
- Theory for strong interactions: QCD

$$\mathscr{L}_{\text{QCD}} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \overline{\psi}(\mathrm{i}\not{D} - \hat{M})\psi$$

- non-Abelian gauge group $\mathrm{SU}(3)_{\text{color}}$
  - each quark: color triplet
  - covariant derivative: $\mathrm{D}_\mu = \partial_\mu + \mathrm{i}g\hat{T}_a A^a$ $(a \in \{1,\ldots,8\})$
  - field-strength tensor $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf^a{}_{bc} A^b_\mu A^c_\nu$
  - group structure constants: $[\hat{T}^a, \hat{T}^b] = \mathrm{i}f^a{}_{bc}\hat{T}^b\hat{T}^c$, $\hat{T}^a = (\hat{T}^a)^\dagger \in \mathbb{C}^{3\times 3}$
- Particle content:
  - ψ: Quarks with flavor $(u,d;c,s;t,b)$ (mass eigenstates!)
  - $\hat{M} = \mathrm{diag}(m_u, m_d, m_s, \ldots)$ = current quark masses
  - A^a_μ: gluons, gauge bosons of $\mathrm{SU}(3)_{\text{color}}$
- Symmetries
  - fundamental building block: local $\mathrm{SU}(3)_{\text{color}}$ symmetry
  - in light-quark sector: approximate chiral symmetry ($\hat{M} \to 0$)
  - dilatation symmetry (scale invariance for $\hat{M} \to 0$)

Features of QCD

- asymptotically free: at large momentum transfers $\alpha_s = 4\pi g_s^2 \to 0$
- running from renormalization group (due to self-interactions of gluons!): Nobel prize 2004 for Gross & Wilczek, Politzer (1973)



- quarks and gluons confined in hadrons
- theoretically not fully understood (nonperturbative phenomenon!)
- need of effective hadronic models at low energies: (Chiral) symmetry!

Chiral Symmetry of (massless) QCD

- Consider only light u, d quarks
- iso-spin 1/2 doublet: $\psi = \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$
- NB: ψ has three "indices": Dirac spinor, color, flavor iso-spin!
- γ matrices: $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}\mathbb{1}$, $\gamma_5 := \mathrm{i}\gamma_0\gamma_1\gamma_2\gamma_3$, $\gamma_5\gamma_\mu = -\gamma_\mu\gamma_5$, $\gamma_5^\dagger = \gamma_5$, $\gamma_5^2 = \mathbb{1}$
- Diracology of left and right-handed components

$$\psi_L = \frac{\mathbb{1} - \gamma_5}{2}\psi = P_L\psi, \quad \psi_R = \frac{\mathbb{1} + \gamma_5}{2}\psi = P_R\psi,$$

$$P_{L/R}^2 = P_{L/R}, \quad P_R P_L = P_L P_R = 0, \quad P_{L/R}\gamma_5 = \gamma_5 P_{L/R} = \mp P_{L/R}$$

$$P_{L/R}\gamma_\mu = \gamma_\mu P_{R/L}, \quad \overline{P_L\psi} = \overline{\psi}P_R, \quad \overline{P_R\psi} = \overline{\psi}P_L$$

$$\overline{\psi}\gamma_\mu\psi = \overline{\psi_L}\gamma_\mu\psi_L + \overline{\psi_R}\gamma_\mu\psi_R, \quad \overline{\psi}\psi = \overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L$$

- $\overline{\psi} := \psi^\dagger\gamma_0$, $\overline{\gamma_5\psi} = \psi^\dagger\gamma_5^\dagger\gamma_0 = -\overline{\psi}\gamma_5$
- in the massless limit ($m_u = m_d = 0$)

$$\mathscr{L}_{u,d} = \overline{\psi}\mathrm{i}\not{D}\psi = \overline{\psi_L}\mathrm{i}\not{D}\psi_L + \overline{\psi_R}\mathrm{i}\not{D}\psi_R$$

Chiral Symmetry

- in the massless limit ($m_u = m_d = 0$)
- a lot of global chiral symmetries:
  - change of independent phases for left and right components:

$$\psi_L(x) \to \exp(\mathrm{i}\phi_L)\psi_L(x), \quad \psi_R(x) \to \exp(\mathrm{i}\phi_R)\psi_R(x)$$

  - symmetry group $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$
  - independent "iso-spin rotations"

$$\psi_L(x) \to \exp(\mathrm{i}\vec{\alpha}_L \cdot \vec{T})\psi_L(x), \quad \psi_R(x) \to \exp(\mathrm{i}\vec{\alpha}_R \cdot \vec{T})\psi_R(x)$$

  - $\vec{T} = \vec{\tau}/2$, $\vec{\tau}$: Pauli matrices; symmetry group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$
- alternative notation scalar-pseudoscalar phases/iso-spin rotations

$$\psi \to \exp(\mathrm{i}\phi_s)\psi, \quad \psi \to \exp(\mathrm{i}\gamma_5\phi_a)\psi$$
$$\psi \to \exp(\mathrm{i}\vec{\alpha}_V \cdot \vec{T})\psi, \quad \psi \to \exp(\mathrm{i}\gamma_5\vec{\alpha}_A \cdot \vec{T})\psi$$

- $\mathrm{U}(1)_s$ and $\mathrm{SU}(2)_V$ are subgroups that are symmetries even if $m_u = m_d \neq 0 \Rightarrow$ Heisenberg's iso-spin symmetry!

Currents: relation to mesons

- based on [Koc97, Sch03, Din11]
- Noether: each global symmetry leads to a conserved quantity
- from chiral symmetries

$$j_s^\mu = \overline{\psi}\gamma^\mu \psi, \quad j_a^\mu = \overline{\psi}\gamma^\mu \gamma_5 \psi$$
$$\vec{j}_V^\mu = \overline{\psi}\gamma^\mu \vec{T} \psi, \quad \vec{j}_A^\mu = \overline{\psi}\gamma^\mu \gamma_5 \vec{T} \psi$$

- Link to mesons: Build Lorentz-invariant objects with corresponding quantum numbers
  - σ: $\overline{\psi}\psi$ (scalar and iso-scalar)
  - π's: $i\overline{\psi}\vec{T}\gamma_5\psi$ (pseudoscalar and iso-vector)
  - ρ's: $\overline{\psi}\gamma_\mu\vec{T}\psi$ (vector and iso-vector)
  - a_1's: $\overline{\psi}\gamma_\mu\gamma_5\vec{T}\psi$ (axialvector and iso-axialvector)
- in nature: σ and π's; ρ's and a_1's do not have same mass!
- reason: QCD ground state not symmetric under pseudoscalar and pseudovector trafos since $\langle \text{vac} | \overline{\psi}\psi | \text{vac} \rangle \neq 0$

Spontaneous symmetry breaking

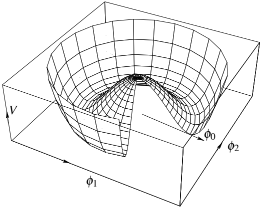
- spontaneously broken symmetry: ground state not symmetric
- vacuum necessarily degenerate
- vacuum invariant under scalar and vector transformations: $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$ broken to $\mathrm{U}(1)_s$; $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ broken to $\mathrm{SU}(2)_V$
- for each broken symmetry massless scalar Goldstone boson
- there are three pions which are very light compared to other hadrons (finite masses due to explicit breaking through m_u, m_d!)
- but no pseudoscalar isoscalar light particle! ($m_\eta \simeq 548$ MeV)
- reason: $\mathrm{U}(1)_a$ anomaly
    - axialscalar symmetry does not survive quantization!
    - good for explanation of correct decay rate for $\pi_0 \rightarrow \gamma\gamma$
    - axialscalar current not conserved $\partial_\mu j_a^\mu = 3/8 \alpha_s \varepsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a$
- explicit breaking due to quark masses
    - can be treated perturbatively $\Rightarrow$ chiral perturbation theory
    - axial-vector current only approximately conserved $\Rightarrow$ PCAC
    - a lot of low-energy properties of hadrons derivable

The minimal linear σ model

- chiral symmetry realized by SO(4): meson fields $\phi \in \mathbb{R}^4$
- describes σ and pions $(\pi^{\pm}, \pi^0)$

$$\mathscr{L}_{\chi\text{limit}} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{\lambda}{4}(\phi^2 - f_\pi^2)^2$$

- spontaneous symmetry breaking: mexican-hat potential



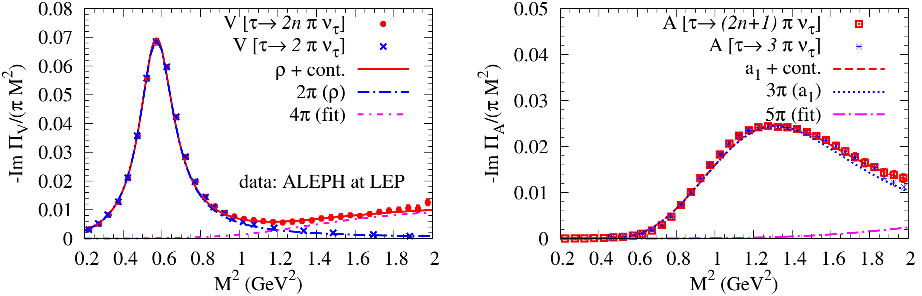
- doesn't cost energy to excite field in direction of the rim
  $\Rightarrow$ massless Nambu-Goldstone bosons (pions)
- vacuum expectation value $\langle \phi^0 \rangle = f_\pi \neq 0$
- symmetry spontaneously broken from SO(4) to SO(3)_V
- particle spectrum: 4 field-degrees of freedom $\Rightarrow$ vacuum inv. 3-dim SO(3)
  $\Rightarrow$ 3 massless pions $\Rightarrow$ $4 - 3 = 1$ massive σ

Explicit symmetry breaking

- explicit χ-symmetry breaking due to m_{quark}: $m_\pi \simeq 140\,\text{MeV}$
- Gell-Mann-Oakes-Renner relation: $m_\pi^2 f_\pi^2 = -m \langle \overline{q} q \rangle$
- vector (isospin) symmetry only fulfilled for $m_u = m_d$
- in reality: $m_u \simeq$ 1.7-3.3 MeV, $m_d \simeq$ 4.1-3.3MeV
- isospin symmetry as strongly broken as χ symmetry!

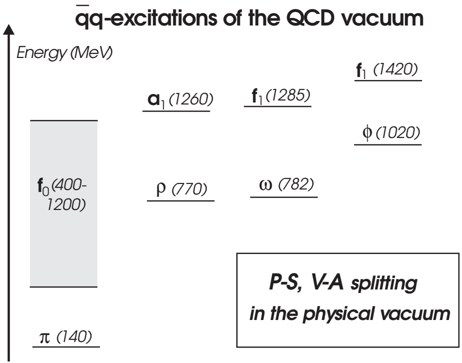
Most accurate experiment related to χSB

- weak decay $\tau \rightarrow \nu_\tau + n \cdot \pi$
- weak interactions: Quantum-Flavor Dynamics (QFD)
- QFD = Glashow-Salam-Weinberg model (Nobel 1979)
  + Higgs, Englert (Nobel 2013) et al
- charged currents $\propto j_V^\mu - j_A^\mu$
- n even: must go through vector current
  n odd: must go through axialvector current



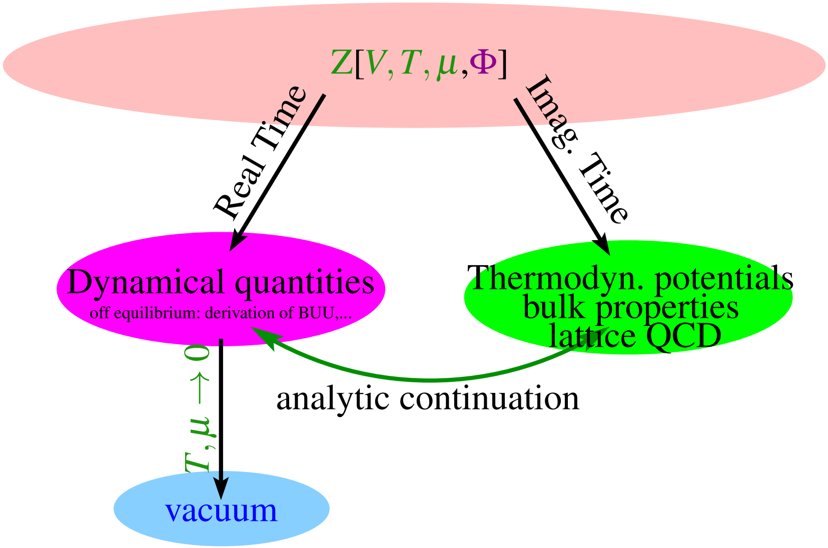
Phenomenology from Chiral Symmetry

- Use (approximate) chiral symmetry to build effective models
- Ward identities
  - PCAC: $\left\langle 0 \left| \partial^\mu j^k_{A\mu} \right| \pi^j(\vec{k}) \right\rangle = \mathrm{i} F_\pi^2 m_\pi^2 \delta^{kj}$
  - $m_\pi^2 F_\pi^2 = -(m_u + m_d) \langle 0 | \overline{u} u | 0 \rangle$
    (Gell-Mann-Oakes-Renner relation)
- Spontaneous breaking causes splitting of chiral partners:



Finite Temperature/Density: Idealized theory picture

- partition sum: $Z(V,T,\mu_q,\Phi) = \mathrm{Tr}\{\exp[-(\mathrm{H}[\Phi]-\mu_q \mathrm{N})/T]\}$



[CSHY85, Lv87, LeB96, KG06]

Finite Temperature

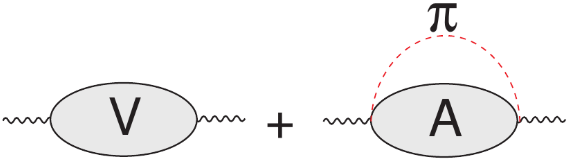
- Asymptotic freedom
  - quark condensate melts at high enough temperatures/densities
- all bulk properties from partition sum:

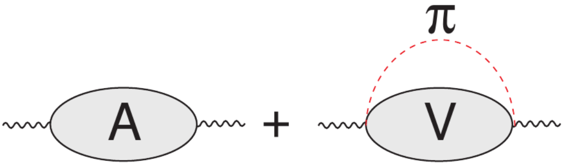
$$Z(V,T,\mu_q) = \mathrm{Tr}\{\exp[-(\mathrm{H} - \mu_q \mathrm{N})/T]\}$$

- Free energy: $\Omega = -\frac{T}{V} \ln Z = -P$
- Quark condensate: $\left\langle \overline{\psi}_q \psi_q \right\rangle_{T,\mu_q} = \frac{V}{T} \frac{\partial P}{\partial m_q}$
- Lattice QCD (at $\mu_q = 0$)
  - chiral symmetry $\Leftrightarrow \langle \overline{\psi} \psi \rangle$
  - deconfinement transition $\Leftrightarrow$ Polyakov Loop $\mathrm{tr} \left\langle P \exp(\mathrm{i} \int_0^\beta \mathrm{d}\tau A^0) \right\rangle$
  - Chiral symmetry restoration and deconfinement transition at same T_c

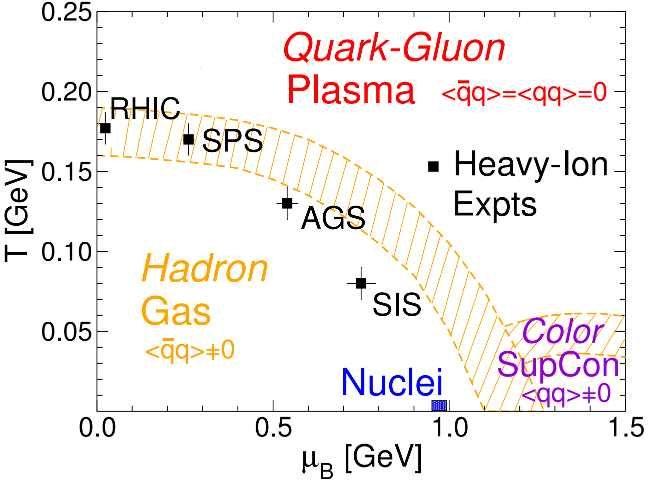
Vector-Axialvector Mixing in the Medium

- in the medium: vector-axialvector currents mix
- due to thermal pions
- possible mechanism for χSR!
- in low-density/temperature approximation: model independent
- see [DEI90a, DEI90b, UBW02, SYZ96, SYZ97]





The QCD Phase Diagram



What can we learn from em. probes in heavy-ion collisions?

- only penetrating probe
  - leptons and photons leave hot and dense fireball unaffected
  - they are produced during the entire fireball evolution
  - dileptons provide information on in-medium spectral properties of hadrons
- theoretical challenge
  - need an understanding of QCD medium at all stages of its evolution
    $\Rightarrow$ transport models, hydrodynamics
  - need to identify all sources of dileptons and photons
  - perturbative QCD not applicable
    $\Rightarrow$ non-perturbative QCD, effective hadronic models
  - evaluate dilepton and photon rates $\Rightarrow$ QFT at finite T and μ_{B}

Vector Mesons and electromagnetic Probes

- photon and dilepton thermal emission rates given by same electromagnetic-current-correlation function ($J_\mu = \sum_f Q_f \overline{\psi_f} \gamma_\mu \psi_f$)
- McLerran-Toimela formula

$$\Pi^{<}_{\mu\nu}(q) = \int \mathrm{d}^4 x \exp(\mathrm{i} q \cdot x) \left\langle J_\mu(0) J_\nu(x) \right\rangle_T = -2 n_{\mathrm{B}}(q_0) \, \mathrm{Im} \, \Pi^{(\mathrm{ret})}_{\mu\nu}(q)$$

$$q_0 \frac{\mathrm{d}N_\gamma}{\mathrm{d}^4 x \mathrm{d}^3 \vec{q}} = -\frac{\alpha_{\mathrm{em}}}{2\pi^2} g^{\mu\nu} \, \mathrm{Im} \, \Pi^{(\mathrm{ret})}_{\mu\nu}(q,u) \Big|_{q_0 = |\vec{q}|} f_B(p \cdot u)$$

$$\frac{\mathrm{d}N_{e^+e^-}}{\mathrm{d}^4 x \mathrm{d}^4 k} = -g^{\mu\nu} \frac{\alpha^2}{3 q^2 \pi^3} \, \mathrm{Im} \, \Pi^{(\mathrm{ret})}_{\mu\nu}(q,u) \Big|_{q^2 = M^2_{e^+e^-}} f_B(p \cdot u)$$

- manifestly Lorentz covariant (dependent on four-velocity of fluid cell, u)
- to lowest order in α: $4\pi\alpha \Pi_{\mu\nu} \simeq \Sigma^{(\gamma)}_{\mu\nu}$
- derivable from underlying thermodynamic potential, Ω!

Vector Mesons and chiral symmetry

- vector and axial-vector mesons $\leftrightarrow$ respective current correlators

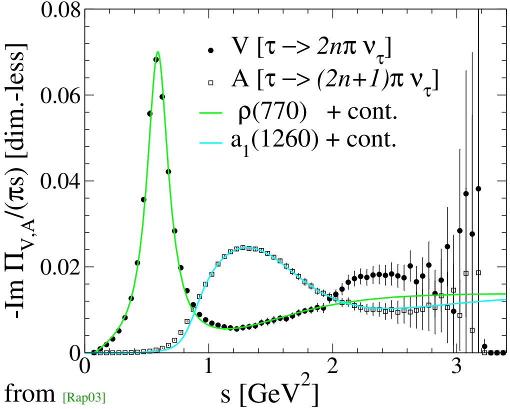
$$\Pi^{\mu\nu}_{V/A}(p) := \int \mathrm{d}^4 x \exp(\mathrm{i}px) \left\langle J^{\nu}_{V/A}(0) J^{\mu}_{V/A}(x) \right\rangle_{\text{ret}}$$

- Ward-Takahashi Identities of χ symmetry $\Rightarrow$ Weinberg-sum rules

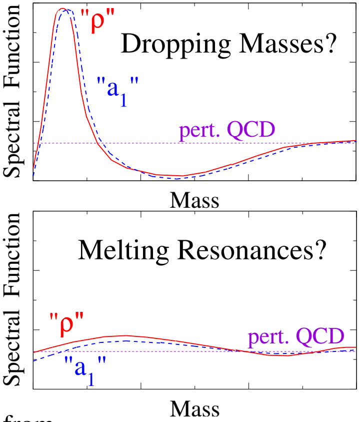
$$f_\pi^2 = -\int_0^\infty \frac{\mathrm{d}p_0^2}{\pi p_0^2} \left[\mathrm{Im}\,\Pi_V(p_0,0) - \mathrm{Im}\,\Pi_A(p_0,0)\right]$$

- spectral functions of vector (e.g. ρ) and axial vector (e.g. a_1) directly related to order parameter of chiral symmetry!

Vector Mesons and chiral symmetry



from [Rap03]



from [Rap05]

- at high enough temperatures and or densities: melting of $\langle \bar{q} q \rangle$
- $\Rightarrow$ spontaneous breaking of chiral symmetry supended
- $\Rightarrow$ chiral phase transition; chiral-symmetry restoration (χSR)
- which scenario is right? microscopic mechanisms behind χSR?

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