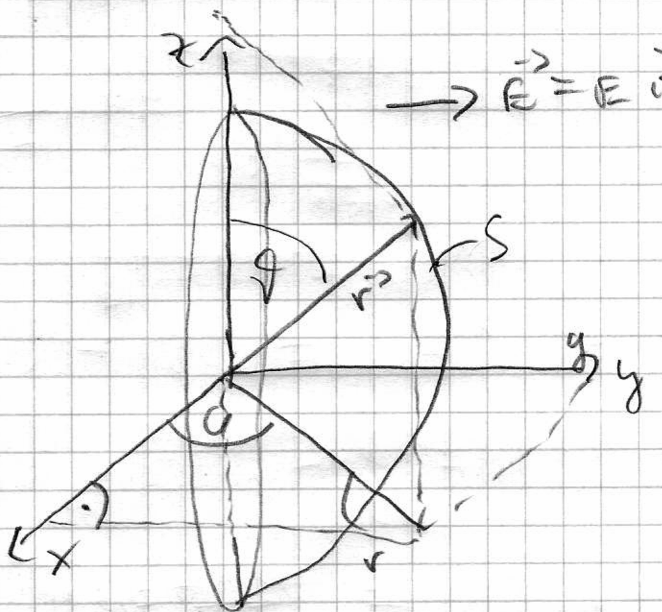
Chpt. 4 Problem 4 in standard spherical coordinates



$$\vec{r} = r\cos\varphi\sin\vartheta\,\vec{i}_x + r\sin\varphi\sin\vartheta\,\vec{i}_y + r\cos\vartheta\,\vec{i}_z$$

$$d\vec{S} = R^2\sin\vartheta\,\vec{i}_r\,d\vartheta\,d\varphi$$

For the hemisphere we have $\vartheta \in (0,\pi)$, $\varphi \in (0,\pi)$

$$\Phi_E = \int_S d\vec{S}\cdot\vec{E} = \int_0^\pi d\vartheta \int_0^\pi d\varphi\; R^2\sin\vartheta\,\vec{i}_r\cdot\vec{i}_y\,E$$

$$= R^2 E \int_0^\pi d\vartheta \int_0^\pi d\varphi\; \sin^2\vartheta\sin\varphi$$

$$\int_0^\pi d\varphi\sin\varphi = -\cos\varphi\Big|_0^\pi = 2$$

$$\cos(2\vartheta) = \cos^2\vartheta - \sin^2\vartheta = 1 - 2\sin^2\vartheta$$

$$\Rightarrow \sin^2\vartheta = \frac{1-\cos(2\vartheta)}{2}$$

$$\Rightarrow \int_0^\pi d\vartheta\,\frac{1-\cos(2\vartheta)}{2} = \frac{\pi}{2} - \left[\frac{1}{4}\sin(2\vartheta)\right]_0^\pi = \frac{\pi}{2}$$

$$\Rightarrow \Phi_E = R^2 E\cdot 2\,\frac{\pi}{2} = \pi R^2 E$$