Physics 208 Final Exam

May 05, 2008

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Print your name **neatly:**

Last Name: | | | | | | | | | | | | | | | | | | | | |

First Name: | | | | | | | | | | | | | | | | | | | | |

Sign your name: ....

Stud. ID (UIN): | | | | | | | | | | |

IMPORTANT

- There are 8 problems totaling 200 points and a formula sheet. Check your exam to make sure you have all the pages. Work each problem on the page the problem is on. You may use the back. If you need extra pages, I have plenty up front. Write your name on each page!
- **Indicate what you are doing!** We cannot give full credit for merely writing down the answer. **Neatness counts!** I will give generous partial credit **if** I can tell that you are on the right track. **Especially indicate the directions of all used vectors, currents, integration paths, etc.**
- Each problem is self explanatory. If you must ask a question, then come to the front, being as discrete as possible so as not to disturb others.

1	2	3	4	5	6	7	8	total

Useful formulae

Maxwell's equations (neglecting the displacement current)

$$\text{Gauss's Law for electric fields:} \quad \oint_{\partial V} \mathrm{d}\vec{S} \cdot \vec{E} = \frac{1}{\epsilon_0} \int_V \mathrm{d}V \rho = \frac{Q_{\text{inside}}}{\epsilon_0}$$

$$\text{Gauss's Law for magnetic fields:} \quad \oint_{\partial V} \mathrm{d}\vec{S} \cdot \vec{B} = 0$$

$$\text{Ampère's Circuital Law:} \quad \oint_{\partial S} \mathrm{d}\vec{r} \cdot \vec{B} = \mu_0 \int_S \mathrm{d}\vec{S} \cdot \vec{j} = \mu_0 \, i_{\text{enclosed}}$$

$$\text{Faraday's Law:} \quad \oint_{\partial S} \mathrm{d}\vec{r} \cdot \vec{E} = -\frac{\mathrm{d}}{\mathrm{d}t} \int_S \mathrm{d}\vec{S} \cdot \vec{B} = -\frac{\mathrm{d}\Phi_{\vec{B}}}{\mathrm{d}t}$$

Remember the relative directions of the surface-normal vectors, $\mathrm{d}\vec{S}$: For Gauss's Laws always **out of the volume**; for Ampère's and Faraday's Laws always **corresponding to the direction of the boundary, ∂S, according to the right-hand rule!**

Coulomb force from a point charge, q_1, at position $\vec{r}_1$ on a point charge, q_2, at position $\vec{r}_2$

$$\vec{F}_{12} = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3}$$

Electric potential of a point charge, q, at position $\vec{r}\,'$

$$V(\vec{r}) = \frac{q}{4\pi\epsilon_0} \frac{1}{|\vec{r} - \vec{r}\,'|}$$

Lorentz force for a point charge in an electromagnetic field

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Force on a current-conducting wire in a magnetic field

$$\vec{F} = i\vec{l} \times \vec{B}$$

Biot-Savart Law for the $\vec{B}$ field from a current-conducting wire

$$\vec{B}(\vec{r}) = \frac{\mu_0 i}{4\pi} \int \mathrm{d}\vec{r}\,' \times \frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|^3}$$

The differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} + c_1 x = c_2,$$

where $c_1 \neq 0$ and c_2 are constants, has the **general** solution

$$x(t) = A \exp(-c_1 t) + \frac{c_2}{c_1}, \quad A = \text{const.}$$

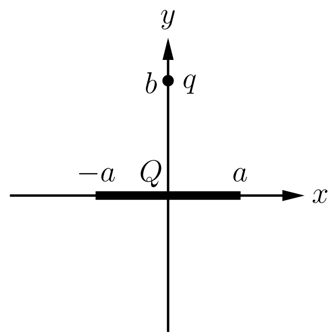
Here x might be a charge, current, or any other quantity!

Potentially useful Integrals

$$\int \mathrm{d}x \frac{1}{[(x+a)^2 + b]^{3/2}} = \frac{x+a}{b\sqrt{(x+a)^2 + b}}, \quad \int \mathrm{d}x \frac{x}{[(x+a)^2 + b]^{3/2}} = -\frac{a(x+a) + b}{b\sqrt{(x+a)^2 + b}}$$

Name: ....

1. (25 points) A charge Q is uniformly spread along the x-axis (from $x = -a$ to $x = a$).

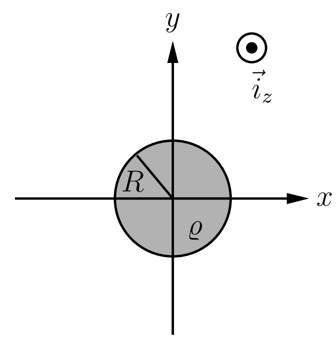


(a) Determine the electric field, $\vec{E}(x, y)$ of this charge distribution.

(b) What is the electric force on a test charge, q, located on the y axis at $y = b$?

Name: ....

2. (25 points) A sphere of radius, R, is charged with a charge distribution such that the charge density $\varrho(r) = \alpha r$, where $\alpha =$ const.

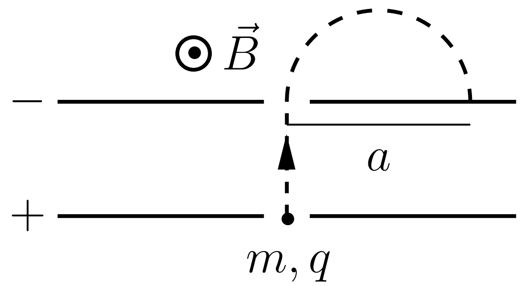


(a) Determine the electric field, $\vec{E}$, everywhere.

(b) Calculate the potential difference between the center of the sphere and a point outside (at a distance $r > R$ from the center).

Name: ....

3. (25 points) Two parallel very large conducting plates are connected to a battery with voltage, V, for a long time. Both have a hole in the middle, and at $t = 0$ a charged particle with mass m, and charge, $q > 0$, enters the hole of the lower plate with negligible velocity. Outside of the plates is a homogeneous magnetic field. Gravity can be neglected.

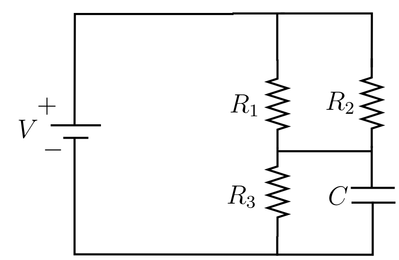


(a) What is the particle's velocity when it leaves the capacitor at the upper plate?

(b) Determine the distance, a, of the point where the particle hits the upper plate again.

Name: ....

4. (25 points) The circuit is hooked up to the battery as shown for a very long time.
(a) What are the currents through each resistor?

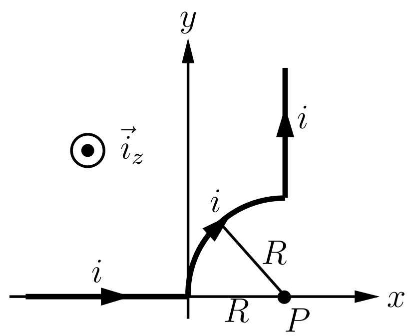


(b) What is the voltage across the capacitor? What is the charge at its positively charged plate?

Hint: Label all currents and the signs of charges at the capacitor in the circuit diagram!

Name: ....

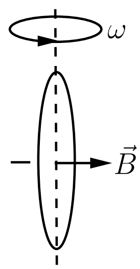
5. (25 points) A thin wire in the xy plane of a Cartesian coordinate system is shaped as shown in the figure: an infinitely long piece lies along the x axis from $x \to -\infty$ to $x = 0$, then a piece is shaped as a quarter of a circle of radius, R. Finally, another infinitely long piece is placed at $x = R$ parallel to the y axis from $y = R$ to $y \to \infty$.



Calculate the magnetic field, $\vec{B}$, at the point P, located at $x = R$, $y = z = 0$, which is the center of the quarter circle!

Name: ....

6. (25 points) A circular wire of radius, b, of cross-sectional area, A, is made of a material with resistivity ρ. For a very long time, it rotates around one of its diameters with constant angular velocity, ω. A homogeneous magnetic field, $\vec{B}$, is pointing perpendicular to the rotation axis.

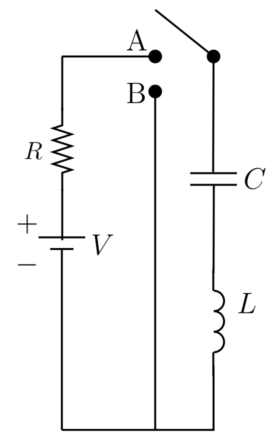


(a) Calculate the current induced in the wire.

(b) How much energy is used during one period of rotation $T = 2\pi/\omega$?

Name: ....

7. (25 points) (a) In the circuit shown in the figure, the switch has been in position A for a long time.
(a) What is the charge at the upper plate of the capacitor?

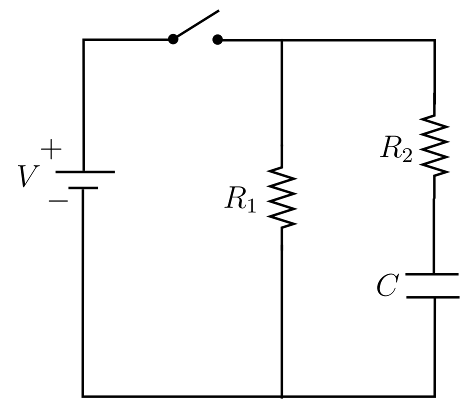


(b) At $t = 0$ the switch is set into position B. Calculate the charge, $Q(t)$, at the upper plate of the capacitor as a function of time!

(c) What is the current, $i(t)$, through the coil, as a function of time (for $t > 0$)?

Name: ....

8.(25 points) (a) Suppose the switch has been open for a very long time. At $t = 0$ it is closed. Calculate the currents through the resistors as a function of time! All self-inductances can be neglected.



(b) Show that after a long time ($t \to \infty$) the currents reach the values to be expected from a steady-state situation.