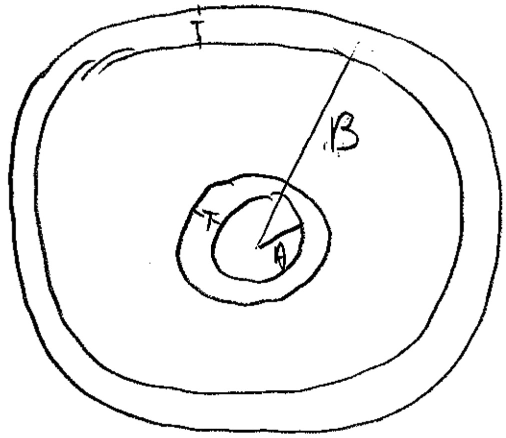
Exam II : More detailed solution to problem 1



(a+b) Since the shells are conductors, in the static state the $\vec{E}$ field is 0 inside the conductors. From spherical symmetry, we know that the electric potential can be only a function of r, the distance from the center:

$$V(\vec{r}) = V(r)$$

Then we know (see lecture notes about spherical coordinates):

$$\vec{E} = -\frac{\partial V}{\partial r}\vec{i}_r = E_r(r)\,\vec{i}_r$$

Now we can apply Gauss's law, taking into account

$$E_r(r) = 0 \quad \text{for} \quad A < r < A+T \ ; \ B < r < B+T$$

As Gaussian surfaces we choose spheres of radius r. Then:

$$\int_{\partial V} d\vec{S}\, E_r(r)\,\vec{i}_r = 4\pi r^2 E_r(r) = \frac{Q_{\text{inside}}}{\varepsilon_0}$$

For $r < A$, always $Q_{inside} = 0$. Thus

$$E_r(r) = 0 \quad \text{for } r < A$$

Now inside the shell $E_r = 0$. So for $A < r < A+T$ also $Q_{inside} = 0$. Thus there is no charge on the inside of the inner shell:

$$\sigma = 0 \quad \text{for } r = A$$

Now for $A+T < r < B$, the total charge Q is inside the Gaussian surface, and thus

$$E_r(r) = \frac{Q}{4\pi \varepsilon_0 r^2} \quad \text{for } A+T < r < B$$

If $B < r < B+T$, E_r vanishes (no field inside the conductor!). From Gauss's law we thus now that for these r there is no charge inside the Gaussian surface. This can only be, because there sits the charge $-Q$ on the inside surface of the outer shell. Thus we have

$$\sigma = -\frac{Q}{4\pi B^2}$$

and $$E_r(r) = 0 \quad \text{for } B < r < B+T$$

For $r > B+T$ there is a field, and all the charge, Q, is inside the Gaussian volume. Thus there must be the charge Q on the outside surface:

$$\sigma = \frac{Q}{4\pi (B+T)^2}$$

and $$E_r(r) = \frac{Q}{4\pi \varepsilon_0 r^2} \quad \text{for } r > B+T$$

To calculate the voltage difference we choose a path radially <u>in</u> from $r = B+T$ to $r = A$

$$\Delta V = V(A) - V(B+T) = -\int_{\ell} d\vec{r}\, \vec{E}(\vec{r}) = -\int_{B+T}^{A} dr\, E_r(r)$$

Now $E_r(r)$ is 0 inside the conductors, and thus we have

$$\Delta V = -\int_{B}^{A+T} dr\, E_r(r) = +\int_{A+T}^{B} dr\, E_r(r) = \frac{Q}{4\pi\varepsilon_0}\int_{A+T}^{B} dr\, \frac{1}{r}$$

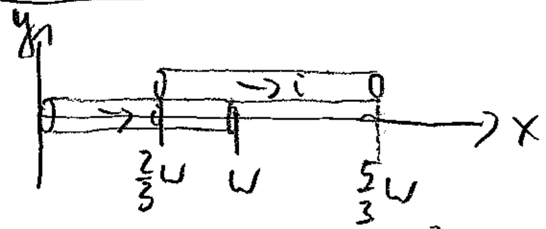
$$= \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r^2}\right]_{A+T}^{B} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{A+T} - \frac{1}{B}\right]$$

(c) The capacitance is defined as

$$C = \frac{Q}{\Delta V} = \frac{4\pi\varepsilon_0}{\frac{1}{A+T} - \frac{1}{B}} = \frac{4\pi\varepsilon_0 (A+T) B}{B-(A+T)}$$

<u>Alternative solution to problem 4</u>

(a)



$$\vec{j} = \frac{i}{A}\begin{cases} \vec{i}_x & \text{for } 0 < x < \frac{2}{3}w \text{ or } w < x < \frac{5}{3}w \\ \frac{1}{2}\vec{i}_x & \text{for } \frac{2}{3}w < x < w \end{cases}$$

(b) $\vec{E} = \rho\,\vec{j}$

$$\Delta V = V(x=0) - V(x=\tfrac{5}{3}w) = -\int_{5/3w}^{0} dx\, \vec{i}_x \cdot \vec{E}$$

$$\Delta V = + \int_0^{5/3 w} dx\, \vec{i}_x \cdot \vec{E} = \frac{\rho i}{A} \left[\int_0^{2/3 w} dx + \frac{1}{2} \int_{2/3 w}^{w} dx + \int_{w}^{5/3 w} dx \right]$$ (4)

$$= \frac{\rho i}{A} \left[\frac{2}{3} w + \frac{1}{6} w + \frac{2}{3} w \right] = \frac{3 \rho w}{2A}\, i$$

(c) $R = \dfrac{\Delta V}{i} = \dfrac{3 \rho w}{2A}$