Kinetics of the chiral phase transition in a quark-meson σ model

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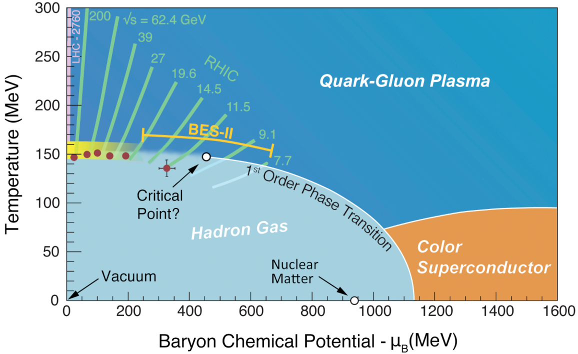
Outline

Chiral Symmetry and the QCD Phase Diagram

$$\mathscr{L}_{\text{QCD}} = \mathscr{L}_{\psi,A_\mu} + \mathscr{L}_G := \sum_{i \in \{u,d,s,c,b,t\}} \bar{\psi}_{i,j} \left(i\gamma^\mu (D_\mu)^j_k - m_i \delta^j_k \right) \psi^k_i - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a,$$

$$\mathscr{L}_{\psi,A_\mu} = \sum_{i \in \{u,d\}} \left[\bar{\psi}_{i,R} (i\gamma^\mu D_\mu) \psi_{i,R} + \bar{\psi}_{i,L} (i\gamma^\mu D_\mu) \psi_{i,L} \right] - \sum_{i \in \{u,d\}} m_i \left[\bar{\psi}_{i,R} \psi_{i,L} + \bar{\psi}_{i,L} \psi_{i,R} \right]$$

- light-quark sector (u+d quarks): approximate chiral symmetry $SU(2)_L \times SU(2)_R$



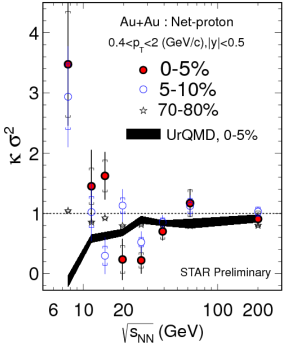
[Fig. from A. Aprahamian et al. Reaching for the horizon]

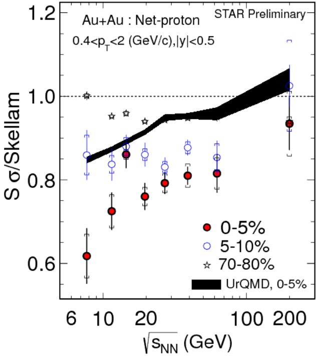
Cumulants of net-baryon number fluctuations

Quark-number susceptibilities

$$c_1 = \frac{N_{q,\text{net}}}{VT^3}, \quad c_2 = \frac{1}{VT^3}\left\langle \left(N_{q,\text{net}} - \langle N_{q,\text{net}}\rangle\right)^2\right\rangle \equiv \frac{1}{VT^3}\sigma^2_{q,\text{net}}$$

$$c_3 = \frac{1}{VT^3}\left\langle \left(N_{q,\text{net}} - \langle N_{q,\text{net}}\rangle\right)^3\right\rangle, \quad c_4 = \frac{1}{VT^3}\left[\left\langle \left(N_{q,\text{net}} - \langle N_{q,\text{net}}\rangle\right)^4\right\rangle - 3\sigma^4_{q,\text{net}}\right],$$





$$\kappa\sigma^2 = c_4/c_2$$
$$S\sigma = c_3/c_2$$

[X. Luo (STAR collab.) NPA **956**, 75 (2016)]

Quark-Meson linear-σ Model

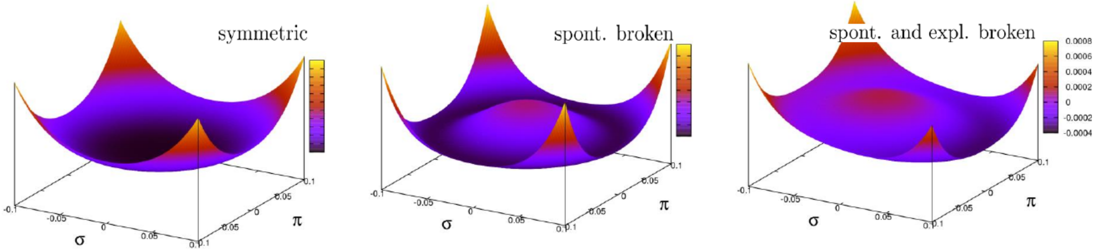
- $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$ linear-σ model
- mesons: σ, $\vec{\pi}$, quarks: $\psi = (u, d)$

$$\mathscr{L} = \bar{\psi}\left[\mathrm{i}\not{\partial} - g\left(\sigma + i\gamma_5 \vec{\pi}\cdot\vec{\tau}\right)\right]\psi + \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi}\right) - \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 + f_\pi m_\pi^2 \sigma + U_0$$

parameter	value	description
λ	20	coupling constant for σ and $\vec{\pi}$
g	$2-5$	coupling constant between $\sigma, \vec{\pi}$ and ψ
f_π	93 MeV	pion decay constant
m_π	138 MeV	pion mass
ν^2	$f_\pi^2 - m_\pi^2/\lambda$	field shift term
U_0	$m_\pi^4/(4\lambda) - f_\pi^2 m_\pi^2$	ground state

Quark-Meson linear-σ Model: meson potential

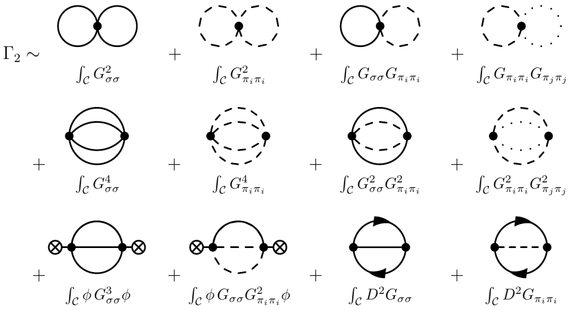
$$\mathscr{L} = \bar{\psi}\left[\mathrm{i}\not{\partial} - g\left(\sigma + i\gamma_5 \vec{\pi}\cdot\vec{\tau}\right)\right]\psi + \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi}\right) - \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2 + f_\pi m_\pi^2 \sigma + U_0$$



Quark-Meson linear-σ Model: 2PI action

$$\mathscr{L} = \bar{\psi}\left[\mathrm{i}\not{\partial} - g\left(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau}\right)\right]\psi + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 + f_\pi m_\pi^2\sigma + U_0,$$

$$\Gamma[\sigma,\vec{\pi},G,D] = S[\sigma,\vec{\pi}] + \frac{\mathrm{i}}{2}\mathrm{Tr}\ln G^{-1} + \frac{\mathrm{i}}{2}G_0^{-1}G - \mathrm{i}\,\mathrm{Tr}\ln D^{-1} - \mathrm{i}\,\mathrm{Tr}\,D_0^{-1}D + \Gamma_2[\sigma,\vec{\pi},G,D]$$

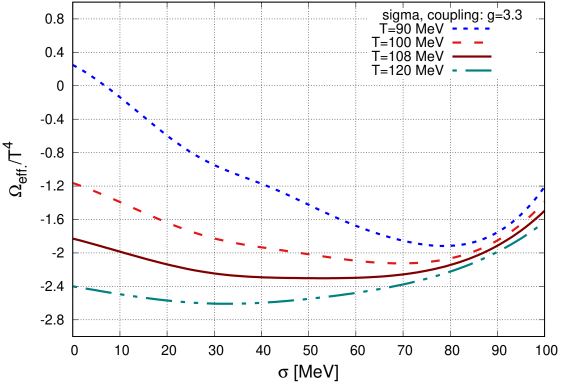


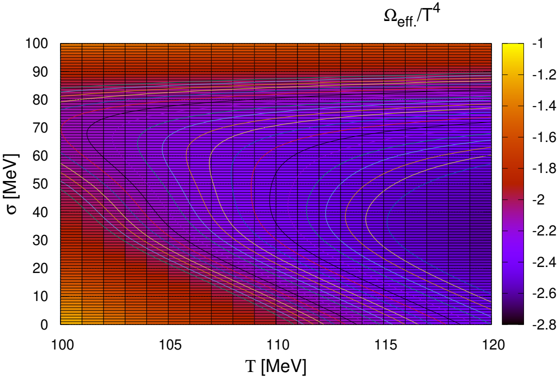
Equations of motion:

$$\frac{\delta\Gamma}{\delta\sigma} = \frac{\delta\Gamma}{\delta\vec{\pi}} = \frac{\delta\Gamma}{\delta G} = \frac{\delta\Gamma}{\delta D} = 0.$$

Quark-Meson linear-σ Model: equilibrium effective potential

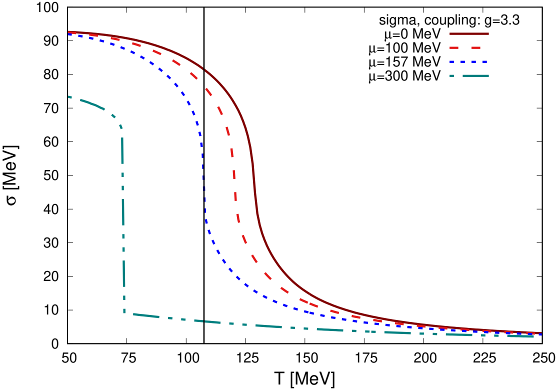
$$\Omega_{\text{eff}}[\sigma,\vec{\pi},G,D] = -\frac{1}{\beta V}\mathrm{i}\Gamma[\sigma,\vec{\pi},G,D]$$

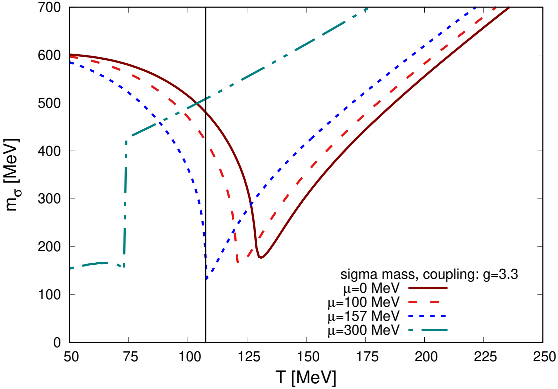




Quark-Meson linear-σ Model: equilibrium order parameter

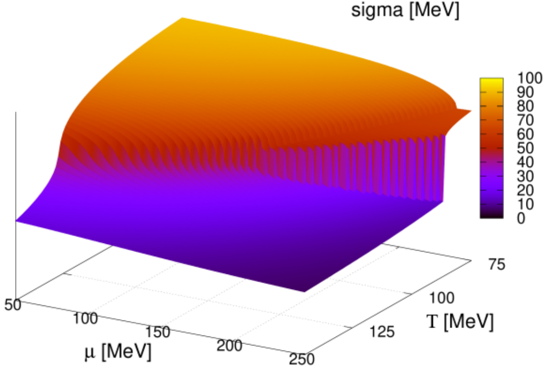
$$\Omega_{\text{eff}}[\sigma,\vec{\pi},G,D] = -\frac{1}{\beta V}\mathrm{i}\Gamma[\sigma,\vec{\pi},G,D], \qquad \frac{\partial\Omega_{\text{eff}}}{\partial\sigma} \stackrel{!}{=} 0, \quad M_\sigma^2 = \frac{\partial^2\Omega_{\text{eff}}}{\partial\sigma^2}, \quad M_\pi^2 = \frac{\partial^2\Omega_{\text{eff}}}{\partial\pi_i^2}$$

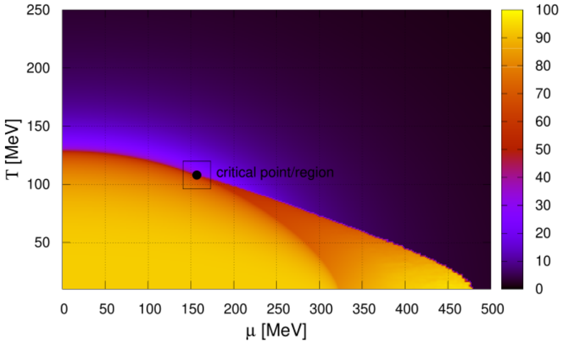




Quark-Meson linear-σ Model: equilibrium order parameter

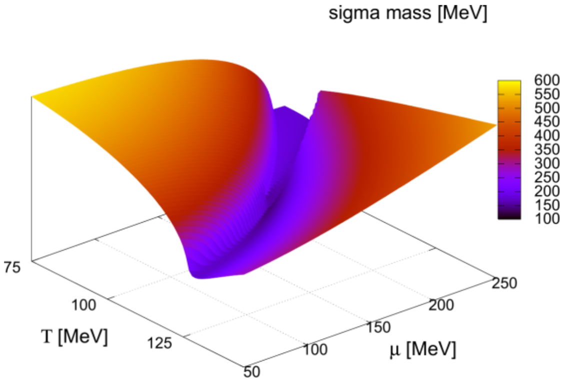
$$\Omega_{\text{eff}}[\sigma,\vec{\pi},G,D] = -\frac{1}{\beta V} \mathrm{i}\Gamma[\sigma,\vec{\pi},G,D], \qquad \frac{\partial \Omega_{\text{eff}}}{\partial \sigma} \stackrel{!}{=} 0, \qquad M_\sigma^2 = \frac{\partial^2 \Omega_{\text{eff}}}{\partial \sigma^2}, \qquad M_\pi^2 = \frac{\partial^2 \Omega_{\text{eff}}}{\partial \pi_i^2}$$

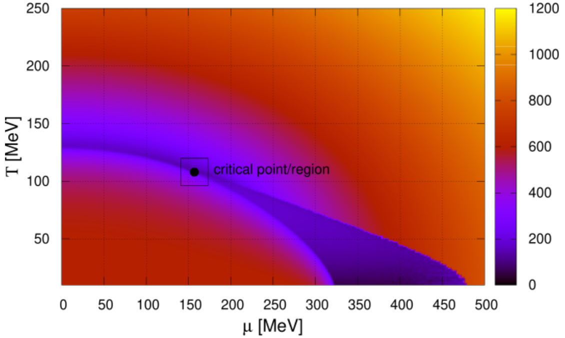




Quark-Meson linear-σ Model: effective σ mass

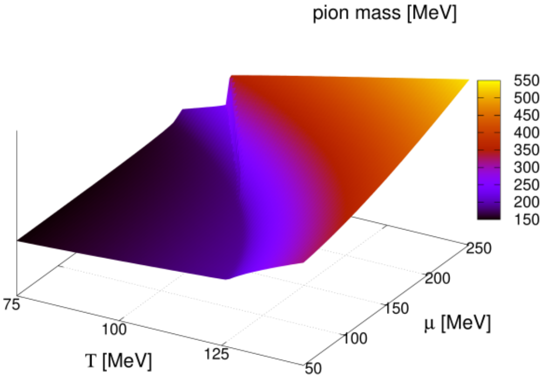
$$\Omega_{\text{eff}}[\sigma,\vec{\pi},G,D] = -\frac{1}{\beta V} \mathrm{i}\Gamma[\sigma,\vec{\pi},G,D], \quad \frac{\partial\Omega_{\text{eff}}}{\partial\sigma} \stackrel{!}{=} 0, \quad M_\sigma^2 = \frac{\partial^2\Omega_{\text{eff}}}{\partial\sigma^2}, \quad M_\pi^2 = \frac{\partial^2\Omega_{\text{eff}}}{\partial\pi_i^2}$$

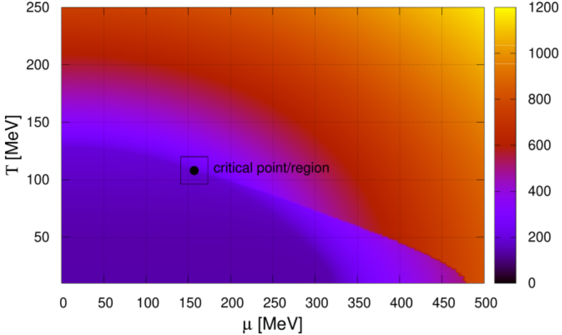




Quark-Meson linear-σ Model: effective π mass

$$\Omega_{\text{eff}}[\sigma,\vec{\pi},G,D] = -\frac{1}{\beta V}\mathrm{i}\Gamma[\sigma,\vec{\pi},G,D], \quad \frac{\partial\Omega_{\text{eff}}}{\partial\sigma} \stackrel{!}{=} 0, \quad M_\sigma^2 = \frac{\partial^2\Omega_{\text{eff}}}{\partial\sigma^2}, \quad M_\pi^2 = \frac{\partial^2\Omega_{\text{eff}}}{\partial\pi_i^2}$$





Quark-Meson linear-σ Model: off-equilibrium equations

- real-time Keldysh contour $\Rightarrow$ 2PI/Kadanoff Baym $\Rightarrow$ transport equation (spatially homogeneous)
- Mean-field equation

$$\partial_t^2 \phi + D(t) + J(t) = 0, J(t) := \lambda\left(\phi^2 - \nu^2 + 3G_{\sigma\sigma}^{11} + \sum_i G_{\pi_i\pi_i}^{11}\right)\phi - f_\pi m_\pi^2 + g\left\langle\bar{\psi}\psi\right\rangle$$

- transport equations for meson- and quark-phase-space-distribution functions

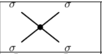
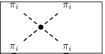
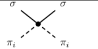
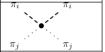
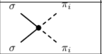
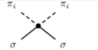
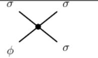
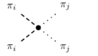
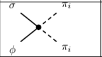
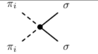
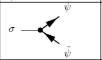
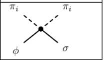
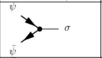
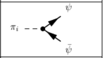
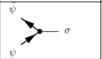
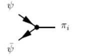
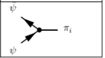
$$\partial_t f^\sigma(t,\vec{p}_1) = \mathscr{C}^{b.}_{\sigma\sigma\leftrightarrow\sigma\sigma} + \sum_i \mathscr{C}^{b.}_{\sigma\pi_i\leftrightarrow\sigma\pi_i} + \sum_i \mathscr{C}^{b.}_{\sigma\sigma\leftrightarrow\pi_i\pi_i} + \mathscr{C}^{b.s.}_{\sigma\phi\leftrightarrow\sigma\sigma} + \sum_i \mathscr{C}^{b.s.}_{\sigma\phi\leftrightarrow\pi_i\pi_i} + \mathscr{C}^{f.s.}_{\sigma\leftrightarrow\psi\bar{\psi}},$$

$$\partial_t f^{\pi_i}(t,\vec{p}_1) = \mathscr{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_i\pi_i} + \sum_{j\neq i} \mathscr{C}^{b.}_{\pi_i\pi_j\leftrightarrow\pi_i\pi_j} + \sum_{j\neq i} \mathscr{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_j\pi_j} + \mathscr{C}^{b.}_{\pi_i\sigma\leftrightarrow\pi_i\sigma} + \mathscr{C}^{b.}_{\pi_i\pi_i\leftrightarrow\sigma\sigma} + \mathscr{C}^{b.s.}_{\pi_i\phi\leftrightarrow\pi_i\sigma} + \mathscr{C}^{f.s.}_{\pi_i\leftrightarrow\psi\bar{\psi}}$$

$$\partial_t f^\psi(t,\vec{p}_1) = \mathscr{C}^{f.s.}_{\psi\bar{\psi}\leftrightarrow\sigma} + \sum_i \mathscr{C}^{f.s.}_{\psi\bar{\psi}\leftrightarrow\pi_i}$$

$$\partial_t f^{\bar{\psi}}(t,\vec{p}_1) = \mathscr{C}^{f.s.}_{\bar{\psi}\psi\leftrightarrow\sigma} + \sum_i \mathscr{C}^{f.s.}_{\bar{\psi}\psi\leftrightarrow\pi_i},$$

Quark-Meson linear-σ Model: collision terms

collision integral	diagram	collision integral	diagram
$C^{b.}_{\sigma\sigma\leftrightarrow\sigma\sigma}$		$C^{b.}_{\pi_i\pi_i\leftrightarrow\pi_i\pi_i}$	
$C^{b.}_{\sigma\pi_i\leftrightarrow\sigma\pi_i}$		$C^{b.}_{\pi_i\pi_j\leftrightarrow\pi_i\pi_j}$	
$C^{b.}_{\sigma\sigma\leftrightarrow\pi_i\pi_i}$		$C^{b.}_{\pi_i\sigma\leftrightarrow\pi_i\sigma}$	
$C^{b.s.}_{\sigma\phi\leftrightarrow\sigma\sigma}$		$C^{b.}_{\pi_i\pi_i\leftrightarrow\pi_j\pi_j}$	
$C^{b.s.}_{\sigma\phi\leftrightarrow\pi_i\pi_i}$		$C^{b.}_{\pi_i\pi_i\leftrightarrow\sigma\sigma}$	
$C^{f.s.}_{\sigma\leftrightarrow\psi\bar{\psi}}$		$C^{b.s.}_{\pi_i\phi\leftrightarrow\pi_i\sigma}$	
$C^{f.s.}_{\psi\bar{\psi}\leftrightarrow\sigma}$		$C^{f.s.}_{\pi_i\leftrightarrow\psi\bar{\psi}}$	
$C^{f.s.}_{\bar{\psi}\psi\leftrightarrow\sigma}$		$C^{f.s.}_{\psi\bar{\psi}\leftrightarrow\pi_i}$	
		$C^{f.s.}_{\bar{\psi}\psi\leftrightarrow\pi_i}$	

Quark-Meson linear-σ Model: non-Markovian dissipation

- memory kernel in mean-field dissipation term

$$D(t) = 6\lambda^2 \left[\Gamma(t, \Delta t = t - t_0)\phi(t_0) - \Gamma(t, \Delta t = 0)\phi(t)\right] + 6\lambda^2 \int_{t_0}^{t} \mathrm{d}t' \dot{\phi}(t')\Gamma(t, t - t'),$$

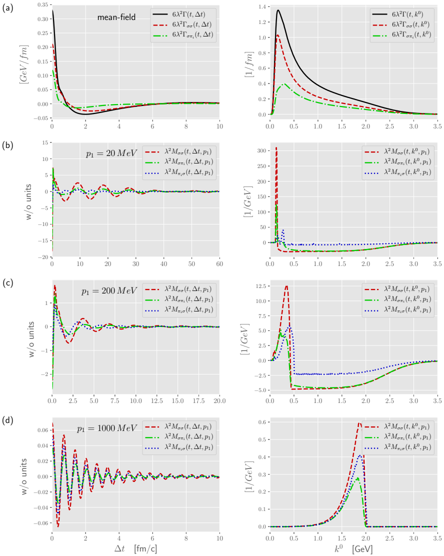
$$\Gamma(t, \Delta t) = \int \frac{\mathrm{d}k^0}{(2\pi)} \mathrm{e}^{-\mathrm{i}k^0 \Delta t} \frac{1}{k^0} \left[\tilde{\mathscr{M}}_{\sigma\sigma}(t, k^0) + \frac{1}{3}\sum_i \tilde{\mathscr{M}}_{\sigma\pi_i}(t, k^0)\right],$$

- memory kernels in Boltzmann equation

$$\begin{aligned}\mathscr{I}_{\sigma}^{b.s.}(t, \vec{p}_1) := {} & \frac{9}{2}\pi\lambda^2\phi(t) \int_0^\infty \mathrm{d}\Delta t\, \phi(t - \Delta t) \int \frac{\mathrm{d}k^0}{(2\pi)} \mathrm{e}^{-\mathrm{i}k^0 \Delta t} \mathscr{M}_{\sigma\sigma}(t, k^0, p_1) \\ & + \frac{1}{2}\pi\lambda^2\phi(t) \sum_i \int_0^\infty \mathrm{d}\Delta t\, \phi(t - \Delta t) \int \frac{\mathrm{d}k^0}{(2\pi)} \mathrm{e}^{-\mathrm{i}k^0 \Delta t} \mathscr{M}_{\sigma\pi_i}(t, k^0, p_1),\end{aligned}$$

$$\mathscr{I}_{\pi_i}^{b.s.}(t, \vec{p}_1) := \pi\lambda^2\phi(t) \int_0^\infty \mathrm{d}\Delta t\, \phi(t - \Delta t) \int \frac{\mathrm{d}k^0}{(2\pi)} \mathrm{e}^{-\mathrm{i}k^0 \Delta t} \mathscr{M}_{\pi_i\sigma}(t, k^0, p_1),$$

Quark-Meson linear-σ Model: Memory kernels



Quark-Meson linear-σ Model: expanding-fireball geometry

- Friedmann-Lemaître-Robertson-Walker metric (spatially flat)

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)(\mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2), \quad H = \dot{a}/a$$

- expanding fireball with radius $R(t) = R_0 + v_e t$, $\dot{a}/a = \dot{R}/R$
- mean-field equation

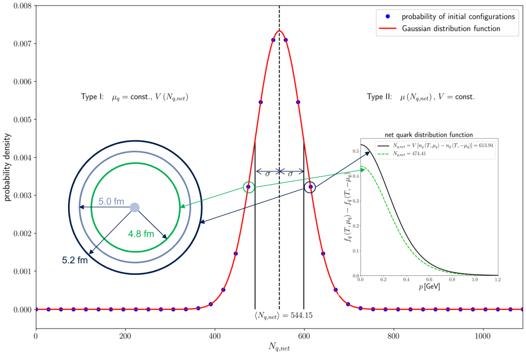
$$\partial_t^2 \phi + 3H\partial_t \phi + D(t) + J(t) = 0$$

- Boltzmann equation

$$\left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right) f = \mathscr{I}$$

Initialization of net-quark numbers

- goal: time-evolution of net-quark number fluctuations
- ensembles with fluctuating initial conditions



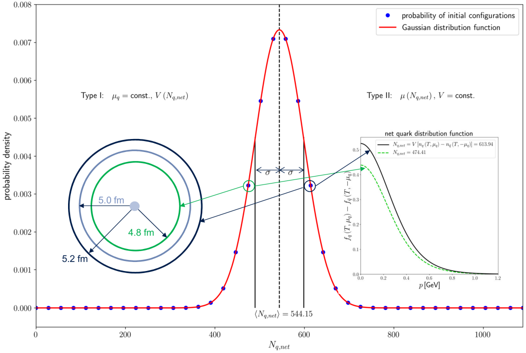
- mean net-quark number

$$\langle N_{\text{q,net}}\rangle = \frac{4\pi}{3} R_0^2 \int \frac{\mathrm{d}^3 p}{(2\pi)^3} [f_q(T,\mu_q) - f_q(T,-\mu_q)]$$

- standard deviation: $\sigma_{\text{q,net}} = \langle N_{\text{q,net}}\rangle / 10$
- choose $M = 200$-1000 values for $N_{\text{q,net}}$
- initialize type I or type II for each $N_{\text{q,net}}$

Observables

- goal: time-evolution of net-quark number fluctuations
- ensembles with fluctuating initial conditions

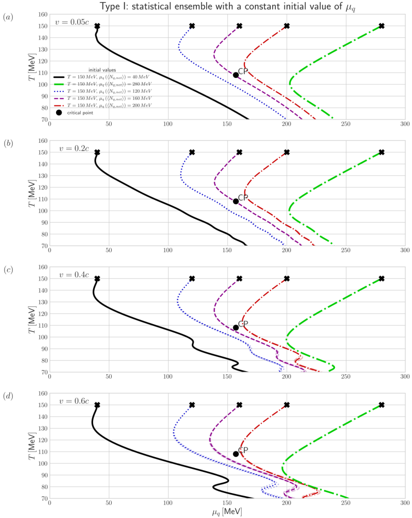


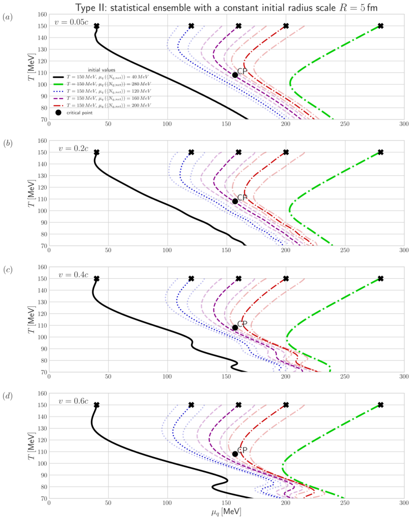
- ensembles with fluctuating initial conditions

$$\langle O \rangle = \frac{p_0 O_0 + p_M O_M}{2} + \sum_{k=1}^{M-1} p_k O_k$$

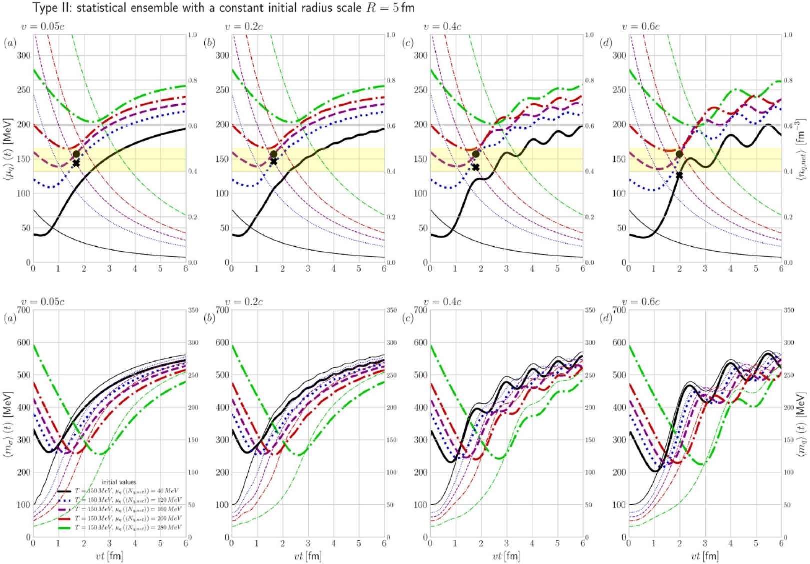
- cumulant ratios $R_{3,1} = c_3/c_1$,
  $R_{4,2} = c_1/c_2 = \kappa\sigma^2$,
  $c_1 = \langle m \rangle$,
  $c_2 = \tilde{m}_2 = \sigma^2$,
  $c_3 = \tilde{m}_3$,
  $c_4 = \tilde{m}_4 - 3\tilde{m}_2^2$

“Trajectories” in phase diagram

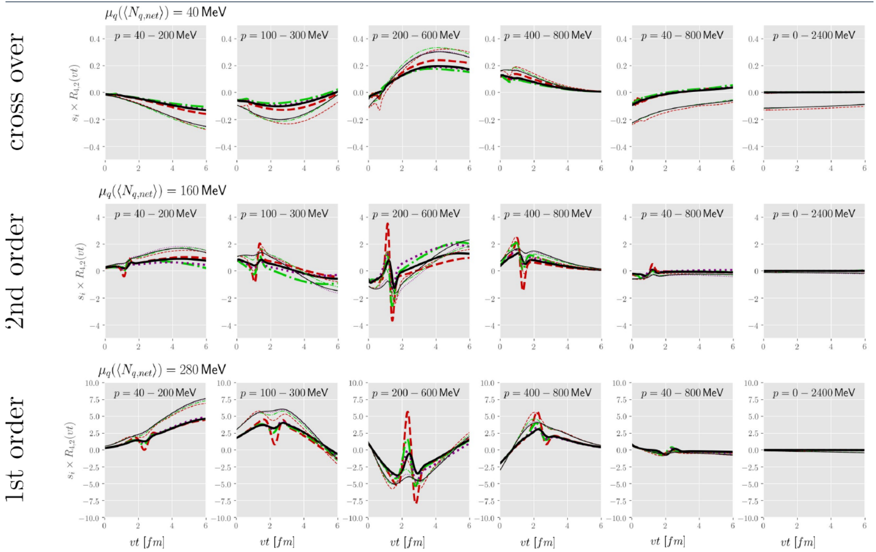




“Trajectories” in phase diagram

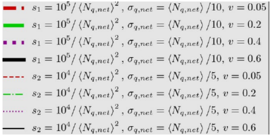


"Trajectories" in phase diagram



$T_{\text{ini}} = 160$ MeV

$\langle N \rangle_{q,\text{net}} = 123, 544, 1158$



Conclusions

- Phase diagram of strongly interacting matter and chiral symmetry
- Quark-meson linear σ model
- 2PI effective action in and off equilibrium
- coupled mean-field and transport equations with memory
  - ensembles with fluctuating initial conditions for net-quark number
  - trajectory through chiral phase transition $\Rightarrow$ non-trivial fluctuations $\Rightarrow$ higher cumulants
  - differences between crossover and 1^{st} or 2^{nd}-order transitions
  - depends on expansion rate and observed momentum bins
  - fluctuations may survive full fireball evolution
- A. Meistrenko, HvH, C. Greiner, Ann. Phys. **431**, 168555 (2021); arXiv:2007.09929 [hep-ph]