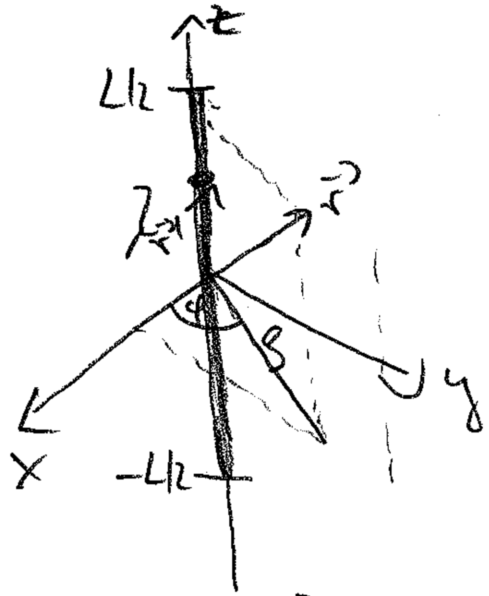
Electric field of a finite uniformly charged wire



$$\vec{r} = \varrho\,\vec{i}_\varrho + z\,\vec{i}_z \quad \Rightarrow |\vec{r}-\vec{r}'| = |\varrho\,\vec{i}_\varrho + (z-z')\,\vec{i}_z|$$

$$\vec{r}' = z'\,\vec{i}_z \qquad\qquad = \sqrt{\varrho^2+(z-z')^2}$$

The potential is

$$V(\varrho, z) = \int_{-L/2}^{L/2} dz' \frac{\lambda}{4\pi\varepsilon_0} \frac{1}{|\vec{r}-\vec{r}'|}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \int_{-L/2}^{L/2} \frac{dz'}{\sqrt{\varrho^2+(z-z')^2}}$$

To find the integral we substitute hyperbolic sine:

$$z'-z = \varrho \sinh u$$

$$dz' = \varrho \cosh u \, du$$

$$\int dz' \frac{1}{\sqrt{\varrho^2+(z'-z)^2}} = \int du \frac{\varrho \cosh u}{\sqrt{\varrho^2(1+\sinh^2 u)}}$$

$$= u = \operatorname{arsinh} \frac{z'-z}{\rho}$$

$$= \ln\left(\frac{z'-z}{\rho} + \sqrt{1 + \frac{(z'-z)^2}{\rho^2}} \right)$$

$$\boxed{V(\rho,z) = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{L/2 - z + \sqrt{\rho^2 + (L/2-z)^2}}{-L/2 - z + \sqrt{\rho^2 + (L/2+z)^2}} \right)}$$

$$\vec{E} = -\operatorname{grad} V = -\left(\frac{\partial V}{\partial \rho} \vec{i}_\rho + \frac{\partial V}{\partial z} \vec{i}_z \right)$$

$$=$$

$$V(\rho,z) = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{\sqrt{4\rho^2 + (2z-L)^2} - 2z + L}{\sqrt{4\rho^2 + (2z+L)^2} - 2z - L} \right)$$

$$\Rightarrow \boxed{\begin{aligned} E_\rho &= -\frac{\partial V}{\partial \rho} = \frac{\lambda}{4\pi\varepsilon_0 \rho} \left(\frac{L-2z}{\sqrt{4\rho^2 + (L-2z)^2}} + \frac{L+2z}{\sqrt{4\rho^2 + (L+2z)^2}} \right) \\ E_z &= \frac{\lambda}{2\pi\varepsilon_0} \left[\frac{1}{\sqrt{4\rho^2 + (L-2z)^2}} - \frac{1}{\sqrt{4\rho^2 + (L+2z)^2}} \right] \end{aligned}}$$

The derivatives I found with help of the computer-algebra program Mathematica. It's rather cumbersome to do them by hand ☺