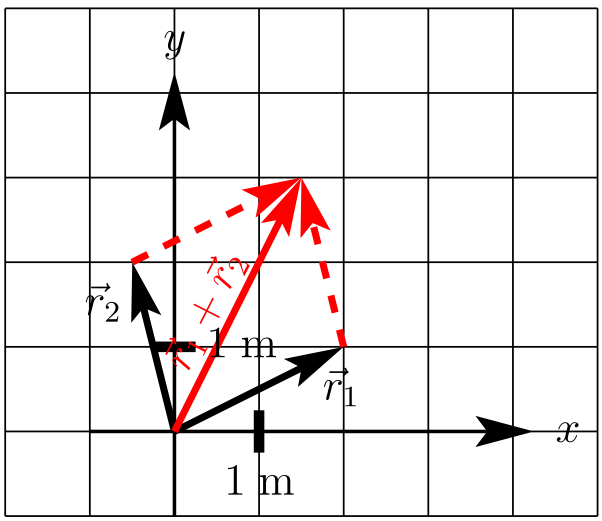
Problem 1 (30 points)

Physics 208 Quiz 1
Solution

January 23, 2008



(a) In the figure above, add the vectors $\vec{r}_1$ and $\vec{r}_2$ geometrically.

See Figure.

(b) What are the components of these vectors, (x_1, y_1) and (x_2, y_2), and the sum $\vec{r}_1 + \vec{r}_2$.

$(x_1, y_1) = (2, 1)$ m, $(x_1, y_1) = (-1/2, 2)$ m, $(x_1 + x_2, y_1 + y_2) = (3/2, 3)$ m.

Do not forget to write the units!

(c) If at the end points of the vectors are particles with masses m_1 and m_2, what is the gravitational force, $\vec{F}_{12}$, exerted by particle 1 on particle 2? [You do not need to give numbers, just the expression in terms of $\vec{r}_1$ and $\vec{r}_2$, the masses etc.]

$$\vec{F}_{12} = \gamma \frac{m_1 m_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$$

Always remember that a force is a **vector**. You must have a difference of "position vectors" of the points and then take the magnitude of this vector to calculate the distance of the two points. In general $|\vec{r}_1 - \vec{r}_2| \neq r_1 - r_2$! The gravitational force is always attractive, γ is Newton's Gravitation Constant. Do not mix it up with $1/(4\pi\epsilon)$ in Coulombs Law, which applies to the electric force between to charges at rest!

Problem 2 (70 points)

(a) A particle with mass, m, moves along the trajectory

$$\vec{r}(t) = R\cos(\omega t)\,\vec{i}_x + R\sin(\omega t)\,\vec{i}_y, \tag{1}$$

where $\vec{i}_x$ and $\vec{i}_y$ are unit vectors, perpendicular to each other (giving a Cartesian coordinate system); t is time and $\omega = \text{const}$. Show that the particle moves along a circle with the center in the origin of the coordinate system. What is the radius of this circle? **Hint:** Calculate the distance of the particle from the origin to show that it is constant with time!

$$r(t) = |\vec{r}(t)| = \sqrt{[R\cos(\omega t)]^2 + [R\sin(\omega t)]^2} = R = \text{const}, \tag{2}$$

because $(\cos\alpha)^2 + (\sin\alpha)^2 = 1$ for all α, i.e., the particle has always the same distance, R, from the origin. This means it runs along a circle of radius, R, with the center in the origin of the coordinate system.

(b) Calculate the velocity, $\vec{v}(t)$, and the acceleration, $\vec{a}(t)$, of the particle. Determine the magnitude of these quantities.

$$\begin{aligned} \vec{v}(t) &= -R\omega\sin(\omega t)\vec{i}_x + R\omega\cos(\omega t)\vec{i}_y, \quad v(t) = |\vec{v}(t)| = R\omega, \\ \vec{a}(t) &= -R\omega^2\cos(\omega t)\vec{i}_x - R\omega^2\sin(\omega t)\vec{i}_y, \quad a(t) = |\vec{a}(t)| = R\omega^2. \end{aligned} \tag{3}$$

(c) What is the force $\vec{F}(t)$ exerted on the particle?

According to Newton's 2nd law

$$\vec{F}(t) = m\vec{a}(t) = -mR\omega^2\cos(\omega t)\vec{i}_x - mR\omega^2\sin(\omega t)\vec{i}_y. \tag{4}$$

(d) Can you express the force in terms of $\vec{r}$? What is its direction and magnitude?

$$\vec{F} = -m\omega^2[R\cos(\omega t)\vec{i}_x + R\sin(\omega t)\vec{i}_y] = -m\omega^2\vec{r}. \tag{5}$$

To make the particle run on a circle of radius, R, one has to exert a force of magnitude

$$F = |\vec{F}| = m\omega^2|\vec{r}| = mR\omega^2 = \frac{mv^2}{R}, \tag{6}$$

pointing towards the center of the circle. That force is known as the centripetal force.