

Coordinates

Hendrik van Hees

Texas A&M University

February 20, 2008



Hendrik van Hees

Coordinates

Cartesian Coordinates

- ▶ **Cartesian Coordinates**, x, y, z : Given are three arbitrary **unit vectors** $\vec{i}_x, \vec{i}_y, \vec{i}_z$, which are pairwise perpendicular to each other and oriented in this order by the “right-hand rule”
- ▶ each point in space is **determined uniquely** by its position vector, pointing from the origin of the coordinate system to the point: $\vec{r} = x\vec{i}_x + y\vec{i}_y + z\vec{i}_z$
- ▶ **dot product**: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$
- ▶ **cross product**

$$\begin{aligned}\vec{a} \times \vec{b} &= (a_y b_z - a_z b_y) \vec{i}_x \\ &\quad + (a_z b_x - a_x b_z) \vec{i}_y \\ &\quad + (a_x b_y - a_y b_x) \vec{i}_z\end{aligned}$$

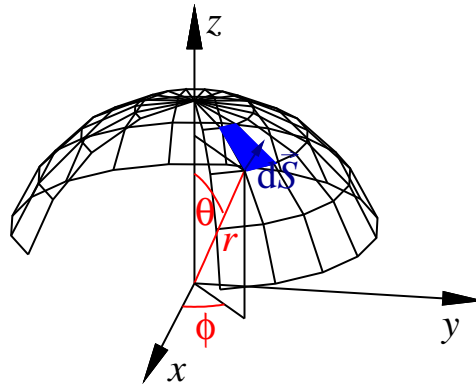
- ▶ **Gradient**: $\vec{E} = -\text{grad } V$

$$\vec{E} = -\nabla V = -\left(\frac{\partial V}{\partial x} \vec{i}_x + \frac{\partial V}{\partial y} \vec{i}_y + \frac{\partial V}{\partial z} \vec{i}_z \right)$$

Hendrik van Hees

Coordinates

Spherical Coordinates (Definition)



- ▶ **spherical coordinates**, r , θ , ϕ
- ▶ determines any point uniquely, **not located on the z axis**
- ▶ Relation to Cartesian Coordinates
(**read them off from the figure!**):

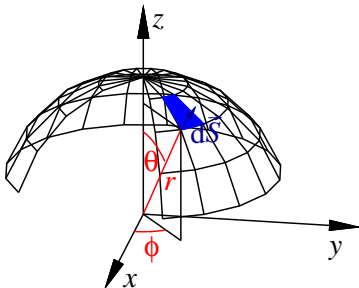
$$\vec{r}(r, \theta, \phi) = r(\sin \theta \cos \phi \vec{i}_x + \sin \theta \sin \phi \vec{i}_y + \cos \theta \vec{i}_z)$$

- ▶ coordinate ranges: $r > 0$, $\theta \in (0, \pi)$, $\phi \in [0, 2\pi)$

Hendrik van Hees

Coordinates

Spherical Coordinates (important formulae)



$$\vec{i}_r = \sin \theta \cos \phi \vec{i}_x + \sin \theta \sin \phi \vec{i}_y + \cos \theta \vec{i}_z,$$

$$\vec{i}_\theta = \cos \theta \cos \phi \vec{i}_x + \cos \theta \sin \phi \vec{i}_y - \sin \theta \vec{i}_z,$$

$$\vec{i}_\phi = -\sin \phi \vec{i}_x + \cos \phi \vec{i}_y.$$

- ▶ each vector, **not pointing into the z -direction**, is then uniquely determined by its components with respect to these unit vectors: $\vec{E} = E_r \vec{i}_r + E_\theta \vec{i}_\theta + E_\phi \vec{i}_\phi$.
- ▶ **these basis vectors depend on θ and ϕ !**
- ▶ **surface element for sphere of radius r : $d\vec{S} = r^2 \sin \theta d\theta d\phi \vec{i}_r$.**
- ▶ **Gradient:**

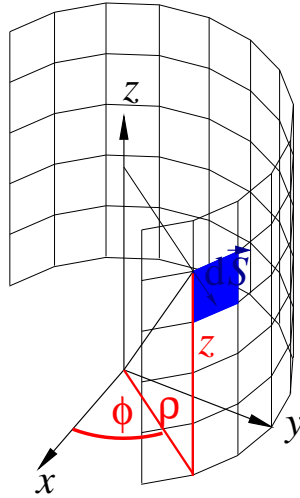
$$\vec{E} = -\nabla V = -\left(\frac{\partial V}{\partial r} \vec{i}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \vec{i}_\theta + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \vec{i}_\phi \right)$$

- ▶ vector products: $\vec{i}_r \times \vec{i}_\theta = \vec{i}_\phi$, $\vec{i}_\theta \times \vec{i}_\phi = \vec{i}_r$, $\vec{i}_\phi \times \vec{i}_r = \vec{i}_\theta$.

Hendrik van Hees

Coordinates

Cylinder Coordinates (Definition)



- ▶ cylinder coordinates, ρ , ϕ , z
- ▶ determines any point uniquely, **not located on the z axis**
- ▶ Relation to Cartesian Coordinates
(read them off from the figure!):

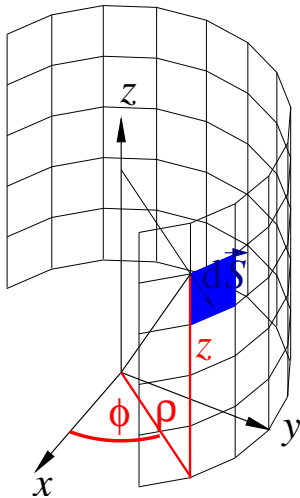
$$\vec{r}(\rho, \phi, z) = \rho \cos \phi \vec{i}_x + \rho \sin \phi \vec{i}_y + z \vec{i}_z$$

- ▶ coordinate ranges: $\rho > 0$, $\phi \in [0, 2\pi)$, $z \in \mathbb{R}$

Hendrik van Hees

Coordinates

Cylinder Coordinates (important formulae I)



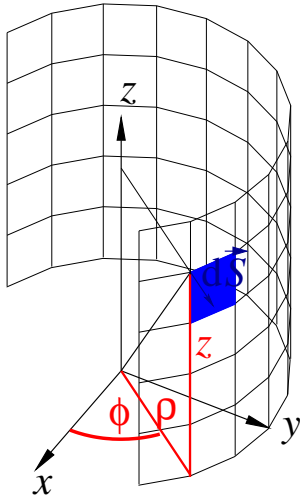
$$\begin{aligned}\vec{i}_\rho &= \cos \phi \vec{i}_x + \sin \phi \vec{i}_y, \\ \vec{i}_\phi &= -\sin \phi \vec{i}_x + \cos \phi \vec{i}_y, \\ \vec{i}_z &= \vec{i}_z\end{aligned}$$

- ▶ each vector, **not pointing into the z -direction**, is then uniquely determined by its components with respect to these unit vectors: $\vec{E} = E_\rho \vec{i}_\rho + E_\phi \vec{i}_\phi + E_z \vec{i}_z$.
- ▶ \vec{i}_ρ and \vec{i}_ϕ depend on ϕ !
- ▶ surface element for cylinder envelope $\rho = \text{const}$:
 $d\vec{S} = \rho d\phi dz \vec{i}_\rho$.
- ▶ surface element for upper cylinder cap: $d\vec{S} = \rho d\rho d\phi \vec{i}_z$.

Hendrik van Hees

Coordinates

Cylinder Coordinates (important formulae II)



$$\begin{aligned}\vec{i}_\rho &= \cos \phi \vec{i}_x + \sin \phi \vec{i}_y, \\ \vec{i}_\phi &= -\sin \phi \vec{i}_x + \cos \phi \vec{i}_y, \\ \vec{i}_z &= \vec{i}_z\end{aligned}$$

► Gradient:

$$\vec{E} = -\nabla V = -\left(\frac{\partial V}{\partial \rho} \vec{i}_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \vec{i}_\phi + \frac{\partial V}{\partial z} \vec{i}_z\right)$$

► vector products: $\vec{i}_\rho \times \vec{i}_\phi = \vec{i}_z$, $\vec{i}_\phi \times \vec{i}_z = \vec{i}_\rho$, $\vec{i}_z \times \vec{i}_\rho = \vec{i}_\phi$.