Comment on: Non-perturbative finite T broadening of the rho meson and dilepton emission in heavy ion-collisions
by J. Ruppert and T. Renk

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In this comment we point out several problems concerning kinematical singularities which are encountered in the calculation of the dilepton rates in [1]. We also comment on the method introduced in [6] and further used in refs. [1, 3, 4, 7].

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In this publication [1] the authors find a surprising broadening of the ρ-meson solely due to its interactions with pions. Since this is in contrast with expectations from low-energy QCD and the implied Goldstone-boson nature of the pion [2] we tried to find the reason for this behavior.

Our reanalysis leads us to the conclusion that the projection method introduced in [1] in order to restore the four transversality of the ρ-meson spectral function, $A^{\mu\nu}$, suffers from serious problems with kinematical singularities. As shown below these singularities lead to a *spurious and unphysical* massless mode of the ρ-meson, in turn leading to a further broadening of the pion modes and through the self-consistence finally to a strong broadening of the ρ-meson. In recent studies [3, 4] the authors alternatively used the projection method introduced by two of us [6] and found a good agreement with the dimuon data of NA60 [5] on the basis of their collision-dynamic model [1]. We point out that also in this projection method there are ambiguities in the calculation.

The analysis in detail: The authors use a Φ-derivable self-consistent Dyson-resummation scheme to evaluate self energies of vector mesons, which, a priori, is a promising method. However, it suffers from the violation of Ward-Takahashi identities at the two-point and higher-order vertex functions level, leading to a violation of current conservation within the self-consistent propagators although the expectation value of the current is conserved. This leads to the artificial excitation of the unphysical four-dimensionally longitudinal mode of the vector meson and thus to a violation of unitarity.

In order to cure this defect the authors employed a naive projection scheme. In any iteration step of the self-consistent scheme it simply cuts off the undesired four-longitudinal components of the polarization tensor,

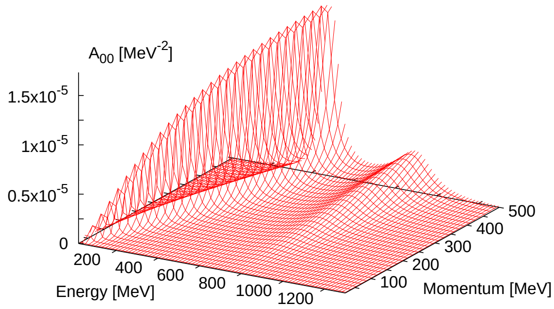

FIG. 1: (Color online) Time-time component $A^{00}(q)$ of the ρ-meson spectral function at $T = 160$ MeV as a function of energy and three momentum for the Ruppert-Renk projection method.

$\Pi^{\mu\nu}(q)$, of the ρ-meson. Since the projectors, however, are singular on the light-cone, the spatially longitudinal component of the polarization tensor,

$$\operatorname{Im} \Pi_L^{\mu\nu}(q) \stackrel{q^2 \to 0}{\longrightarrow} \epsilon(q) \frac{q^\mu q^\nu}{(q^2)^2}, \qquad (1)$$

becomes evidently divergent on the light cone. Here $\epsilon(q)$ is a measure of the violation of four transversality on the light cone, since proper four transversality requires $\lim_{q^2 \to 0} \epsilon(q) = 0$! The occurrence of this singularity was already stated by the authors themselves [1], though qualified as harmless! However, it strongly violates analyticity requirements, since

a) $\Pi^{\mu\nu}(q)$ is given by the space-time Fourier transformation of the corresponding current-current correlator $\langle J^\mu(x) J^\nu(0)\rangle$. Thus, apart from UV regularizations the four-momentum Fourier transformation of $\Pi^{\mu\nu}(q)$ must exist, not to mention serious value constraints on $\Pi^{\mu\nu}$ arising from sum rules such as the Thomas-Reiche-Kuhn sum rule, the f-sum rule, or Weinberg's sum rule which all would diverge by this construction!

b) the corresponding physically relevant Lorentz components of the ρ-meson spectral-function can sim-

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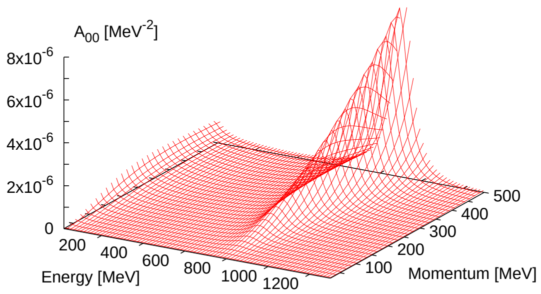

FIG. 2: (Color online) Same as in Fig. 1 for the method by van Hees and Knoll.

ply be estimated analytically,

$$A_L^{\mu\nu}(q) \stackrel{q^2\to 0}{\longrightarrow} \frac{2\epsilon q^\mu q^\nu}{4m_\rho^4 \vec{q}^2(q^0-|\vec{q}|)^2+\epsilon^2}, \qquad (2)$$

in the vicinity of the light-cone. There these components show a strong peak, c.f. the result for A^{00} in Figs. 1 obtained from our numerical repetition of the Ruppert-Renk method.

Note that $A^{00}(q) = \frac{\vec{q}^2}{q^2} A_L(q)$ drops to zero at vanishing spatial momentum $\vec{q}$. This light-cone structure represents a zero-mass mode with amazing stamina. This fictitious mode always emerges unless the unprojected tensor is *exactly* four-transversal on the light-cone. It has the remarkable feature that for any given momentum $\vec{q}$, its energy-weighted integral strength (obtained from the residue) is about identical to the resonance strength integrated across m_ρ! Similar conclusions hold for the spatial components of A_L.

Two of us (HvH and JK) [6] suggested an alternative method for the construction of a four-transversal polarization tensor which definitely avoids the above stated light-cone singularity, since there $\Pi_L(q)$ vanishes by construction. For details we refer to refs. [6, 7]. Here one discards the self-consistently obtained time components $\Pi^{00}(q)$ and Π^{0i}, Π^{i0}, since due to the conservation law they involve an infinite relaxation time which is known to escape a reliable treatment in self-consistent schemes at finite loop order. Therefore the full tensor is constructed solely from the self-consistently obtained spatial components Π^{ik} such that $\Pi^{\mu\nu}$ becomes exactly four-dimensionally transversal. It should be mentioned though that due to the $1/q_0^2$ factor in the construction of $\Pi^{00}(q) = q_i q_k \Pi^{ik}/q_0^2$, this method may lead to a less controlled determination of A^{00} close to vanishing energy $q_0 = 0$. Even though we expect contributions arising from classical random scattering [8] (the hight T limit of Landau damping), which indeed strongly peak close to $q_0 = 0$ we point out that they are essentially uncontrolled. Comparing the numerical result given in Fig. 2, this component may look tiny (possibly due to the anti-symmetry Im $A^{00}(q) = -$Im $A^{00}(-q)$, which suppresses the components near $q_0 = 0$) as compared to the artificial light-cone mode of the Ruppert-Renk method, Fig. 1 (note the differences in the ordinate scales) but additional clarification is mandatory.

Conclusions

The zero-mass mode of the ρ-meson produced by the Ruppert-Renk projection method rests on a kinematical singularity and is therefore completely unphysical. In the self-consistent scheme it provides a strong new decay mode for the $\pi\pi\rho$-coupling, which in turn significantly broadens the pion spectral function, and finally leads to the stated broadening of the ρ-meson! Since the projection strategy of van Hees and Knoll was also used in the studies [3, 4, 7] we will carefully reinvestigate the scheme and hopefully achieve a concept that also complies with the sum-rule constraints for the polarization tensor, before we subject the method to a quantitative comparison with data. We strongly support the statement made by the authors in [4] that the current status of the model does not allow to conclude that nonperturbative $\rho - \pi$ interactions are the main mechanism for the broadening observed in the NA60 data.

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Note added after the response of the authors
In inspecting and recomputing the authors' new numerical results we found that the spurious modes indeed strongly influence the physical results. Beyond this we now do suspect that the authors deal with unnormalized spectral functions for the pion which through the spurious modes get so seriously spoiled that unphysical results emerge. Furthermore the analytic expressions for the self-energy, c.f. Eqs. (27) and (29) in [1], are found to be by a factor two too large. Thus we further suspect that also in their selfconsistent numerics they work with this false factor of two, since otherwise we do not come even close to their results. Thus additional clarifications by the authors are mandatory.

Detailed explanations of our investigations were submitted to the authors.

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