GOING FULL CIRCLE: QCD TO EFT TO DYNAMICALLY GENERATED RESONANCES AND BACK TO (LATTICE) QCD

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UNIVERSITY OF BERN (main)

THE GEORGE WASHINGTON UNIVERSITY

Nuclear Physics Kolloquium 30.01.2025

Goethe University Frankfurt (Institute for Theoretical Physics)

Heisenberg-Programm

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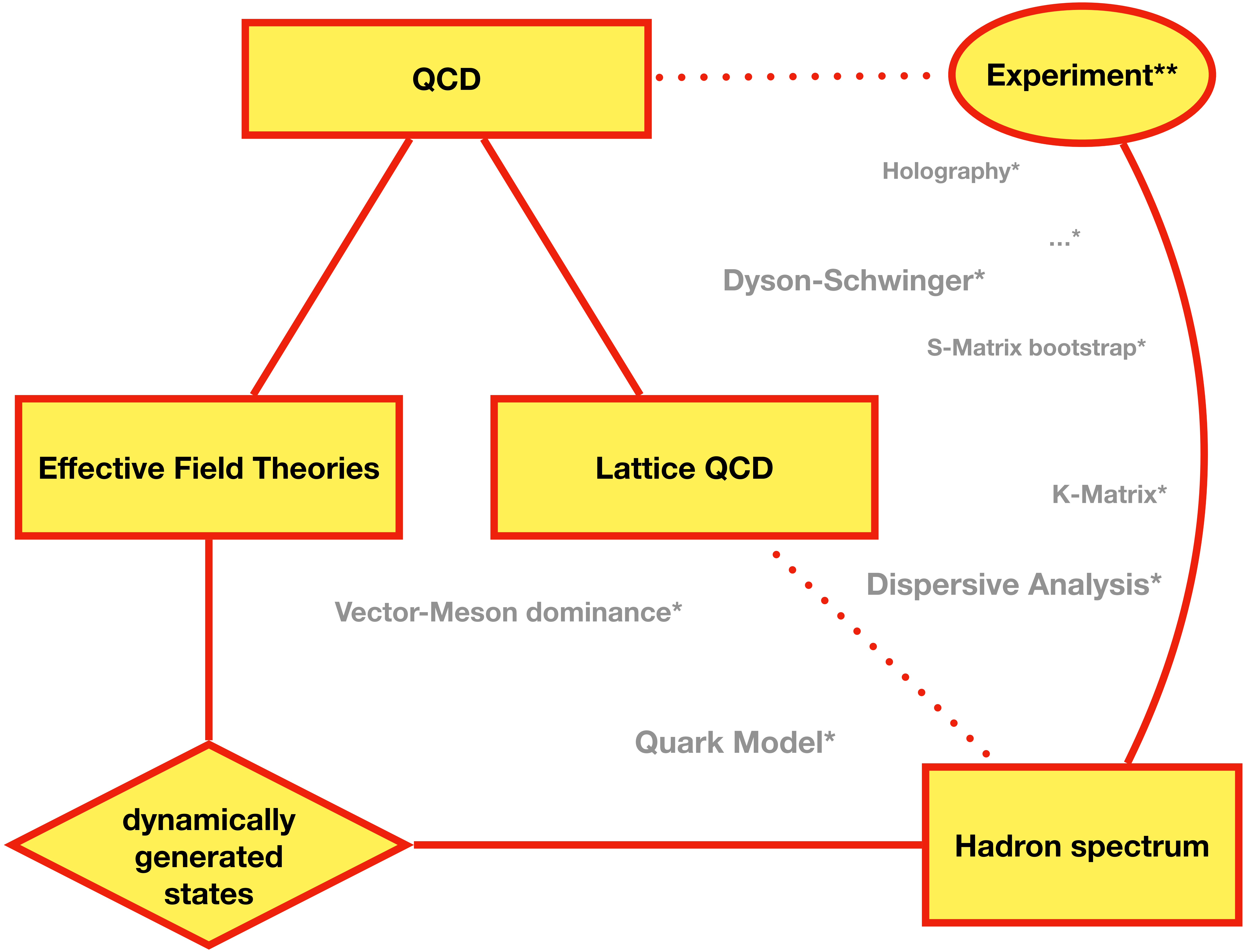
TRR110: NSFC Grant No. 12070131001, DFG Project-ID 196253076 DOE: DE-SC0016582, DE-AC05-06OR23177, DE-FG02-95ER40907 , NSF: PHY-2012289



*) not part of the talk **) low-energy

OUTLINE

1. **Motivation**

   Observation, Theory, …

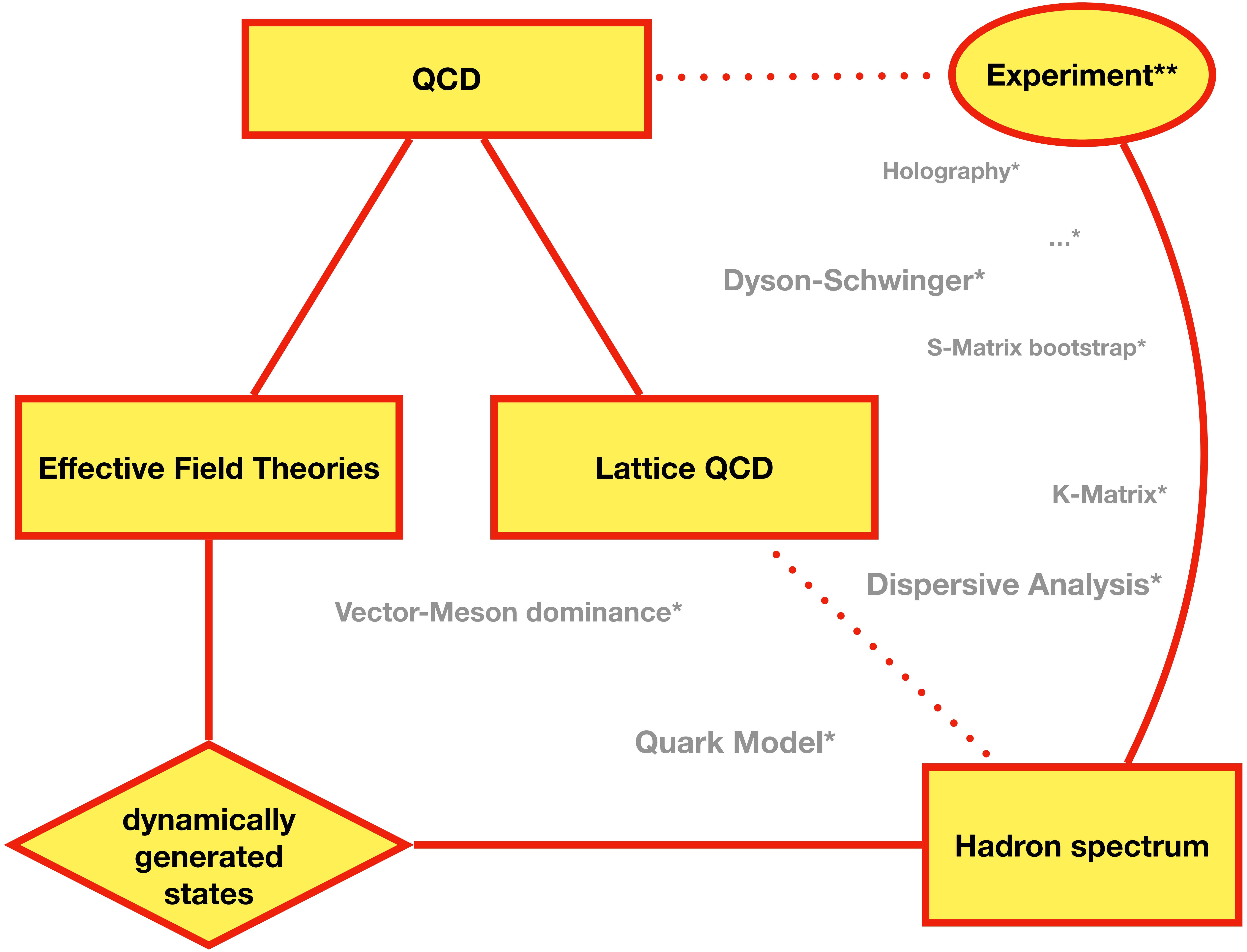
2. **Dynamically Generated Resonances**

   Methodology, Examples, $\Lambda(1405)$, …

3. **Applications to LQCD**

   Chiral extrapolations,
   Quantization conditions…

4. **Summary/Outlook**



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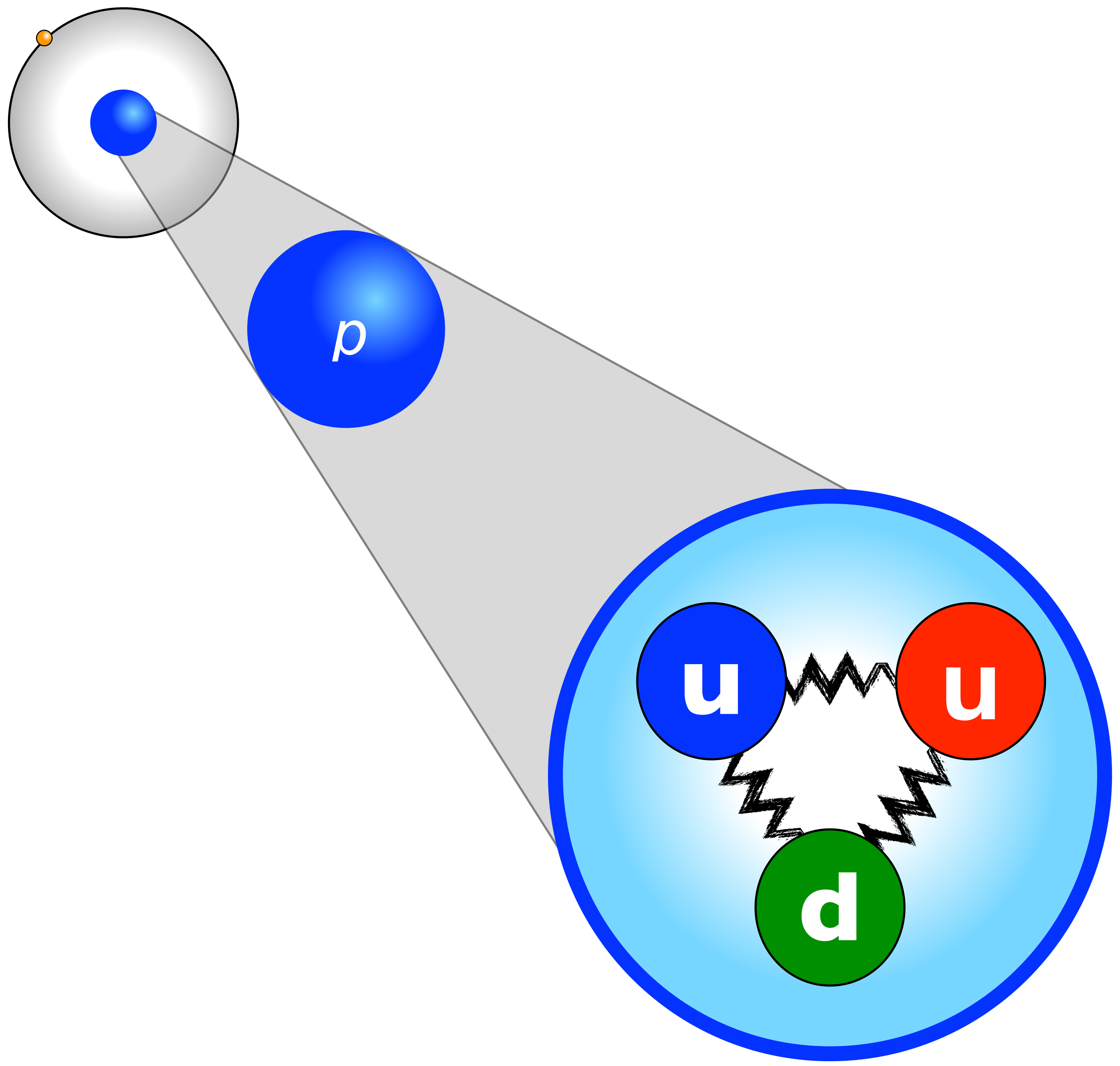
BIG PICTURE

Protons/neutrons

- 99% of the mass of visible matter in the universe
- Building blocks: **quarks & gluons (strong force)**
- Part of a large class of particles: **hadrons**



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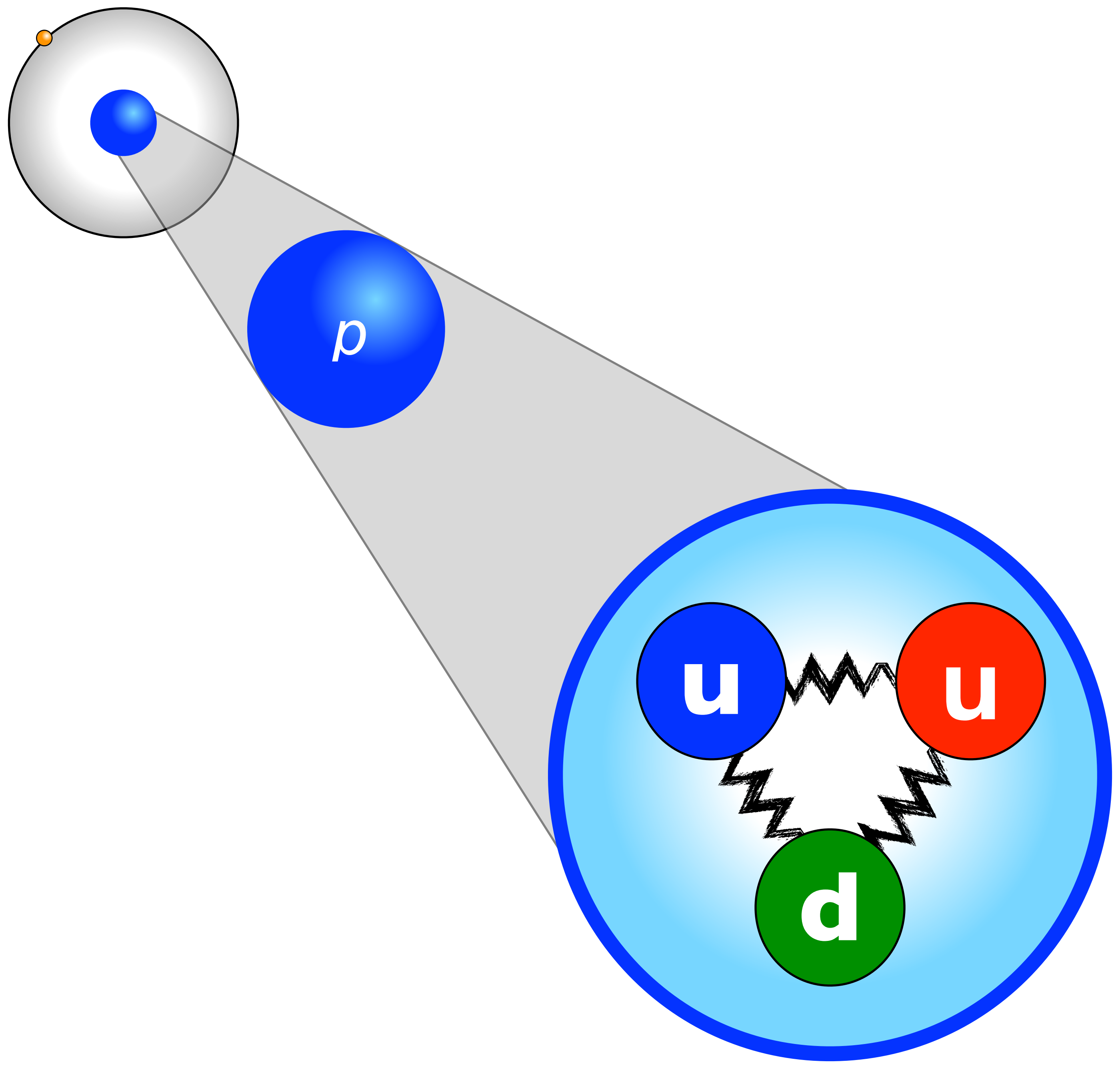

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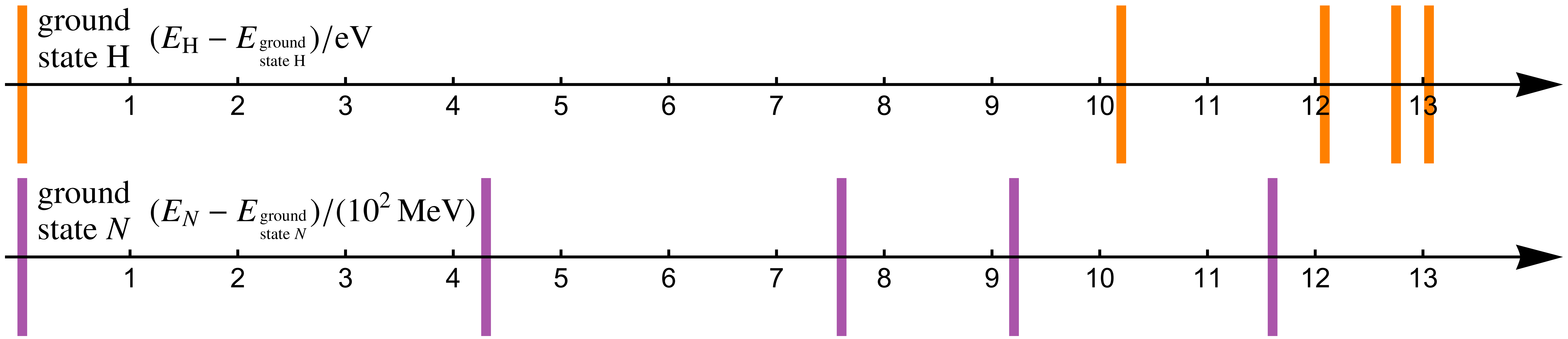


Hydrogen spectrum **(✓)**

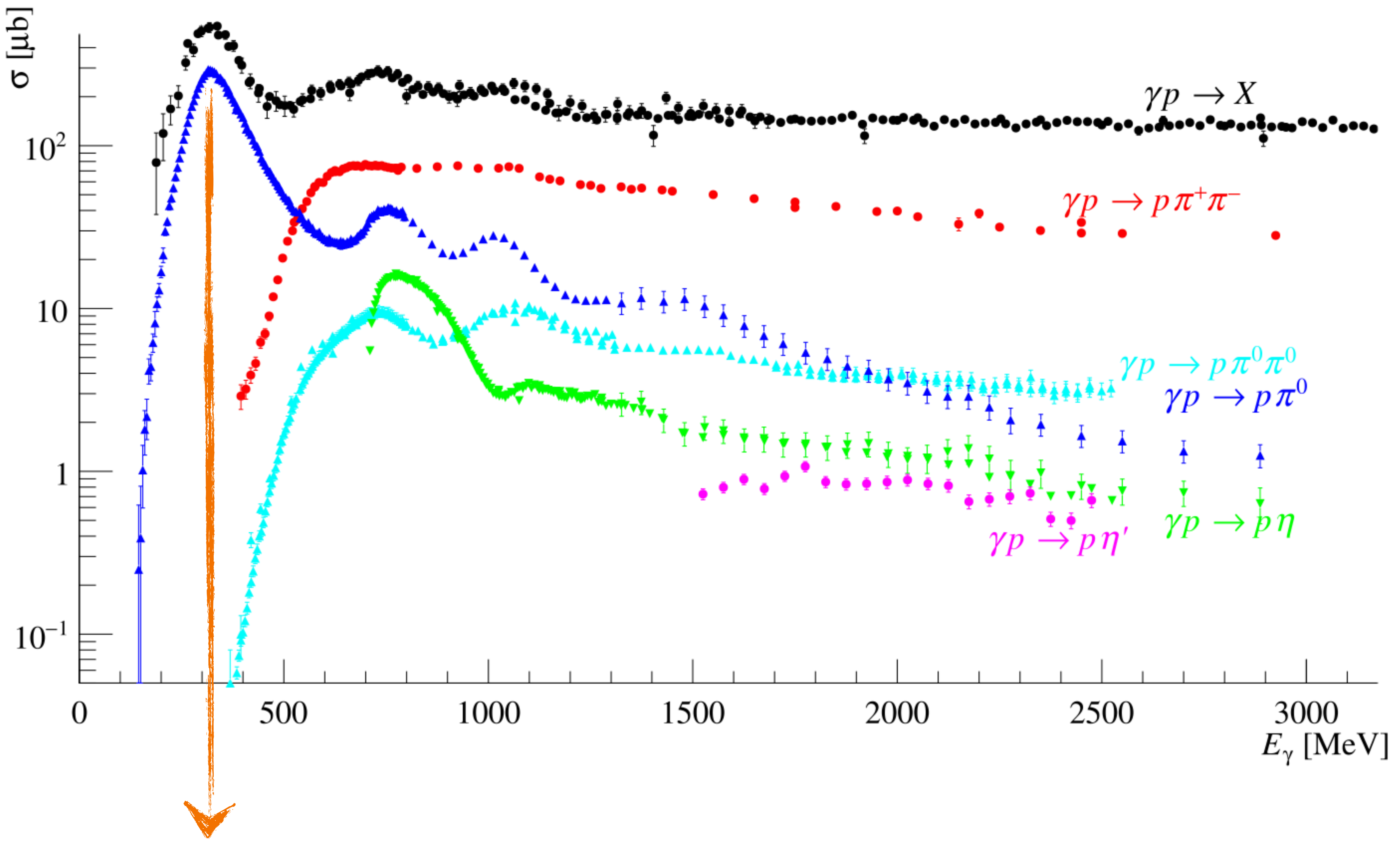
Proton spectrum **(?)**

EXPERIMENTS / RESONANCES

Data: JLAB, ELSA, MAMI CLAS12, GlueX, ...
Plot: Thiel, Afzal, Wunderlich, Light Baryon Spectroscopy, Prog. Part. Nucl. Phys. 125 (2022) 103949

Observations

- many available data and ongoing experiments
- resonances:
  - increased interaction rates (bumps)



$\Delta(1232)$

Anderson/Fermi/...PhysRev.85.934

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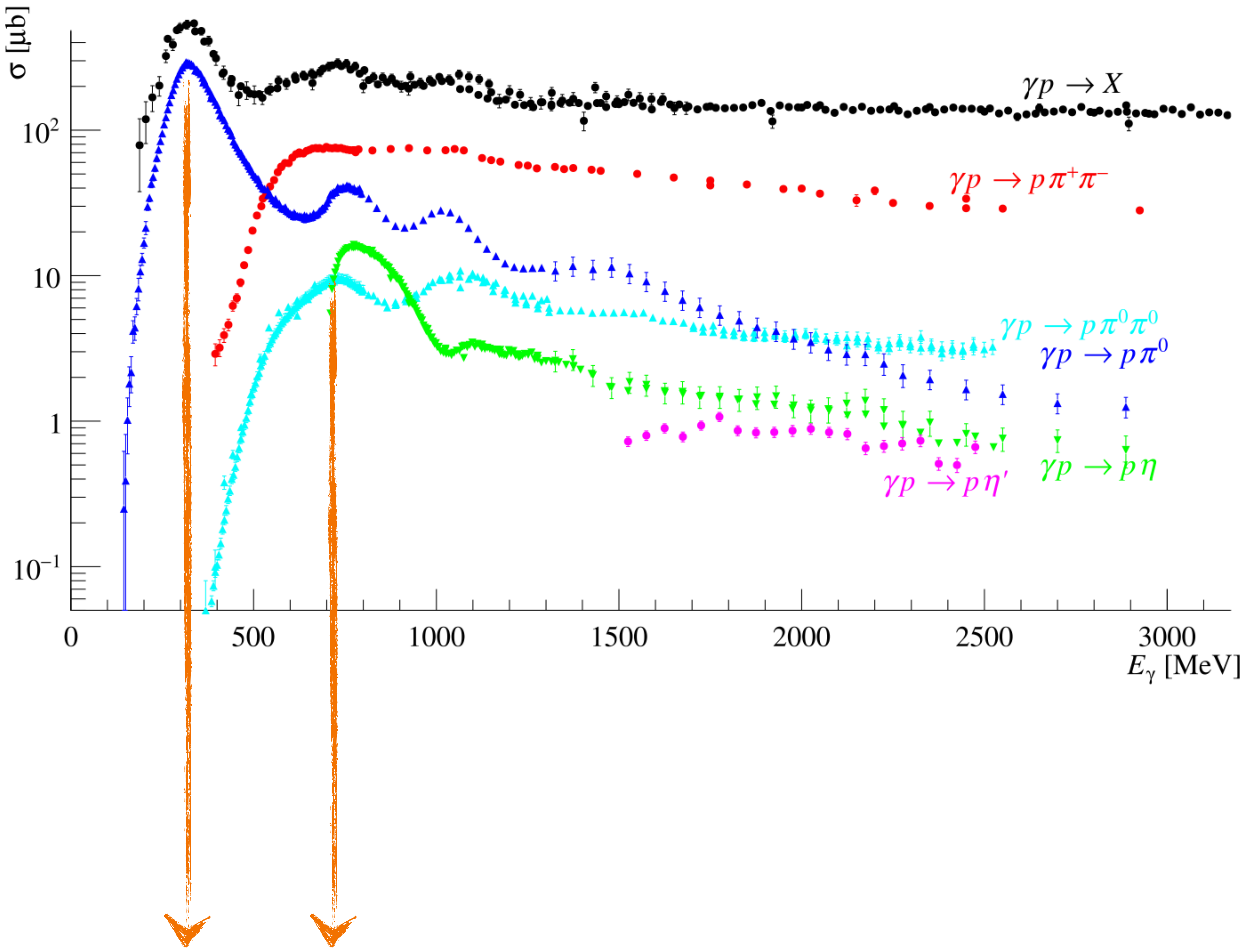

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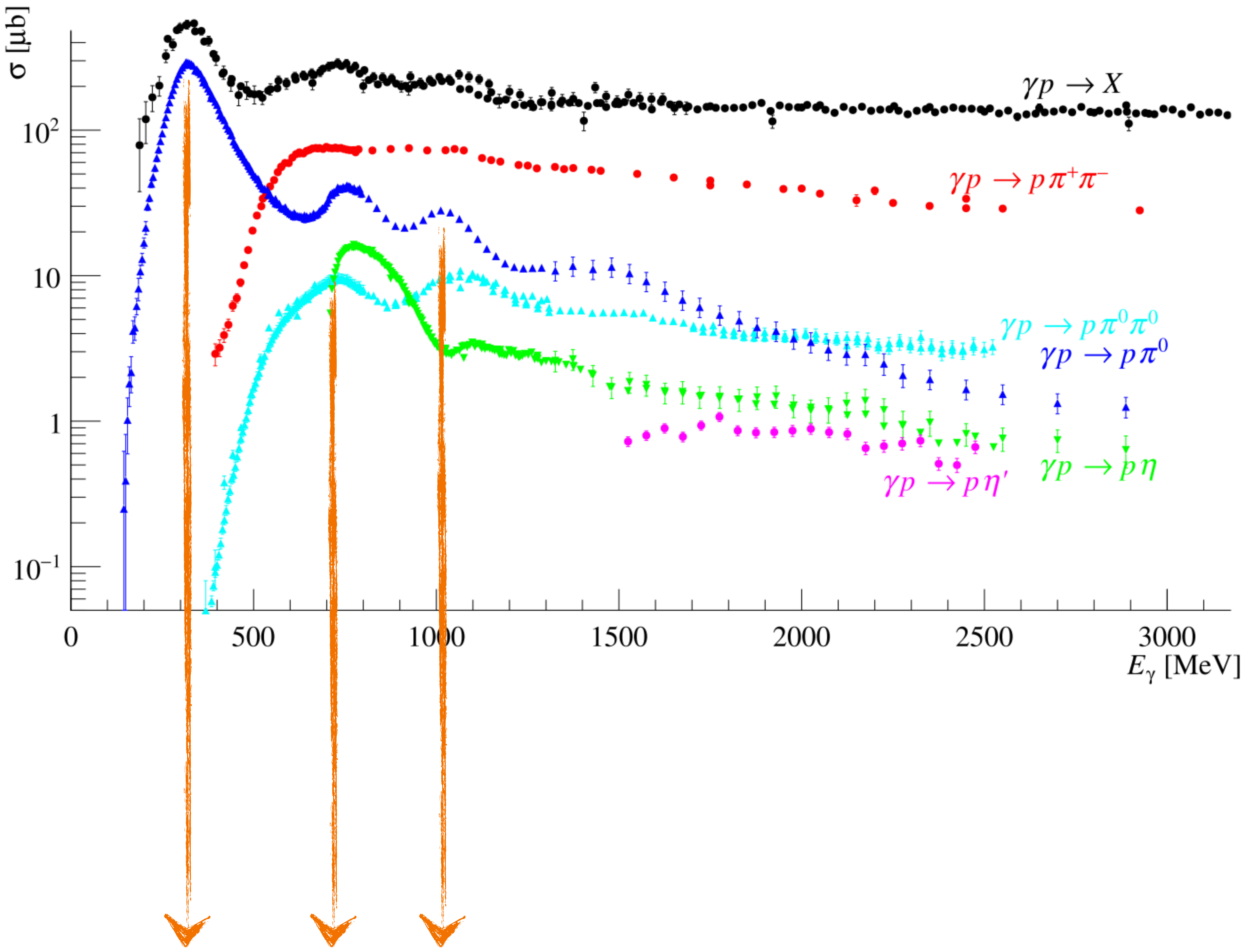

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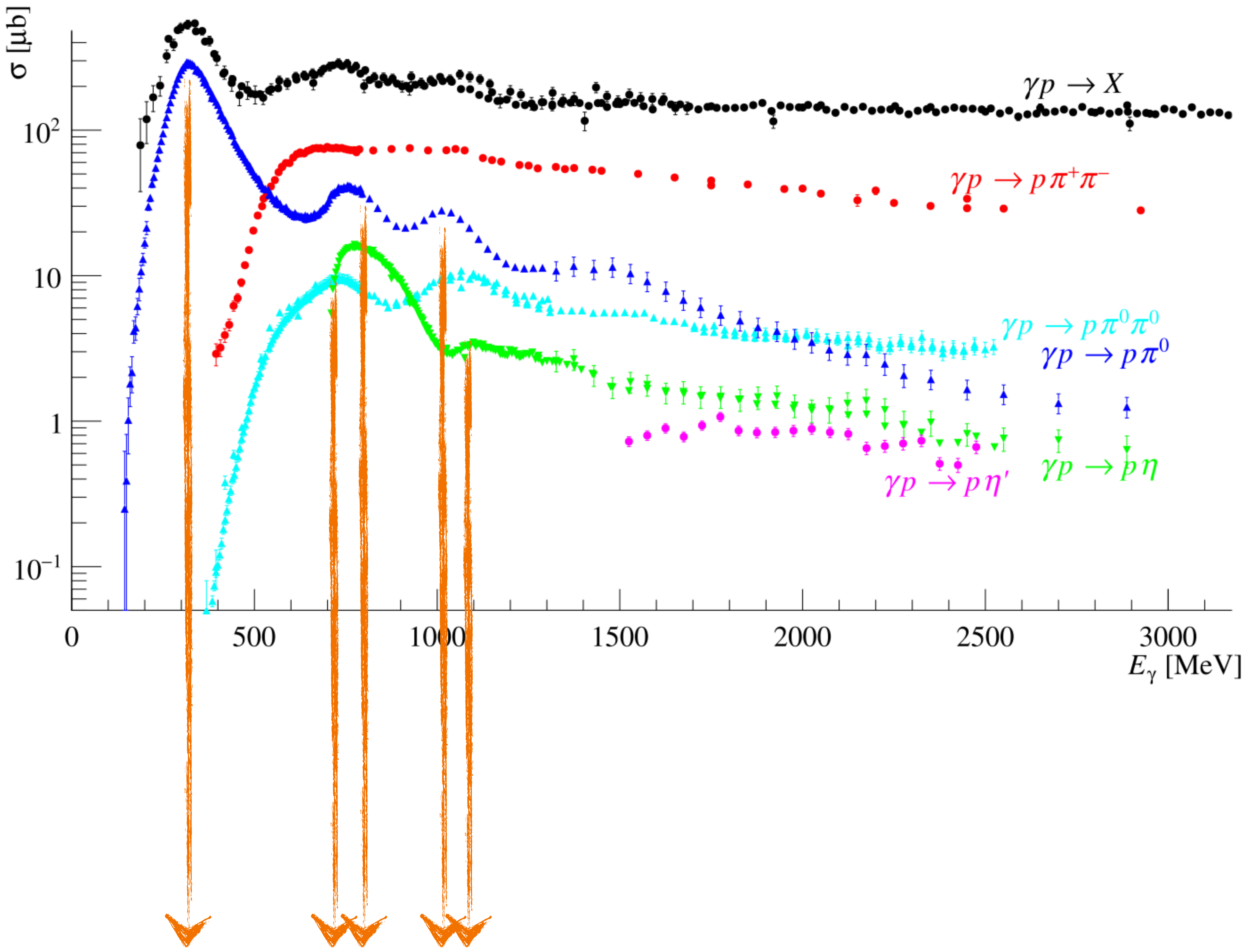

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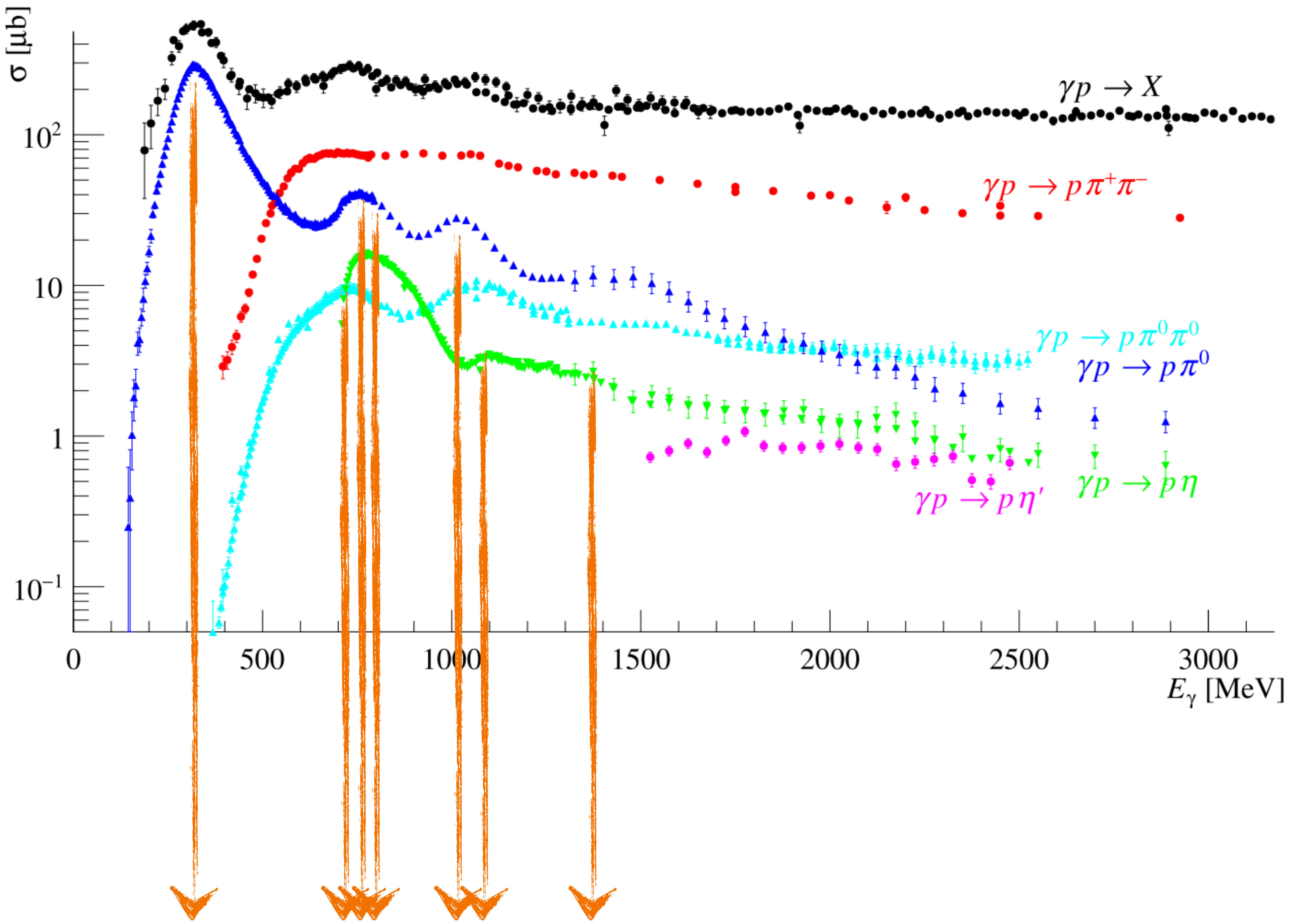

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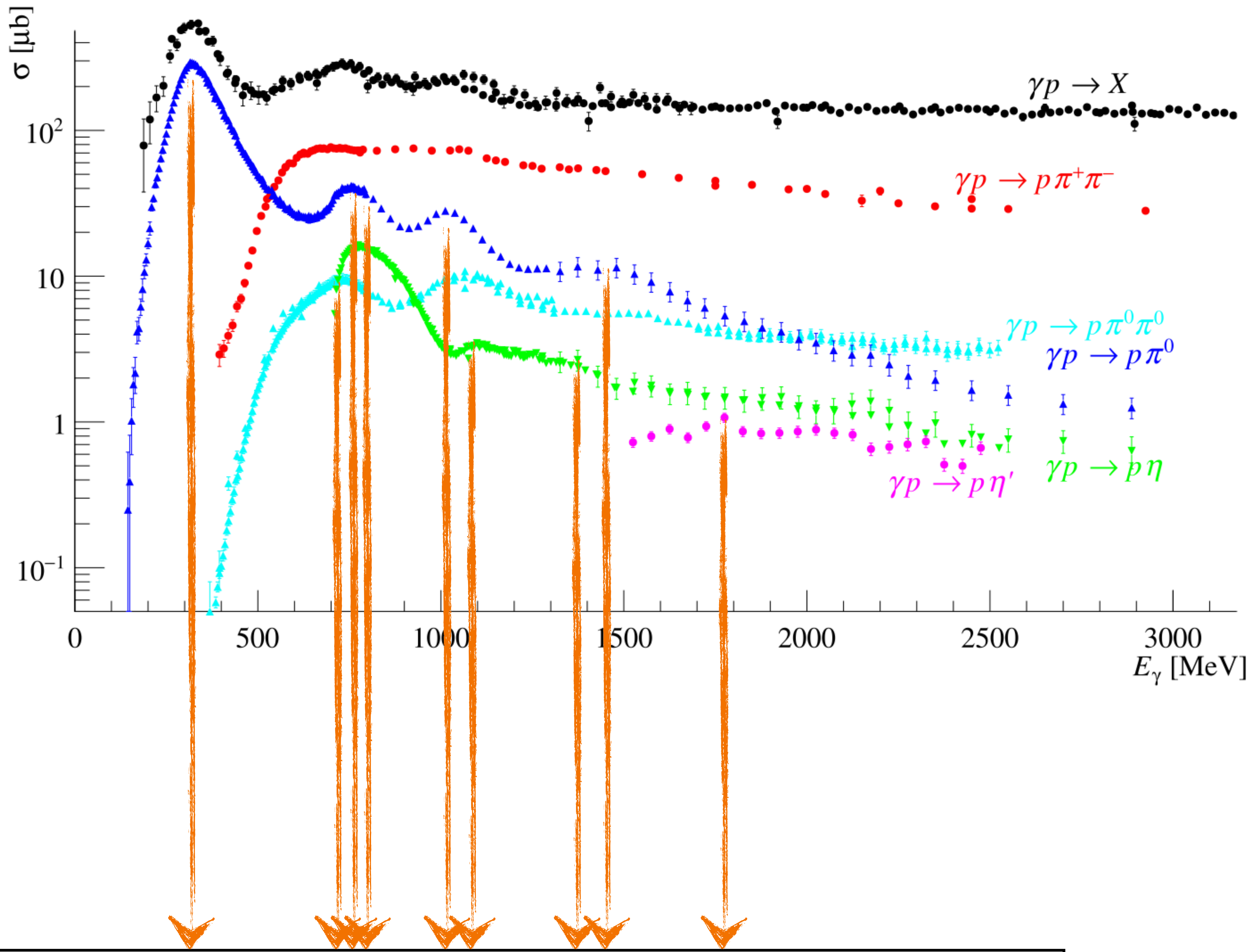

Observations

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- resonances:
  - increased interaction rates (bumps)

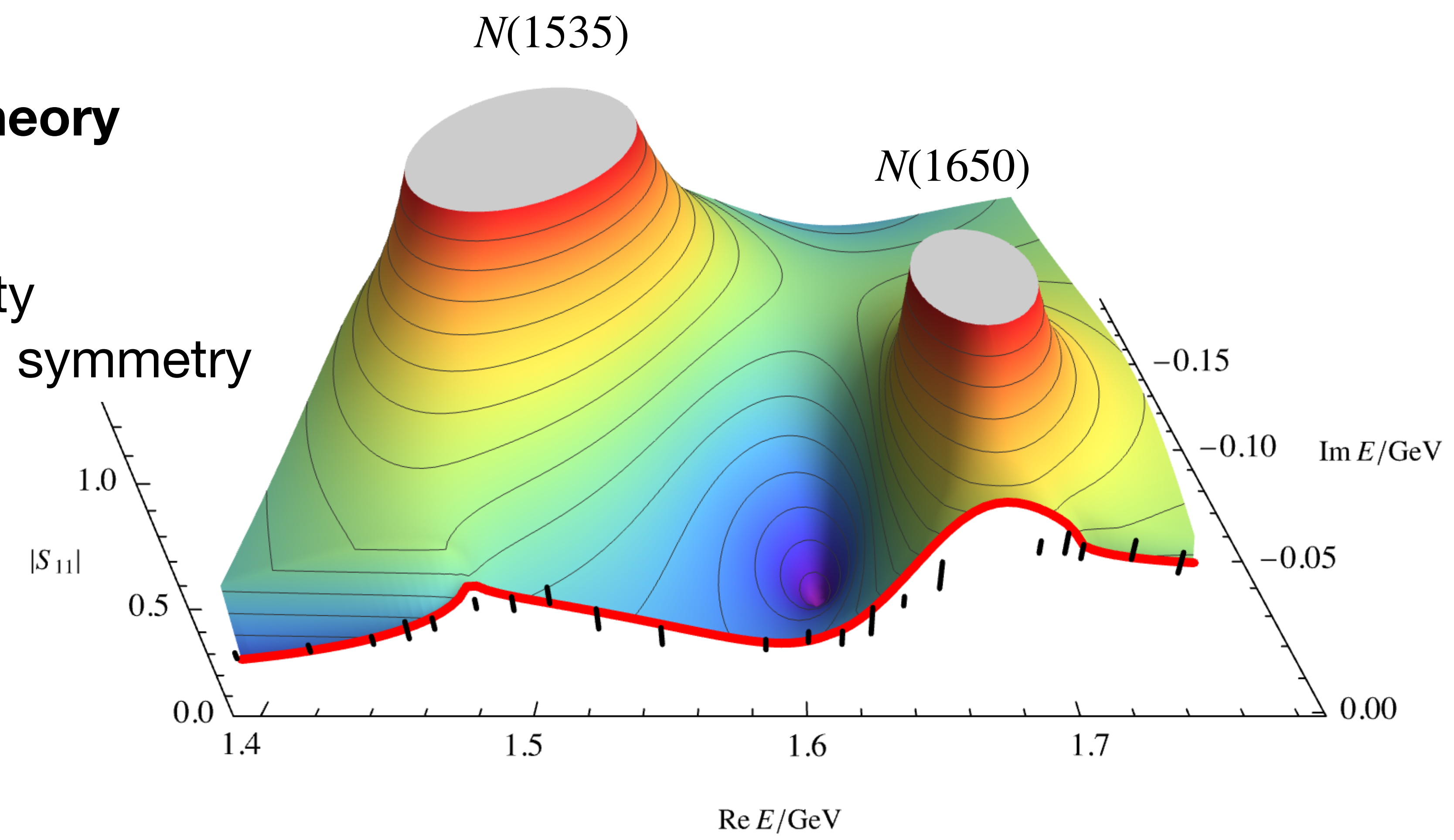
- reaction-type dependence
- overlapping resonances
- kinematical effects (cusps/triangle singularities/…)

B
P ~~D~~ G=“*particle bump group*”

TRANSITION AMPLITUDES

S-matrix theory

- Unitarity
- Analyticity
- Crossing symmetry



Data: SAID: Phys. Rev. C 74 (2006) 045205
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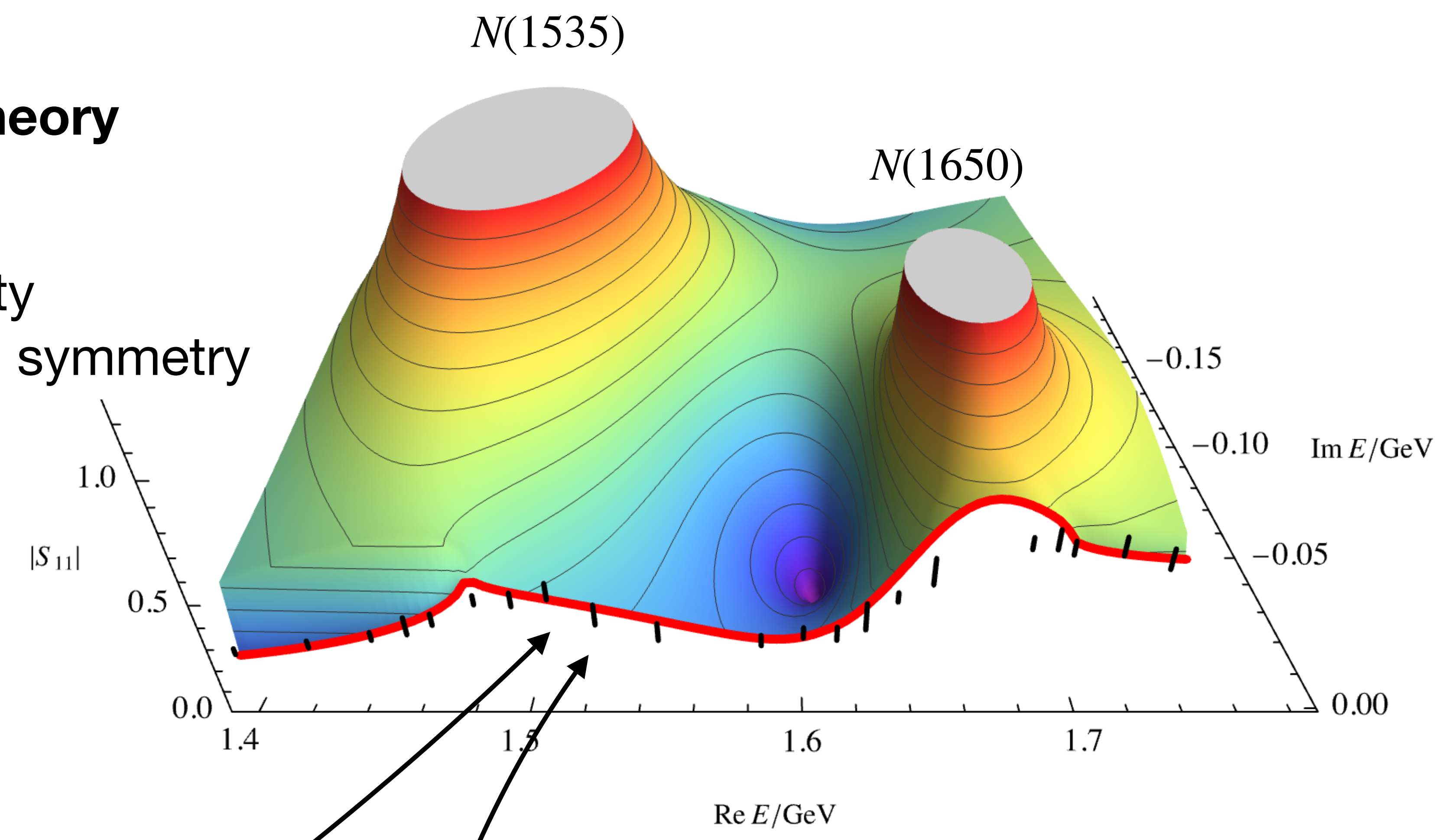
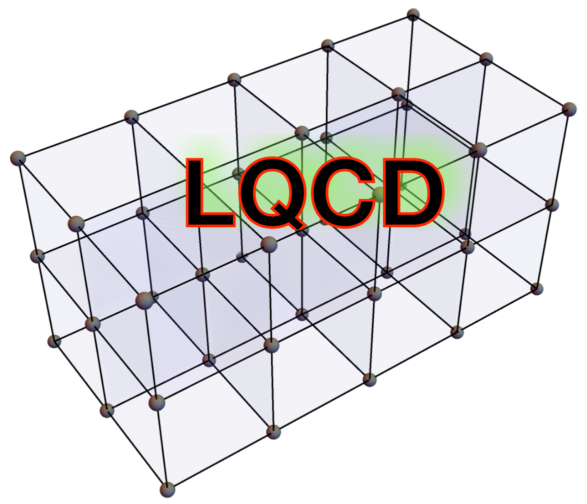
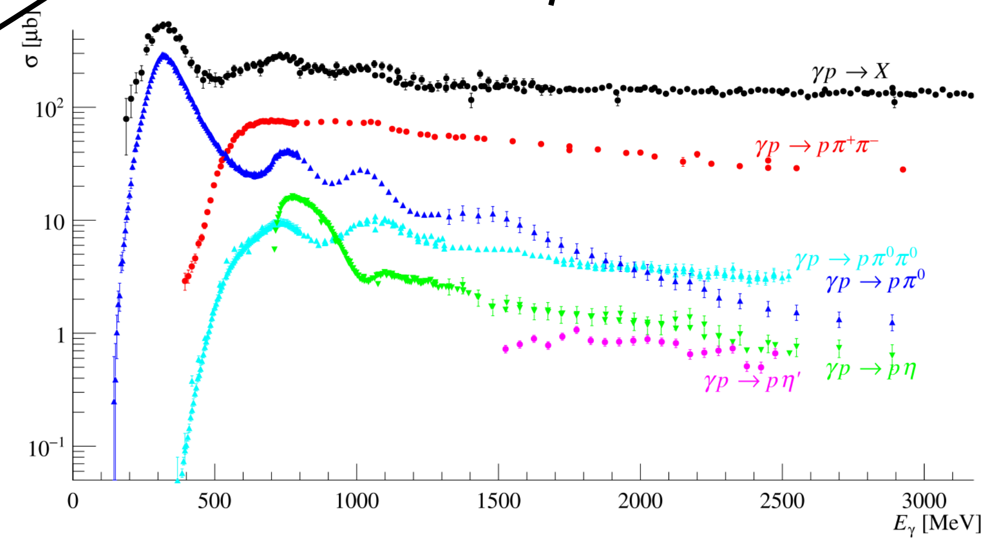


Boundary ($E \in \mathbb{R}$):

- Experiment
- Lattice QCD
- Effective Field Theories





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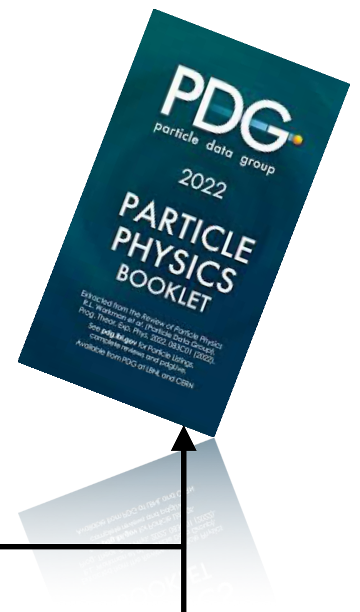
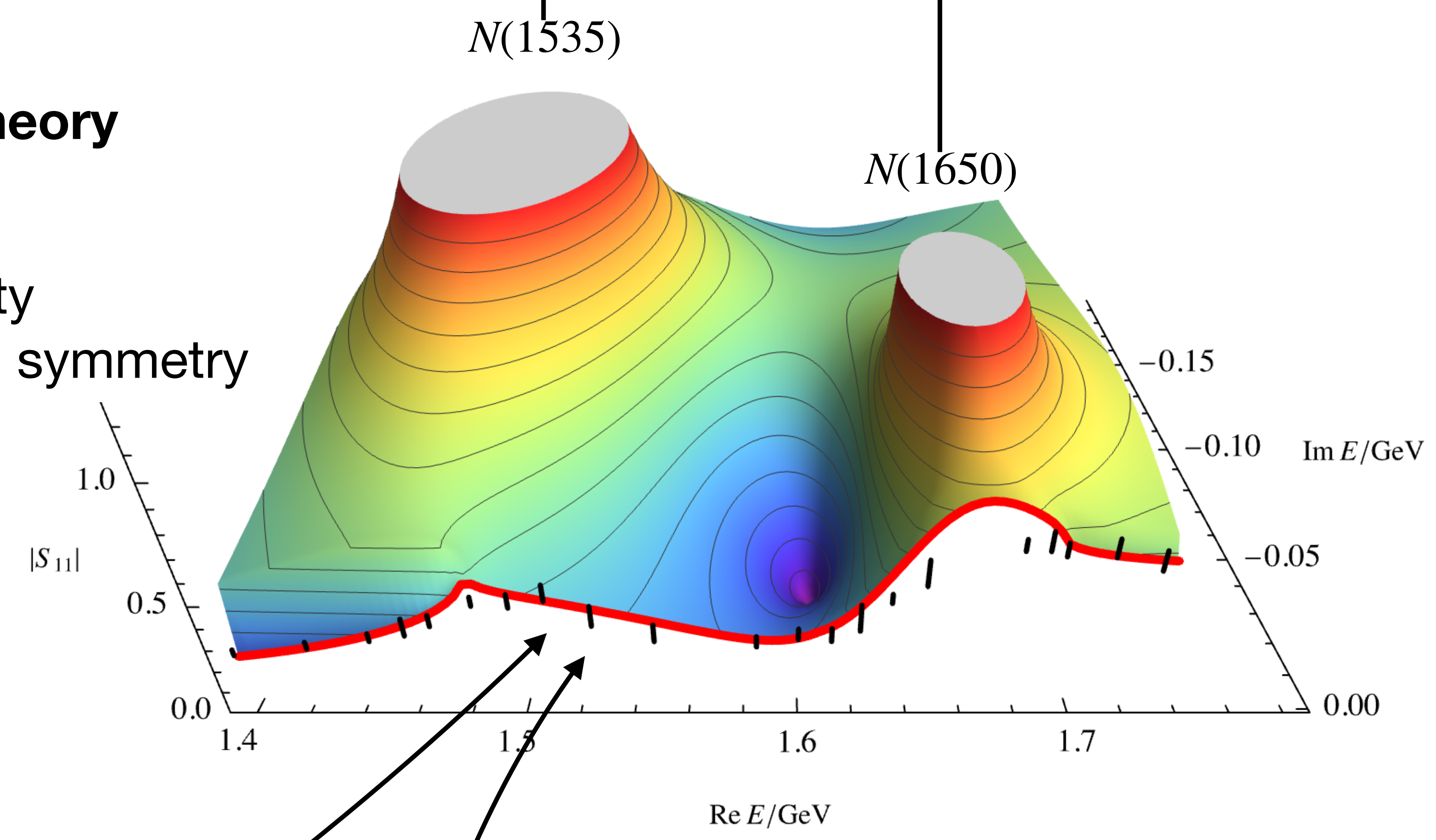


Poles on unphysical Riemann Sheets

- Universal resonance parameter

S-matrix theory

- Unitarity
- Analyticity
- Crossing symmetry



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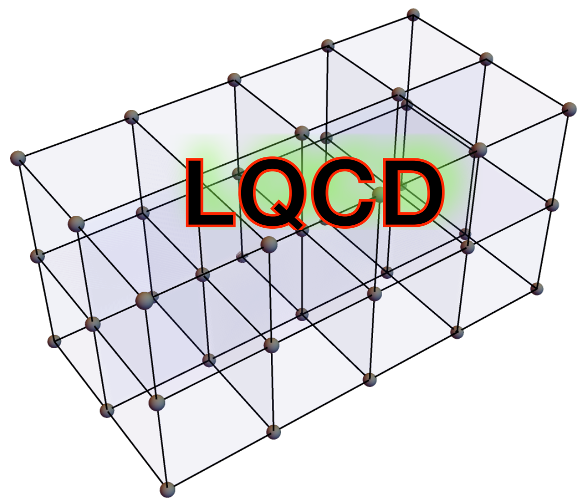

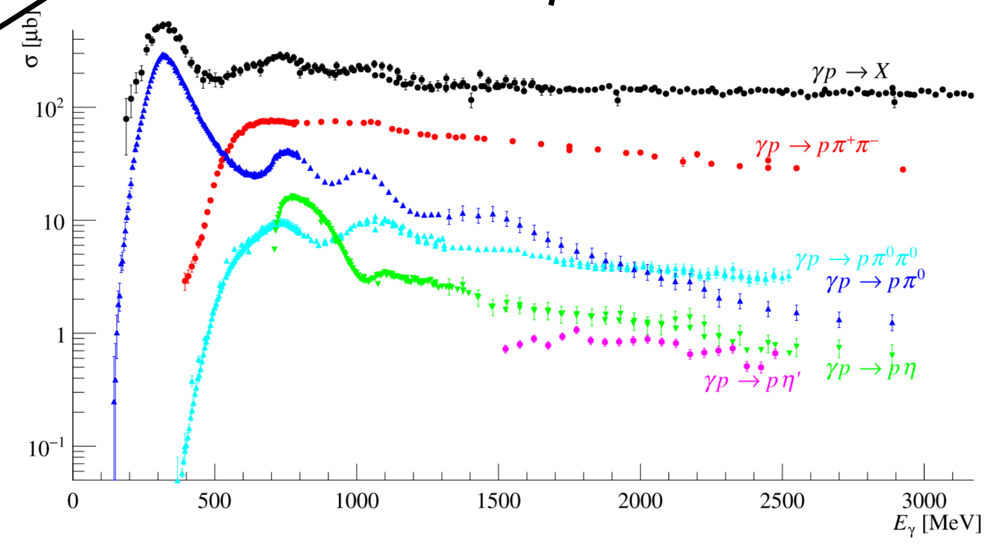

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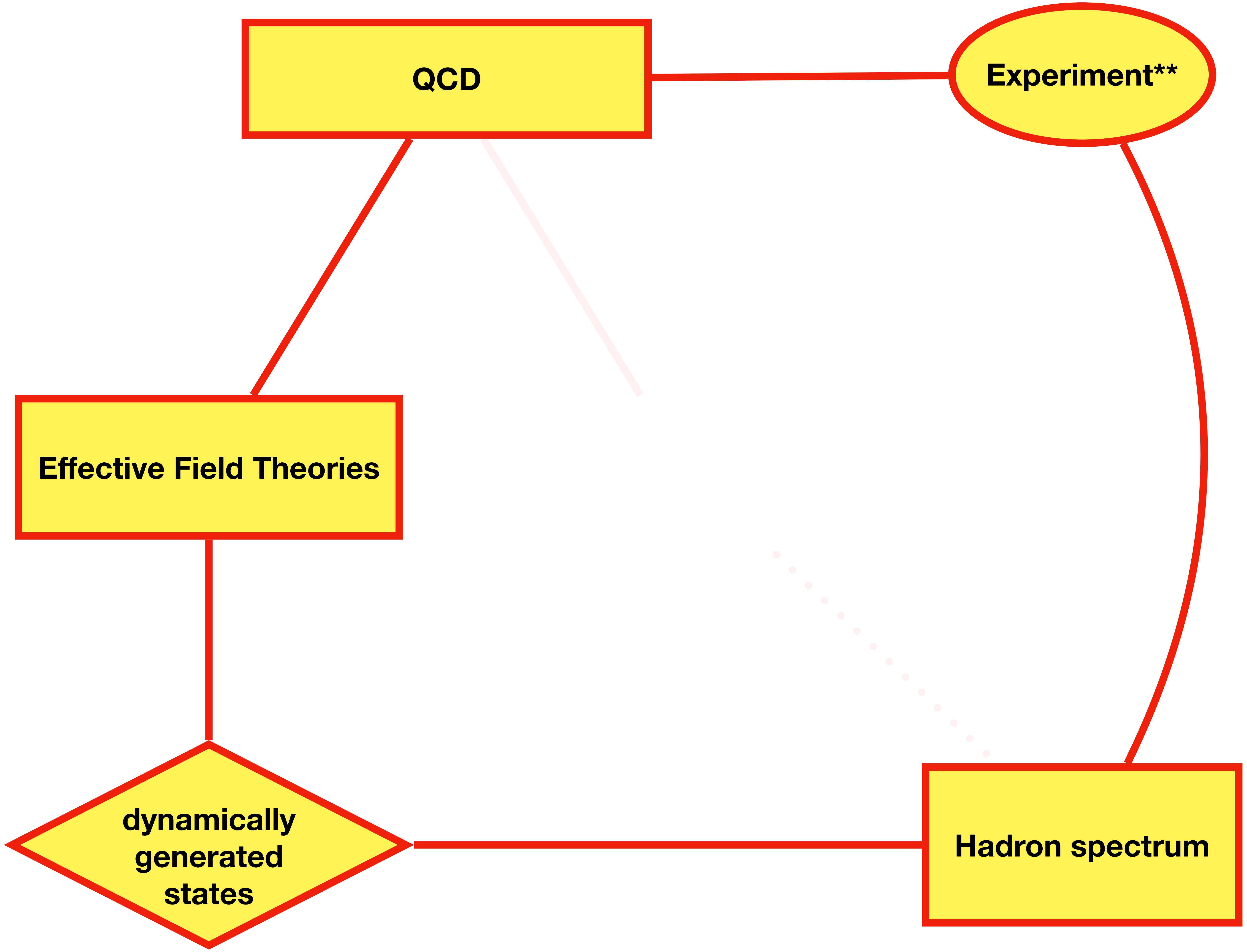

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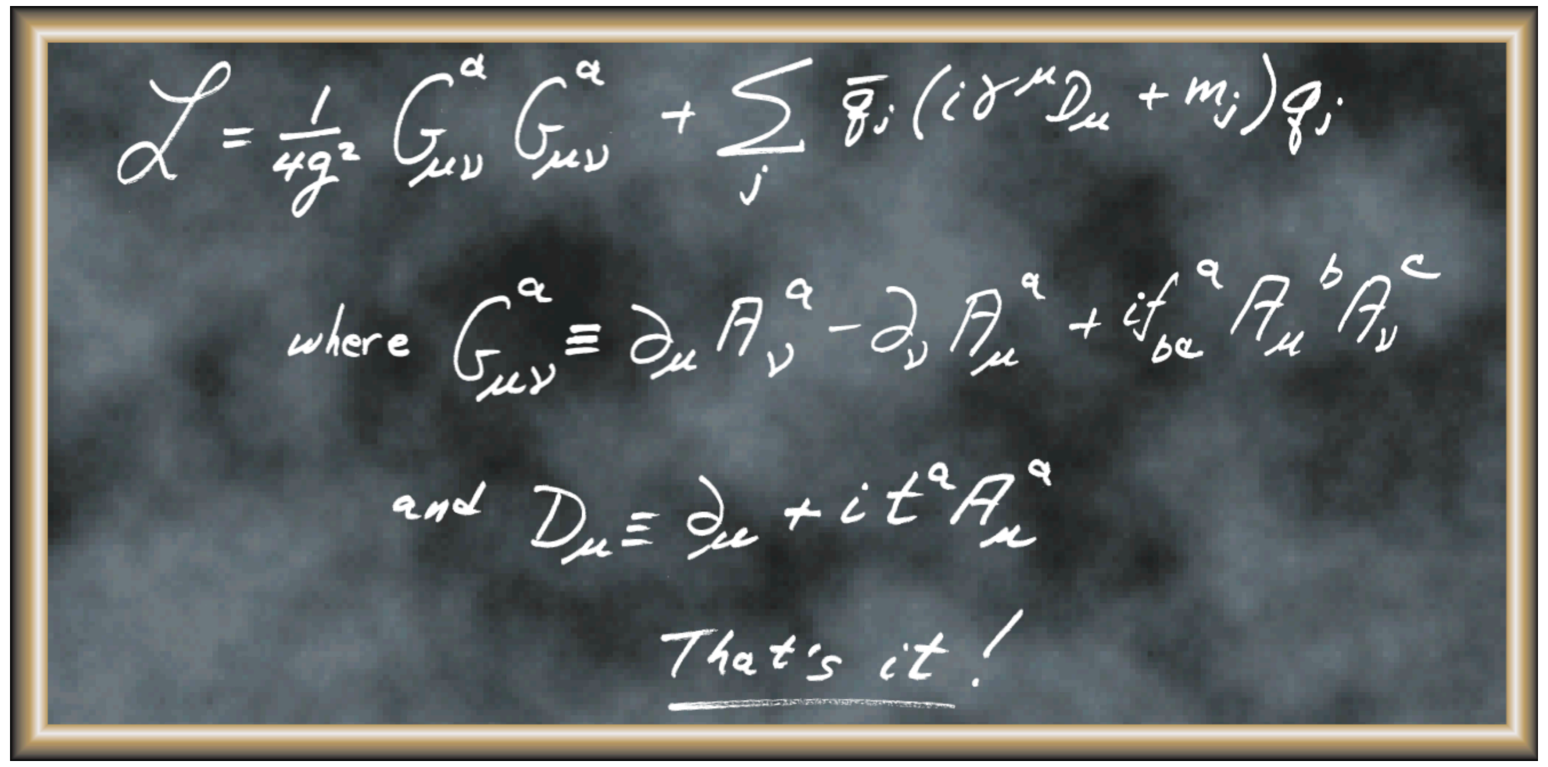
EXCITED HADRONS AND QCD

Low-energy regime of QCD = double trouble

- small relative momenta
- non-perturbative energy regime
- need to evaluate <u>infinitely</u> many diagrams



http://frankwilczek.com/Wilczek_Easy_Pieces/298_QCD_Made_Simple.pdf

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- need to evaluate infinitely many diagrams

$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

where $G^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i f^a_{bc} A^b_\mu A^c_\nu$

and $D_\mu \equiv \partial_\mu + i t^a A^a_\mu$

That's it!

http://frankwilczek.com/Wilczek_Easy_Pieces/298_QCD_Made_Simple.pdf

Effective Field Theory (CHPT)

- Effective/Hadronic degrees of freedom
- Infinitely many low-energy constants
- Well-defined power counting
- Benchmark for many low-energy hadronic interactions

$$Z[J] = \int [DU] e^{\int i d^4x\, \mathscr{L}_{\text{eff}}(U,v,a,s,p)}$$

$$\mathscr{L}_{\phi} = \mathscr{L}^{(2)}_{\phi} + \mathscr{L}^{(4)}_{\phi} + \dots$$

$$\mathscr{L}_{\phi B} = \mathscr{L}^{(1)}_{\phi B} + \mathscr{L}^{(2)}_{\phi B} + \mathscr{L}^{(3)}_{\phi B} + \dots$$

Weinberg (1979) Gasser, Leutwyler (1981)

Reviews:
V. Bernard and U.-G. Meißner, Ann. Rev. Nucl. Part. Sci. 57, 33 (2007)
V. Bernard, Prog. Part. Nucl. Phys. 60, 82 (2008)
S. Scherer, Adv. Nucl. Phys. 27, 277 (2003)

EXAMPLE: BARYON CHPT

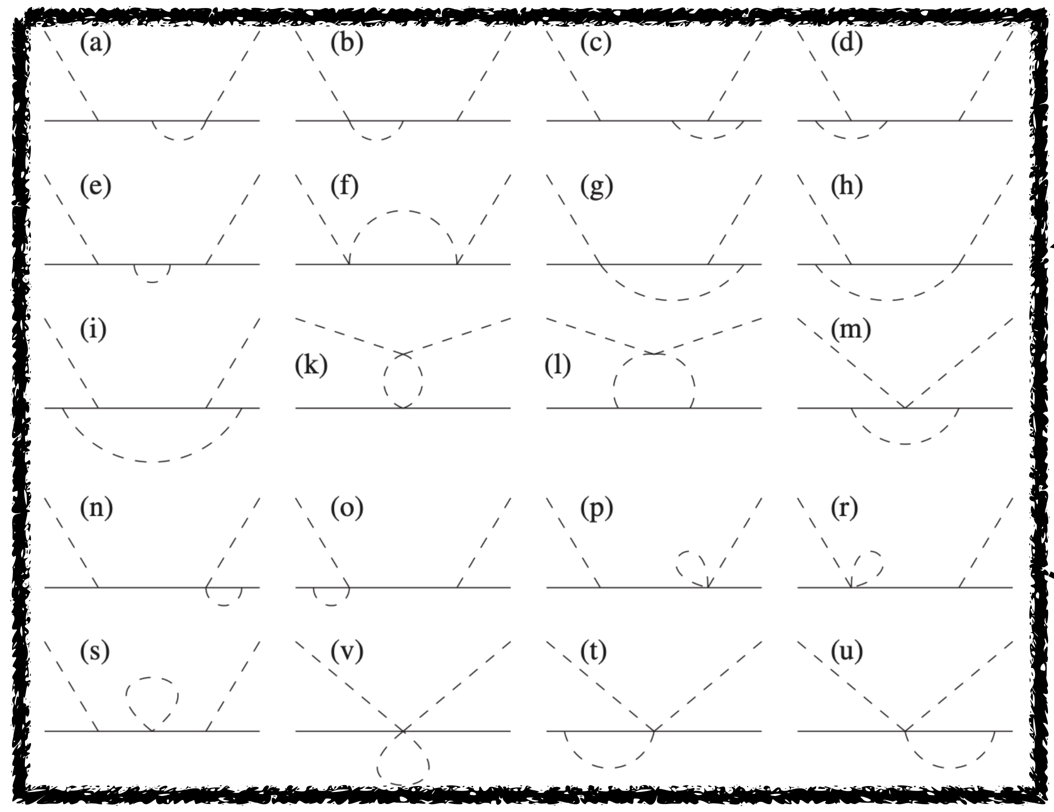
$$
\begin{aligned}
\mathcal{L}^{(2)}_{\phi B} = {} & b_{D/F}\langle \bar{B}[\chi_+, B]_\pm\rangle + b_0\langle \bar{B}B\rangle\langle\chi_+\rangle + b_{1/2}\langle \bar{B}[u_\mu,[u^\mu,B]_\mp]\rangle + b_3\langle \bar{B}\{u_\mu,\{u^\mu,B\}\}\rangle + b_4\langle \bar{B}B\rangle\langle u_\mu u^\mu\rangle \\
& + i\sigma^{\mu\nu}(b_{5/6}\langle \bar{B}[[u_\mu,u_\nu],B]_\mp\rangle + b_7\langle \bar{B}u_\mu\rangle\langle u_\nu B\rangle) + \frac{ib_{8/9}}{2m_0}(\langle \bar{B}\gamma^\mu[u_\mu,[u_\nu,[D^\nu,B]]_\mp]\rangle + \langle \bar{B}\gamma^\mu[D_\nu,[u^\nu,[u_\mu,B]]_\mp]\rangle) \\
& + \frac{ib_{10}}{2m_0}(\langle \bar{B}\gamma^\mu\{u_\mu,\{u_\nu,[D^\nu,B]\}\}\rangle + \langle \bar{B}\gamma^\mu[D_\nu,\{u^\nu,\{u_\mu,B\}\}]\rangle) + \frac{ib_{11}}{2m_0}(2\langle \bar{B}\gamma^\mu[D_\nu,B]\rangle\langle u_\mu u^\nu\rangle \\
& + \langle \bar{B}\gamma^\mu B\rangle\langle[D_\nu,u_\mu]u^\nu + u_\mu[D_\nu,u^\nu]\rangle), \qquad (8)
\end{aligned}
$$

Meson-baryon scattering from CHPT

MM/P.C.Bruns/Ulf-G. Meißner/B.Kubis Phys.Rev.D 80 (2009) 094006

- full SU(3) dynamics near threshold
- agrees with experiment in many cases
- provides predictions for not measured channels

$$u_\mu = -i\frac{\partial_\mu\phi}{F} + \mathcal{O}(\phi^3)$$



Channel =	$\mathcal{O}(q^1)$	$+\mathcal{O}(q^2)$	$+\mathcal{O}(q^3)_{\text{HB}}$	Σ_{HB}
$a^{(3/2)}_{\pi N}$ =	-0.12	$+0.05^{+0.02}_{-0.03}$	$-0.06^{+0.00}_{+0.00}$	$-0.13^{+0.03}_{-0.03}$
$a^{(1/2)}_{\pi N}$ =	$+0.21$	$+0.05^{+0.02}_{-0.03}$	$+0.00^{+0.00}_{+0.00}$	$+0.26^{+0.03}_{-0.03}$
$a^{(3/2)}_{\pi \Xi}$ =	-0.12	$+0.04^{+0.03}_{-0.03}$	$-0.09^{+0.00}_{+0.00}$	$-0.17^{+0.03}_{-0.03}$
$a^{(1/2)}_{\pi \Xi}$ =	$+0.23$	$+0.04^{+0.03}_{-0.03}$	$-0.03^{+0.00}_{+0.00}$	$+0.23^{+0.03}_{-0.03}$
$a^{(2)}_{\pi \Sigma}$ =	-0.24	$+0.07^{+0.01}_{-0.01}$	$-0.07^{+0.00}_{+0.00}$	$-0.24^{+0.01}_{-0.01}$

EXAMPLE: BARYON CHPT

Meson-baryon scattering from CHPT

MM/P.C.Bruns/Ulf-G. Meißner/B.Kubis Phys.Rev.D 80 (2009) 094006

- Fails for resonant (strangeness) channel
  - Kaon mass is large → convergence
  - Relevant thresholds are widely separated → convergence
  - Resonance just below $\bar{K}N$ threshold → non-perturbative effect

$$\begin{aligned}
\mathcal{L}^{(2)}_{\phi B} = {} & b_{D/F}\langle \bar{B}[\chi_+, B]_\pm\rangle + b_0\langle \bar{B}B\rangle\langle\chi_+\rangle + b_{1/2}\langle \bar{B}[u_\mu,[u^\mu,B]_\mp]\rangle + b_3\langle\bar{B}\{u_\mu,\{u^\mu,B\}\}\rangle + b_4\langle\bar{B}B\rangle\langle u_\mu u^\mu\rangle \\
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\end{aligned}$$

$$\begin{aligned}
a^{I=0}_{\bar{K}N} &= \Big((+0.53)_{\rm LO} + (+0.97)_{\rm NLO} + (-0.40+0.22i)_{\rm NNLO} + \ldots\Big)\ \text{fm}\,, \\
a^{I=1}_{\bar{K}N} &= \Big((+0.20)_{\rm LO} + (+0.22)_{\rm NLO} + (-0.26+0.18i)_{\rm NNLO} + \ldots\Big)\ \text{fm}\,.
\end{aligned}$$

$\bar{K}N$ INTERACTION

Overarching impact

- Test of our understanding of QCD
  Modern/Upcoming experiments: CLAS12, Klong, SIS100

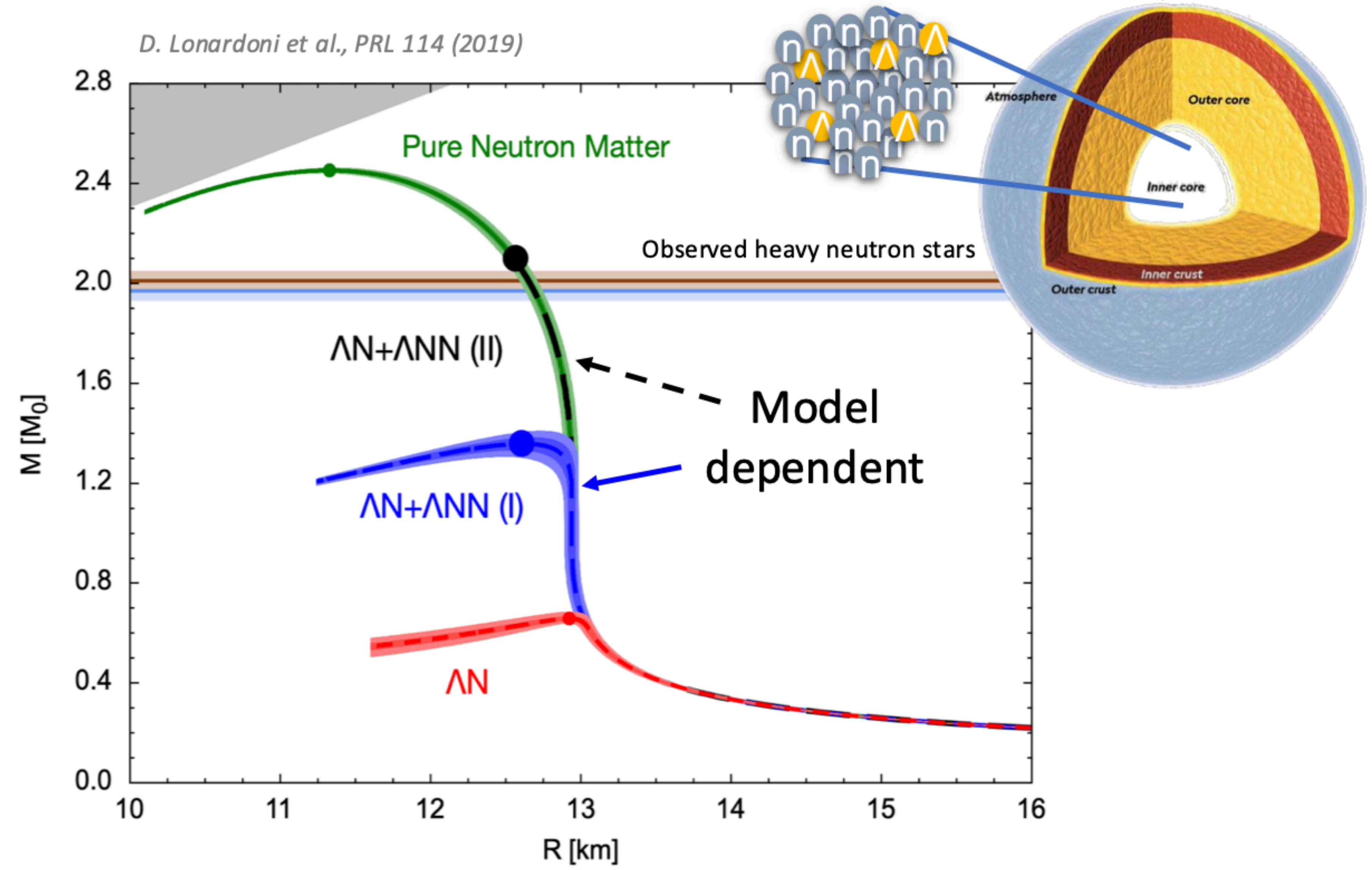
- Kaonic hydrogen/deuterium energy shift
  DAPHNE/DEAR…

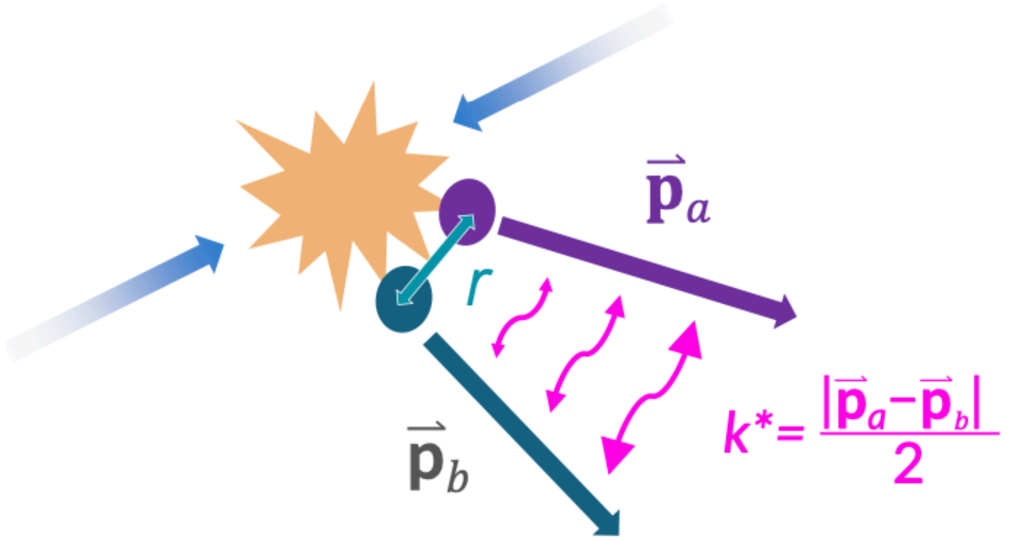
- $\bar{K}NN$ & $\bar{K}NNN$ bound states (JPARC/…)
  Review: Gal/Hungerford/Millener (2016); Iwasaki et al. Phys.Rev.C 110 (2024) 1, 014002, …

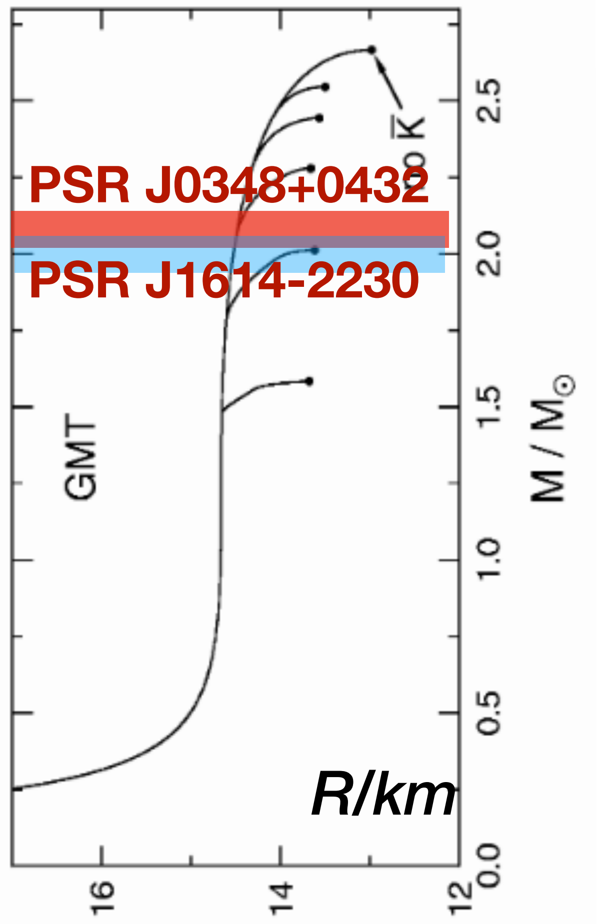
- K^- in medium
  Mareš et al. Acta Phys. Polon. B 51, 129 (2020), Hrtánkova et al. Phys.Lett. B 785, 90 (2018), …

  >> K^--condensate in NS >> Equation of State

- Femtoscopy/Correlations
  Michael Annan Lisa et al, Ann.Rev.Nucl.Part.Sci. 55 (2005) 357-402, L. Fabbietti et al., ARNPS 71 (2021), 377-402







Pal/Greiner…,
Nucl. Phys. A 674, 553 (2000)

UNITARISATION

Extension to resonant channels/higher energies — Chiral Unitary Approach

- **Good**
  - Non-perturbative scheme
  - Record complex pole-positions (*II Riemann Sheet*)
  - Often works: $N(1535), N(1650), \Lambda(1405), \Lambda(1380), \dots$

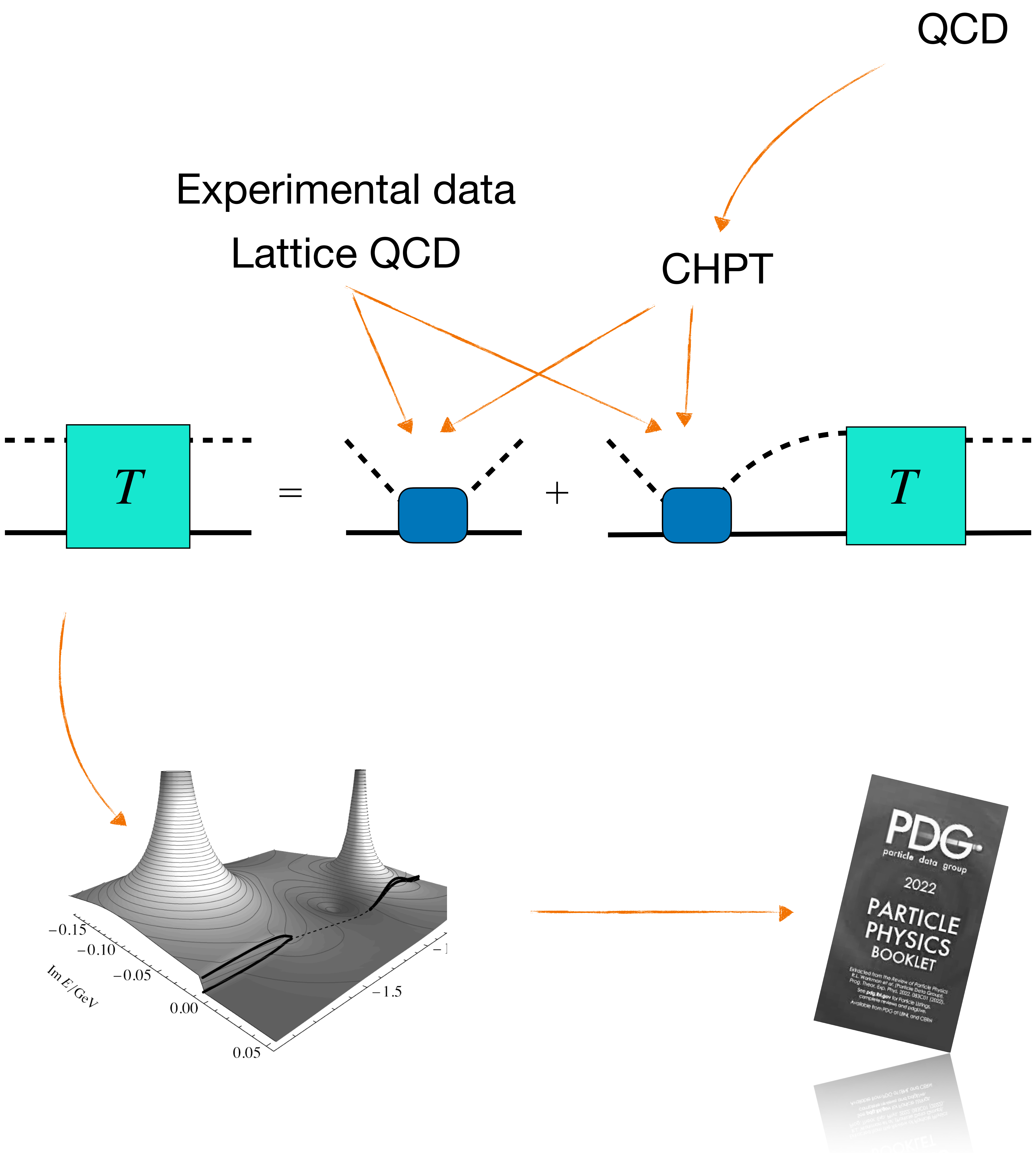
Kaiser/Siegel/Weise Phys.Lett.B 362 (1995)
Lutz/Soyeur Nucl.Phys.A 773 (2006);
MM et al. Phys.Lett.B 697 (2011); …

Attention (model dependence)
Review: MM, *Eur.Phys.J.ST* 230 (2021) 6, 1593-1607

- Renormalisation
- Crossing symmetry
- Power counting

only perturbatively

- Choice of the interaction kernel

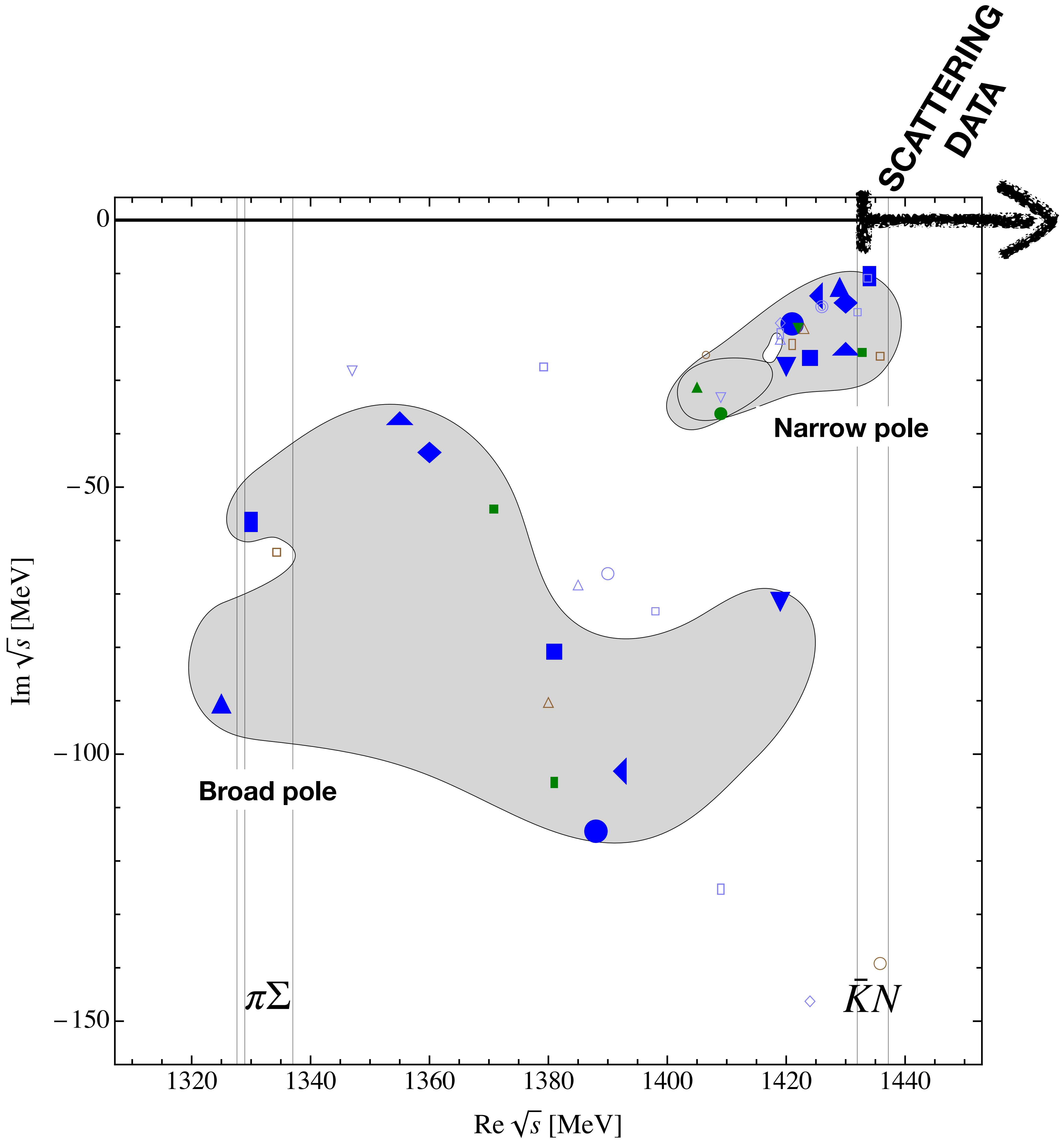


STATUS: $\Lambda(1405)\ldots\Lambda(1380)$

"A curious case of a strangeness resonance" *

MM, Eur.Phys.J.ST 230 (2021) 6, 1593-1607

- Sub-($\bar{K}N$)-threshold $\Lambda(1405)$ resonance
- second state $\Lambda(1380)$ predicted from UCHPT
  - no direct experimental verification
    - indirectly through photoproduction experiments
      [CLAS] Moriya et al. Phys.Rev.Lett. 112 (2014) 8
      MM/Meißner Eur.Phys.J.A 51 (2015) 3, 30
    - confirmed by many critical tests & LQCD
      Bulava et al. [BaSc] Phys.Rev.Lett. 132 (2024) 5, 051901



Models:
Ikeda/Weise/Feijoo/MM/Meißner/Ramos/Hyodo/...

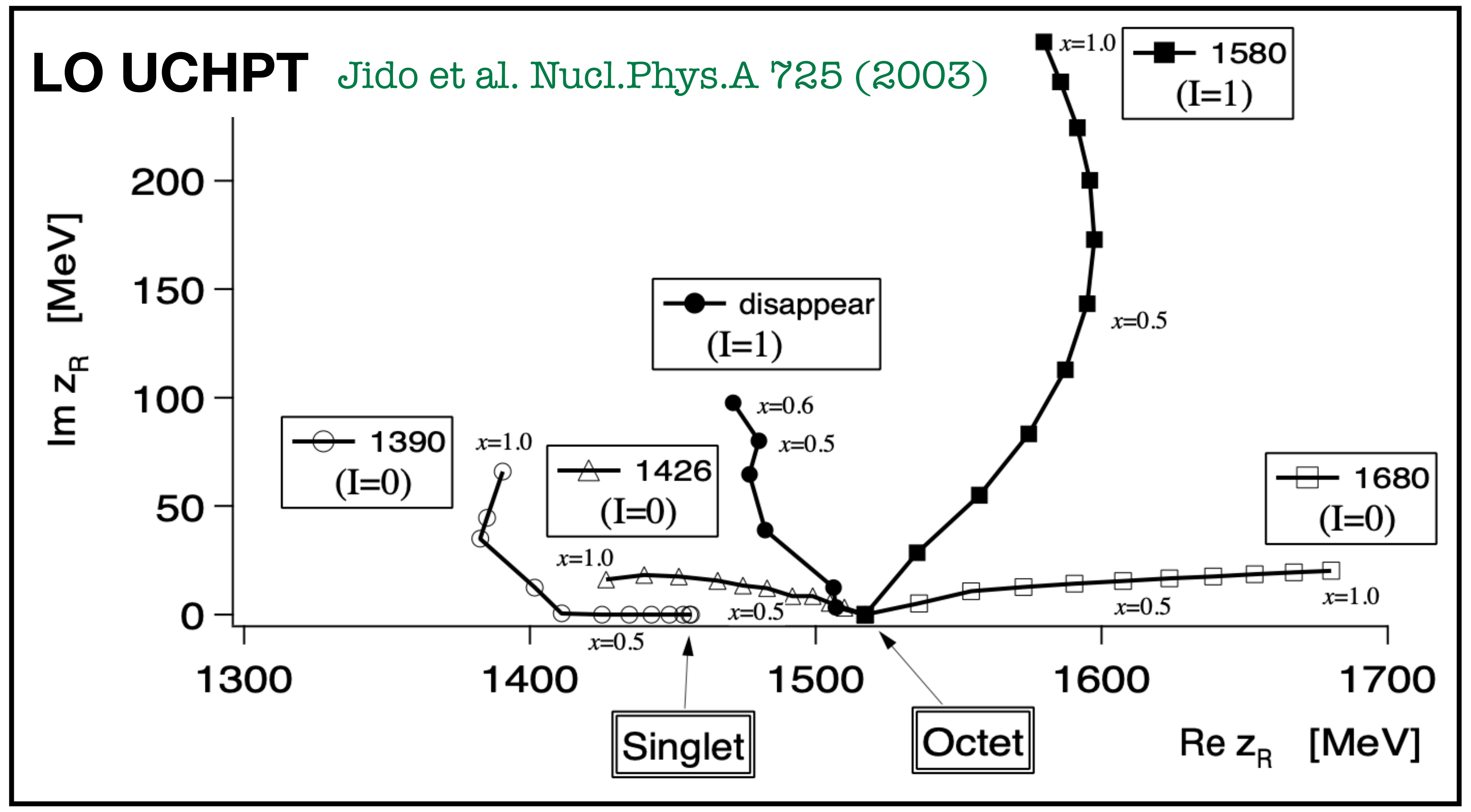
*) After "The Curious Incident of the Dog in the Night-Time" — Mark Haddon

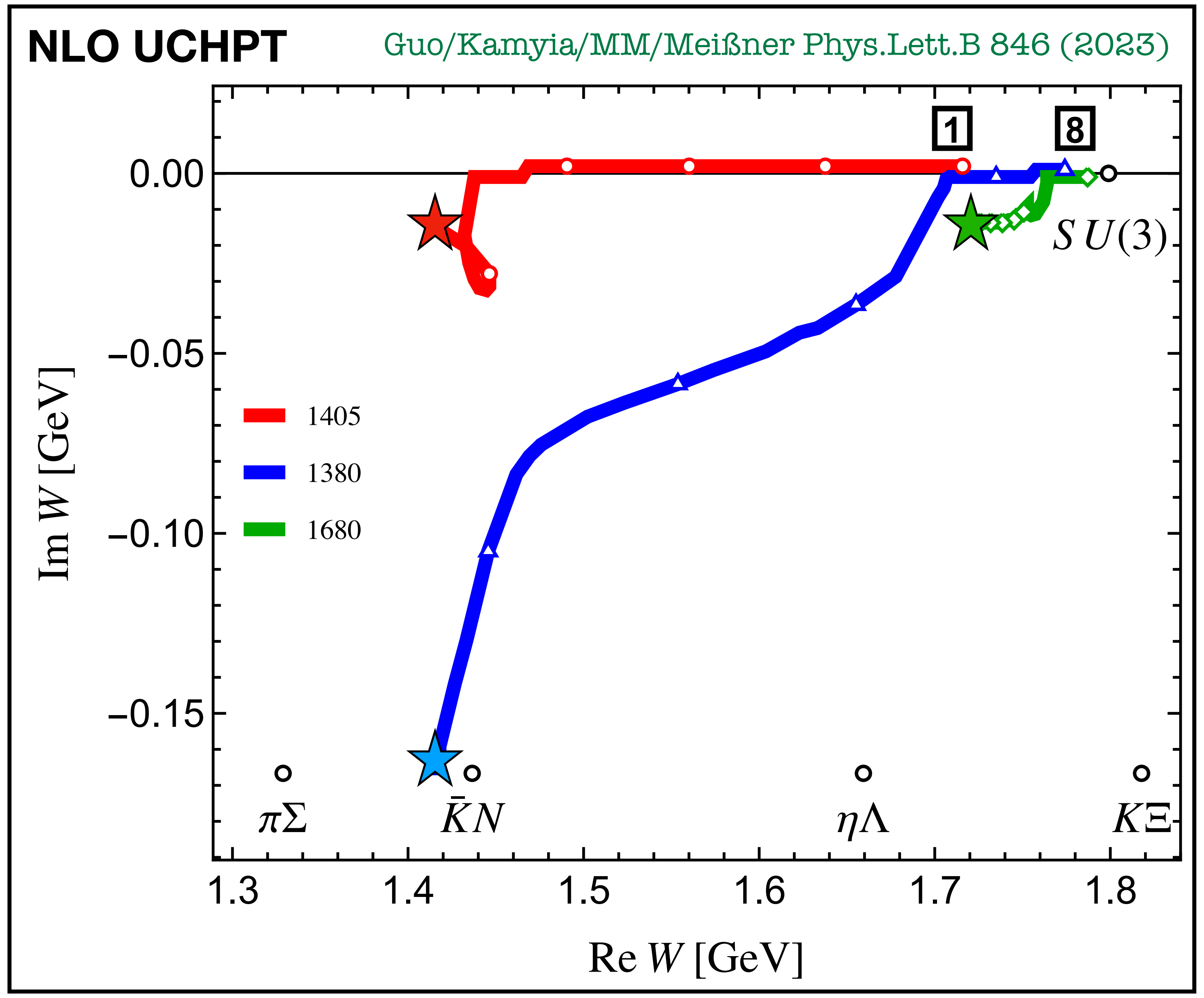
QUARK MASS DEPENDENCE

CHPT encodes quark mass dependence

- SU(3) limit provides a simpler resonance structure
  Jido et al. Nucl.Phys.A 725 (2003); Garcia-Recio/Lutz/Nieves Phys.Lett.B 582 (2004) 49-54;
  - 1 singlet + 2 octet poles
  - LO/NLO UCHPT *pole-"tracks"* differ
    Guo/Kamyia/MM/Meißner Phys.Lett.B 846 (2023)
- Resonance ⇜⇝ virtual bound state ⇜⇝ bound state

  (?) Lattice QCD





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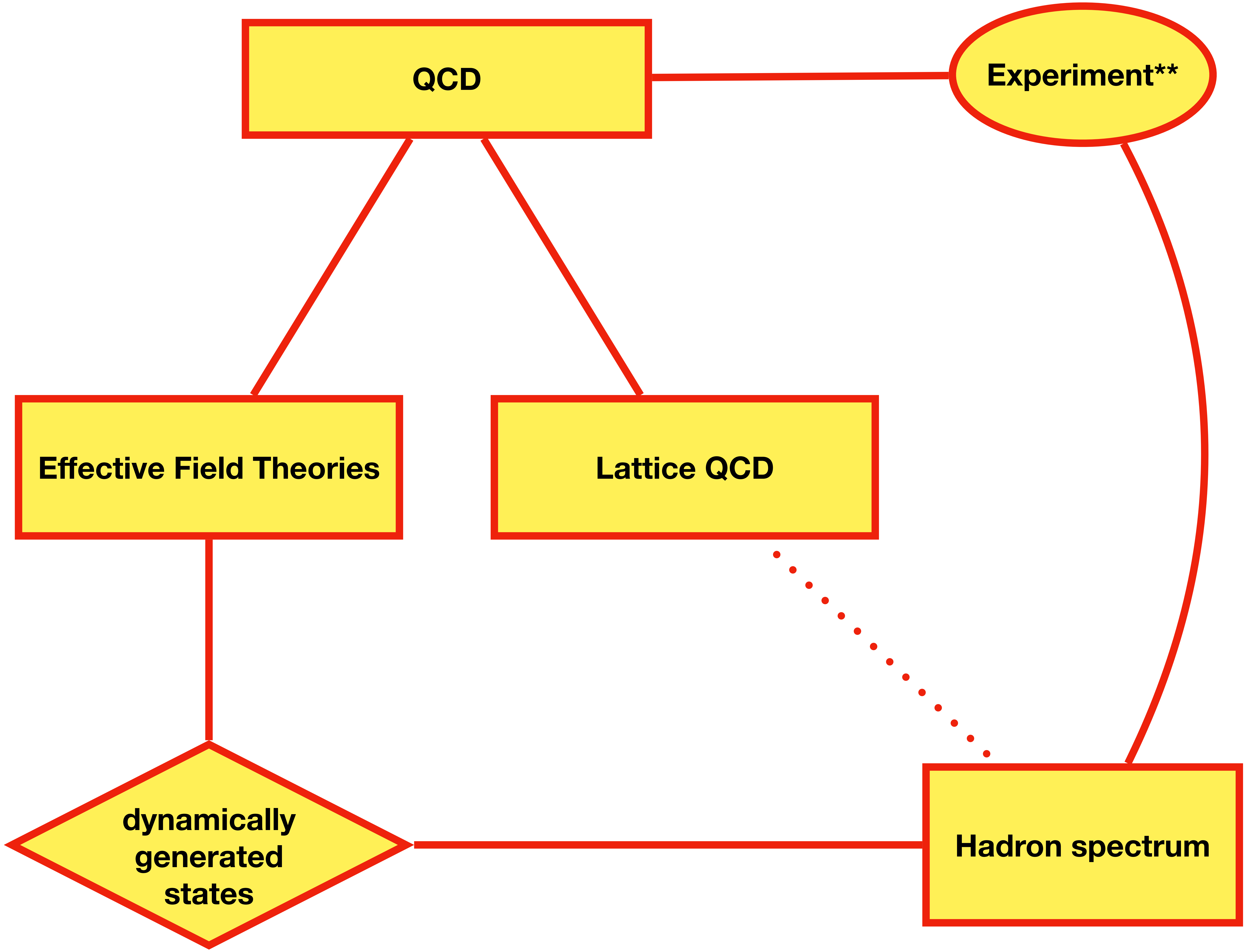

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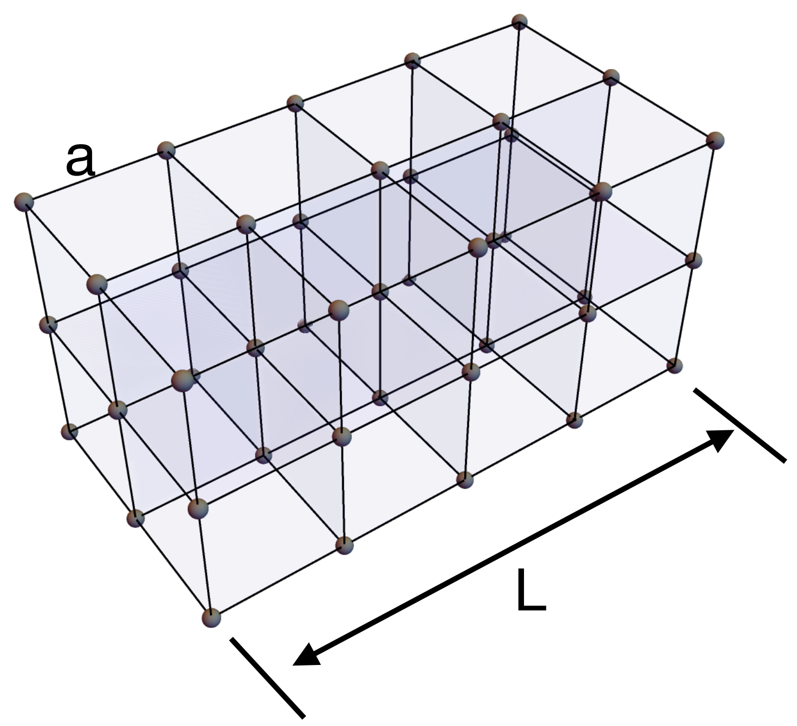
LATTICE QCD (SPECTROSCOPY)



Discretization of space-time

Euclidean space-time

Boundary conditions

Gauge and fermion degrees of freedom

links

plaquettes

Nielsen–Ninomiya theorem

Fermion doublers

Lattice QCD action

Construction of the action

Wick's theorem

measure of integration in the path integral.

Generating functional

Hybrid Monte-Carlo simulation

The transcription of the operators used to probe the physics

Operator construction

Correlation functions

Generalized eigenvalue problem

scale setting

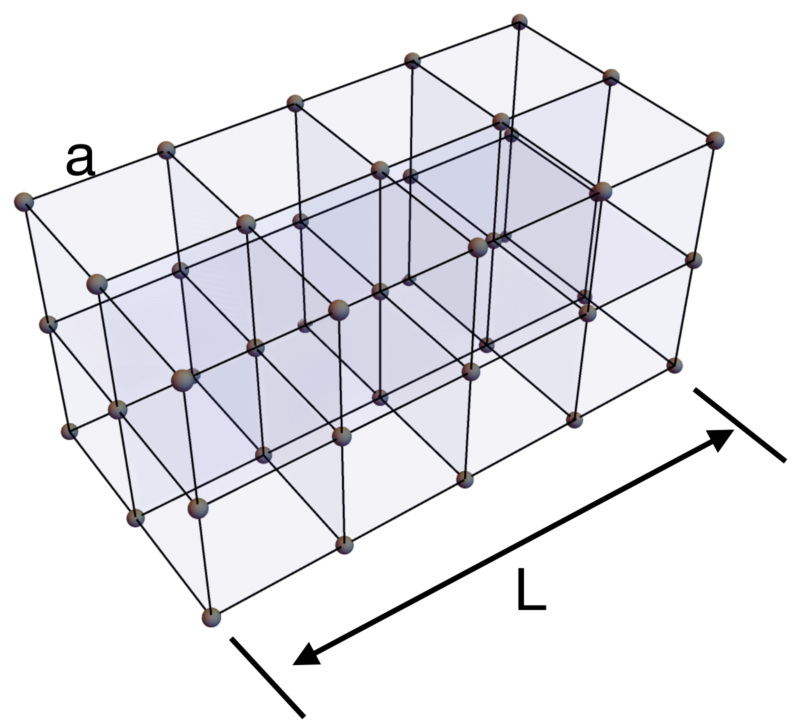
Energy eigenvalues ($\mathbb{R}$)

K. Wilson, Phys. Rev. D10 (1974) 2445 , ...
Introduction to lattice QCD: Course Rajan Gupta hep-lat/9807028 [hep-lat] ....

LATTICE QCD (SPECTROSCOPY)

Roadblocks

- discretized (Euclidean) space-time — continuum extrapolation
- unphysical quark mass — extrapolations tools from CHPT
- finite volume — quantization conditions needed



Discretization of space-time

Euclidean space-time

Boundary conditions

Gauge and fermion degrees of freedom

links

plaquettes

Nielsen–Ninomiya theorem

Fermion doublers

Lattice QCD action

Construction of the action

Wick's theorem

measure of integration in the path integral.

Generating functional

Hybrid Monte-Carlo simulation

The transcription of the operators used to probe the physics

Operator construction

Correlation functions

Generalized eigenvalue problem

scale setting

Energy eigenvalues ($\mathbb{R}$)

K. Wilson, Phys. Rev. D10 (1974) 2445 , ...
Introduction to lattice QCD: Course Rajan Gupta hep-lat/9807028 [hep-lat] ....

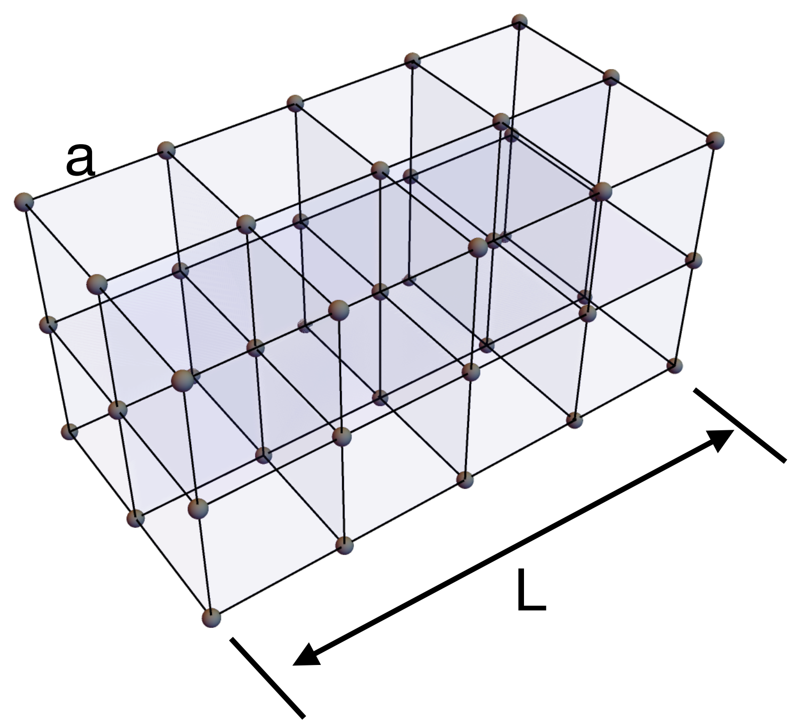
LATTICE QCD (SPECTROSCOPY)

Roadblocks

- discretized (Euclidean) space-time — continuum extrapolation
- unphysical quark mass — extrapolations tools from CHPT
- finite volume — quantization conditions needed

Advantages

- QCD degrees of freedom (first principles)
- Experimentally inaccessible scenarios:
  - → Unconventional quantum numbers (later…)
  - → Three-body scattering/… (later…)
  - → Chiral trajectory (later …)



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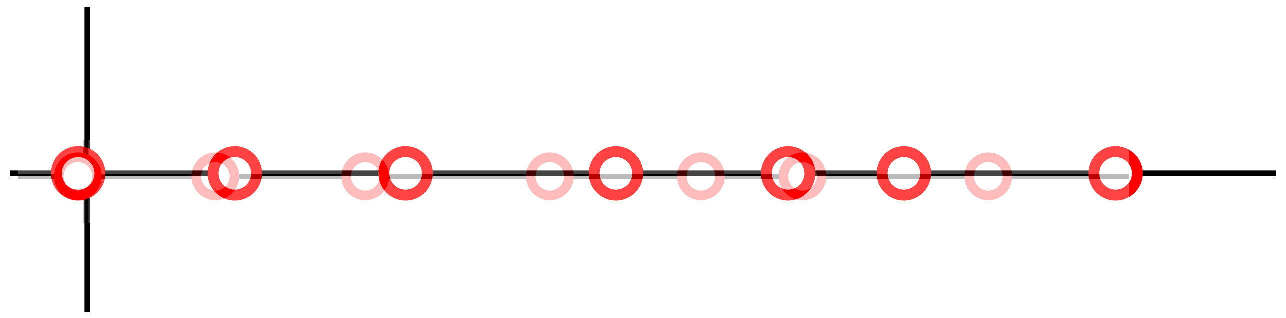
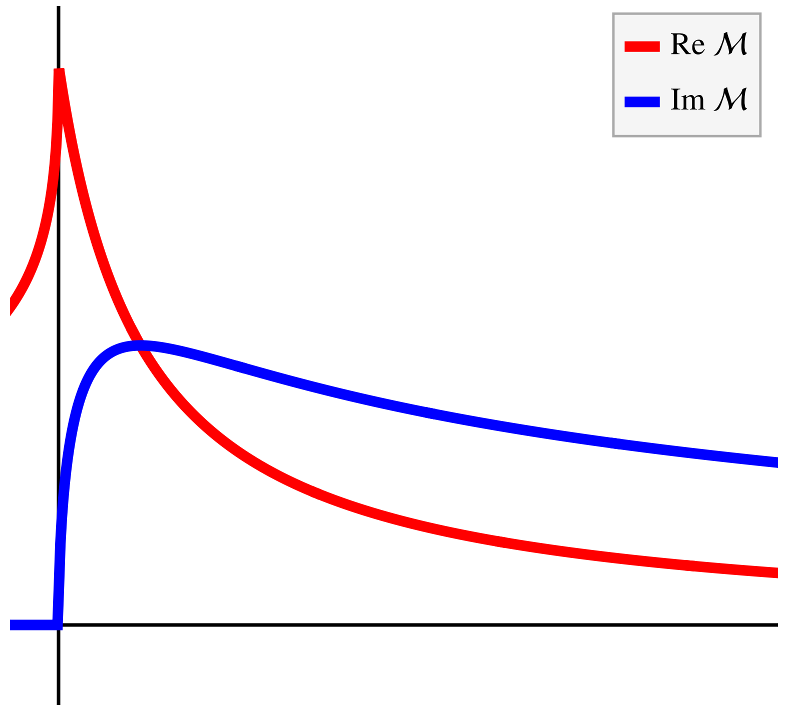


- Finite volume calculations: no direct access to scattering quantities
- Real-valued energy eigenvalues
  - Shifted from free energies — physical information
  - Relation to observables = **Quantization condition**

Lattice QCD

continuum QFT



Review: MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021);

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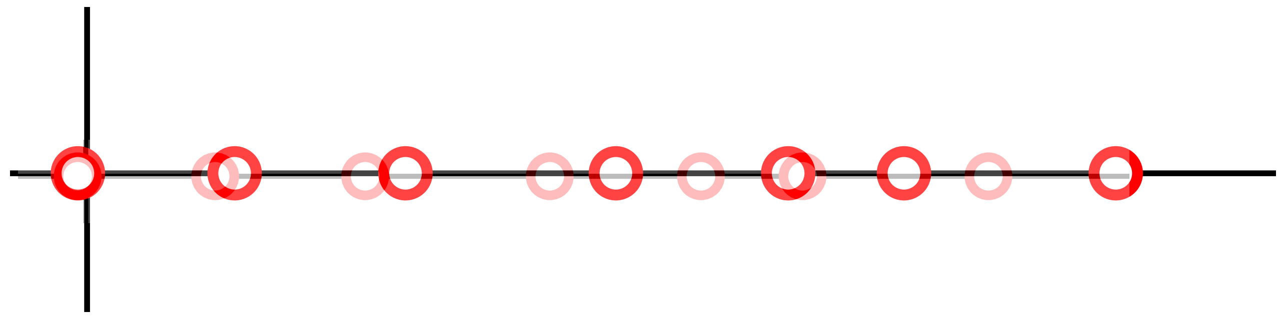

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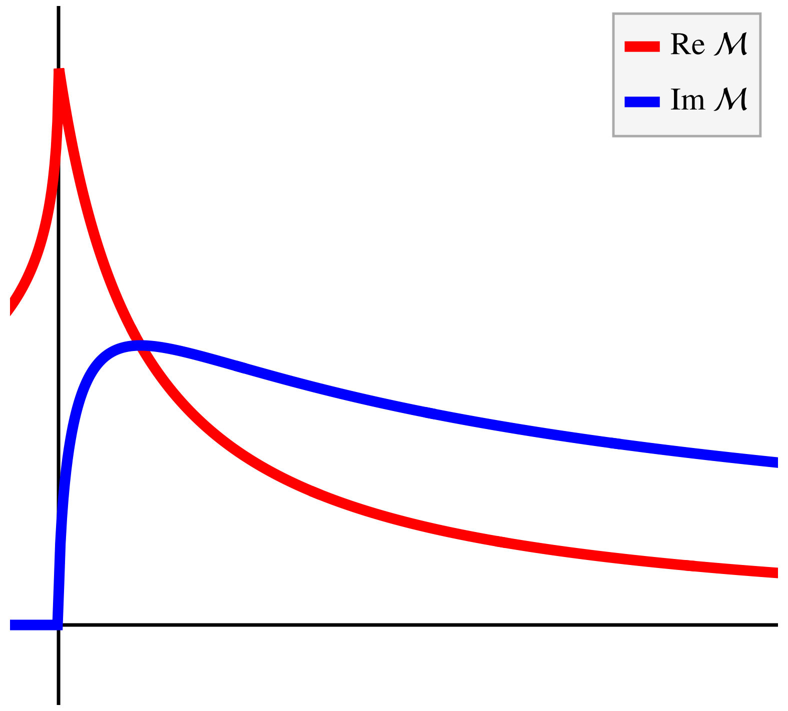
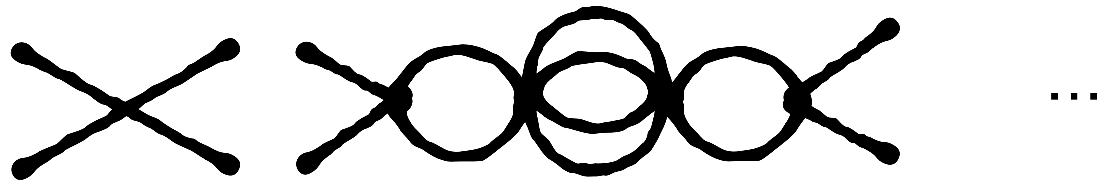
continuum QFT

- **one-way of thinking:**
  - on-shell states “feel” the box-size $\sim (ML)^n$
  - off-shell configurations decay exponentially $\sim e^{-ML}$





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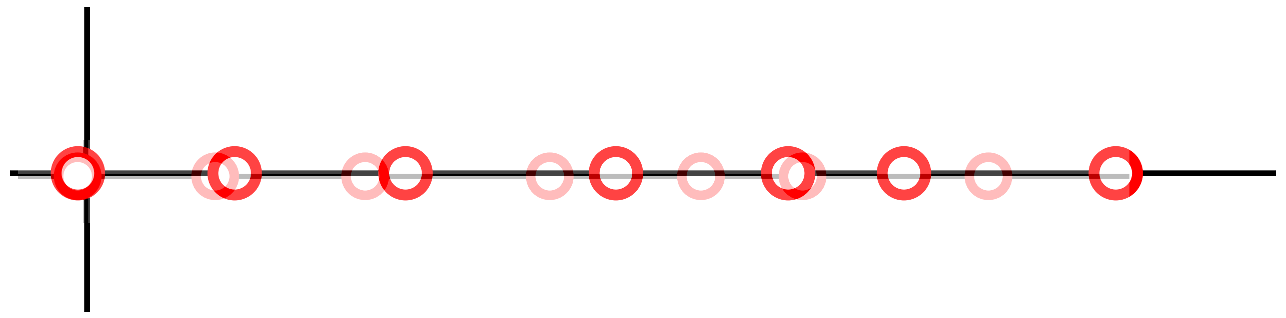

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Lattice QCD

continuum QFT

$$M_\infty^{-1} = p\cot\delta - \left(\int \ldots - Re\int \ldots\right)$$

➡ Unitarity $\quad M_\infty^{-1} = \tilde{K}^{-1} - \int \frac{d^3l}{(2\pi)^3}\frac{1}{2E_l(s-4E_l^2+i\epsilon)}$

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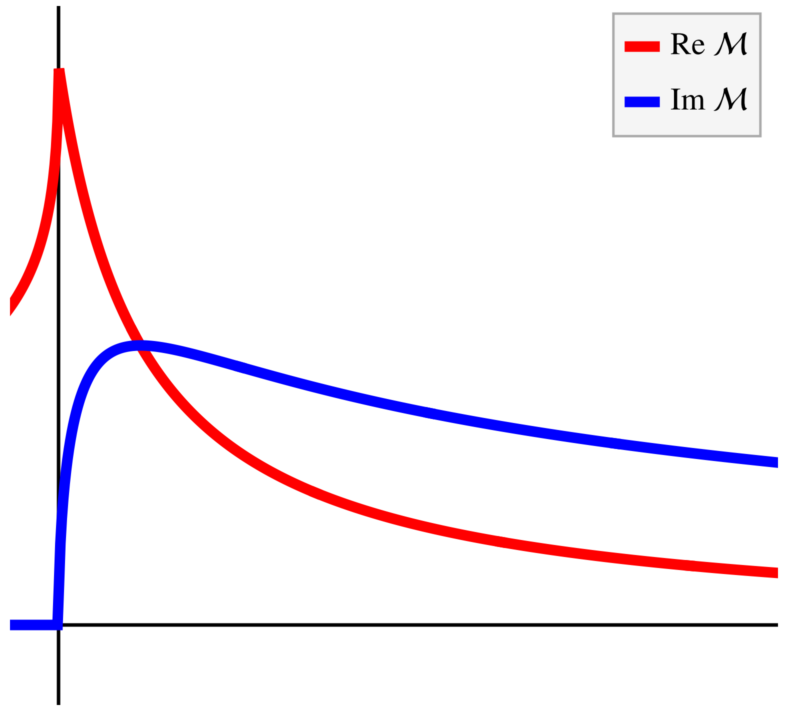

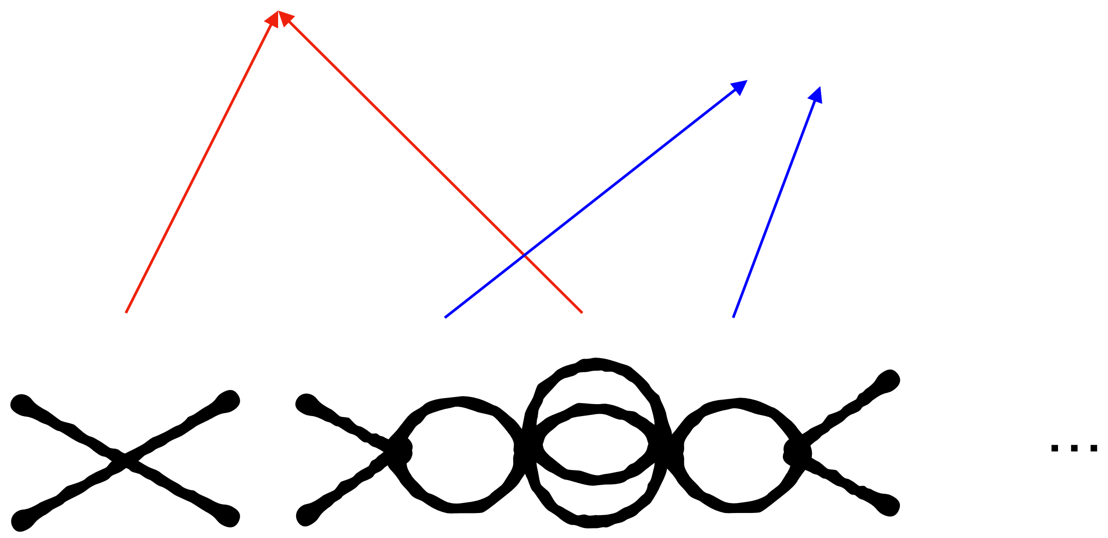

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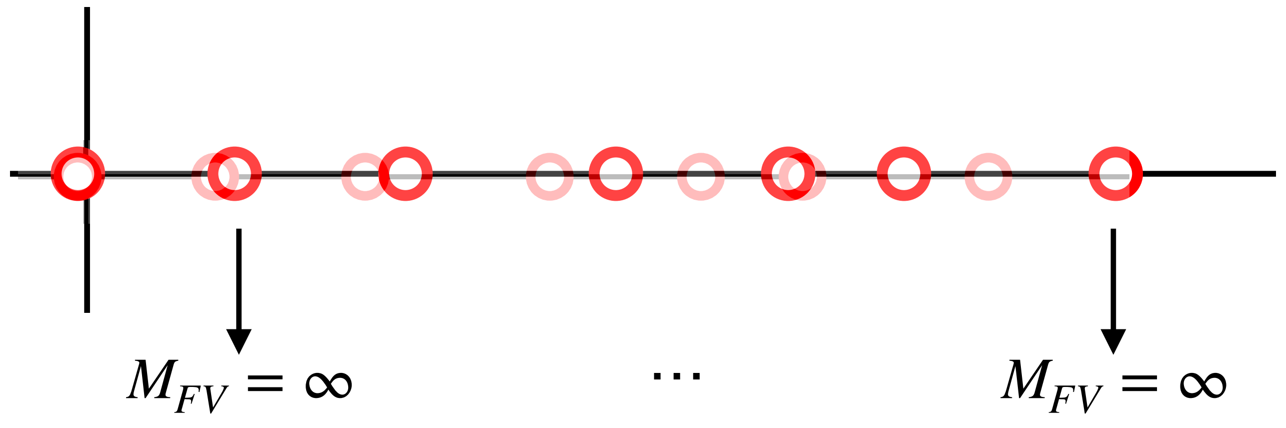

$Z_{00} + \mathcal{O}(e^{-ML})$

Lüscher Zeta-function

$$M_{FV}^{-1} = p\cot\delta - \left(\frac{1}{L^3}\sum_{\vec p}\ldots - Re\int_{\vec l}\ldots\right)$$

Lattice QCD

continuum QFT

$$M_{\infty}^{-1} = p\cot\delta - \left(\int\ldots - Re\int\ldots\right)$$

- **one-way of thinking:**
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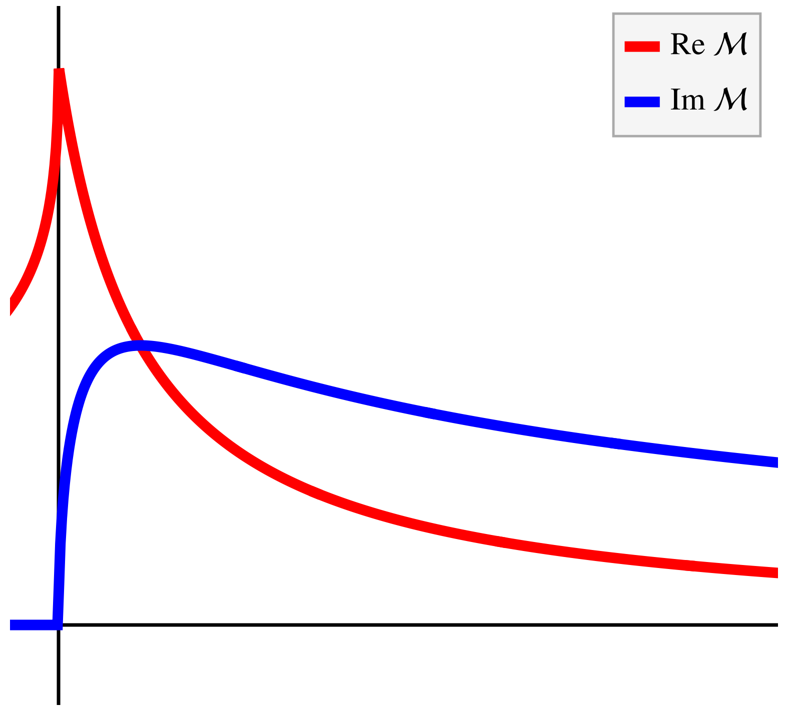

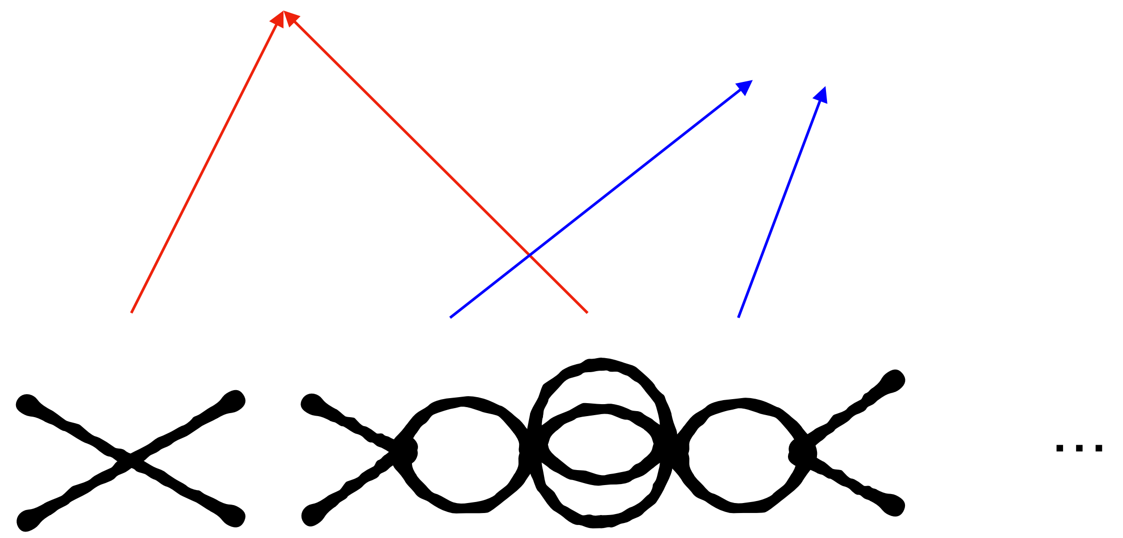

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Review: MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021);

3-BODY

Generalization to 3-body states — Finite Volume Unitarity (FVU) approach

- 3-body unitarity accounts for all on-shell states
- genuine determinant condition

- Alternatives: RFT, NREFT

  RFT(Hansen/Sharpe 2014) NREFT(Rusetsky/Hammer/Pang 2017)

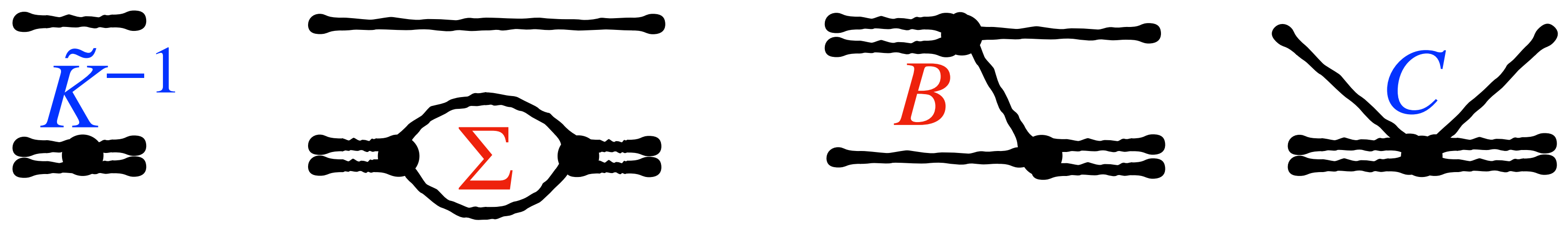
  - equivalence shown in different regimes

    Jackura et al. Phys.Rev.D 100 (2019) 3, 034508, Garofalo et al. JHEP 02 (2023) 252

FVU

$$\det\left[\,2L^3E_{\mathbf{p}}\left(\tilde{K}^{-1}-\Sigma^L\right)-B-C\,\right]^{\Lambda}\equiv 0$$

MM/Döring
Eur.Phys.J.A 53 (2017) 12, 240



Many new applications

- proof of concepts and spin-less repulsive systems

  MM/Doring Phys.Rev.Lett. 122 (2019) 6, Fischer et al. Eur.Phys.J.C 81 (2021) 5, Blanton, Lopez, Hansen, Briceno, ...

- systems with left-hand cut

  Hansen et al. JHEP 06 (2024) 051, Dawid et al. JHEP 01 (2025) 060, Rusetsky, ...

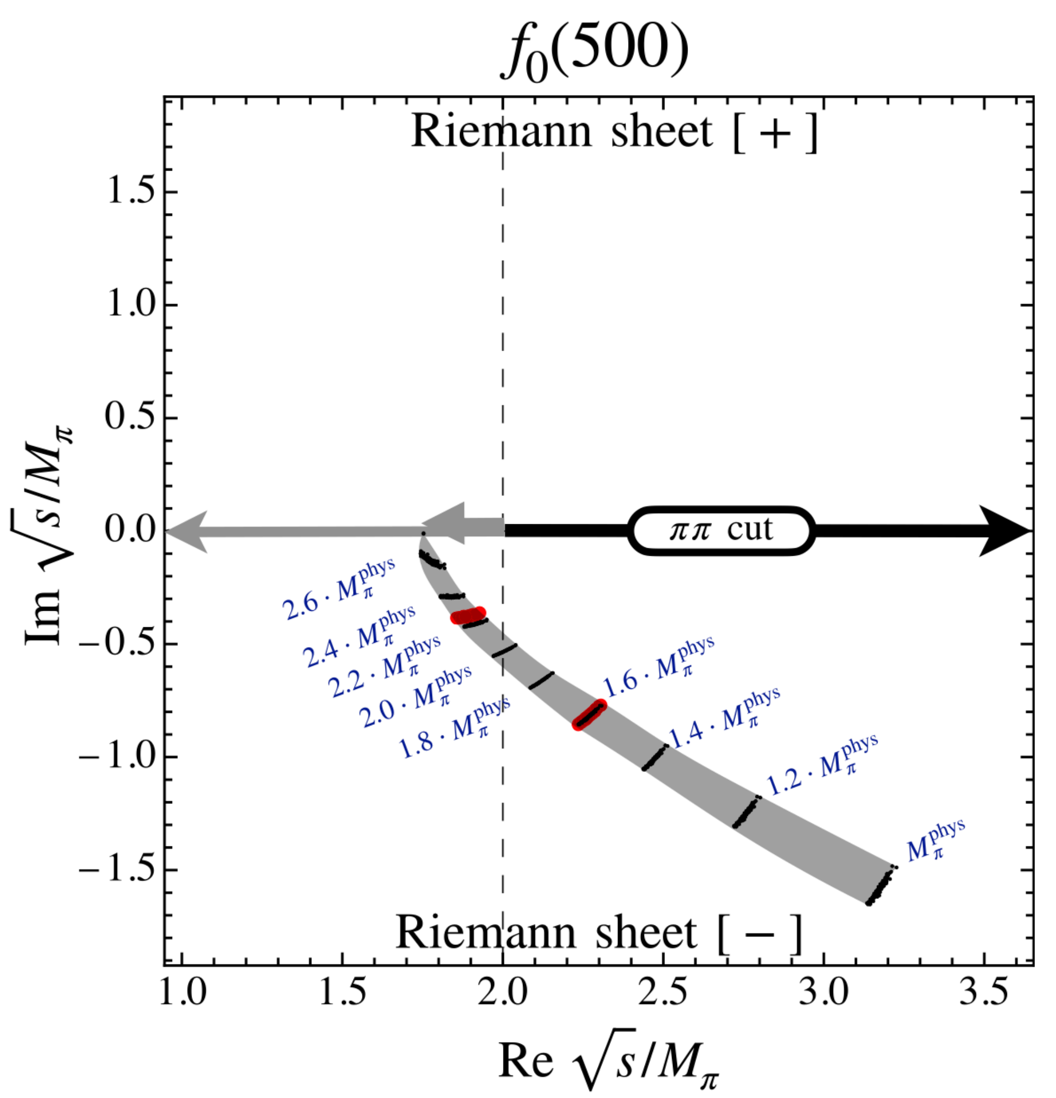
- 3-body resonant systems (later ...)

  MM/Culver Phys.Rev.Lett. 127 (2021) 22
  Yan et al. Phys.Rev.Lett. 133 (2024) 21

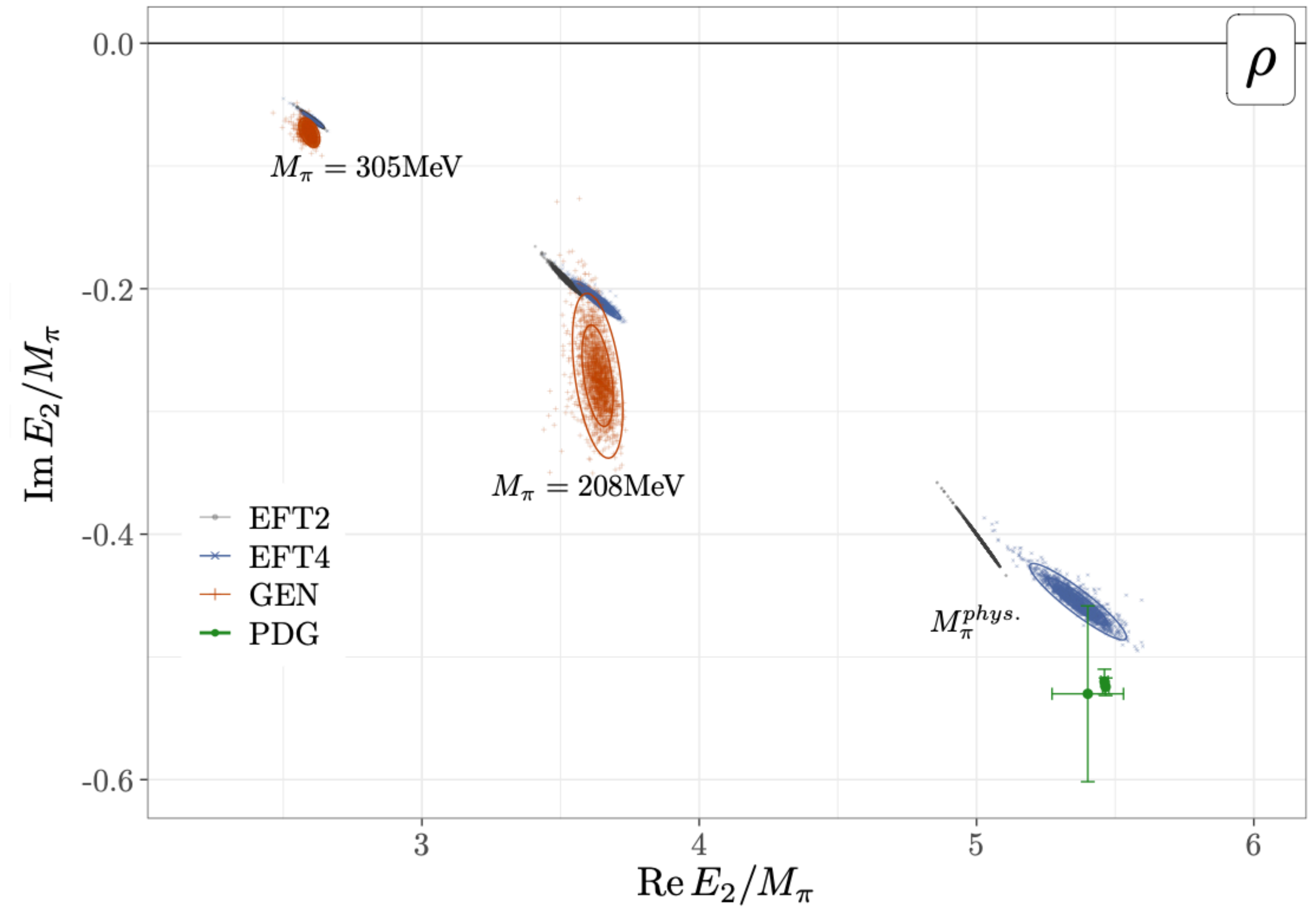
APPLICATION I

Two pion system

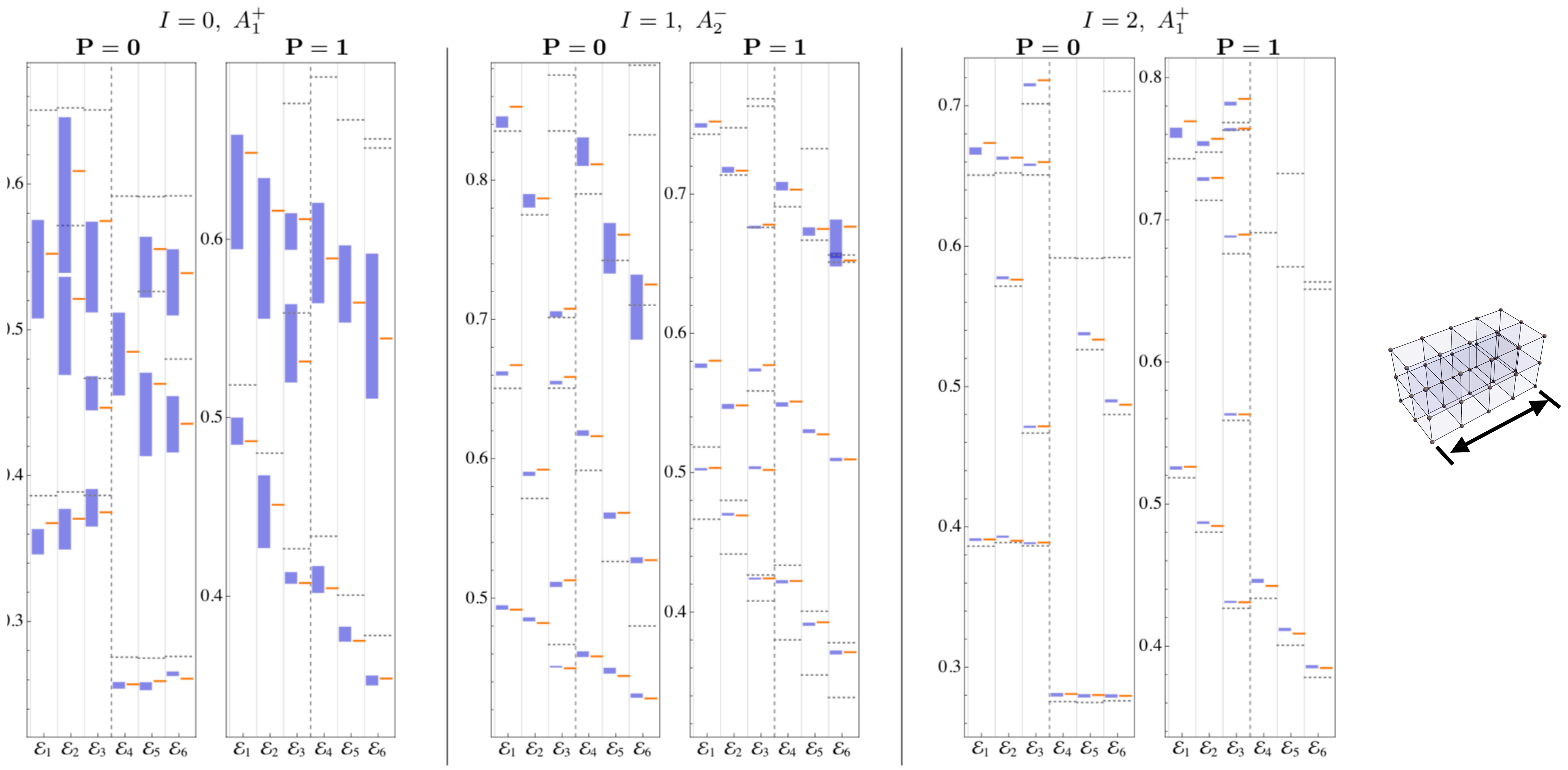
- simplest 2-hadron system
- many LQCD results
  NPLQCD; HadSpec; ETMC; GW-lattice; CP-PACS;….
- simultaneous description of all $\pi\pi$ interaction channels through CHPT — UCHPT
  GWQCD: Guo et al. (2016) Guo et al. (2018) Culver et al. (2019) MM et al.(2019)

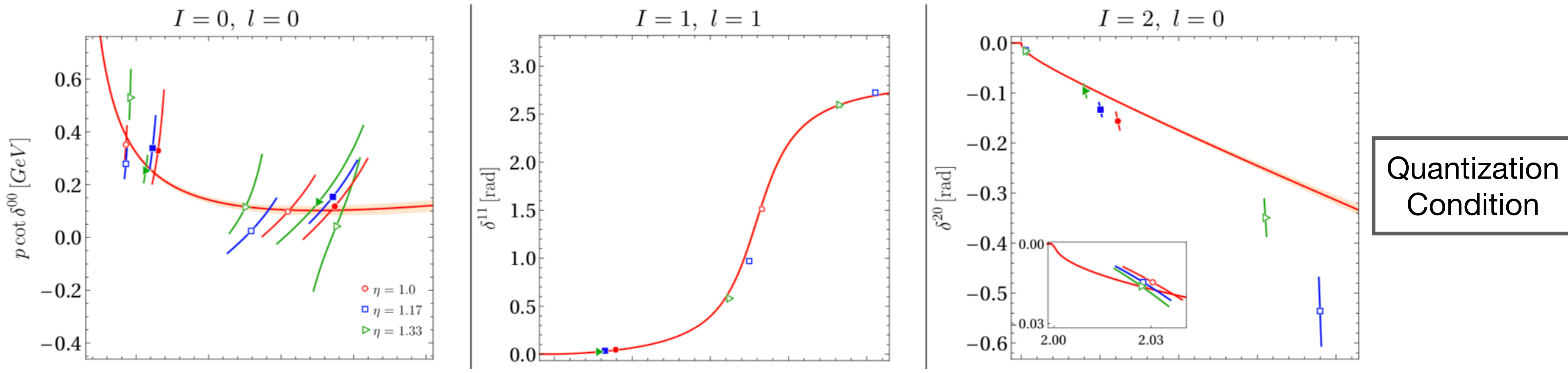


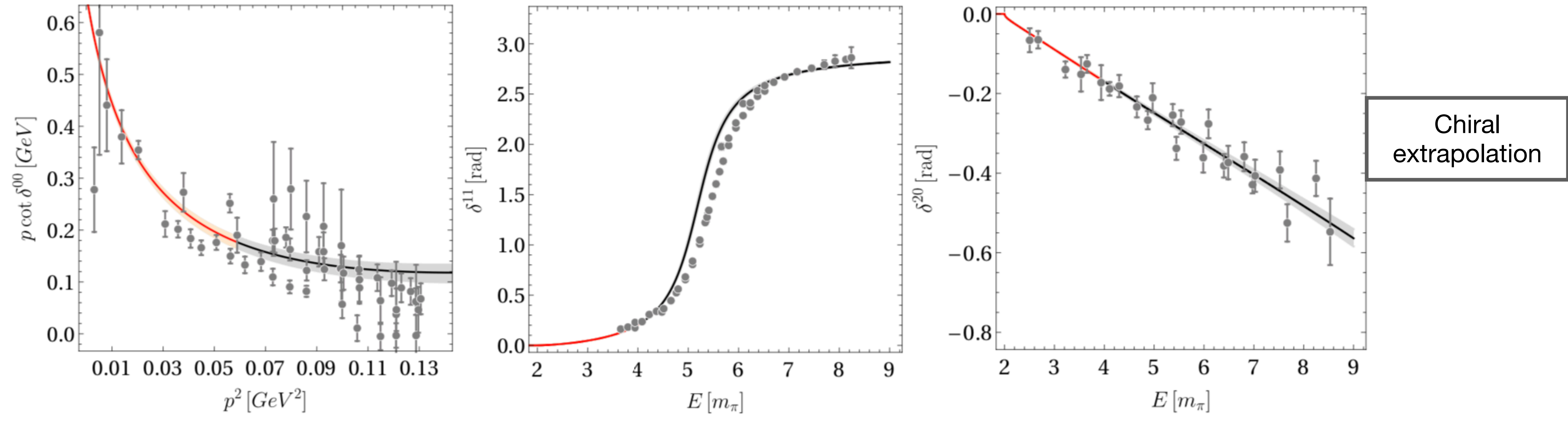
MM/Urbach/Meißner Phys.Rept. 1001 (2023) 1-66



Yan/MM/... Phys.Rev.Lett. 133 (2024) 21

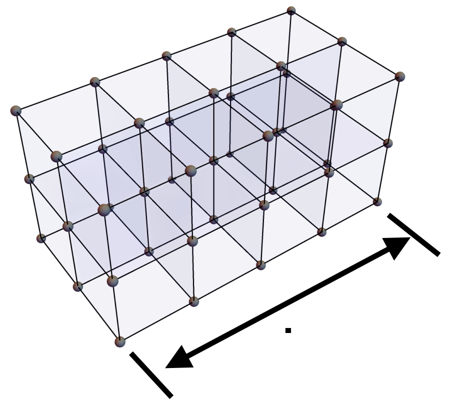


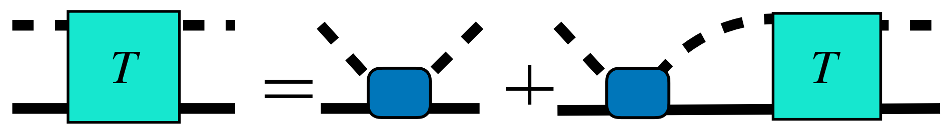




[GWQCD] MM/Culver/... Phys.Rev.D100(11)(2019) 114514

APPLICATION II





Meson-baryon systems ($\bar{K}N/\pi\Sigma/\pi\Lambda/K\Xi$)

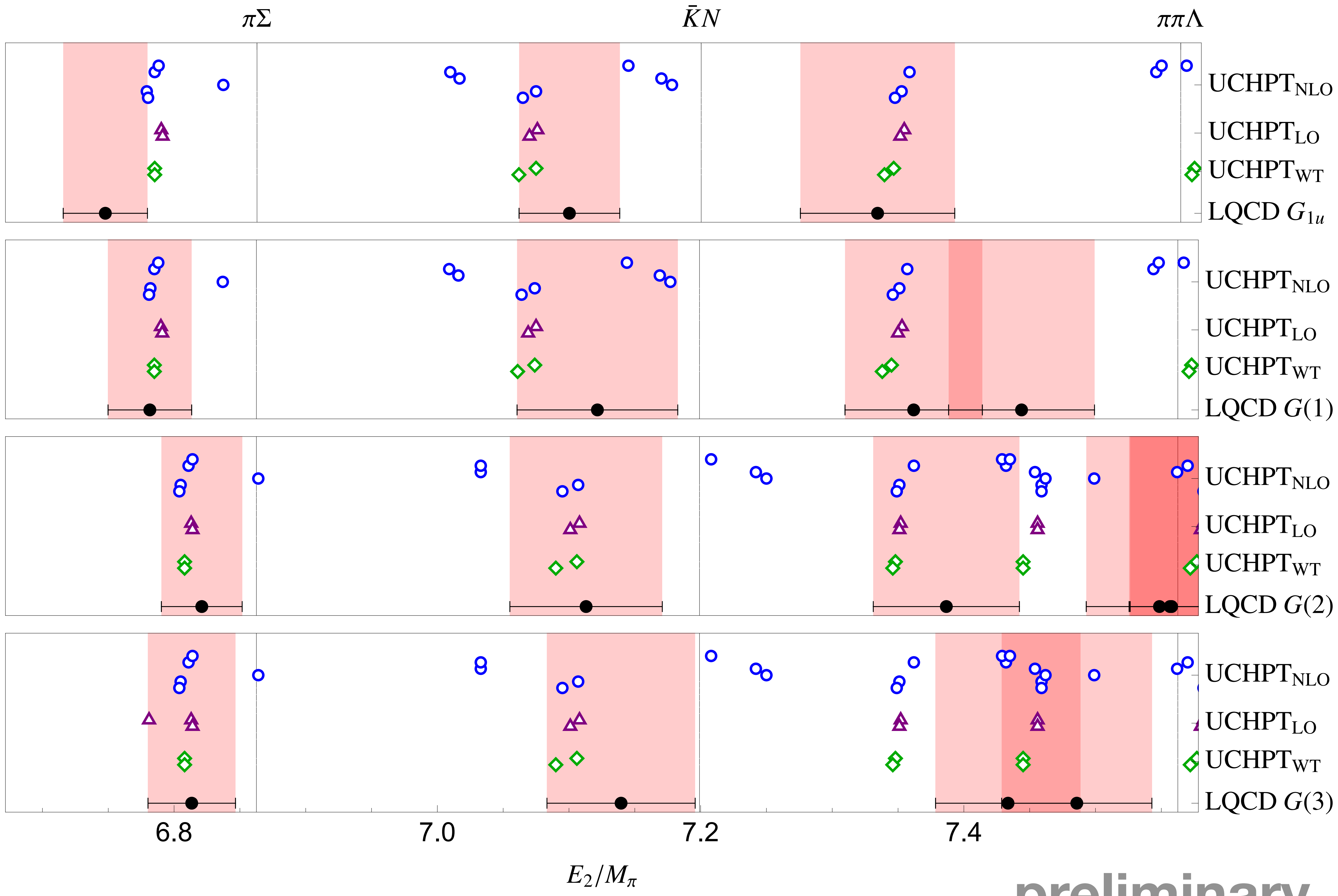
- Available Lattice spectrum
  [BaSc] Bulava et al. Phys.Rev.Lett. 132 (2024) 5; 2307.13471

  $M_\pi \approx 200\,\mathrm{MeV}$ $M_K \approx 487\,\mathrm{MeV}$
  $M_\pi L = 4.181(16)$ $a = 0.0633(4)(6)\,\mathrm{fm}$

- **Compare to UCHPT**
  - Unified analysis LQCD+UCHPT+EXPERIMENT

  … mostly ok, but not always

  … ongoing work



preliminary

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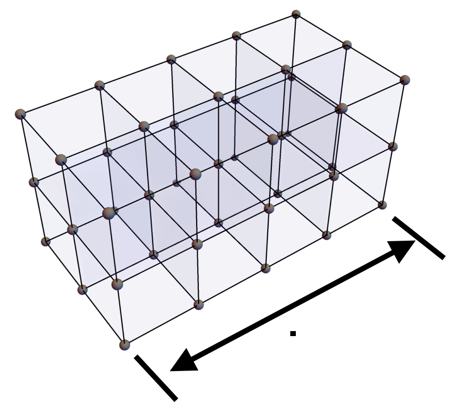

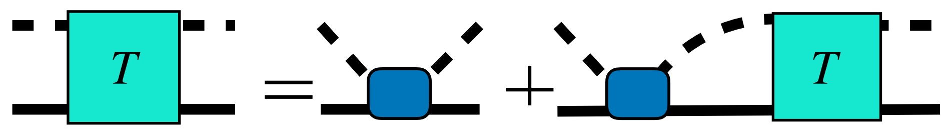

CHPT encodes quark mass dependence

- Available Lattice spectrum
  [BaSc] Bulava et al. Phys.Rev.Lett. 132 (2024) 5; 2307.13471

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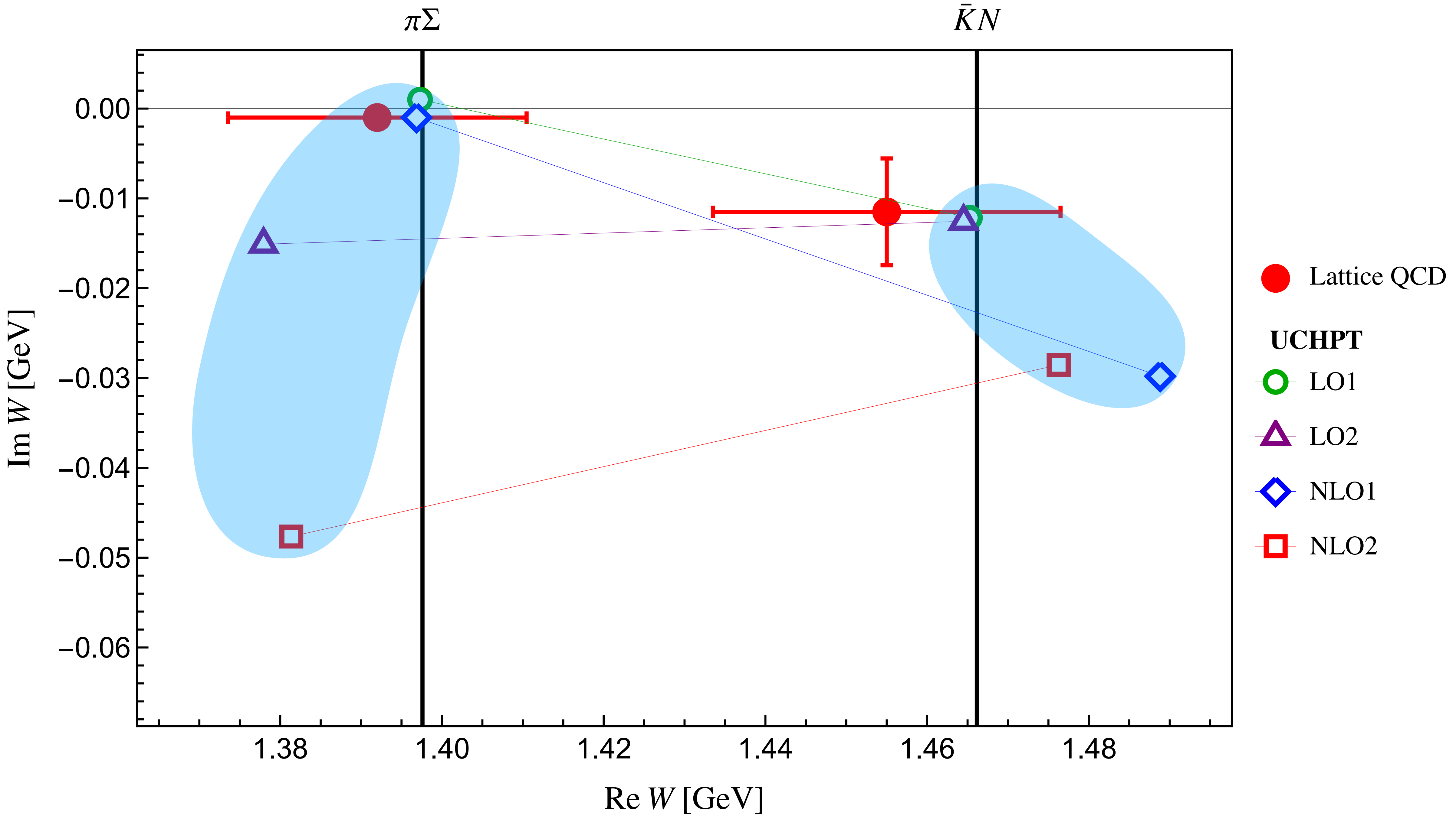
- pole positions from available UCHPT approaches

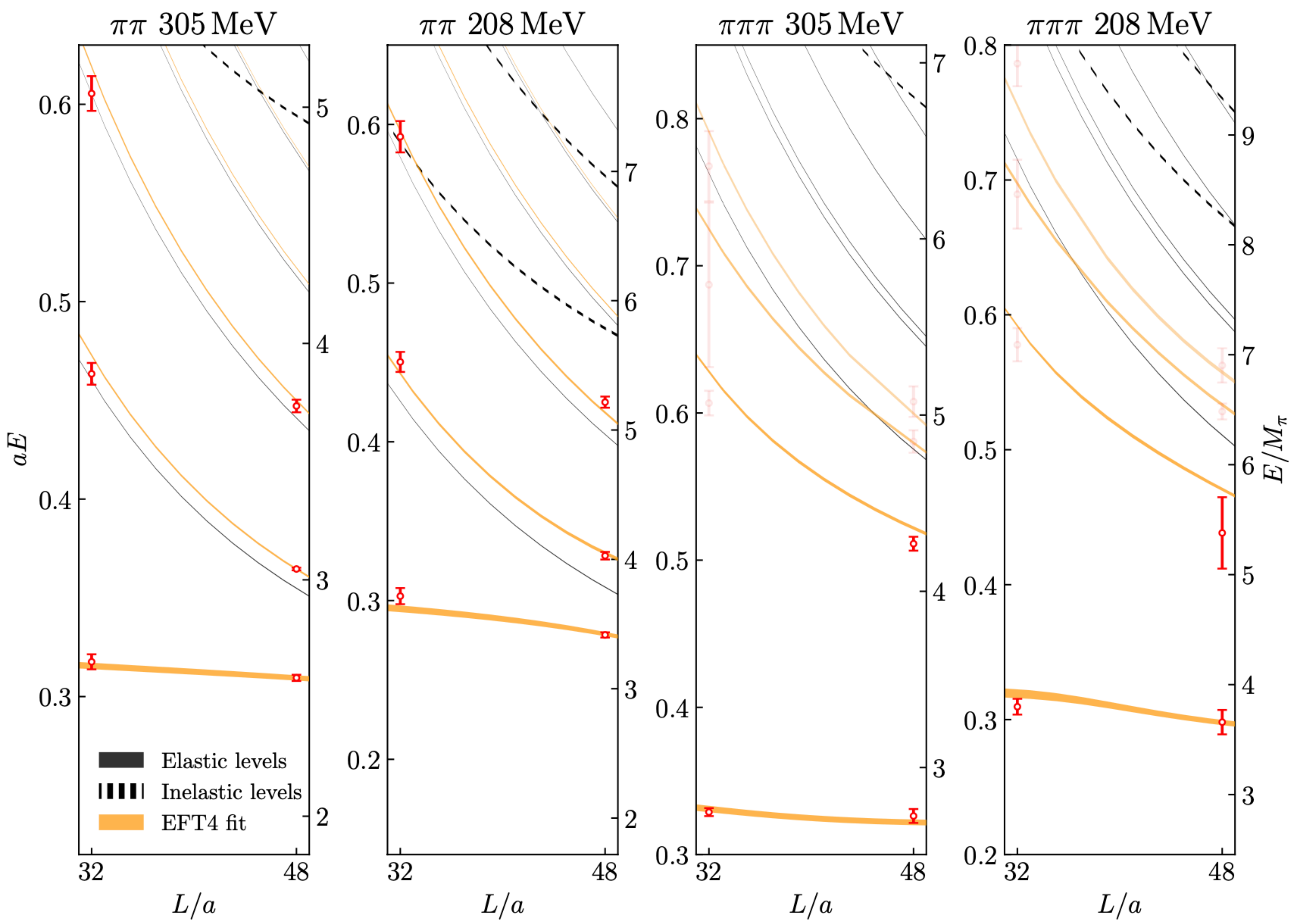


Guo/Kamyia/MM/Meißner Phys.Lett.B 846 (2023)

APPLICATION III $\omega \to \pi\pi\pi$

Lattice QCD

- Nf = 2 + 1 Clover fermions
- 2/3 particle operators
- 2 pion masses ($\approx 210, 305$ MeV) 2 volumes ($L^3 = 32^3, 48^3$)



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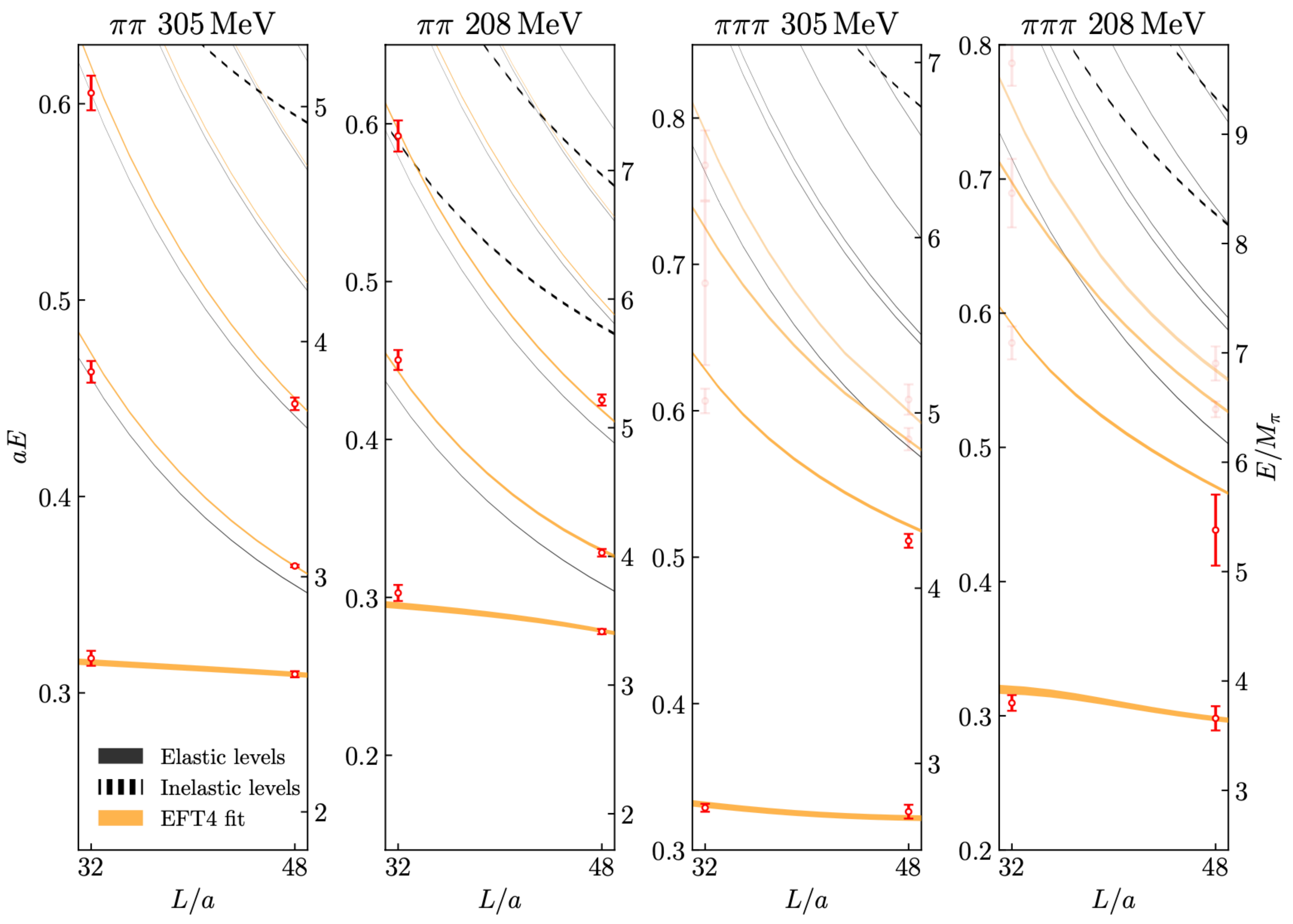
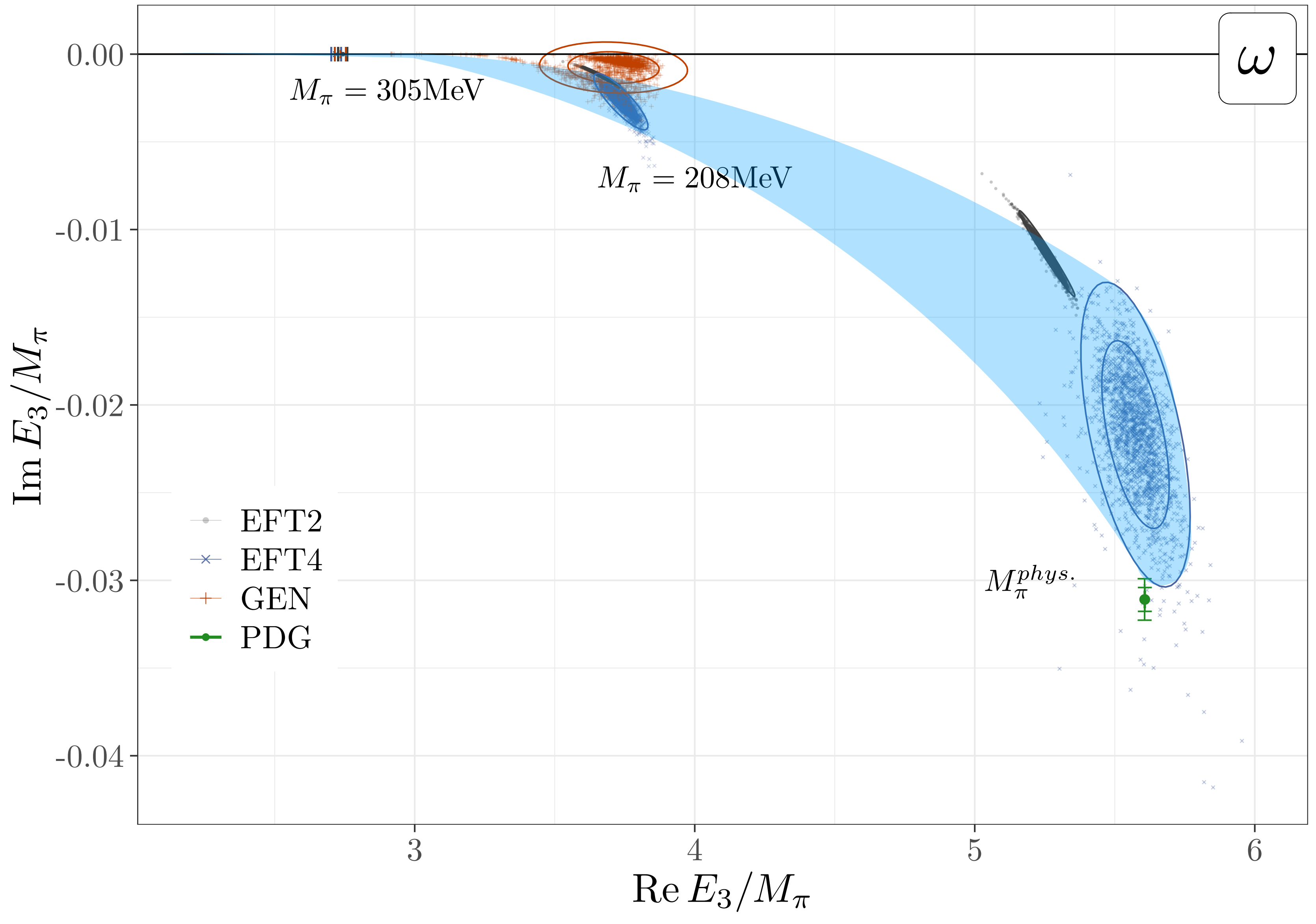


FVU

$$\det\left[\,2L^3E_{\mathbf{p}}\left(\tilde{K}^{-1}-\Sigma^L\right)-B-C\,\right]^{\Lambda}\equiv 0$$

Result

- Various EFT based ansatzes
- $\omega(782)$ becomes abound state at ~300 MeV
- at the physical point very close to the EXP value



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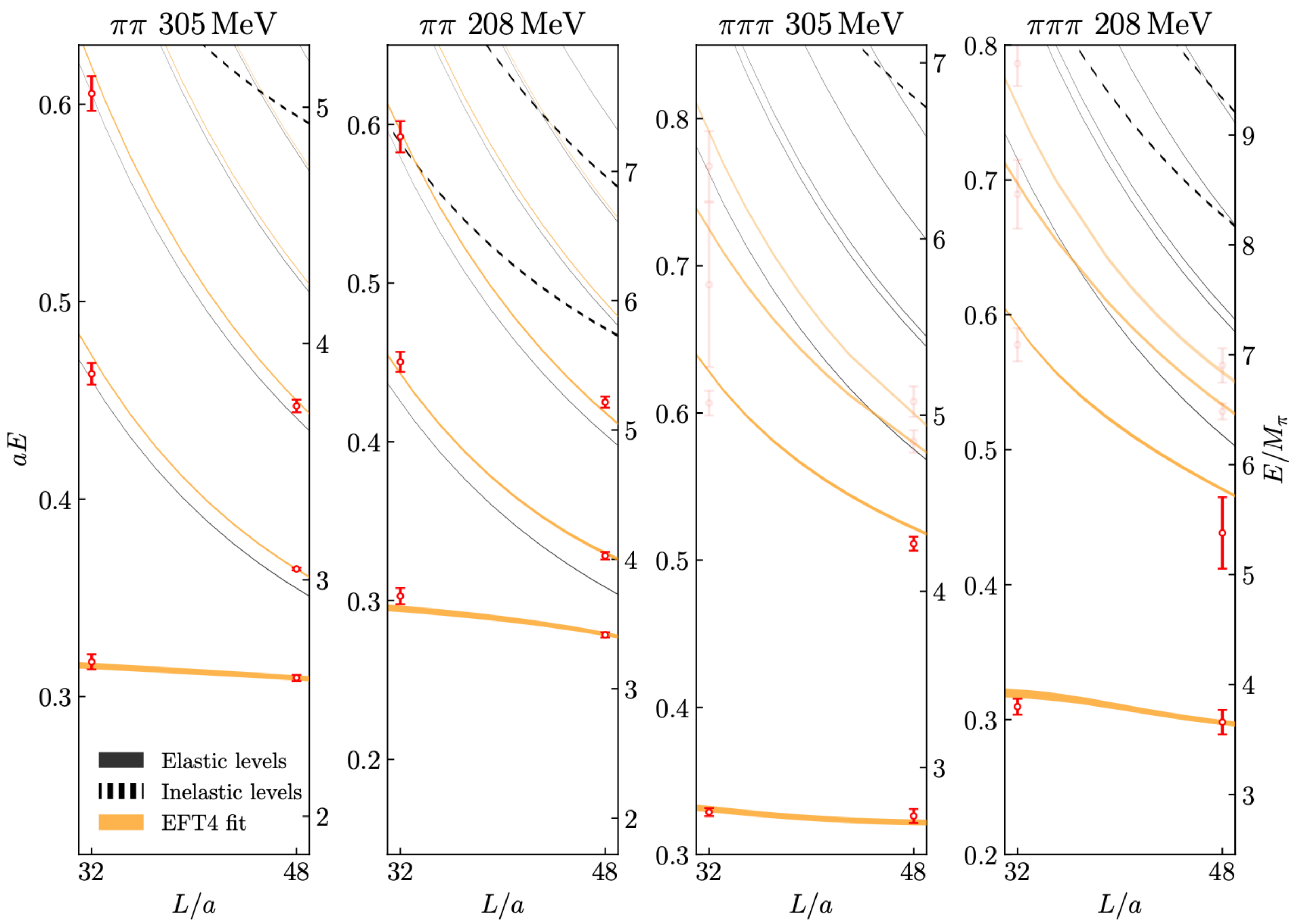

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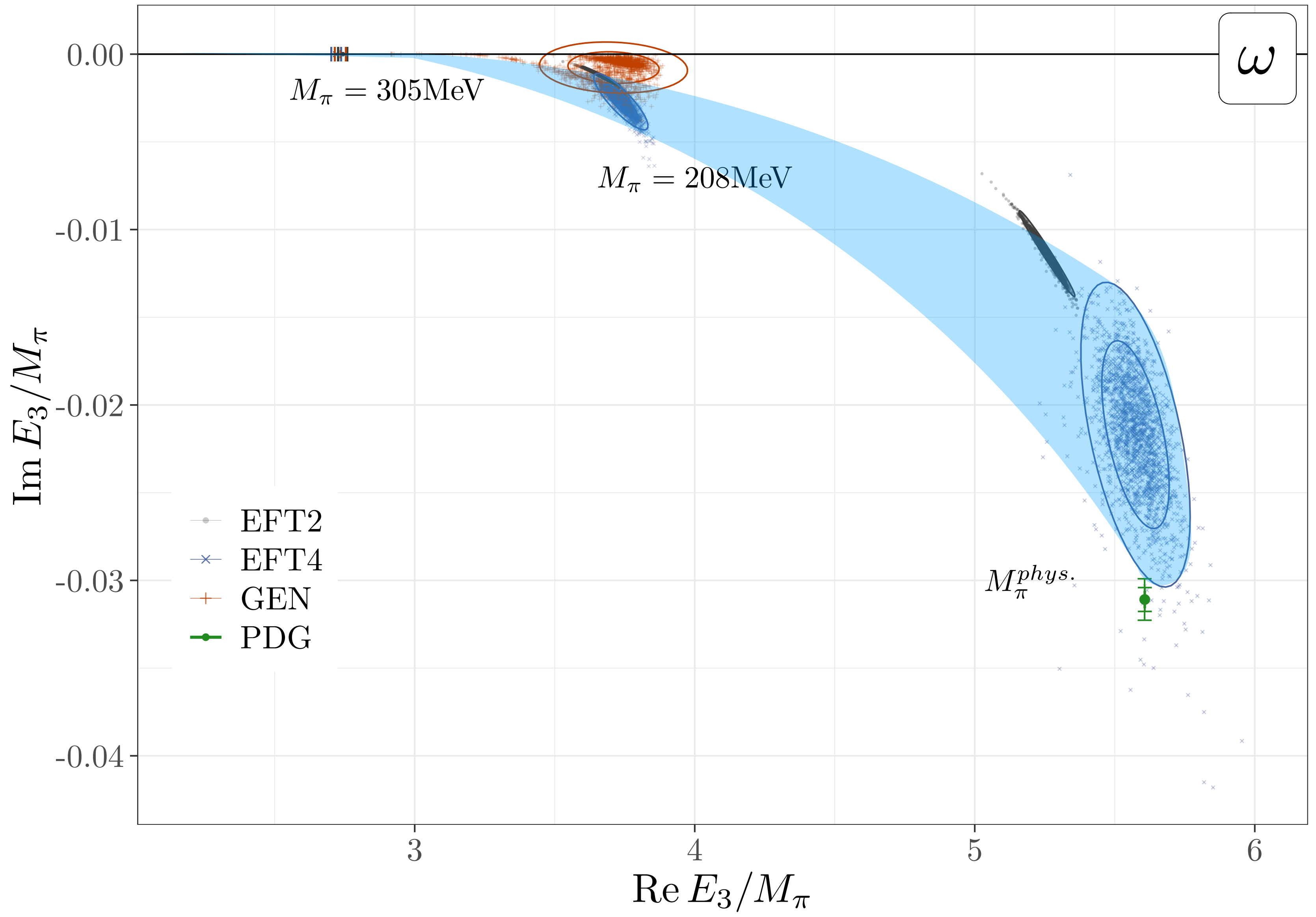
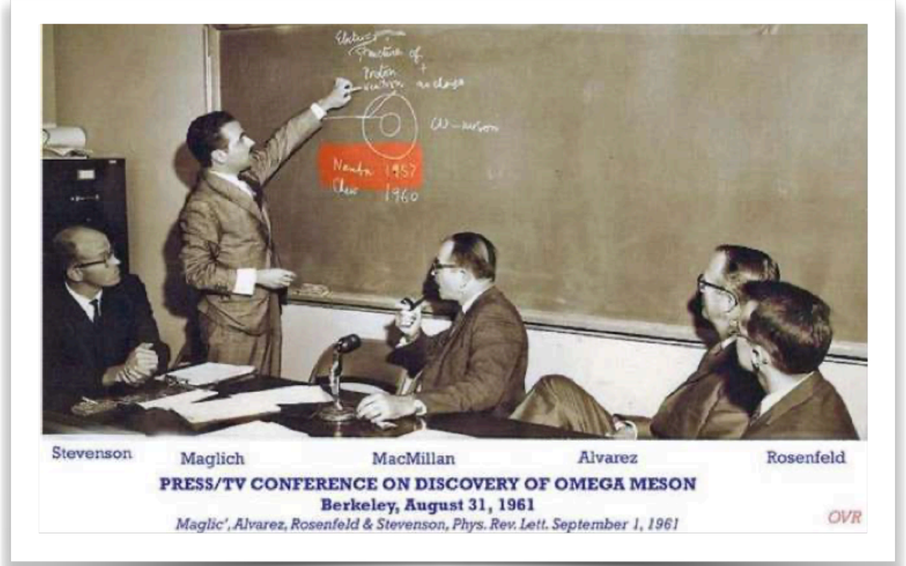
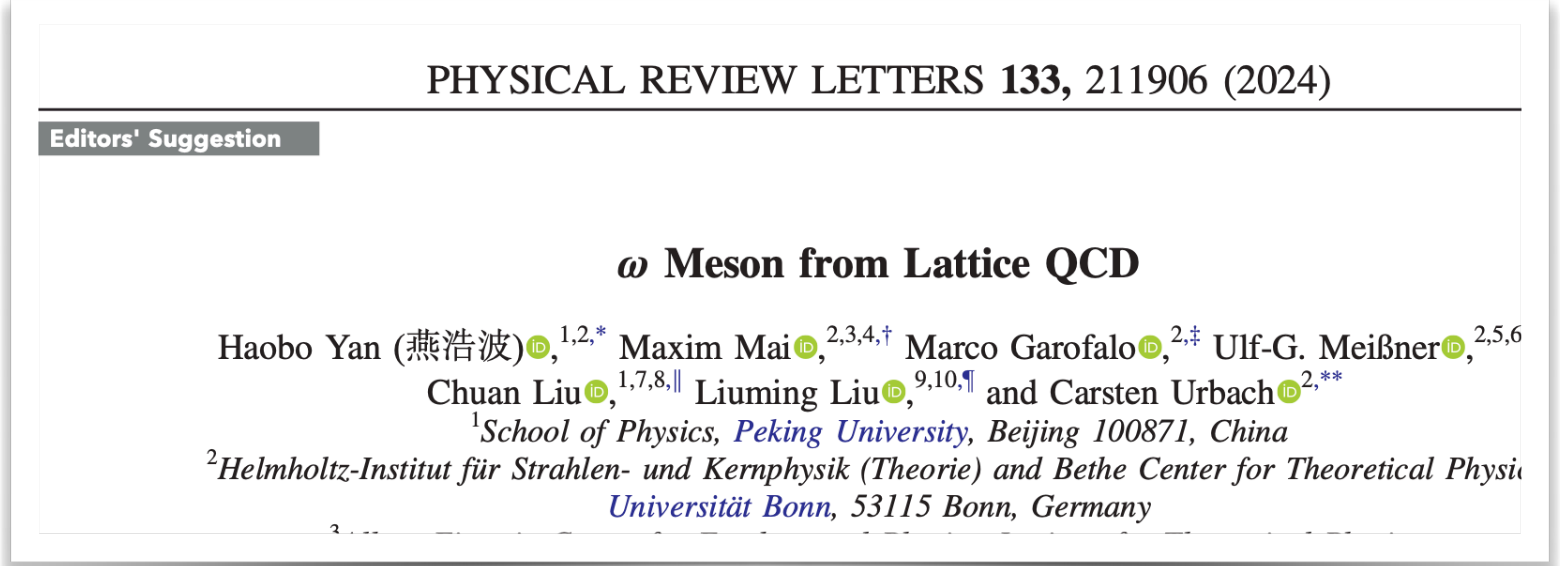




63 years



PHYSICAL REVIEW LETTERS **133,** 211906 (2024)

Editors' Suggestion

ω Meson from Lattice QCD

Haobo Yan (燕浩波),[1,2,*] Maxim Mai,[2,3,4,†] Marco Garofalo,[2,‡] Ulf-G. Meißner,[2,5,6] Chuan Liu,[1,7,8,‖] Liuming Liu,[9,10,¶] and Carsten Urbach[2,**]

[1]*School of Physics, Peking University, Beijing 100871, China*

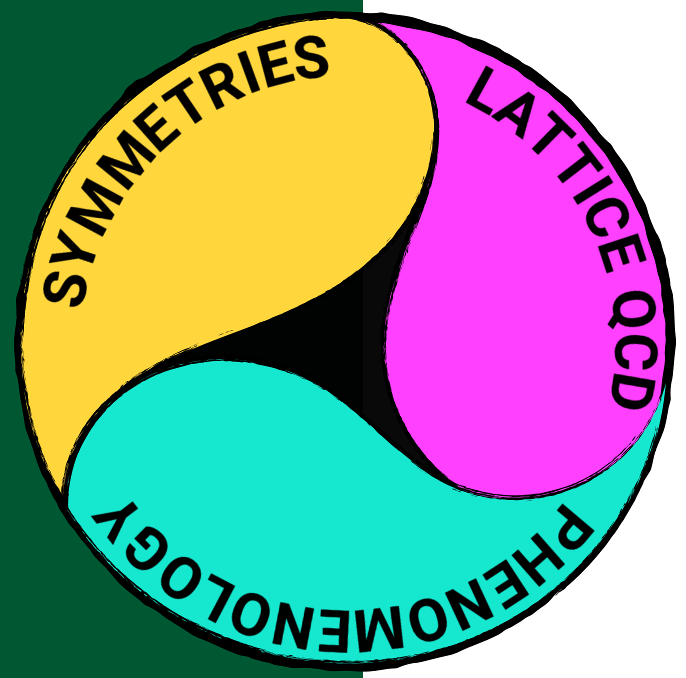
[2]*Helmholtz-Institut für Strahlen- und Kernphysik (Theorie) and Bethe Center for Theoretical Physi… Universität Bonn, 53115 Bonn, Germany*

SUMMARY / OUTLOOK

Effective Field Theories
- quark-mass dependence
- analytical tools
- dynamically generated resonances

S-matrix
- Mathematical constraints on transitions
- Universal resonance parameter

Lattice QCD:
- ab-initio calculations
- universal tool for physical und unphysical scenarios
- many new advances and results



UCHPT models
- $f_0(500), \rho(770), \ldots$ well established quark-mass dependence
- Two-pole structure: $\Lambda(1405), \Lambda(1380)$ **discovered**

Novel FVU 3b Quantization Condition
- pilot results on $3\pi(I=3,2..), a_1(1260), \phi^4, \ldots$
- **Re-discovered** $\omega(782)$ **from QCD** — pole and chiral trajectories

Outlook
- $N(1440)$**,** $DD\pi$**, ...**
- spin-exotics
- Triangles/Strangeness — $a_1(1420)$ ... *first steps: hys.Rev.D* 110 (2024), *JHEP* 10 (2024) 246
- UCHPT + LQCD $\Lambda(1405), \Lambda(1380)$ ongoing …

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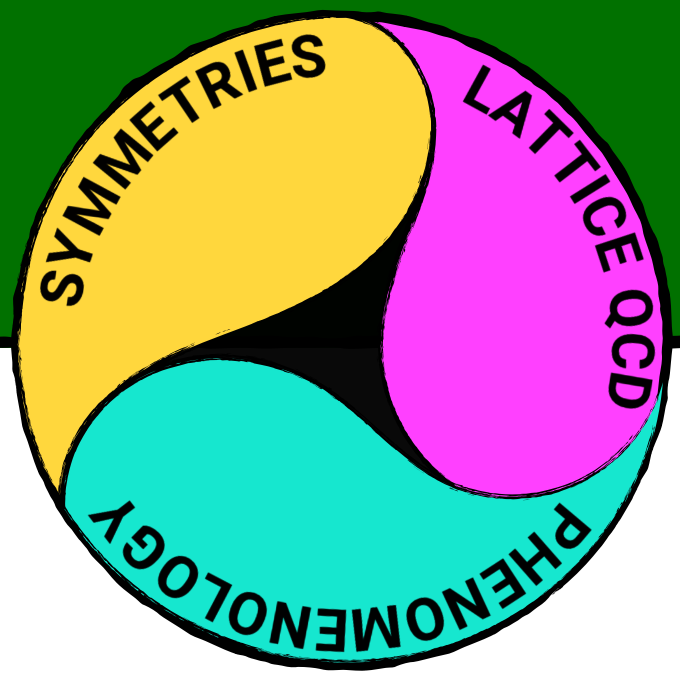

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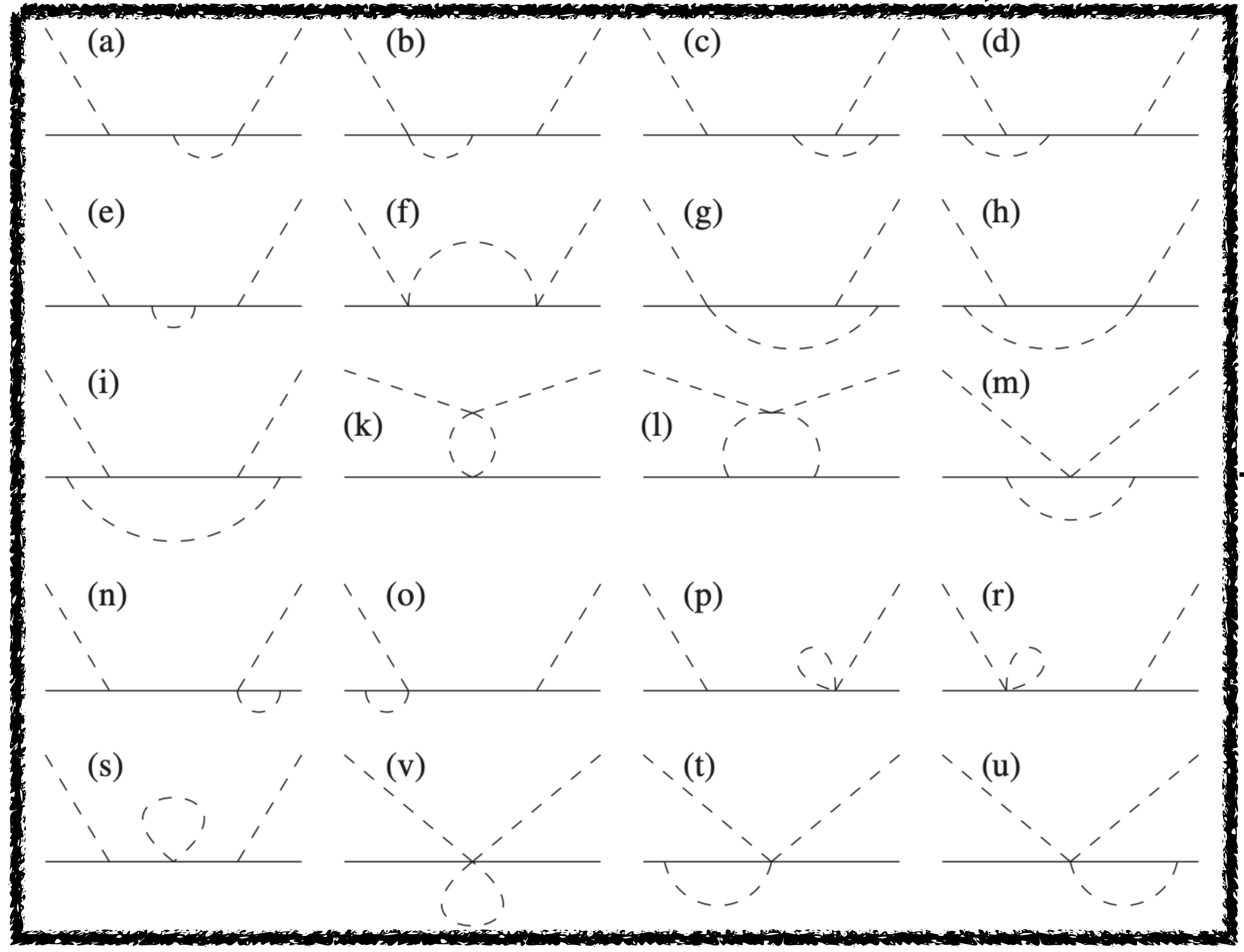
APPLICATIONS I

Meson-baryon scattering from CHPT

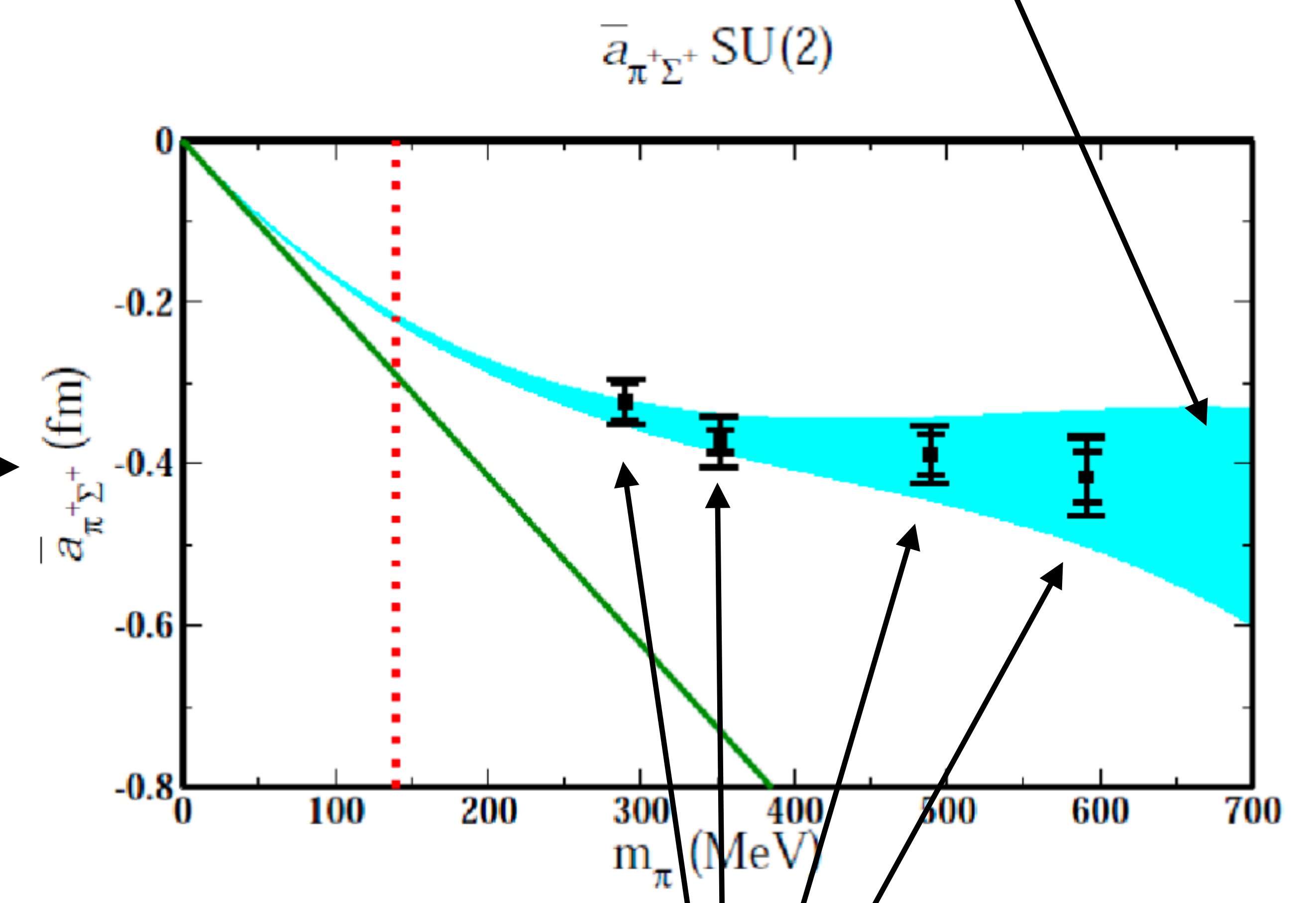
MM/P.C.Bruns/Ulf-G. Meißner/B.Kubis Phys.Rev.D 80 (2009) 094006

- full SU(3) dynamics near threshold
- agrees with experiment in many cases
- well controlled chiral extrapolation of Lattice QCD results ($\pi\Sigma$)

$$
\begin{aligned}
\mathcal{L}^{(2)}_{\phi B} = {} & b_{D/F}\langle \bar{B}[\chi_+, B]_\pm\rangle + b_0\langle \bar{B}B\rangle\langle\chi_+\rangle + b_{1/2}\langle \bar{B}[u_\mu,[u^\mu,B]_\mp]\rangle + b_3\langle \bar{B}\{u_\mu,\{u^\mu,B\}\}\rangle + b_4\langle \bar{B}B\rangle\langle u_\mu u^\mu\rangle \\
& + i\sigma^{\mu\nu}\left(b_{5/6}\langle \bar{B}[[u_\mu,u_\nu],B]_\mp\rangle + b_7\langle \bar{B}u_\mu\rangle\langle u_\nu B\rangle\right) + \frac{ib_{8/9}}{2m_0}\left(\langle \bar{B}\gamma^\mu[u_\mu,[u_\nu,[D^\nu,B]]_\mp]\rangle + \langle \bar{B}\gamma^\mu[D_\nu,[u^\nu,[u_\mu,B]]_\mp]\rangle\right) \\
& + \frac{ib_{10}}{2m_0}\left(\langle \bar{B}\gamma^\mu\{u_\mu,\{u_\nu,[D^\nu,B]\}\}\rangle + \langle \bar{B}\gamma^\mu[D_\nu,\{u^\nu,\{u_\mu,B\}\}]\rangle\right) + \frac{ib_{11}}{2m_0}\left(2\langle \bar{B}\gamma^\mu[D_\nu,B]\rangle\langle u_\mu u^\nu\rangle\right. \\
& \left. + \langle \bar{B}\gamma^\mu B\rangle\langle [D_\nu,u_\mu]u^\nu + u_\mu[D_\nu,u^\nu]\rangle\right), \qquad (8)
\end{aligned}
$$



MM/P.C.Bruns/Ulf-G. Meißner/B.Kubis Phys.Rev.D 80 (2009) 094006



LQCD at unphysical pion masses
Torok/Beane/Detmold/Luu/... Phys.Rev.D 81 (2010) 074506

AXIAL-VECTOR MESON*

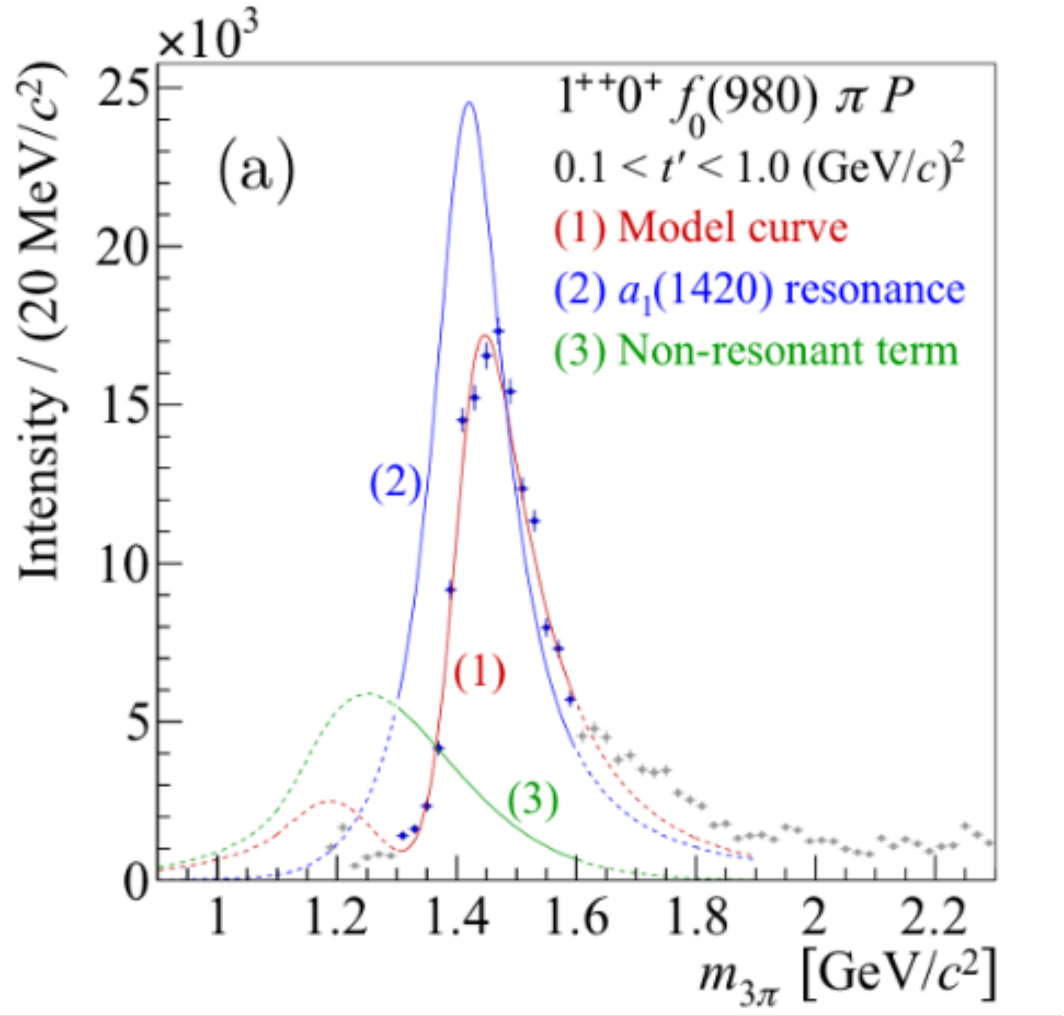
Excited axial-vector meson: $a_1(1420)$

Observed by COMPASS/Belle in $\pi^-\pi^+\pi^-$ final state COMPASS:2015kdx, Rabusov:2023tna

Creation mechanisms:

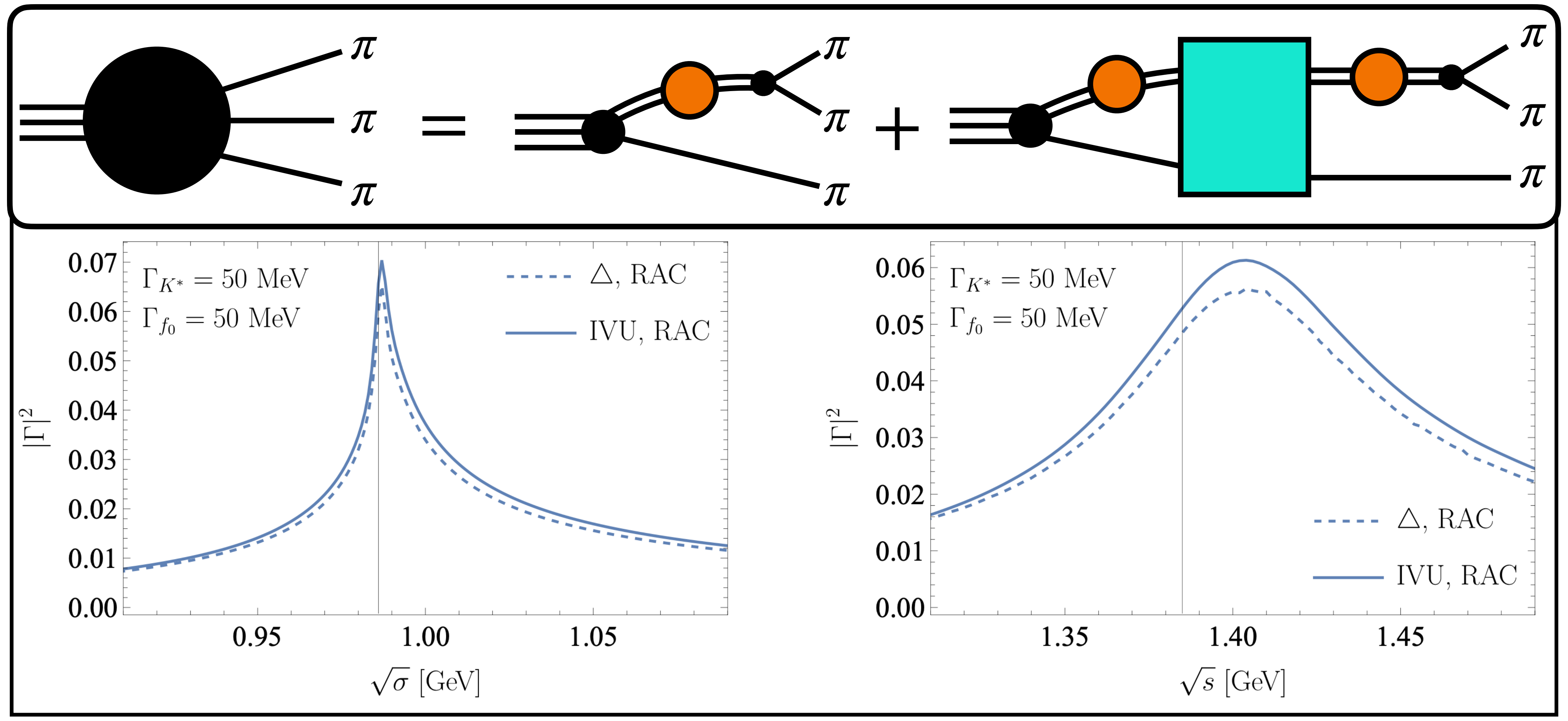
- Excited state of $a_1(1260)$ COMPASS:2020yhb
- "Triangle singularity" $K^*(892)\bar{K} \xrightarrow{K} \pi f_0(980)$

Mikhasenko:2015oxp **Review: Guo:2019twa**
Related: Dai:2018hqb, Dai:2018rra, Liang:2019jtr, Jing:2019cbw, Du:2021zdg, Duan:2023dky, Wang:2016dtb, Nakamura:2023obk, Zhang:2024dth, Achasov:2022onn, Nakamura:2023hbt, arXiv:1609.04133 [hep-ph].
Talks: J.J.Wu — Z.Zhang



3b unitary formalism IVU Ajay Sakthivasan/MM/Döring/Rusetsky 2407.17969 [hep-ph]

- formalism to incorporate both hypothesis
- full 2- and 3-body re-scattering
- **for now:** only kinematic/analytical properties (no spin)



➡ Effect is small but distinguishable

➡ Add spin, fit to the line-shapes … in progress

VECTOR MESON

Mapping to infinite volume

- 3-body quantization condition

FVU

$$\det\left[\, 2L^3E_{\mathbf{p}}\left(\tilde{K}^{-1} - \Sigma^L\right) - B - C \,\right]^{\Lambda} \equiv 0$$

MM/Döring
Eur.Phys.J.A 53 (2017) 12, 240

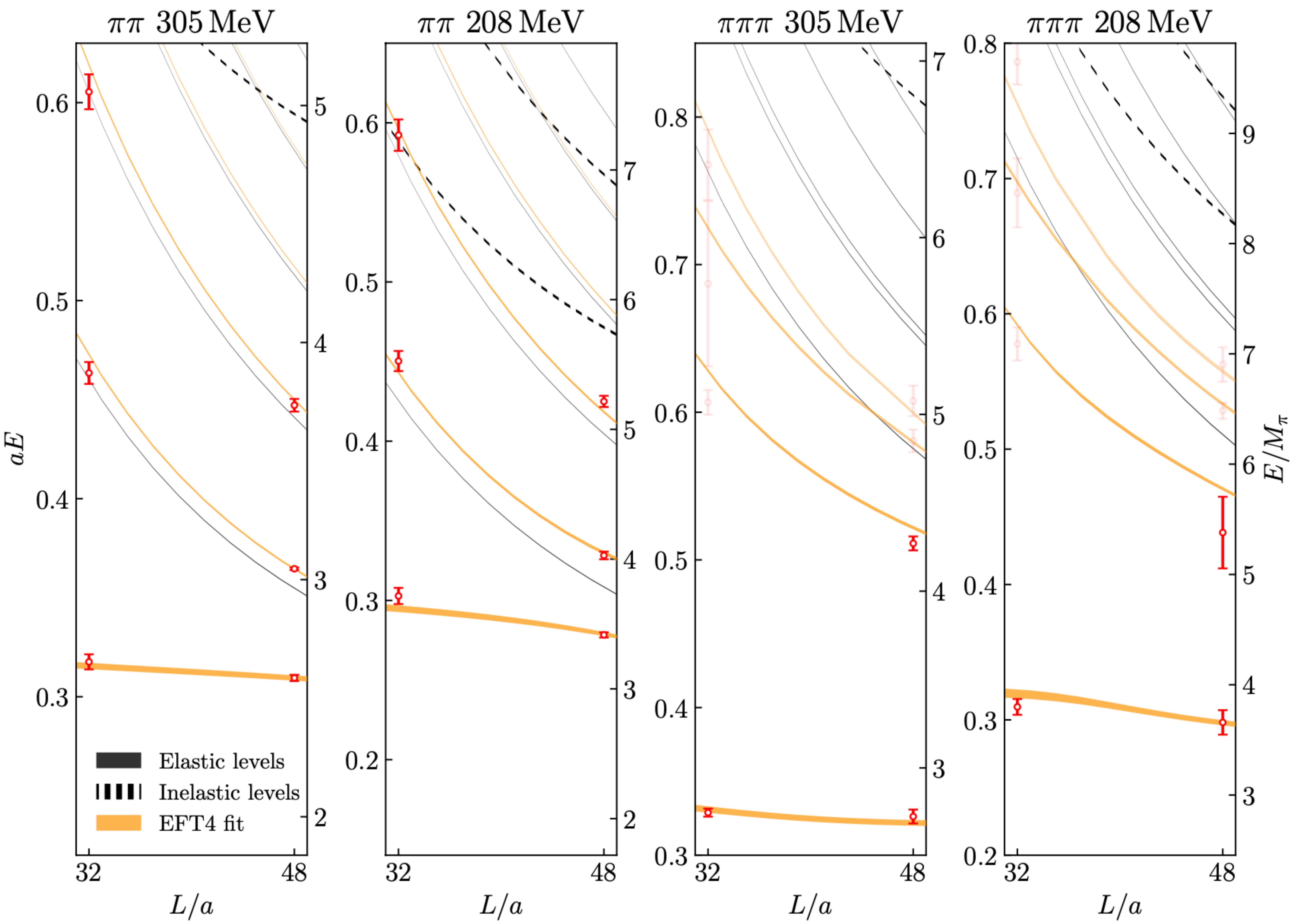
- Volume-independent 2-,3-body force
$C, \tilde{K}$

- saturated by meson s-channel interaction — **EFT form**

$$\left[\tilde{K}^{-1}\right]_{\boldsymbol{p}'\lambda',\boldsymbol{p}\lambda} = \delta_{\lambda'\lambda}\delta_{\boldsymbol{p}'\boldsymbol{p}}\frac{\sigma_p - M_\rho^2}{2g^2}\,,$$

$$\tilde{c}_{11} = \frac{6s(M_\rho^2 - \sigma_q + 6g^2 f_\pi^2)(M_\rho^2 - \sigma_p + 6g^2 f_\pi^2)}{64g^2\pi^3 f_\pi^6(s - M_\omega^2)}\,,$$

Finite-volume spectrum = Energy eigenvalues



H.Yan/MM/Garofalo/Meißner/Lui/Liu/Urbach: 2407.16659 [hep-lat]

Current frontier: 3-body dynamics from LQCD

➥ 3-body Quantization Conditions1

➥ RFT / FVU / NREFT

➥ many perturbatively interacting systems are studied2

$$0 = \det\left(L^3\left(\tilde{F}/3 - \tilde{F}(\tilde{K}_2^{-1} + \tilde{F} + \tilde{G})^{-1}\tilde{F}\right)^{-1} + K_{\mathrm{df},3}\right)$$ RFT

$$0 = \det\left(B_0 + C_0 - E_L\left(K^{-1}/(32\pi) + \Sigma_L\right)\right)$$ FVU

3-body force

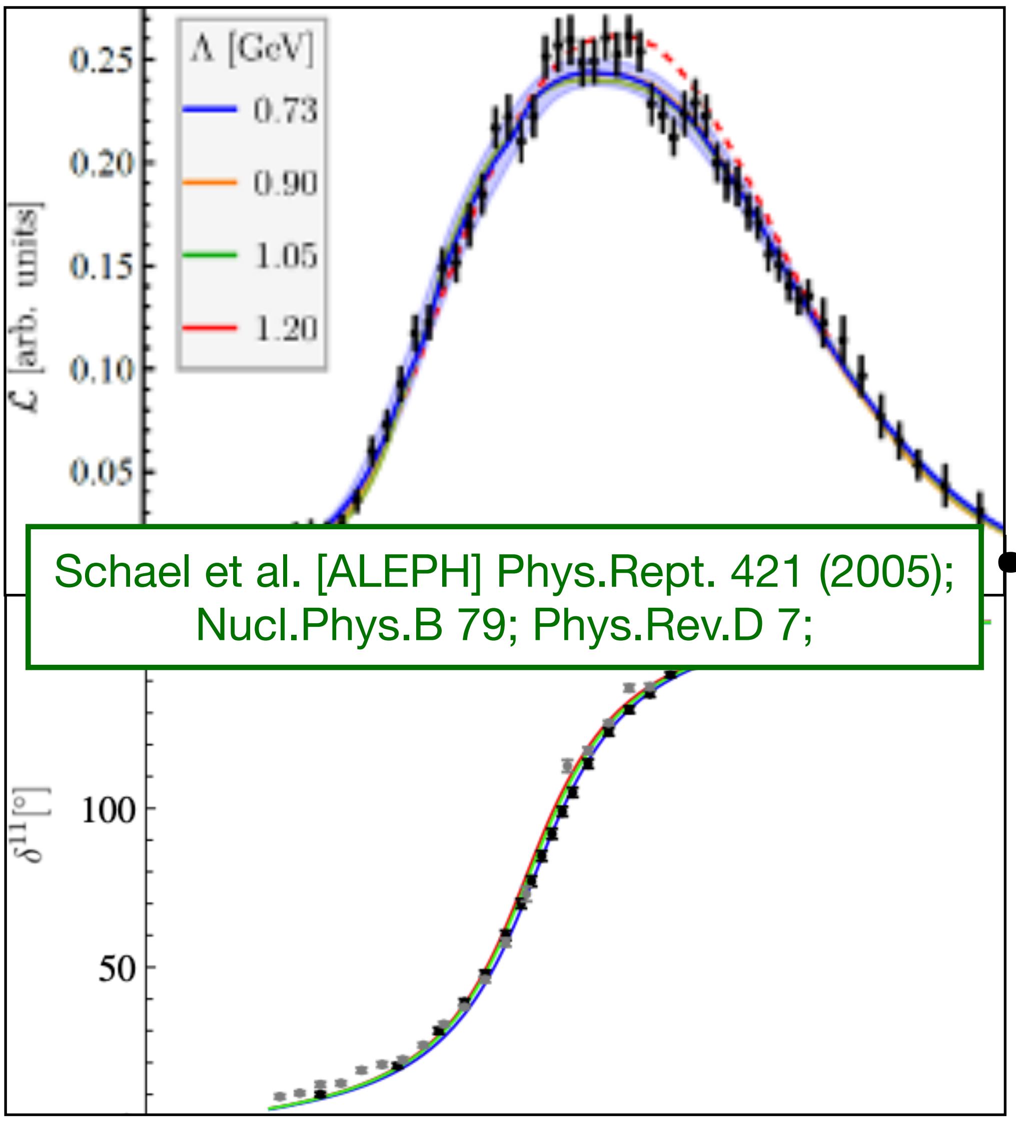
2-body interaction

one-particle exchange

2-body self-energy

1) Rusetsky, Bedaque, Grießhammer, Sharpe, Meißner, Döring, Hansen, Davoudi, Guo....
Reviews:
Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019);
MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);
2) MM/Döring PRL122(2019); Blanton et al. PRL 124 (2020); Hansen et al. PRL 126 (2021); ....

AXIAL-VECTOR MESON



Schael et al. [ALEPH] Phys.Rept. 421 (2005); Nucl.Phys.B 79; Phys.Rev.D 7;

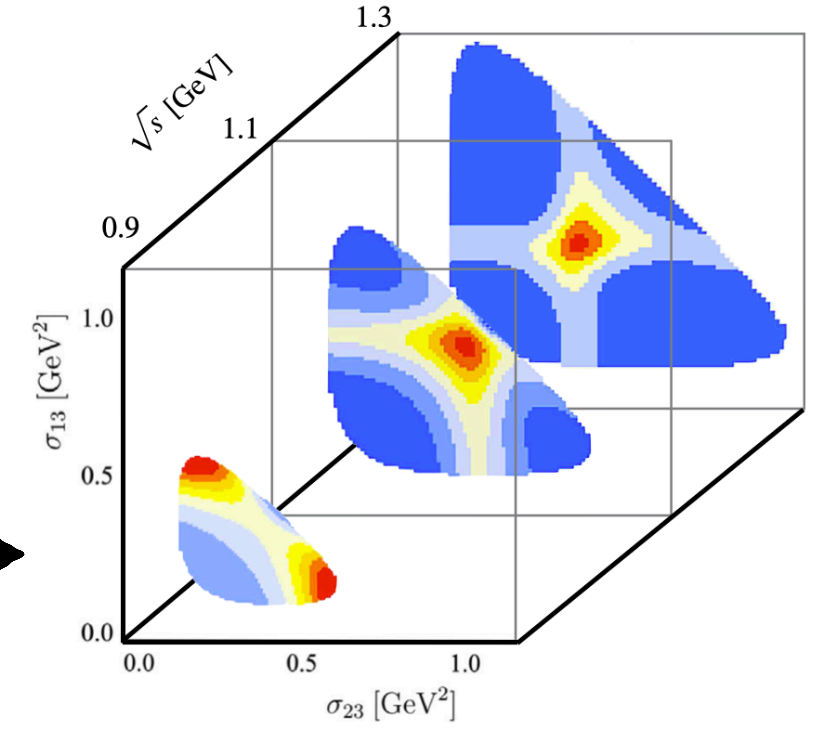
INPUT
EXPERIMENT

IVU

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}^{-1} - \Sigma} T^c$$

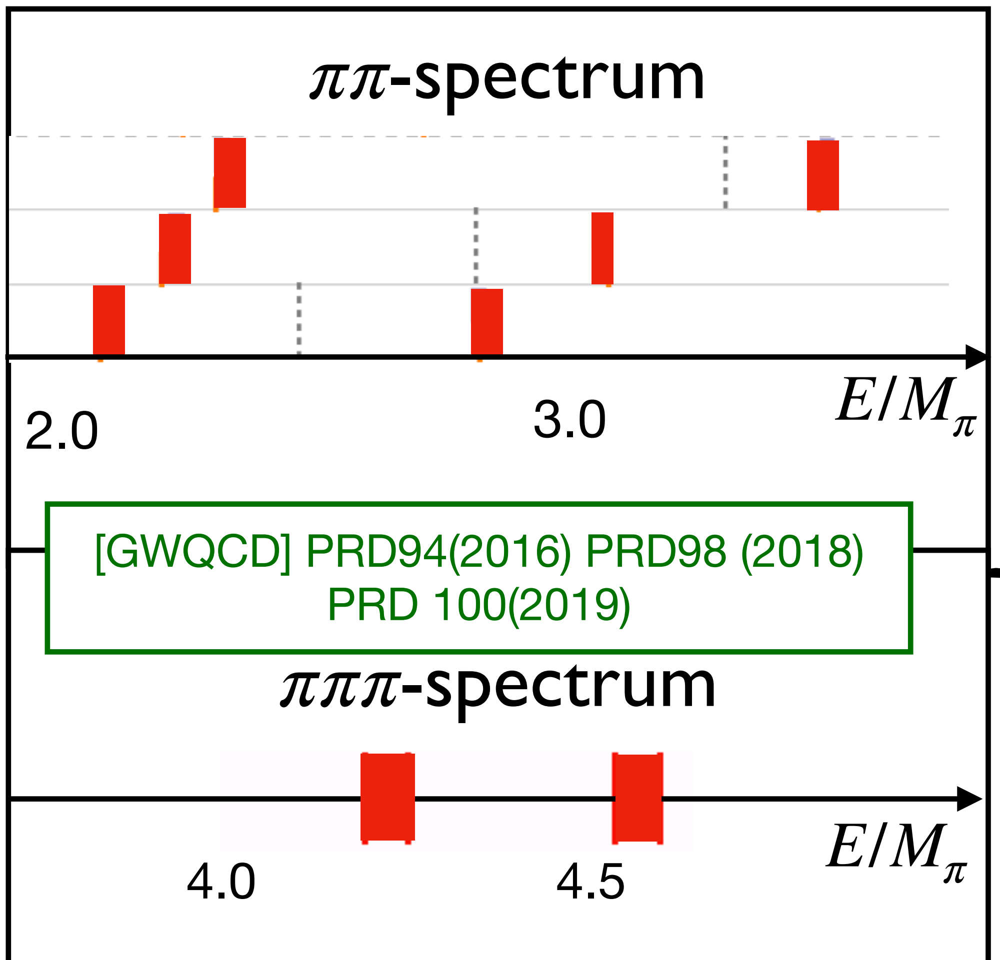
OUTPUT

Sadasivan/MM/... Phys.Rev.D 101 (2020);



$C, \tilde{K}$

volume independent



[GWQCD] PRD94(2016) PRD98 (2018) PRD 100(2019)

LATTICE QCD

$[M_\pi = 315\,\mathrm{MeV}]$

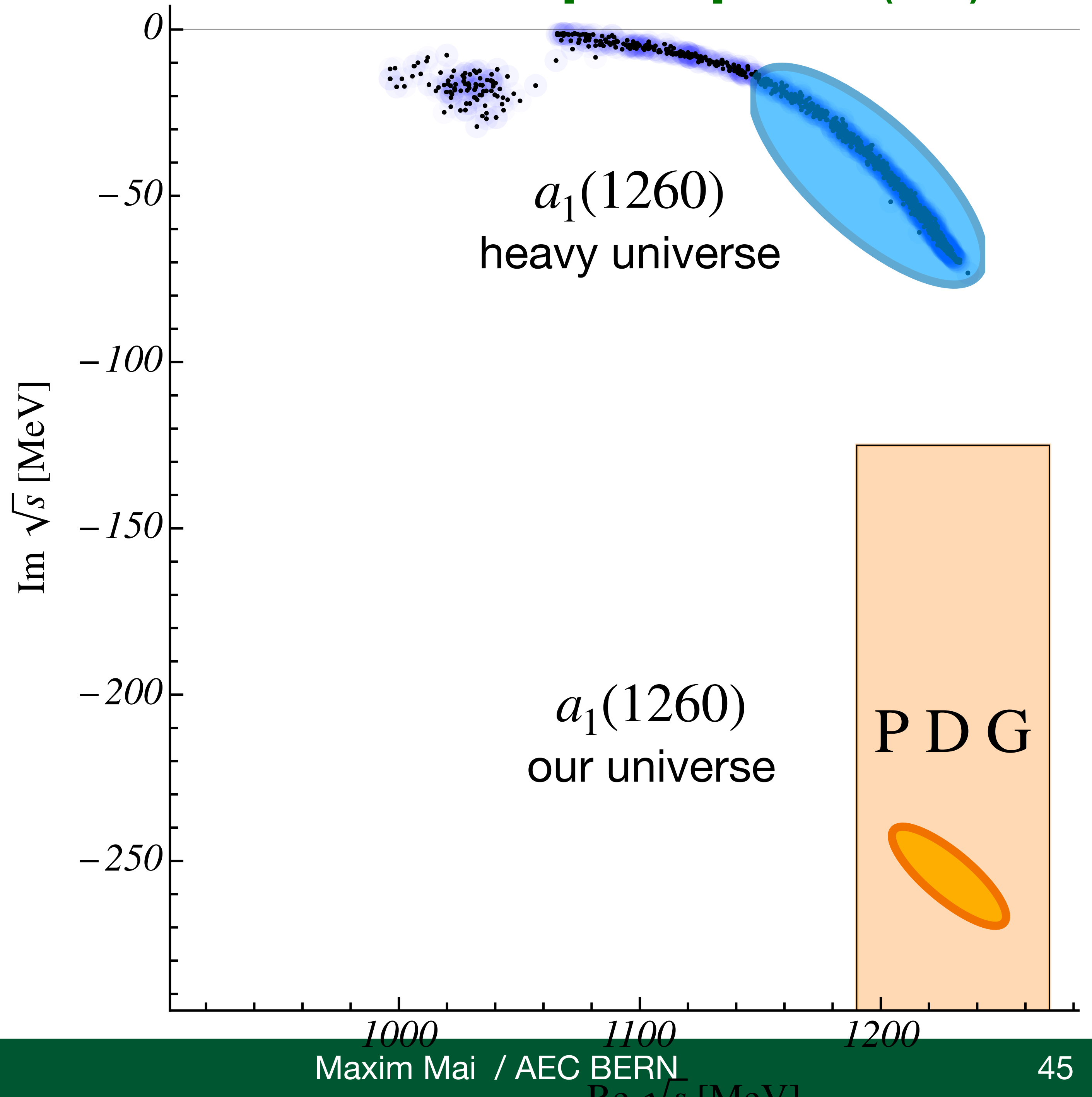
INPUT
Lattice QCD

FVU

$$\det\left[\, 2L^3 E_{\mathbf{p}} \left(\tilde{K}^{-1} - \Sigma^L\right) - B - C \,\right]^{\Lambda} \equiv 0$$

OUTPUT

MM/Culver/... [GWQCD] PRL 127 (2021)



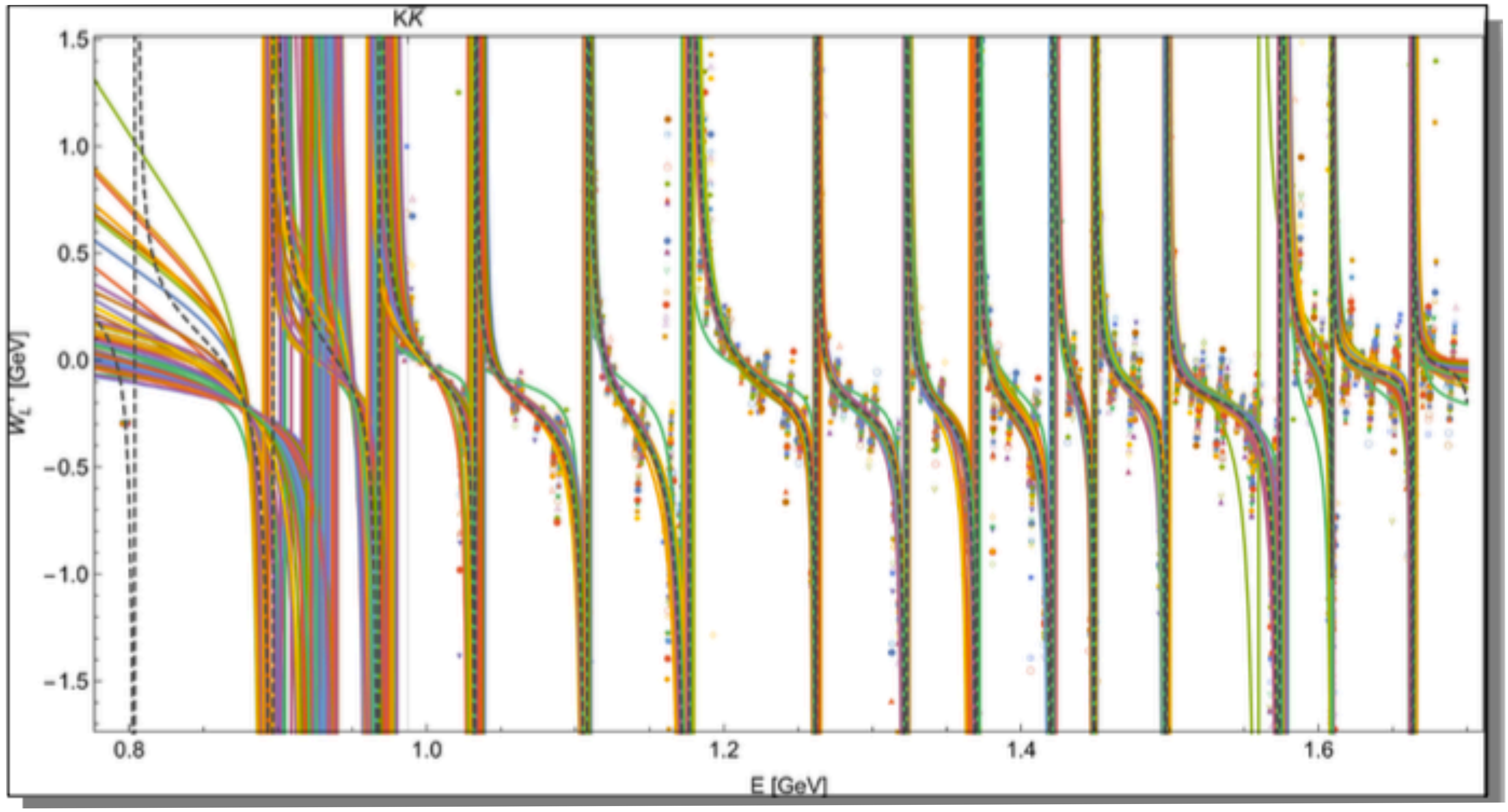
OPTICAL POTENTIAL

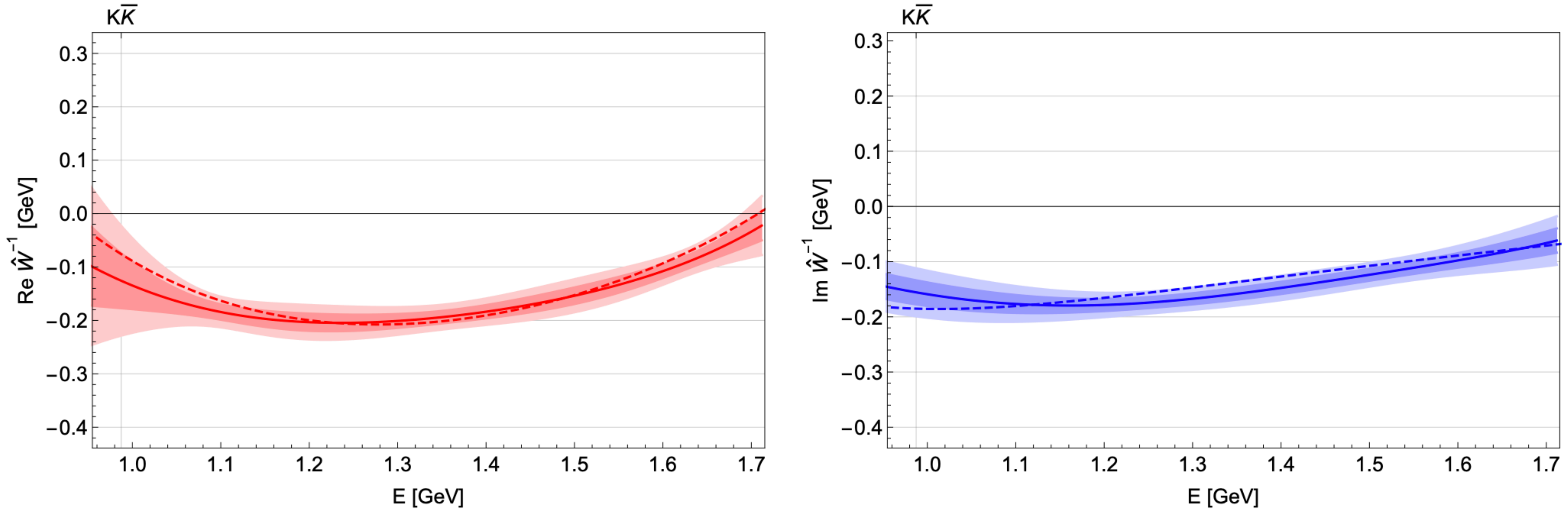
Alternatively: bulk properties of int. spectra

- double-limit prescription[1]

$$W^{-1}(E) = \lim_{\epsilon \to 0} \lim_{L \to \infty} W_L^{-1}(E + i\epsilon)$$

- smoothing and inverse problem[2]
- typically many input (EEVs) required





Agadjanov/MM/.. JHEP 06 (2016)

1) DeWitt, Phys. Rev. 103 (1956)
2) Agadjanov/MM/.. JHEP 06 (2016); Hansen et al. Phys.Rev.D 96 (2017) Bulava/Hansen Phys.Rev.D 100 (2019); Briceño et al. Phys.Rev.D 103 (2021)

STRANGENESS

$$
\begin{aligned}
\mathcal{L}_{\phi B}^{(2)} = {} & b_{D/F}\langle \bar{B}[\chi_+, B]_\pm\rangle + b_0\langle \bar{B}B\rangle\langle\chi_+\rangle + b_{1/2}\langle \bar{B}[u_\mu, [u^\mu, B]_\mp]\rangle + b_3\langle \bar{B}\{u_\mu, \{u^\mu, B\}\}\rangle + b_4\langle \bar{B}B\rangle\langle u_\mu u^\mu\rangle \\
& + i\sigma^{\mu\nu}(b_{5/6}\langle \bar{B}[[u_\mu, u_\nu], B]_\mp\rangle + b_7\langle \bar{B}u_\mu\rangle\langle u_\nu B\rangle) + \frac{ib_{8/9}}{2m_0}(\langle \bar{B}\gamma^\mu[u_\mu, [u_\nu, [D^\nu, B]]_\mp]\rangle + \langle \bar{B}\gamma^\mu[D_\nu, [u^\nu, [u_\mu, B]]_\mp]\rangle) \\
& + \frac{ib_{10}}{2m_0}(\langle \bar{B}\gamma^\mu\{u_\mu, \{u_\nu, [D^\nu, B]\}\}\rangle + \langle \bar{B}\gamma^\mu[D_\nu, \{u^\nu, \{u_\mu, B\}\}]\rangle) + \frac{ib_{11}}{2m_0}(2\langle \bar{B}\gamma^\mu[D_\nu, B]\rangle\langle u_\mu u^\nu\rangle \\
& + \langle \bar{B}\gamma^\mu B\rangle\langle [D_\nu, u_\mu]u^\nu + u_\mu[D_\nu, u^\nu]\rangle), \qquad (8)
\end{aligned}
$$

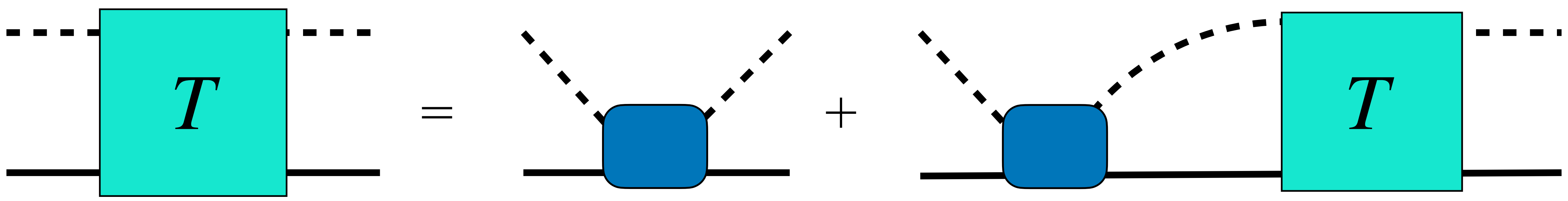
$$
\begin{aligned}
a_{\bar{K}N}^{I=0} &= \Big((+0.53)_{\mathrm{LO}} + (+0.97)_{\mathrm{NLO}} + (-0.40 + 0.22i)_{\mathrm{NNLO}} + \ldots\Big)\ \mathrm{fm}\,, \\
a_{\bar{K}N}^{I=1} &= \Big((+0.20)_{\mathrm{LO}} + (+0.22)_{\mathrm{NLO}} + (-0.26 + 0.18i)_{\mathrm{NNLO}} + \ldots\Big)\ \mathrm{fm}\,.
\end{aligned}
$$

Meson-baryon scattering from CHPT

MM/P.C.Bruns/Ulf-G. Meißner/B.Kubis Phys.Rev.D 80 (2009) 094006

- Fails for resonant (strangeness) channel
  - Kaon mass is large → convergence
  - Relevant thresholds are widely separated → convergence
  - Resonance just below $\bar{K}N$ threshold → non-perturbative effect

- Extension to resonant channels/higher energies — Chiral Unitary Approach (**UCHPT**)



EXAMPLE $\Lambda(1405)\ldots\Lambda(1380)$

Long history of experimental and theoretical efforts

- Sub-($\bar{K}N$)-threshold $\Lambda(1405)$ resonance
- second state $\Lambda(1380)$ predicted from UCHPT
- no direct experimental verification
- confirmed by many critical tests & LQCD