The soft-photon puzzle: Review

Hendrik van Hees

Goethe University Frankfurt

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based on R. Bailhache et al, Phys. Rept. 1097, 1 (2024) [B+24]

Outline

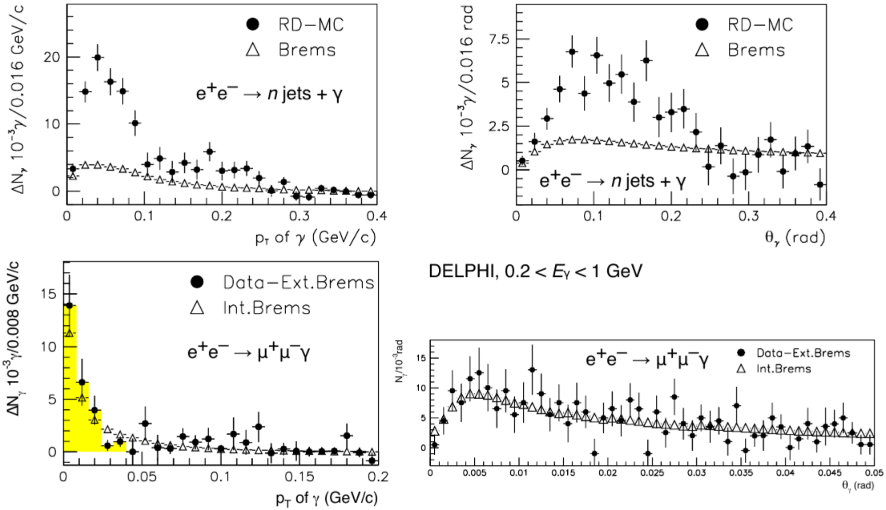
Introduction

- 1958 Francis Low: soft-photon-production cross sections ($2 \rightarrow 2$ scattering) [Low58]

$$\sigma = \frac{\sigma_0}{\omega_k} + \sigma_1 + \omega_k \sigma_2 + \cdots$$

- σ_0: cross section for same processes with charged particles without soft-photon emission
- based on very fundamental properties of local relativistic quantum field theory
- generalized to $n \longleftrightarrow m$ processes of charged particles
- puzzle: experimentally large excess of soft photons compared to expectation from σ_0 order (factors 4-8) in hadronic scattering processes

Introduction



DELPHI Collaboration [A+06, A+08]

Introduction

Exp.	year	p_{beam} or $\sqrt{s}$	photon k_T	$\gamma_{\text{meas}}/\gamma_{\text{brems}}$	method	Ref.
π^+p	1979	$10.5\,\text{GeV}/c$	$p_T < 20\,\text{MeV}/c$	1.25 ± 0.25	bubble chamber	Goshaw et al. [10]
K$^+$p WA27, CERN	1984	$70\,\text{GeV}/c$	$k_T < 60\,\text{MeV}/c$	4.0 ± 0.8	bubble chamber (BEBC)	Chliap-nikov et al. [11]
π^+p CERN, EHS, NA22	1991	$250\,\text{GeV}/c$	$k_T < 40\,\text{MeV}/c$	6.4 ± 1.6	bubble chamber (RCBC)	Botterweck et al. [12]
K$^+$p CERN, EHS, NA22	1991	$250\,\text{GeV}/c$	$k_T < 40\,\text{MeV}/c$	6.9 ± 1.3	bubble chamber (RCBC)	Botterweck et al. [12]
π^-p CERN, WA83, OMEGA	1993	$280\,\text{GeV}/c$	$k_T < 10\,\text{MeV}/c$ $(0.2 < E_\gamma < 1\,\text{GeV})$	7.9 ± 1.4	calorimeter	Banerjee et al. [13]
p–Be	1993	$450\,\text{GeV}/c$	$k_T < 20\,\text{MeV}/c$	< 2	pair conversion, calorimeter	Antos et al. [14]
p–Be, p–W	1996	$18\,\text{GeV}/c$	$k_T < 50\,\text{MeV}/c$	< 2.65	calorimeter	Tincknell et al. [15]
π^-p CERN, WA91, OMEGA	1997	$280\,\text{GeV}/c$	$k_T < 20\,\text{MeV}/c$ $(0.2 < E_\gamma < 1\,\text{GeV})$	7.8 ± 1.5	pair conversion	Belogianni at al. [16]
π^-p CERN, WA91, OMEGA	2002	$280\,\text{GeV}/c$	$k_T < 20\,\text{MeV}/c$ $(0.2 < E_\gamma < 1\,\text{GeV})$	5.3 ± 1.0	pair conversion	Belogianni at al. [17]
pp CERN, WA102, OMEGA	2002	$450\,\text{GeV}/c$	$k_T < 20\,\text{MeV}/c$ $(0.2 < E_\gamma < 1\,\text{GeV})$	4.1 ± 0.8	pair conversion	Belogianni at al. [6]
e$^+$e$^-$ $\rightarrow n$ jets CERN, DELPHI	2006	$91\,\text{GeV}$ $(\sqrt{s})$	$k_T < 80\,\text{MeV}/c$ $(0.2 < E_\gamma < 1\,\text{GeV})$	$4.0{\pm}0.3{\pm}1.0$	pair conversion	DELPHI [7, 19]
e$^+$e$^-$ $\rightarrow \mu^+\mu^-$ CERN, DELPHI	2008	$91\,\text{GeV}$ $(\sqrt{s})$	$k_T < 80\,\text{MeV}/c$ $(0.2 < E_\gamma < 1\,\text{GeV})$	~ 1	pair conversion	DELPHI [18]

Fundamental origin of Low's theorem

- relativistic local QFT
  - local observables transform as tensor fields under Poincaré transformations
  - (elementary) particles $\Leftrightarrow$ unitary irreducible representations of Poincaré group
  - local observables obey microcausality constraint

$$\left[\mathbf{O}_1(x), \mathbf{O}_2(y)\right] = 0 \quad \text{for} (x-y)^2 < 0$$

  - consequences: spin-statistics and CPT theorems
- gauge theories
  - Poincaré group irreps: characterized by $m^2 \geq 0$ and spin $s \in \{0, 1/2, 1, \ldots\}$
  - $m^2 > 0$: momentum eigenstates via Lorentz boost of $\vec{p} = 0$-state
  - "little group": trafos leaving $\vec{p} = 0$ invariant $\Rightarrow \mathrm{SO}(3)$ (compact group) $\Rightarrow (2s+1)$ discrete spin/polarization dof's
  - $m^2 = 0$: standard momentum $(\kappa, 0, 0, \kappa)$
  - little group $\mathrm{ISO}(1,2)$ (not compact!) $\Rightarrow$ "continuous spin dofs" (???)
  - way out: realization as gauge field theories that represent "translations" trivially
  - only $\mathrm{SO}(2)$ of little group left (rotations around 3-axis)
  - $s \geq 1/2$: only two helicity/polarization states $\lambda = \pm s$

Photons

- $m^2 = 0$, spin 1 $\Rightarrow$ gauge field A_μ equivalent to $A_\mu + \partial_\mu \chi$ with arbitrary scalar field χ
- interaction with charged-particle fields must obey this gauge invariance
- e.g. Dirac spinor: local gauge trafo $\psi \rightarrow \exp(-\mathrm{i} q \chi)\psi$, $\overline{\psi} \rightarrow \exp(+\mathrm{i} q \chi)\overline{\psi}$
- gauge-invariant Lagrangian: use gauge-invariant field-strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\nu$
- couplings to charged-particle fields: gauge-covariant derivatives $\partial_\mu \rightarrow \mathrm{D}_\mu = \partial_\mu + \mathrm{i} q A_\mu$
- leads to Maxwell's equations $\partial_\mu F^{\mu\nu} = J^\nu$ with $\partial_\mu J^\mu = 0$
- charge conservation $Q = \int_{\mathbb{R}^3} \mathrm{d}^3 x \, J^0(x) = \text{const}$ (Ward identity)
- Ward identity $\Leftrightarrow$ Low theorem: Weinberg [Wei64a, Wei64b]

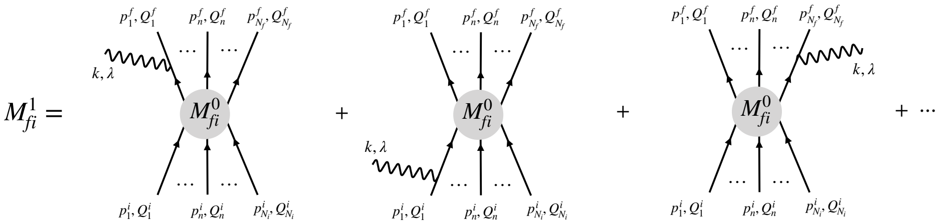
Soft photons and IR problems

- massless photons ⇒ long-ranged interaction
- in addition to usual UV divergences in all QFTs IR divergences
- reason: naive asymptotic free Fock states ("plane waves" of non-interacting "bare" particles) not the true ones!
- instead: "infraparticles"=bare particle + soft-photon coherent state

  [Dir55, Dol64, Chu65, KF70]
- "reorganized perturbation theory" with infraparticle states: IR divergences cancel at any order for the S-matrix elements (amplitudes)
- more "conventional": use naive Fock states and Bloch-Nordsieck argument (IR finite results for cross sections, not amplitudes) [BN37, JR54, JR76, YFS61, Wei65]

Leading-order soft-photon theorem

- at leading order: only photons radiated from external legs of Feynman diagrams



$$M^1_{fi} = \frac{e}{\sqrt{2\omega_k}} \sum_{n=1}^{N_i+N_f} \eta_n Q_n \frac{\epsilon^*(\boldsymbol{k},\lambda)\cdot p_n}{k\cdot p_n} M^0_{fi} + \mathscr{O}(\omega_k^0)$$

- soft-photon factor × non-radiative scattering amplitude of charged particles
- $\eta_n = +1(-1)$ for incoming (outgoing) charged-particle lines with charges Q_n
- as in classical bremsstrahlung calculation
- independent of specific structure (like spin) of charged particles
- soft photons only “resolve” total charges going in and out

Leading-order soft-photon theorem

- independent of specific structure (like spin) of charged particles
- independent of internal loops (without soft-photon line ⇒ NLO effect!)
- soft photons only "resolve" total charges going in and out
- factorization generalizes to radiation of arbitrary number N_γ of soft photons

$$M_{fi}^{N_\gamma} = \prod_{r=1}^{N_\gamma} \left\{ \frac{\epsilon_\mu^*(\boldsymbol{k}_r, \lambda_r)}{\sqrt{2\omega_k}} e \sum_{n=1}^{N_i+N_f} \eta_n Q_n \frac{p_n^\mu}{k_r \cdot p_n} \right\} M_{fi}^0 + \mathcal{O}(\omega_k^0)$$

Leading-order soft-photon theorem

- "soft" photon: "soft" with respect to what?
- pole from $k \cdot p = \omega(E - |\boldsymbol{p}| \cos\vartheta)$
  (NB: additional "collinear divergence" for massless charged particles)
- in frame, where charged particle is ultra-relativistic (e.g., in the "beam frame") in forward direction:

$$k \cdot p \simeq \omega E(1-\cos\vartheta) \simeq \omega E \vartheta^2/2 = k_{\mathrm{T}} E \vartheta/2$$

- expansion parameter ω/Λ (scale Λ?)
- necessary $\Lambda \simeq m_{\text{charges}}$ but is it sufficient?
- neglected radiation from inner lines!
- if "collision time" $\Delta\tau$ large $\Rightarrow \Lambda \simeq 1/\Delta\tau$ (long-lived virtual particles; "in-medium" interactions (Landau-Pomeranchuk-Midal effect!)

Soft-photon distribution

- single-photon emission: squaring the amplitude, summing over photon polarizations

$$\mathrm{d}I = \frac{\alpha}{(2\pi)^2} \frac{\mathrm{d}^3 \boldsymbol{k}}{\omega_k} \sum_{n,m=1}^{N_i+N_f} \eta_n Q_n \eta_m Q_m \frac{-p_n \cdot p_m}{(k \cdot p_n)(k \cdot p_m)} |M_{fi}^0|^2$$

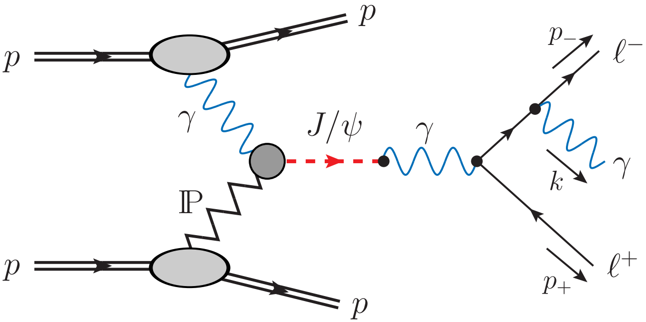
- integrating over $\mathrm{d}\Omega_k$

$$\frac{\mathrm{d}I}{\mathrm{d}\omega_k} = |M_{fi}^0|^2 \frac{1}{\omega_k} \frac{\alpha}{2\pi} \sum_{n,m=1}^{N_i+N_f} \eta_n Q_n \eta_m Q_m \frac{1}{v_{nm}} \log \frac{1+v_{nm}}{1-v_{nm}} \text{ with } v_{nm} = \sqrt{1 - \left(\frac{m_n m_m}{p_n \cdot p_m}\right)^2}$$

- v_{nm} relative velocity between charges n and m
- total probability logarithmically IR divergent
- total energy IR finite
- Bloch, Nordsieck: IR divergences cancel to all orders when summing also over virtual soft photons

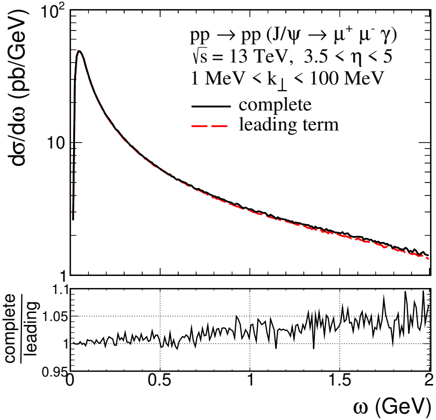
Example: $pp \to pp(J/\psi \to \mu^+\mu^-\gamma)$

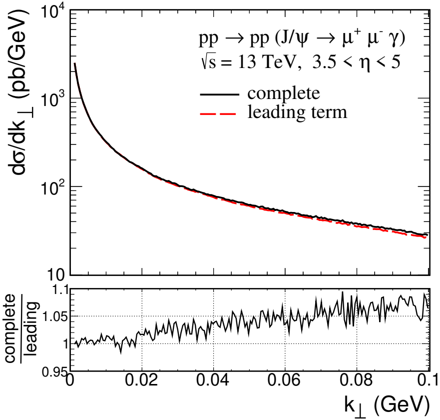
- Regge-inspired toy model: photon-pomeron ($\gamma\mathbb{P}$) fusion



Example: $pp \to pp(J/\psi \to \mu^+\mu^-\gamma)$

- Regge-inspired toy model: photon-pomeron ($\gamma\mathbb{P}$) fusion
- $1\ \text{MeV} < k_\perp < 100\ \text{MeV}$





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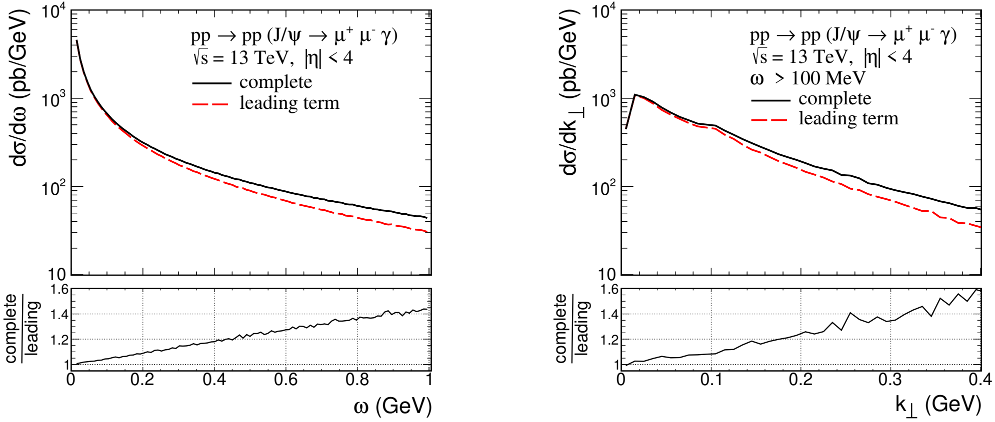

Conclusions

- soft-photon theorem consequence of fundamental properties of relativistic QFT
  - Poincaré symmetry + locality (microcausality)
  - gauge theory for massless particles with $s \geq 1 \Rightarrow$ Ward(-Takahashi) identities
  - IR structure: "true" asymptotic states of charged particles: infra-particles
    $\Rightarrow$ IR divergences cancel for S-matrix amplitudes
  - NB: related to asymptotic symmetries and
    symmetry group of large gauge transformations [Str]
- in leading soft-photon order: factorization in universal soft-photon and non-radiative charged-particle scattering amplitude
- higher orders $\Rightarrow$ next talk!

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