Name: .... Section: ....

Physics 208 Quiz 8

April 11, 2008; due April 18, 2008

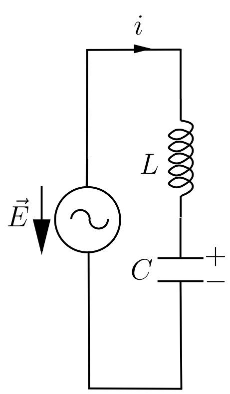
Problem 1 (60 Points)

Consider the AC circuit consisting of an AC voltage in series with a coil of self-inductance, L, and a capacitor of capacitance, C. The voltage has the time dependence

$$V(t) = V_0 \cos(\omega_0 t). \tag{1}$$

Assume that

$$\omega_0 \neq \frac{1}{\sqrt{LC}}. \tag{2}$$



(a) Derive the differential equation for the charge on the upper plate (labeled + in the diagram) of the capacitor.

Hint: The solution for this part is

$$L\frac{\mathrm{d}^2Q}{\mathrm{d}t^2} + Q/C = V(t). \tag{3}$$

(b) Find the solution for Q and $i = \frac{\mathrm{d}Q}{\mathrm{d}t}$ for the initial condition $Q(t=0) = Q_0$, $i(t=0) = 0$.

(c) Assume now that there is a small resistance, R[1], such that after a long time the "transient part" (i.e., the part of the solution which is given by the solution for the homogeneous equation) is damped out, and i obeys the the "steady-state" solution

$$i(t) = \hat{i}\cos(\omega_0 t - \phi_0). \tag{4}$$

Calculate the amplitude for the current, $\hat{i}$, and the phase shift, ϕ_0, as a function of L, C, V_0, and ω_0. Show that this case is contained in the general solution for the damped RLC circuit discussed in the lecture for $R \to 0$.

(d) [for extra credit] Work out the solutions for questions (a) and (b) in the "resonance case", i.e., for $\omega_0 \to \omega_R$, where $\omega_R = 1/\sqrt{LC}$.

Hint: To find a particular solution of the inhomogeneous equation, make the ansatz

$$Q_I(t) = q(t)\sin(\omega_0 t) \tag{5}$$

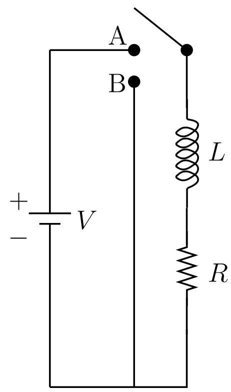
and solve for q.

See Problem 2 on next page!

[1] You do not need to give solutions for this more general case which is treated in detail in the lecture. You can look it up in the lecture notes on the course home page.

Problem 2 (40 Points)

(2) Consider the following circuit with a coil of self-inductance, L, and a resistor with resistance, R (which is assumed to account also for the resistance of the coil).



(a) Assume that the switch has been set to position A for a very long time so that a constant current, i_0, runs through coil and resistor. What is this current, i_0?

(b) At $t = 0$ the switch is brought into position B. Derive the differential equation for the current through the resistor and solve for $i(t)$, assuming the appropriate initial condition.

(c) What is the power (energy per time) dissipated into heat in the resistor as a function of time (for $t \geq 0$)?

(d) What is the total energy dissipated into heat?

Hint: Integrate your result for the power from part (c) with respect to time from $t = 0$ to $t \to \infty$!

(e) What has been the total energy content of the coil immediately before switching to position B? Explain briefly, in which form this energy has been "stored" within the coil!