# Classical Langevin Approach to Heavy Quarkonia

B05

**Nadja Krenz**, H. van Hees and C. Greiner

1st CRC-TR 211 Retreat, ZiF, Bielefeld, March 13-15, 2018

March 15, 2018

Universität Bielefeld

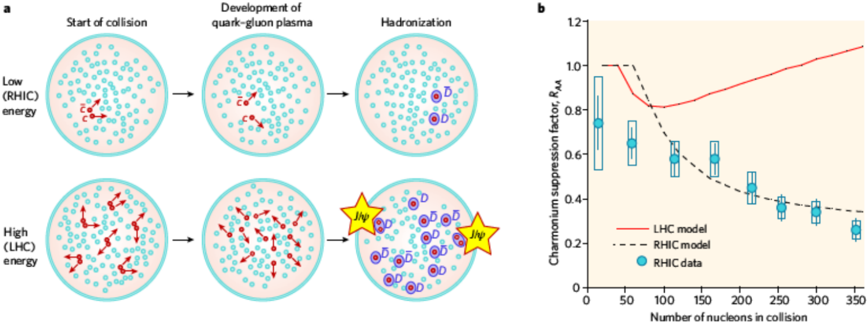
TECHNISCHE UNIVERSITÄT DARMSTADT

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

# Outline

# Motivation

- $J/\psi$ suppression in heavy-ion collisions as signal for deconfinement [MS86]
- heavy quarkonia produced in primordial hard collisions
- melt/dissociate in QGP due to color screening/collisions
- at higher beam energies: also regeneration




[from J. Stachel, Talk at EMMI workshop Feb/13/18]

# Langevin Equation for $Q\bar{Q}$ pair

- Fokker-Planck equation for c and $\bar{c}$ quarks
- single as well as many pairs

$$\dot{\boldsymbol{r}} = \frac{1}{2M}\boldsymbol{p},$$
$$\dot{\boldsymbol{p}} = \boldsymbol{F}(\boldsymbol{r}-\bar{\boldsymbol{r}}) - \gamma\boldsymbol{p} + \sqrt{2MT\gamma\Delta t}\,\boldsymbol{\rho}$$

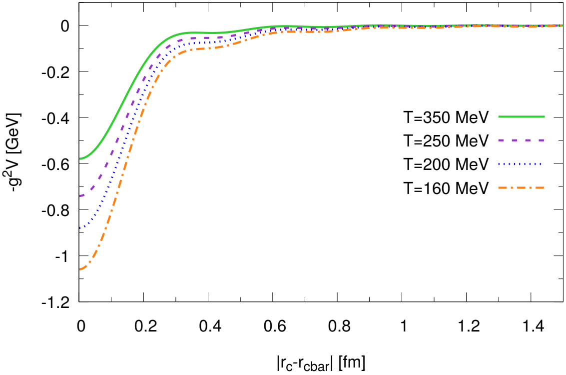
and analogous for $\bar{c}$

- $\gamma$: drag coefficient from [BDBFG16] (Abelian plasma model)
- $\boldsymbol{\rho}$: Gaussian-distributed white noise
- $\boldsymbol{F} = -\nabla V$ from same model!

# Langevin Equation for $Q\bar{Q}$ pair

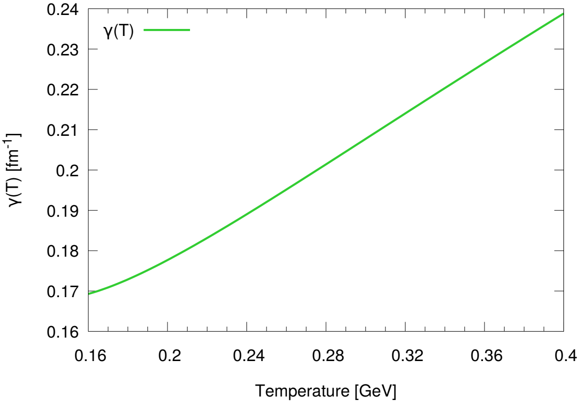
- UV-regularized screened Coulomb potential: $\Lambda = 4\,\mathrm{GeV}$
- running coupling

$$g^2 = 4\pi\alpha_s = \frac{4\pi\alpha_s(T_c)}{1 + C\ln(T/T_c)}, \quad C = 0.76, \quad T_c = 160\,\mathrm{MeV}, \quad \alpha_s(T_c) = 0.5$$



- UV-regularized screened Coulomb potential: $\Lambda = 4\,\mathrm{GeV}$
- drag coefficient

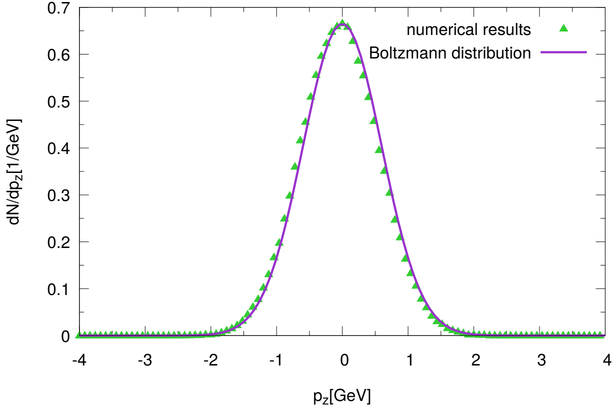
$$\gamma = \frac{m_\mathrm{D}^2}{24\pi M}\left[\ln\left(1+\frac{\Lambda^2}{m_\mathrm{D}^2}\right)-\frac{\Lambda^2/m_\mathrm{D}^2}{1+\Lambda^2/m_\mathrm{D}^2}\right]$$



# Numerical tests

- single $c\bar{c}$ pair, $T = 200\,\text{MeV}$, $M = 1.8\,\text{GeV}$ open system
- equlibrium limit: momentum distribution

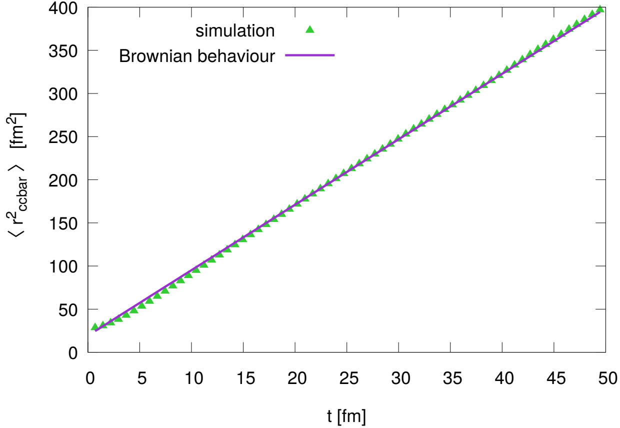
$$f_{\text{eq}}(\boldsymbol{p}) \propto \exp\left(-\frac{\boldsymbol{p}^2}{2MT}\right)$$



# Numerical tests

- single $c\bar{c}$ pair, $T = 200\,\text{MeV}$, $M = 1.8\,\text{GeV}$, open system
- Brownian behavior of distance between $c$ and $\bar{c}$

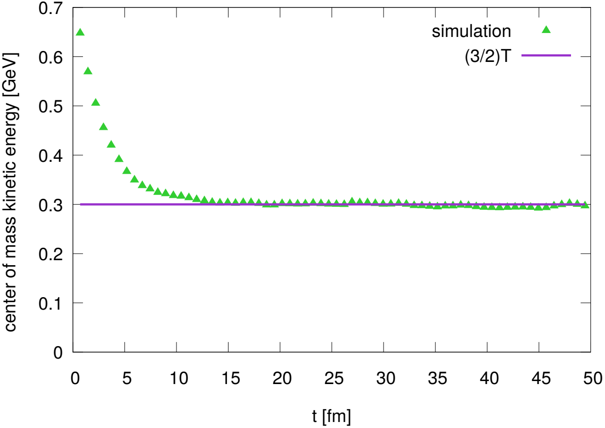
$$\left\langle (\boldsymbol{r}(t) - \boldsymbol{r}(0))^2 \right\rangle \underset{t\to\infty}{\cong} 2 \cdot 6 \cdot D_\text{s}\, t\,, \quad D_\text{s} = \frac{T}{M\gamma}$$



# Numerical tests

- single $c\bar{c}$ pair, $T = 200\,\text{MeV}$, $M = 1.8\,\text{GeV}$, open system
- equipartition theorem for center-mass-pair energy

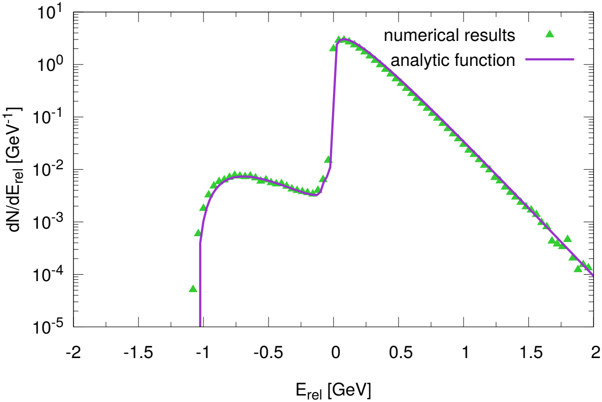
$$\langle E_{\text{cm}} \rangle \underset{t\to\infty}{\cong} \frac{3}{2} T$$



# Numerical tests

- single $c\bar{c}$ pair, $T = 160\,\mathrm{MeV}$, $M = 1.8\,\mathrm{GeV}$, $(8\,\mathrm{fm})^3$ rigid box
- $E_{\mathrm{rel}}$ distribution in equilibrium limit

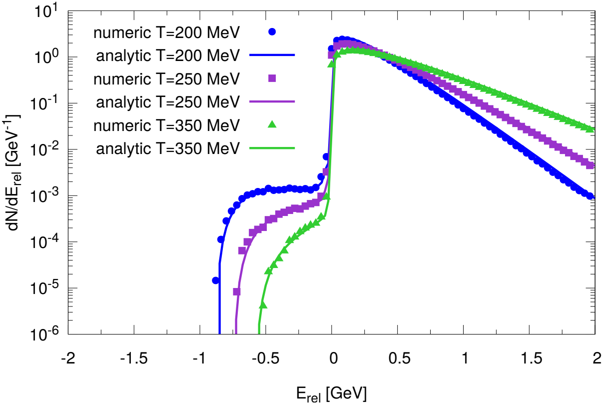
$$\frac{\mathrm{d}N}{\mathrm{d}E_{\mathrm{rel}}} = (4\pi)^2 (2\mu)^{3/2} C \exp\left(-\frac{E_{\mathrm{rel}}}{T}\right) \int_0^R \mathrm{d}r\, \mathrm{d}r^2 \sqrt{E_{\mathrm{rel}} - V(r)}$$



# Numerical tests

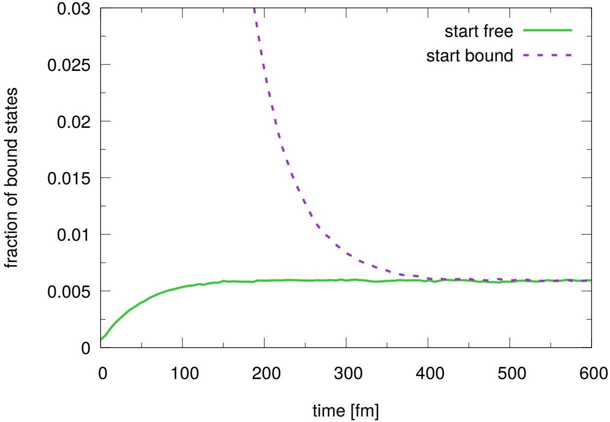
- single $c\bar{c}$ pair, $T = 200, 250, 300\,\mathrm{MeV}$ $M = 1.8\,\mathrm{GeV}$, $(8\,\mathrm{fm})^3$ rigid box
- $E_{\mathrm{rel}}$ distribution in equilibrium limit

$$\frac{\mathrm{d}N}{\mathrm{d}E_{\mathrm{rel}}} = (4\pi)^2 (2\mu)^{3/2} C \exp\left(-\frac{E_{\mathrm{rel}}}{T}\right) \int_0^R \mathrm{d}r\,\mathrm{d}r^2 \sqrt{E_{\mathrm{rel}} - V(r)}$$



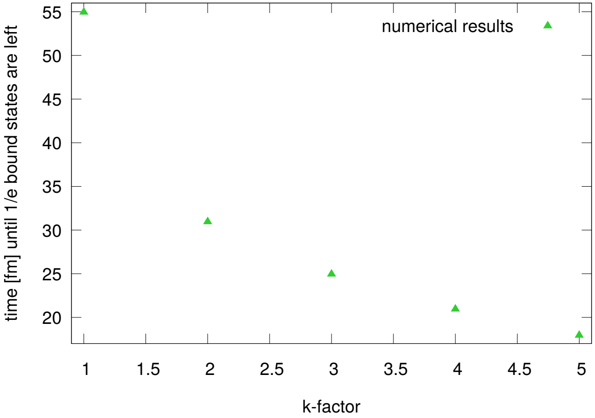
# Numerical tests

- single $c\bar{c}$ pair, $T = 160\,\text{MeV}$, $M = 1.8\,\text{GeV}$, $(8\,\text{fm})^3$ rigid box
- fraction of bound states



# Numerical tests

- single $c\bar{c}$ pair, $T = 160\,\mathrm{MeV}$, $M = 1.8\,\mathrm{GeV}$, $(8\,\mathrm{fm})^3$ rigid box
- relaxation time for bound-state formation/dissociation
- “k factor”: $\gamma \to k\gamma$



# Numerical tests

- single $c\bar{c}$ pair, $T = 160\,\mathrm{MeV}$, rigid boxes
- fraction of bound states:
  Langevin equilibrium limit vs. (grand-)canonical ensemble

| Volume | $(8\,\mathrm{fm})^3$ | $(10\,\mathrm{fm})^3$ | $(12\,\mathrm{fm})^3$ |
|---|---|---|---|
| grand canonical fraction of $J/\psi$ | 0.0066 | 0.0035 | 0.002 |
| Langevin fraction of $J/\psi$ | 0.0059 | 0.0029 | 0.0017 |

# Summary and Outlook

- Summary
  - Langevin simulation of charm-anticharm quarks in QGP
  - based on potential and drag coefficients from Abelian gauge model [BDBFG16]
  - passes all equilibration “box tests”
  - bound-state properties in finite box $\Leftrightarrow$ GC ensemble
- Outlook
  - generalize to expanding fireballs mimicking a heavy-ion collision
  - explore different heavy-quark potentials
  - using quantum Wigner function to analyze phase-space distribution functions [YS09] in terms of quantum bound states
  - long-time goal: full in-medium quantum Langevin treatment

# Backup Slides

# Langevin Equation for $Q\bar{Q}$ pair

- heavy quarks in an Abelian plasma [BDBFG16]
- $Q\bar{Q}$ pair described by set of Langevin equations

$$M\ddot{\boldsymbol{r}} + \frac{\beta g^2}{2}[\mathscr{H}(0)\dot{\boldsymbol{r}} - \mathscr{H}(\boldsymbol{s})\dot{\bar{\boldsymbol{r}}}] - g^2\nabla V(\boldsymbol{s}) = \xi(\boldsymbol{s},t)$$

$$M\ddot{\bar{\boldsymbol{r}}} + \frac{\beta g^2}{2}[\mathscr{H}(0)\dot{\bar{\boldsymbol{r}}} - \mathscr{H}(\boldsymbol{s})\dot{\boldsymbol{r}}] + g^2\nabla V(\boldsymbol{s}) = \bar{\xi}(\boldsymbol{s},t).$$

- Drag and diffusion coefficients derived from complex potential

$$\begin{aligned}\mathscr{V}(\boldsymbol{s}) &= -g^2 V(\boldsymbol{r}) - \mathrm{i}g^2[W(\boldsymbol{s}) - W(0)] \\ &= -\frac{g^2}{4\pi}\frac{\exp(-m_{\mathrm{D}}s)}{s} - \mathrm{i}\frac{g^2 T}{4\pi}\phi(m_{\mathrm{D}}s),\end{aligned}$$

$$\phi(x) = 2\int_0^\infty \mathrm{d}z\, \frac{z}{(z^2+1)^2}\left[1 - \frac{\sin(zx)}{zx}\right],$$

$$\mathscr{H}_{\alpha\beta}(\boldsymbol{s}) = \frac{\partial^2 W(\boldsymbol{s})}{\partial r_\alpha \partial r_\beta}$$

# Langevin Equation for $Q\bar{Q}$ pair

- heavy quarks in an Abelian plasma [BDBFG16]
- $Q\bar{Q}$ pair described by set of Langevin equations

$$M\ddot{\boldsymbol{r}} + \frac{g^2}{2T}[\mathscr{H}(0)\dot{\boldsymbol{r}} - \mathscr{H}(\boldsymbol{s})\dot{\bar{\boldsymbol{r}}} - g^2\nabla V(\boldsymbol{s}) = \xi(\boldsymbol{s},t)$$
$$M\ddot{\bar{\boldsymbol{r}}} + \frac{g^2}{2T}[\mathscr{H}(0)\dot{\bar{\boldsymbol{r}}} - \mathscr{H}(\boldsymbol{s})\dot{\boldsymbol{r}}] + g^2\nabla V(\boldsymbol{s}) = \bar{\xi}(\boldsymbol{s},t).$$

- "Random force" as white noise

$$\left\langle \xi_\alpha(\boldsymbol{s},t)\xi_\beta(\boldsymbol{s},t')\right\rangle = \left\langle \bar{\xi}_\alpha(\boldsymbol{s},t)\bar{\xi}_\beta(\boldsymbol{s},t')\right\rangle = g^2\mathscr{H}(0)\delta_{\alpha\beta}\delta(t-t'),$$
$$\left\langle \xi_\alpha(\boldsymbol{s},t)\bar{\xi}_\beta(\boldsymbol{s},t')\right\rangle = -g^2\mathscr{H}_{\alpha\beta}(\boldsymbol{s})\delta(t-t'),$$
$$g^2\mathscr{H}_{\alpha\beta}(0) = 2MT\gamma\delta_{\alpha\beta}$$

- for $m_{\mathrm{D}}s \gg 1$ center-mass coordinate $\boldsymbol{\rho} = (\boldsymbol{r}+\bar{\boldsymbol{r}})/2$ moves as Brownian particle with mass $2M$, drag $\gamma$, diffusion coefficient $D = 2M\gamma T$

# Bibliography I


[BDBFG16] J.-P. Blaizot, D. De Boni, P. Faccioli, G. Garberoglio, Heavy quark bound states in a quark-gluon plasma: Dissociation and recombination, Nucl. Phys. A **946** (2016) 49. `http://dx.doi.org/10.1016/j.nuclphysa.2015.10.011`

[MS86] T. Matsui, H. Satz, $J/\psi$ suppression by quark-gluon plasma formation, Phys. Lett. B **178** (1986) 416. `http://dx.doi.org/10.1016/0370-2693(86)91404-8`

[YS09] C. Young, E. Shuryak, Charmonium in strongly coupled quark-gluon plasma, Phys. Rev. C **79** (2009) 034907. `http://dx.doi.org/10.1103/PhysRevC.79.034907`