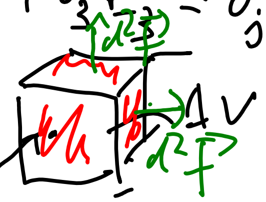
Gaußscher Integralsatz

GIS

$$\operatorname{div} \vec{E} = \vec{\nabla} \cdot \vec{E} = \frac{1}{\varepsilon_0} \rho \quad \text{(Gaußsches Gesetz)}$$

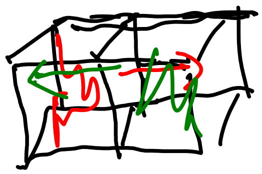
$$\vec{\nabla} \cdot \vec{E} = \begin{pmatrix} \partial_1 \\ \partial_2 \\ \partial_3 \end{pmatrix} \cdot \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} = \underbrace{\partial_1 E_1 + \partial_2 E_2 + \partial_3 E_3}_{\left(\sum_{j=1}^{3}\right)} = \partial_j E_j$$



$$\operatorname{div} \vec{E}(\vec{x}) = \lim_{\Delta V \to \{\vec{x}\}} \frac{1}{\Delta V} \int_{\partial \Delta V} d^2\vec{f} \cdot \vec{E}$$

$\vec{x}$

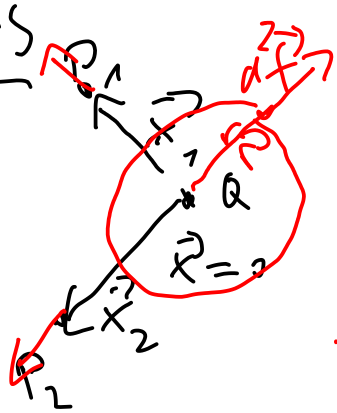
$$\int_V d^3x \, \operatorname{div} \vec{E}(x) = \int_{\partial V} d^2\vec{f} \cdot \vec{E} \quad \text{G. Integralsatz}$$



$$\Rightarrow \int_V d^3x \, \operatorname{div} \vec{E} = \int_{\partial V} d^2\vec{f} \cdot \vec{E} = \frac{1}{\varepsilon_0} \int_V d^3x \, \rho(\vec{x})$$

$$\boxed{\int_{\partial V} d^2\vec{f} \cdot \vec{E} = \frac{Q_V}{\varepsilon_0}}$$

Punktladung



Symmetrie: $\vec{e}_r$

$$\vec{E} = E_r(r) \frac{\vec{x}}{r}$$

$$\int_{\partial V} d^2\vec{f} \cdot \vec{E} = \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta \; r^2 \sin\vartheta \, \vec{e}_r \cdot E_r \vec{e}_r$$

$$\int_{\partial V} d^2\vec{f}\cdot\vec{E} = \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta\; r^2 \sin\vartheta\; E_r(r)$$

$$= r^2\, E_r(r)\, 2\pi \left(-\cos\vartheta\right)\Big|_{\vartheta=0}^{\pi}$$

$$= 4\pi r^2\, E_r(r) = \frac{Q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_V d^3x\; \rho(\vec{x}),$$

$$\rho(\vec{x}) = Q\,\delta^{(3)}(\vec{x})$$

$$E_r(r) = \frac{Q}{4\pi\varepsilon_0 r^2} \Rightarrow \vec{E}(\vec{x}) = \frac{Q}{4\pi\varepsilon_0 r^2}\,\vec{e}_r$$

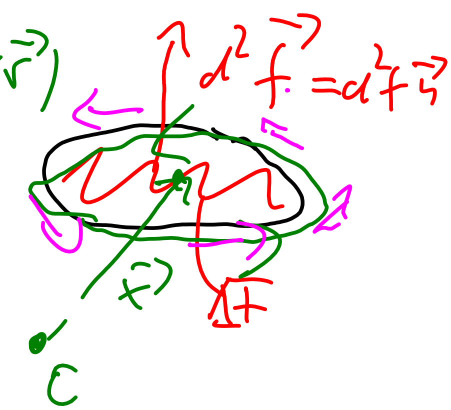
$$\boxed{\vec{E}(\vec{x}) = \frac{Q}{4\pi\varepsilon_0 r^3}\,\vec{x}}$$

$\vec{\nabla}\times\vec{E} = 0$ (Statik; nur für zeitunabhg. Fel(der))

$$\mathrm{rot}\,\vec{E} = \vec{\nabla}\times\vec{E} = \begin{pmatrix}\partial_1\\ \partial_2\\ \partial_3\end{pmatrix}\times\begin{pmatrix}E_1\\ E_2\\ E_3\end{pmatrix} = \begin{pmatrix}\partial_2 E_3 - \partial_3 E_2\\ \partial_3 E_1 - \partial_1 E_3\\ \partial_1 E_2 - \partial_2 E_1\end{pmatrix}$$

$$\vec{n}\cdot\mathrm{rot}\,\vec{E}(\vec{x}) = \lim_{\Delta F\to\{\vec{x}\}} \frac{1}{\Delta F}\int_{\partial\Delta F} d\vec{r}\cdot\vec{E}(\vec{r})$$

$\vec{n} \in \{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$

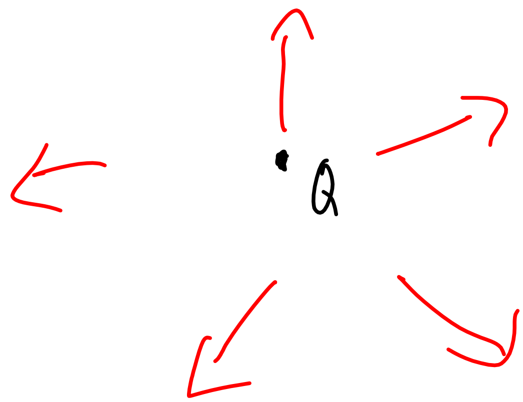


$$\int_F d^2\vec{f}\cdot\mathrm{rot}\,\vec{E} = \int_{\partial F} d\vec{r}\cdot\vec{E}$$

$\vec{E}$-Feld ist reines Quellenfeld

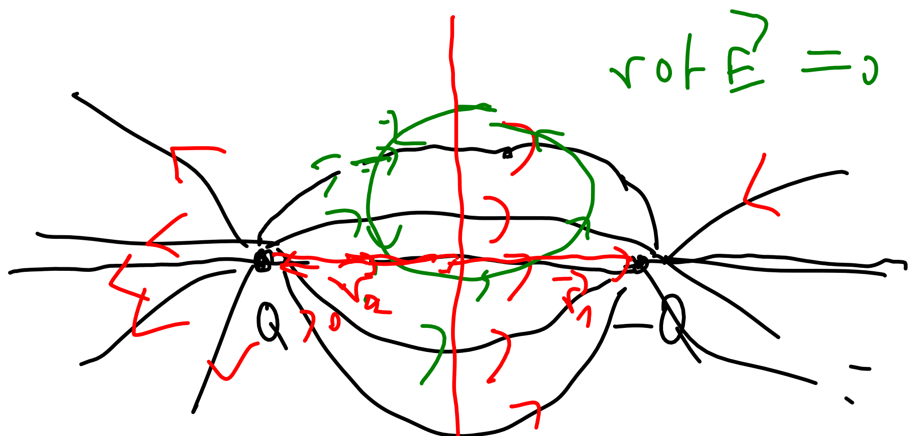
$$\mathrm{rot}\,\vec{E} = 0$$

$$\mathrm{div}\,\vec{E} = \frac{\rho}{\varepsilon_0}$$



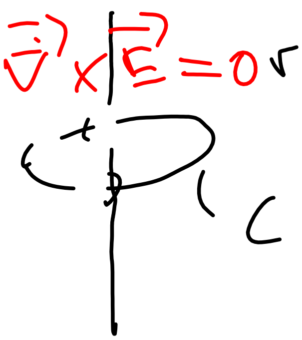
$$\mathrm{rot}\,\vec{E} = 0 \Rightarrow \oint_C d\vec{r}\cdot\vec{E} = 0$$

$$\vec{x} \in \mathbb{R}^3 \setminus \{\vec{r}_1, \vec{r}_2\}$$



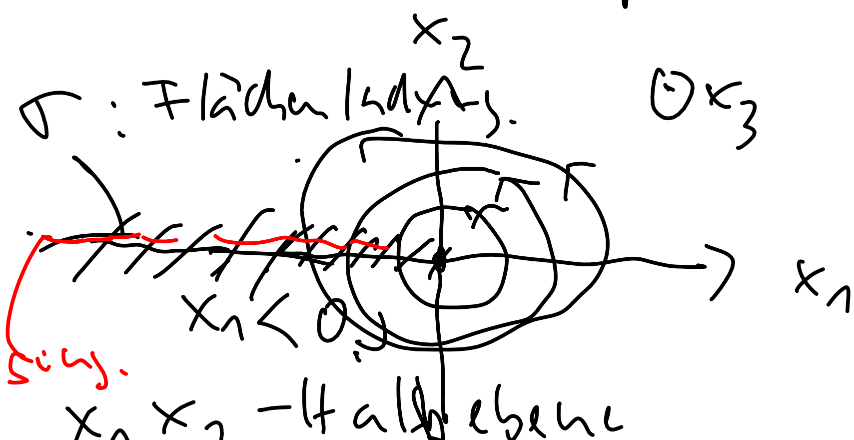
$\vec{j}(\vec{r})$; $\vec{r}$/z-Achse

$$\int_{C=\partial F} d\vec{r}\cdot\vec{E} = \int_F d^2\vec{f}\cdot\vec{\nabla}\times\vec{E} = 0 \qquad \mathrm{rot}\,\vec{V} = 0$$



$$\oint_C d\vec{r}\cdot\vec{E} \neq 0$$

σ : Flächenladung



$\phi(\vec{r})$: Sprung entlang Halbebene