Does η/s depend on the EoS?

Pasi Huovinen

Institute of Physics Belgrade

Nuclear Physics seminar

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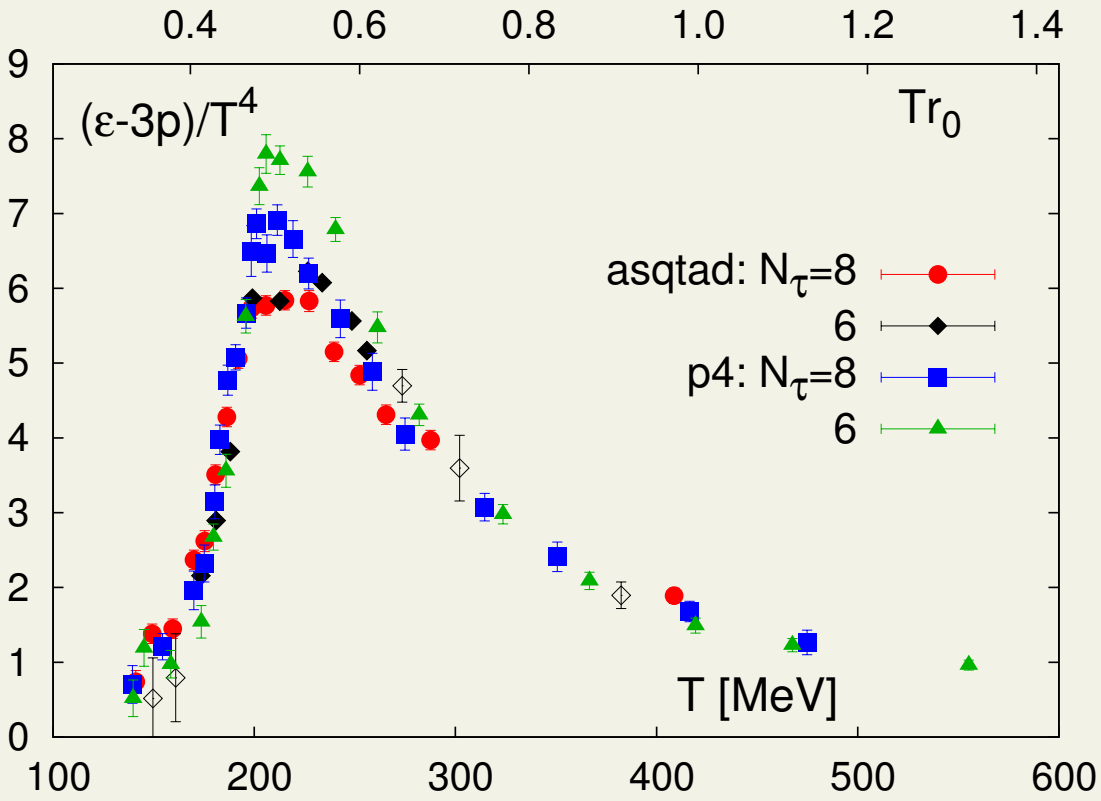
reporting work done by Jussi Auvinen and Harri Niemi

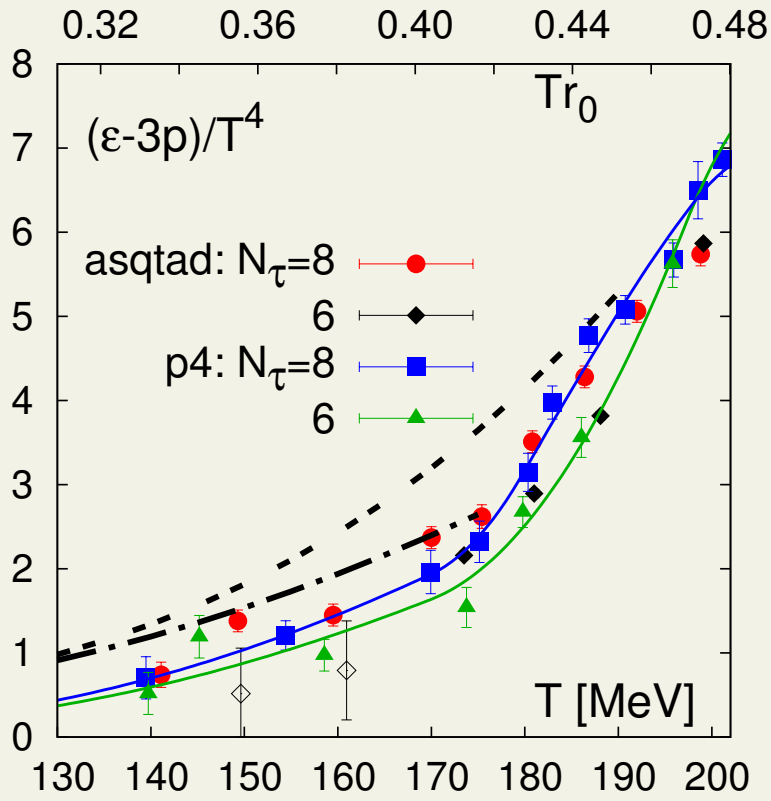
in collaboration with Kari J. Eskola, Risto Paatelainen, and Peter Petreczky

Does η/s extracted from the data depend on the EoS used in the calculations?

Lattice EoS at 2009

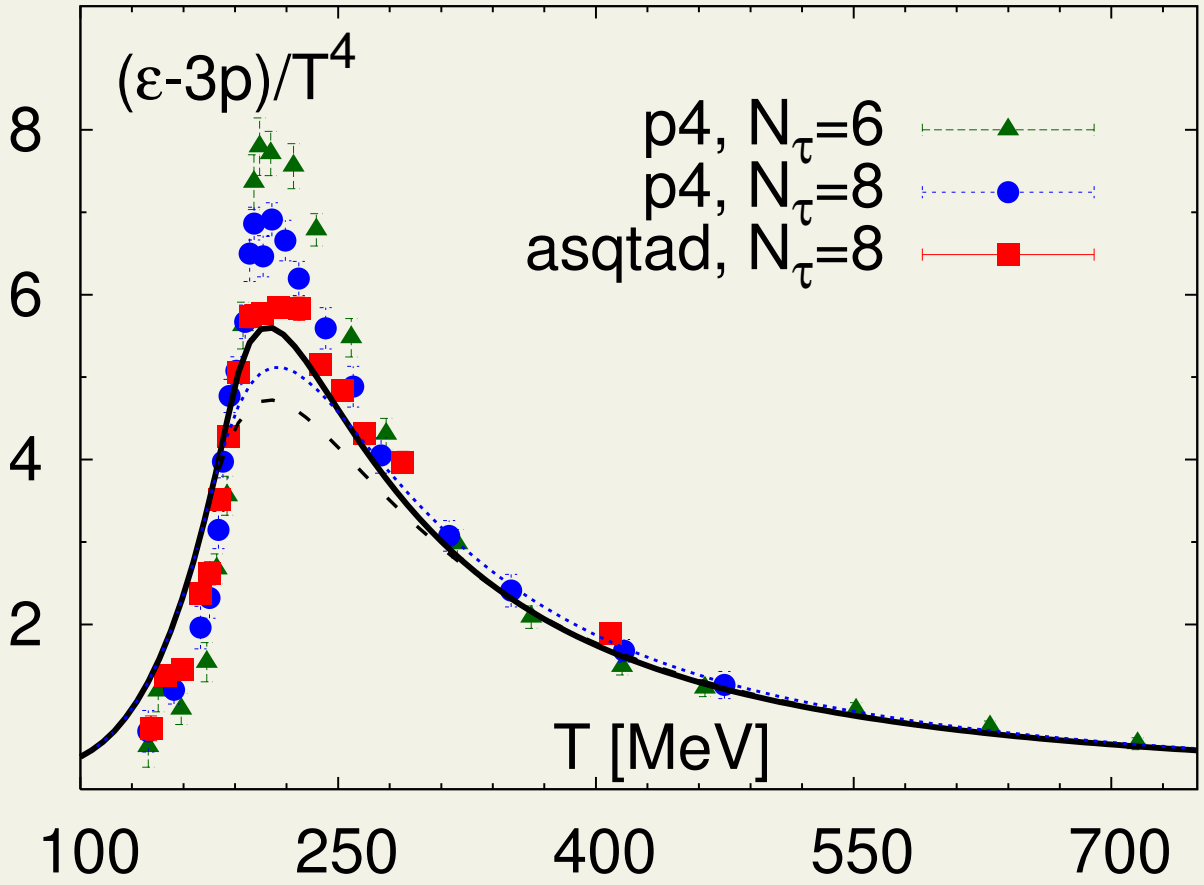
Bazavov *et al.* [hotQCD collaboration] arXiv:0903.4379 [hep-lat]





- **Good at large T, not at low T**

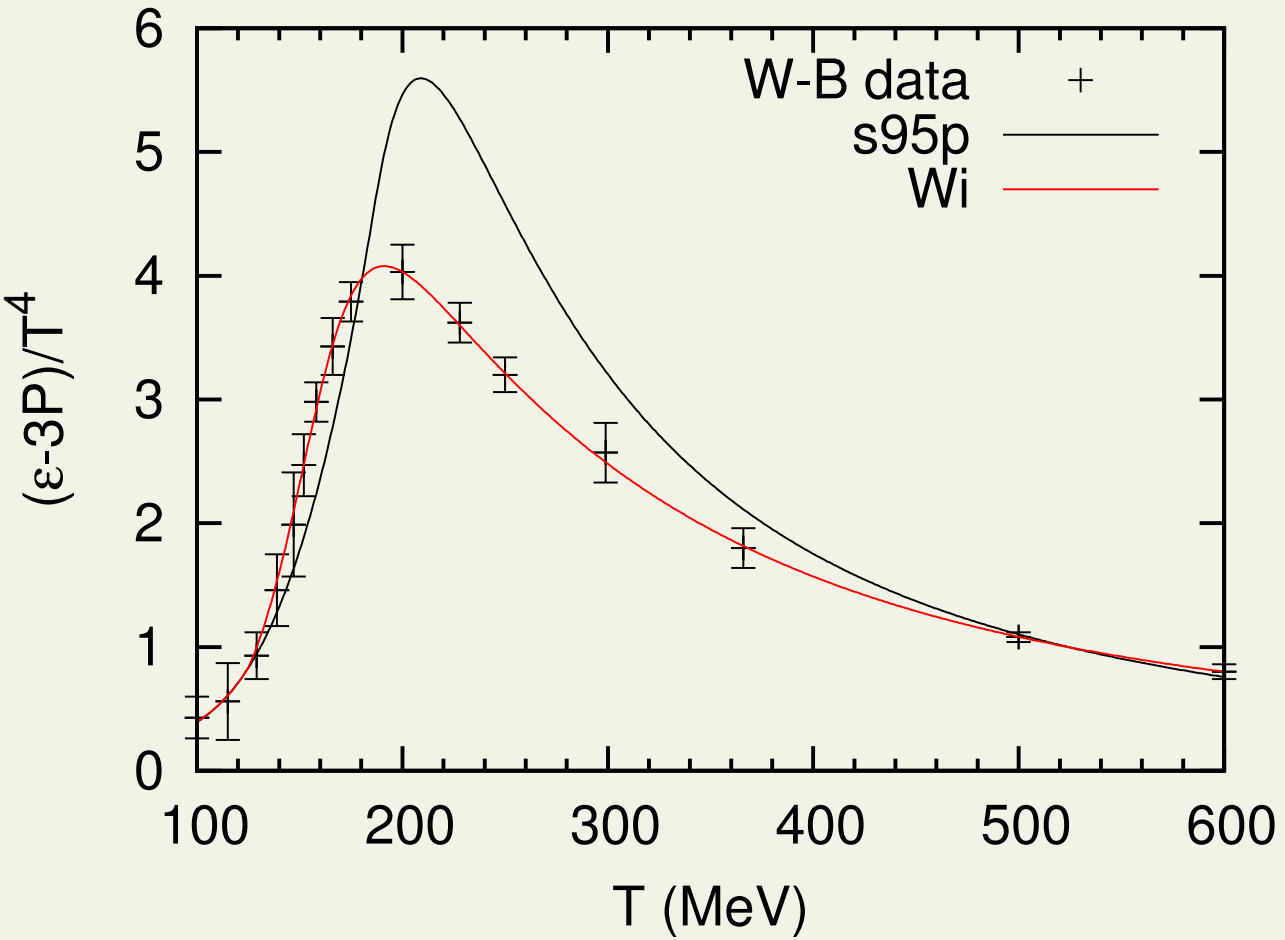
s95p



- **HRG below $T \approx 170$–190 MeV**
- **lattice above $T = 250$ MeV**
- **interpolate between**

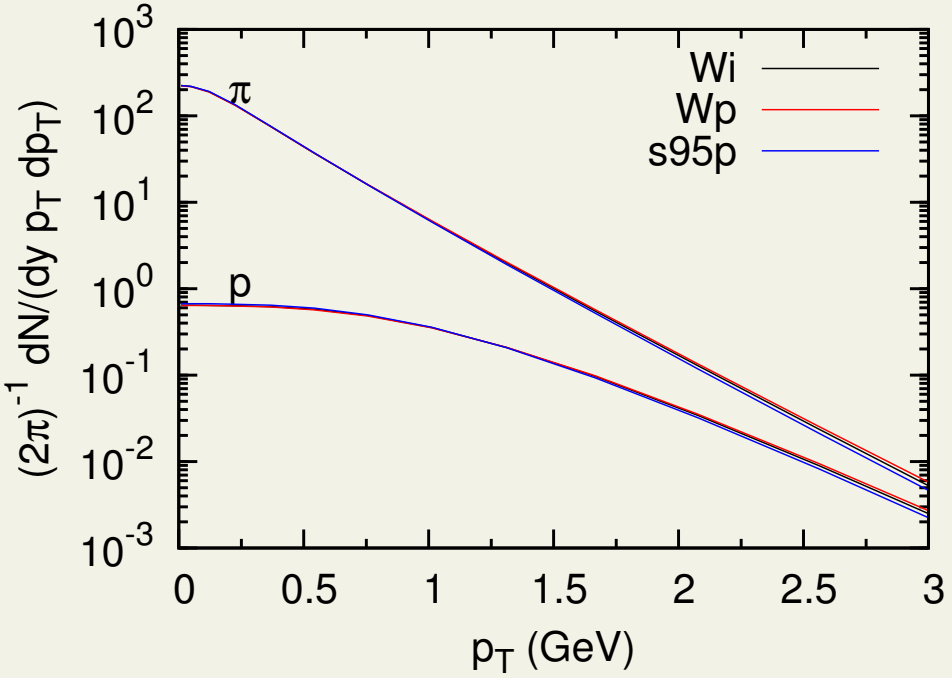
Budapest-Wuppertal trace anomaly

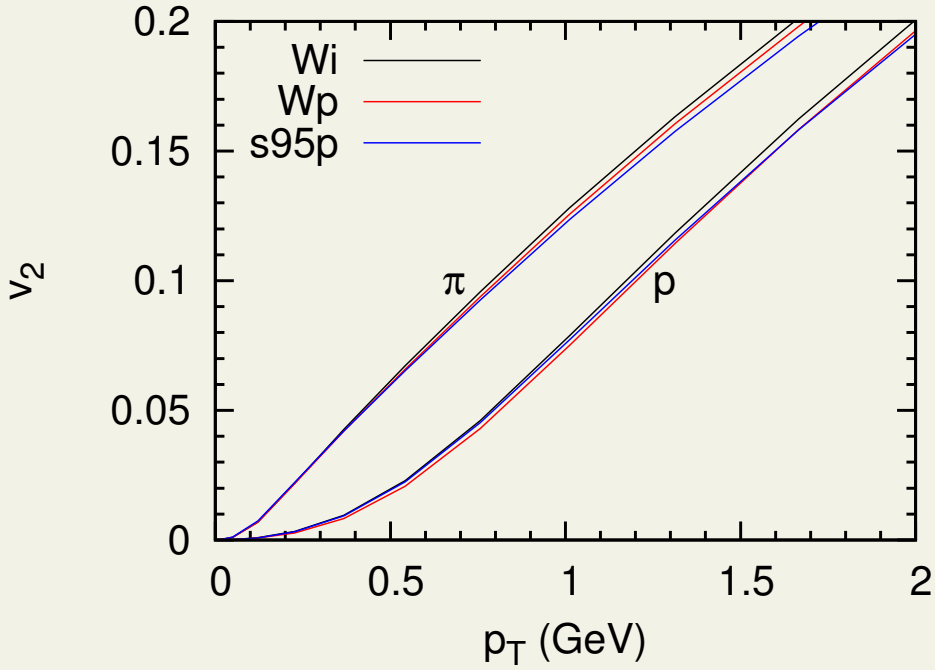
Borsanyi *et al.*, arXiv:1007.2580



Effect on distributions

- **ideal fluid**
- **Au+Au collision at RHIC, $\sqrt{s} = 200$ GeV, b=7 fm**
- $T_{\rm dec} = 124$ **MeV; all EoSs!**

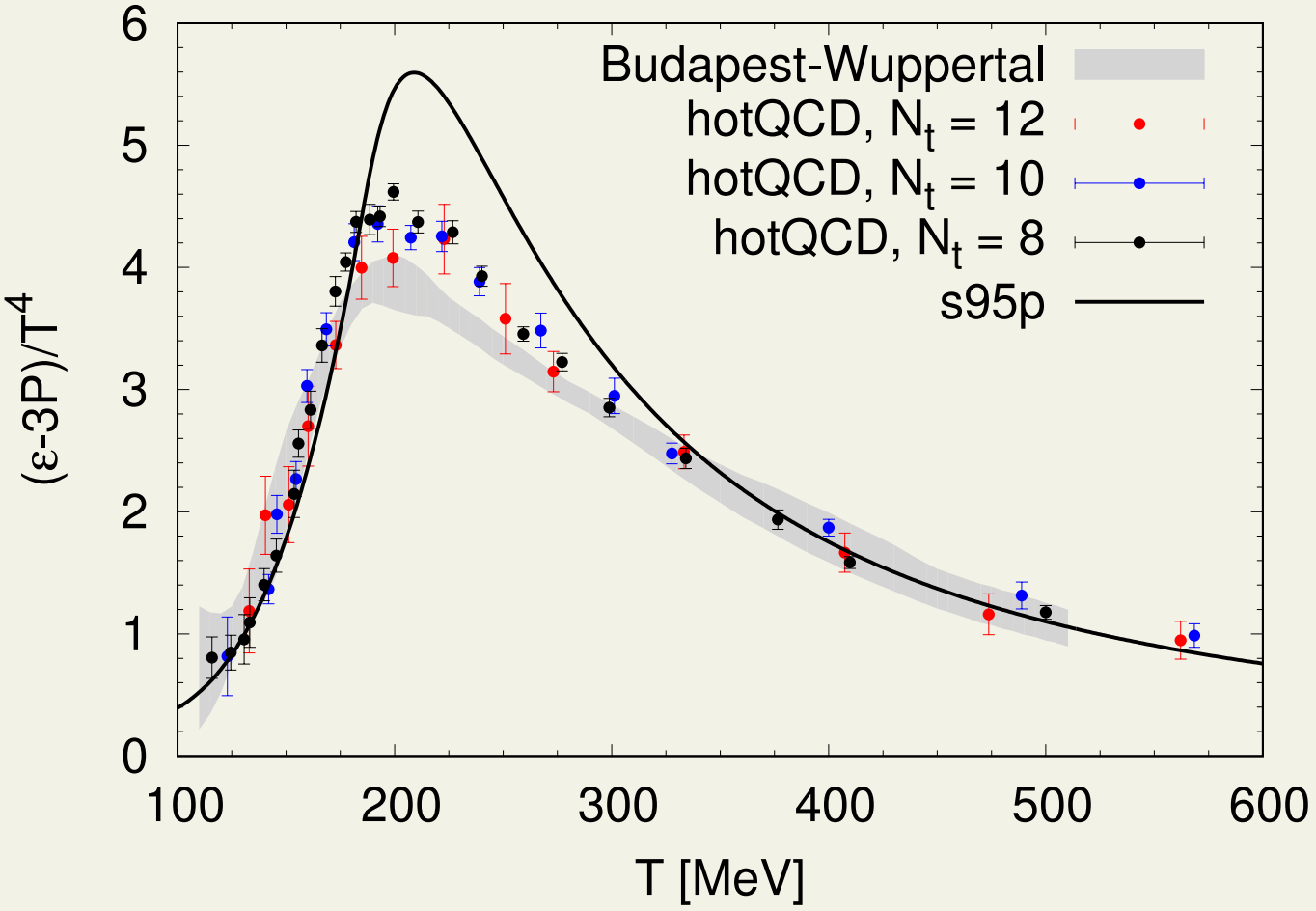




Effect on η/s

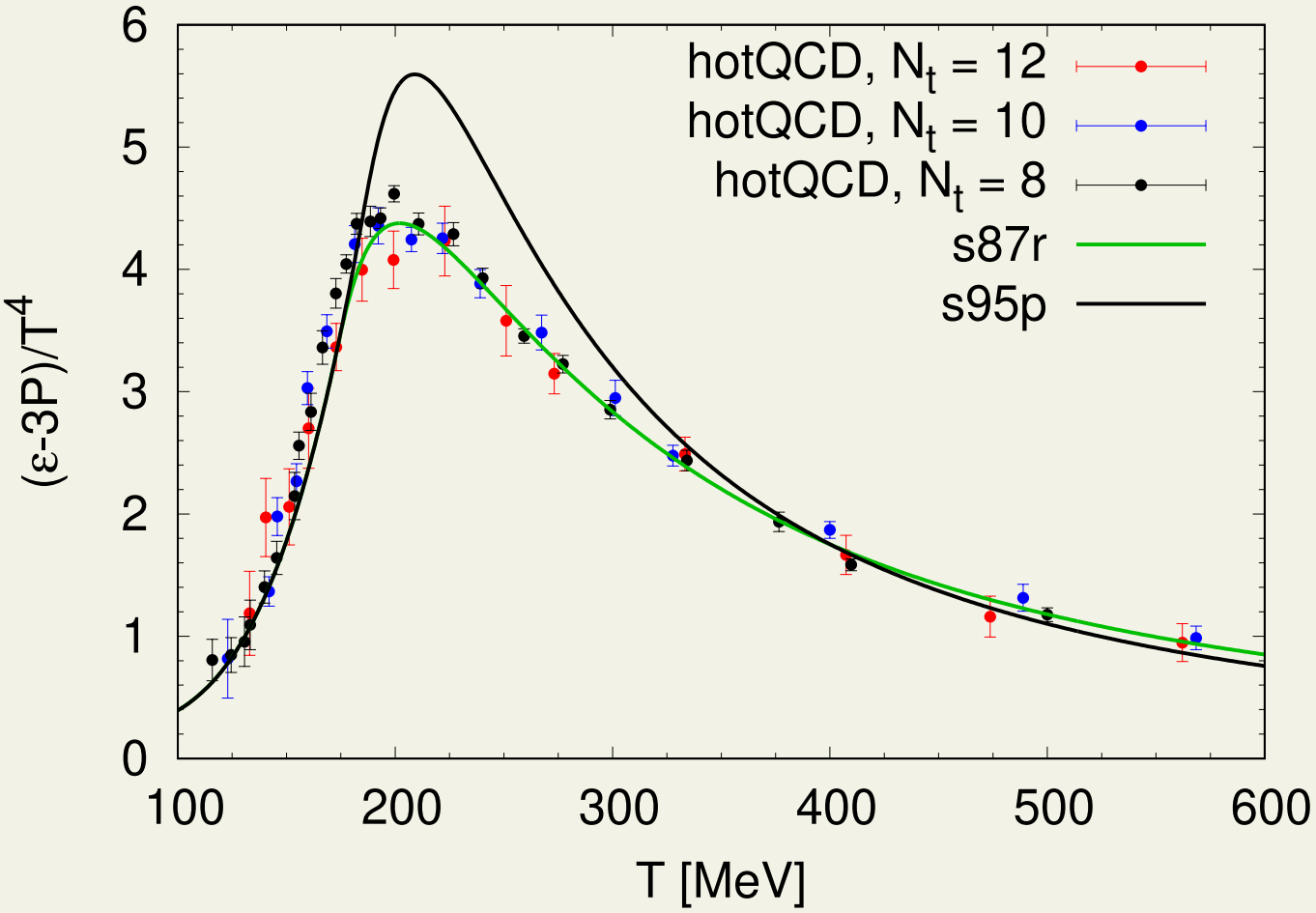
- **Alba $et\ al.$, arXiv:1711.05207**
  - **s95p:** $\eta/s = 0.025$
  - **B-W:** $\eta/s = 0.047$
- **Schenke $et\ al.$, arXiv:1901.04378**
  - **s95p:** $\eta/s = 0.095$
  - **B-W:** $\eta/s = 0.12$

Lattice EoS at 2018



- **s95p: PDG 2005, hotQCD 2008**

New EoS parametrisations



- **s87r: PDG 2005, latest hotQCD data**
- **s95p: PDG 2005, hotQCD 2008**

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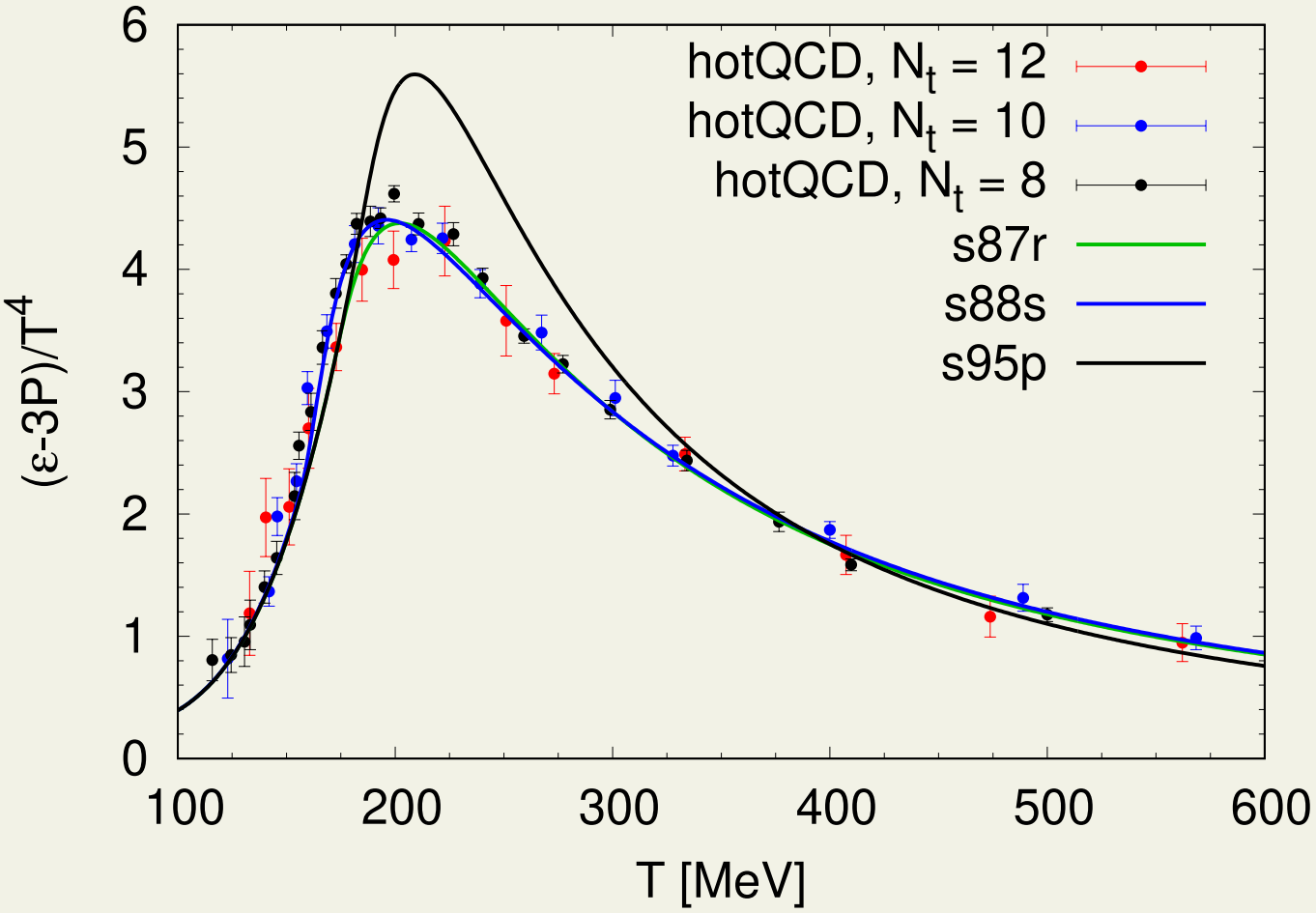

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- **s88s: PDG 2017, latest hotQCD data**
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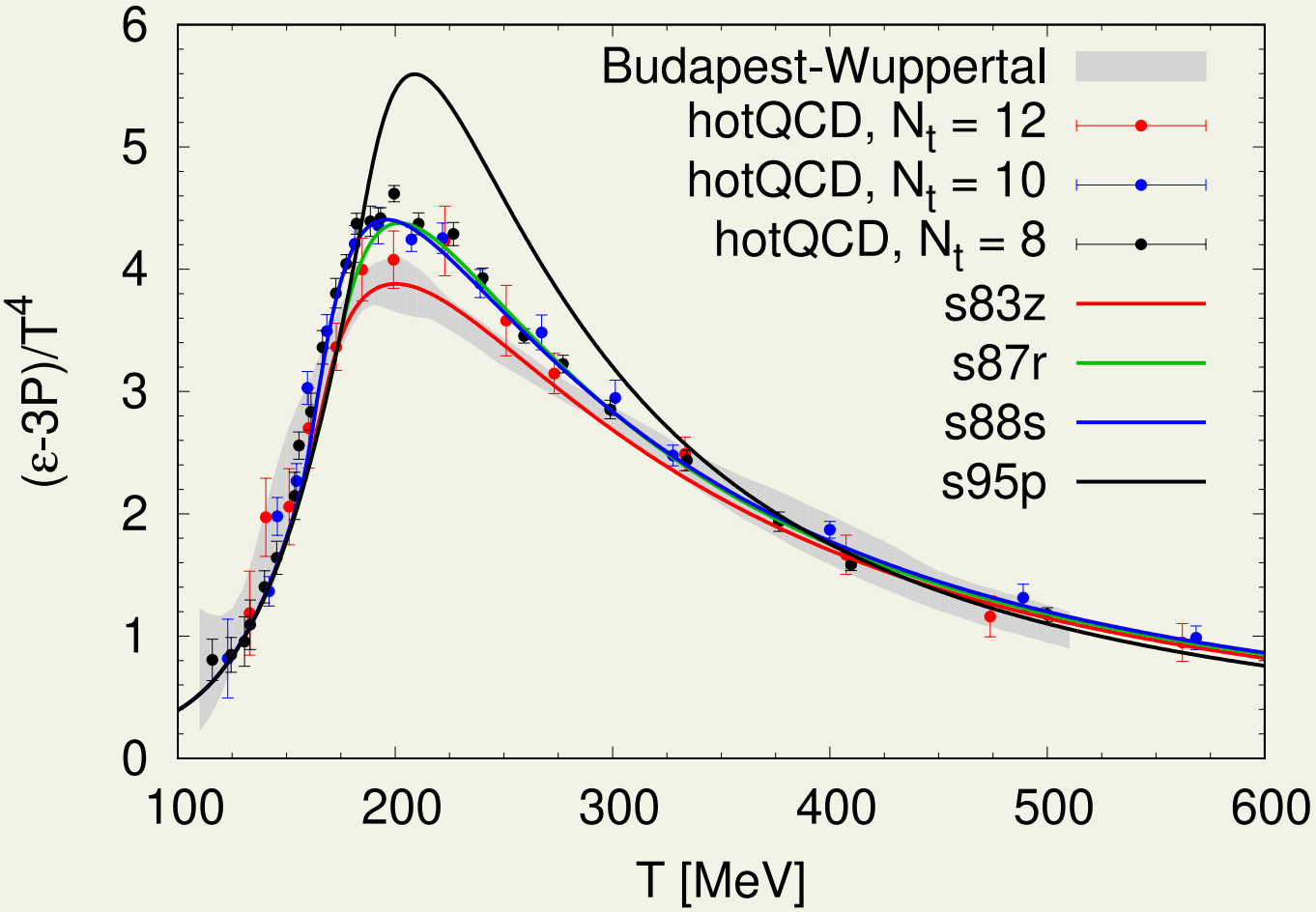

- **s83z: PDG 2017, latest B-W data**
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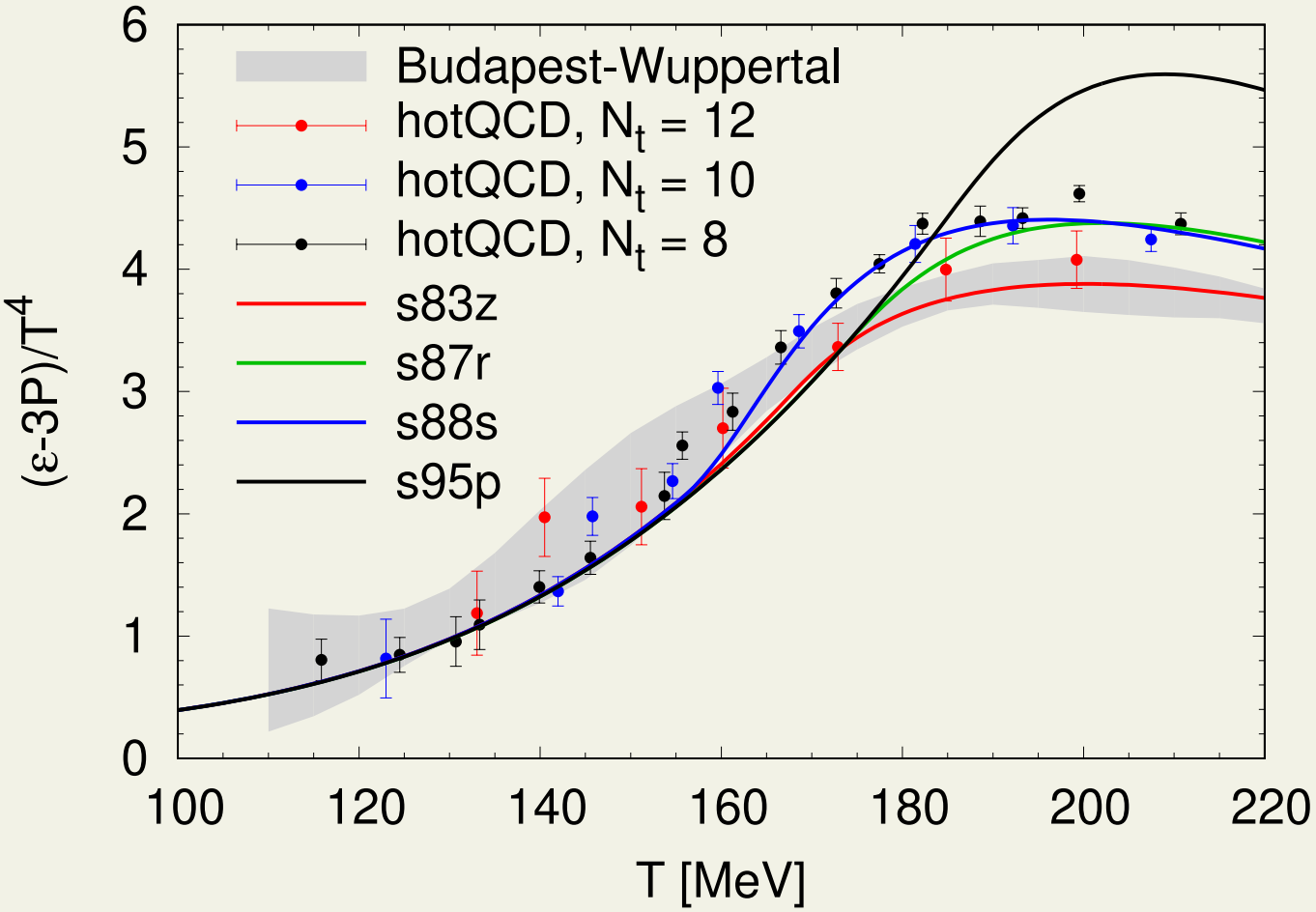

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Modeling problem

Model parameters (input): $\vec{x} = (x_1, ..., x_n)$

$(\tau_0, \epsilon_{\rm init}, \eta/s, T_{\rm dec}, T_{\rm chem}, \ldots)$

$\Downarrow$

Model output $\vec{y} = (y_1, ..., y_m) \Leftrightarrow$ **Experimental values** $\vec{y}^{\rm exp}$

$(\mathrm{d}N/\mathrm{d}y, \langle p_T \rangle, v_n, \ldots)$

- **Which values of input parameters $\vec{x}$ give the best reproduction of experimental output $\vec{y}^{\rm exp}$?**

- **What is the level of uncertainty of these values?**

Bayesian analysis

Model parameters (input): $\vec{x} = (x_1, ..., x_n)$

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Bayes' theorem:

Posterior probability $\propto$ **Likelihood** $\cdot$ **Prior knowledge**

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- **Prior knowledge: Range of parameter values**

Bayesian analysis

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Posterior probability $\propto$ **Likelihood** $\cdot$ **Prior knowledge**

- **Likelihood:** $\mathcal{L}(\vec{x}) \propto \exp\left(-\frac{1}{2}(\vec{y}(\vec{x}) - \vec{y}^{\rm exp})\Sigma^{-1}(\vec{y}(\vec{x}) - \vec{y}^{\rm exp})^T\right)$,
  where Σ is the covariance matrix

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  where Σ **is the covariance matrix**
- **evaluation of the likelihood function** $\mathcal{O}(10^8)$ **runs. . .**
- **use Gaussian emulator instead**
  = stochastic, non-parametric interpolation of the model

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- **evaluation of the likelihood function $\mathcal{O}(10^8)$ runs. . .**
- **use Gaussian emulator instead**
  = stochastic, non-parametric interpolation of the model
- **Sample the likelihood function using Markov chain Monte Carlo**
  = random walk in parameter space constrained to favour high likelihood

  $\rightarrow$ distribution of Markov chain steps $\equiv$ probability distribution

The model

- **2+1D viscous hydro with shear viscosity only**
  - **event averaged EKRT initialisation, normalisation parameter** $K_{\rm sat}$
  - $\tau_0 = 0.2$ **fm fixed**
  - **initial** $v_r = 0$ **and** $\pi^{\mu\nu} = 0$

- $T_{\rm dec}$ **and** $T_{\rm chem}$ **free parameters**

- $(\eta/s)(T)$ **of the form**

$$
\begin{aligned}
(\eta/s)(T) &= S_{\rm HG}(T_{\rm min} - T) + (\eta/s)_{\rm min}, && T < T_{\rm min} \\
(\eta/s)(T) &= (\eta/s)_{\rm min}, && T_{\rm min} < T < T_{\rm min} + W \\
(\eta/s)(T) &= S_{\rm QGP}(T - T_{\rm min} - W) + (\eta/s)_{\rm min}, && T > T_{\rm min} + W
\end{aligned}
$$

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(\eta/s)(T) &= S_{\rm QGP}(T - T_{\rm min} - W) + (\eta/s)_{\rm min}, && T > T_{\rm min} + W
\end{aligned}
$$

- **Free parameters** $K_{\rm sat}$, $T_{\rm min}$, $(\eta/s)_{\rm min}$, $S_{\rm HG}$, $S_{\rm QGP}$, W, $T_{\rm dec}$, $T_{\rm chem}$

The data

- $\frac{\mathrm{d}N_{ch}}{\mathrm{d}\eta}$, $\frac{\mathrm{d}N_{\pi}}{\mathrm{d}y}$, $\frac{\mathrm{d}N_{K}}{\mathrm{d}y}$ **and** $\frac{\mathrm{d}N_{p}}{\mathrm{d}\eta}$

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- $v_2^{ch}\{4\}$

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Au+Au	**Pb+Pb**	**Pb+Pb**
$\sqrt{s_{\mathrm{NN}}} = 200$ **GeV**	$\sqrt{s_{\mathrm{NN}}} = 2.76$ **TeV**	$\sqrt{s_{\mathrm{NN}}} = 5.02$ **TeV**

The data

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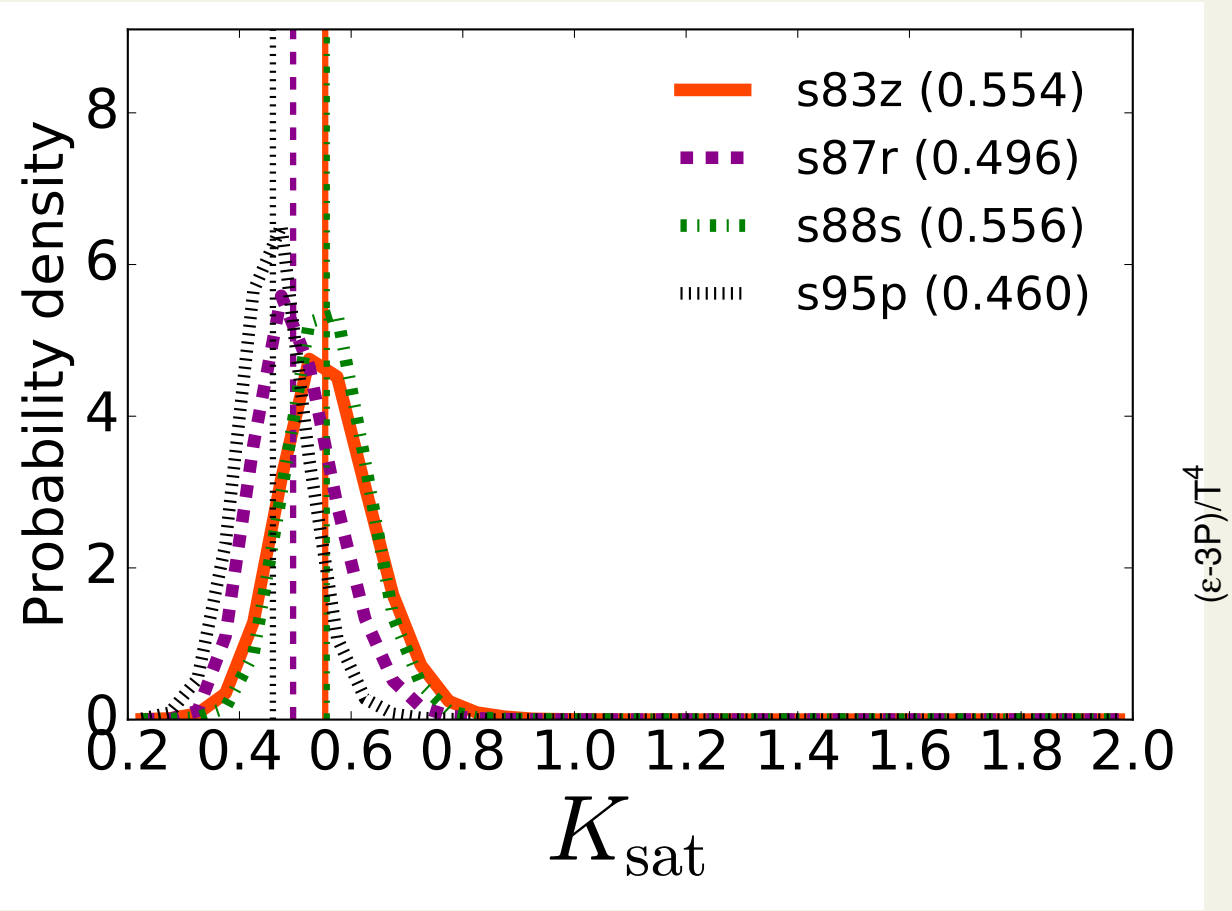
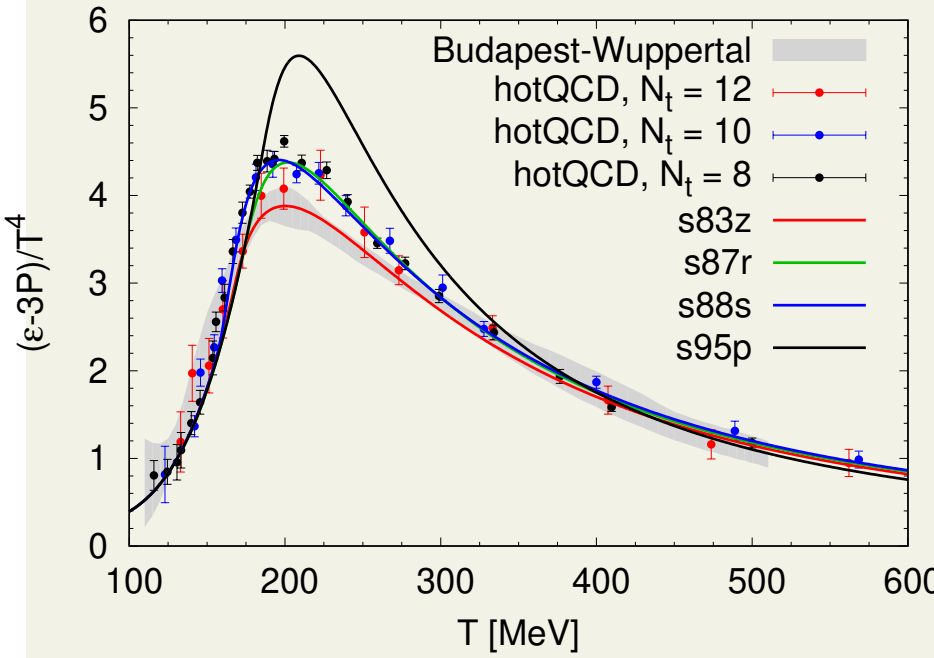
	Au+Au $\sqrt{s_{\mathrm{NN}}} = 200$ **GeV**	**Pb+Pb** $\sqrt{s_{\mathrm{NN}}} = 2.76$ **TeV**	**Pb+Pb** $\sqrt{s_{\mathrm{NN}}} = 5.02$ **TeV**
10-20%	●●●●	●●●●	● ●
20-30%	●●●●	●●●●	● ●
30-40%	●●●●	●●●●	● ●
40-50%	●●●●	●●●●	● ●
50-60%	●●●●	●●●●	● ●

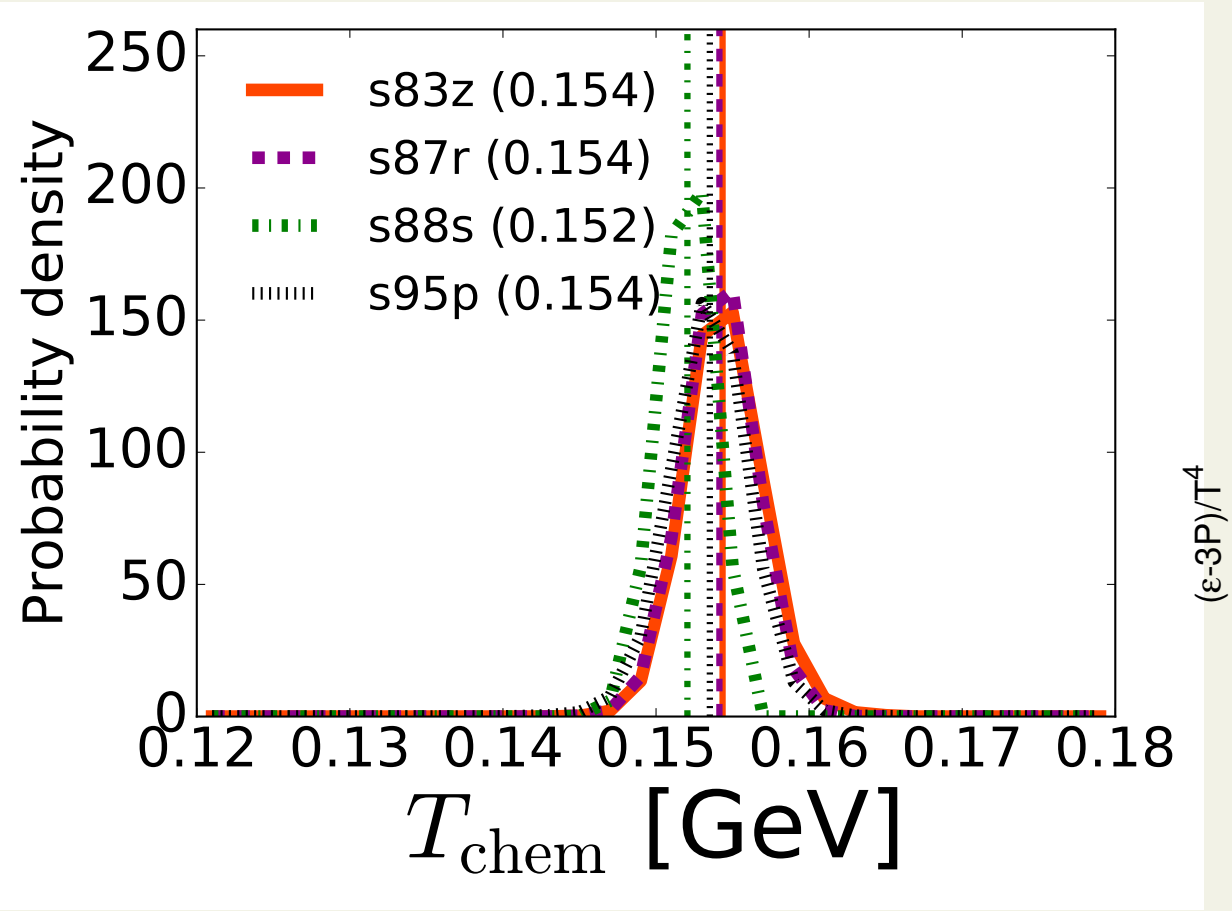
- **RHIC data by STAR**
- **LHC data by ALICE**

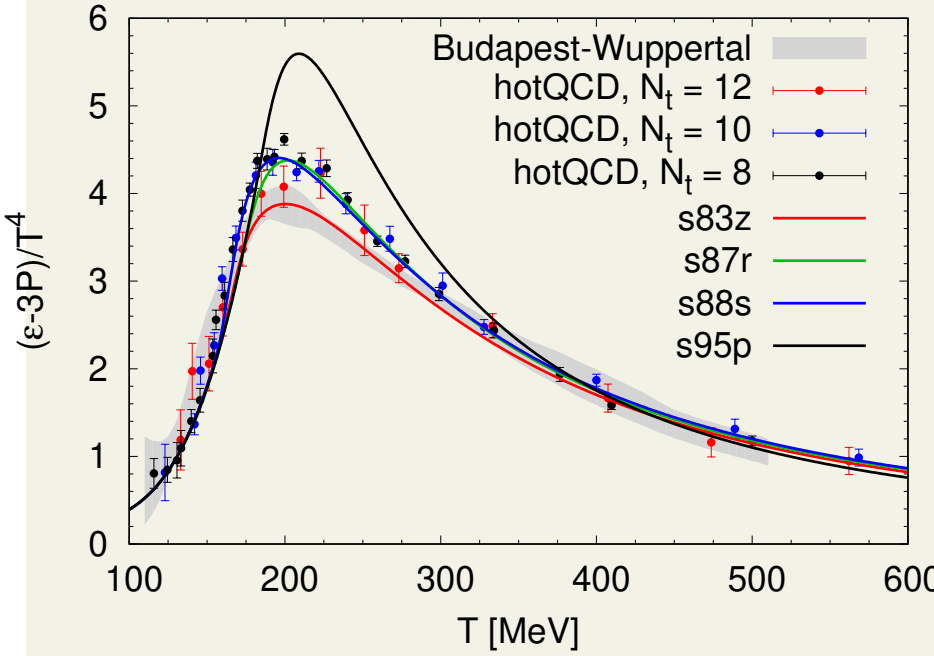
$K_{\rm sat}$


s83z (0.554)
s87r (0.496)
s88s (0.556)
s95p (0.460)
Probability density
0
2
4
6
8
0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
$K_{\rm sat}$


Budapest-Wuppertal
hotQCD, N$_t$ = 12
hotQCD, N$_t$ = 10
hotQCD, N$_t$ = 8
s83z
s87r
s88s
s95p
(ε-3P)/T^4
0
1
2
3
4
5
6
100 200 300 400 500 600
T [MeV]

$T_{\rm chem}$


Probability density
250
200
150
100
50
0
0.12 0.13 0.14 0.15 0.16 0.17 0.18
$T_{\rm chem}$ [GeV]
s83z (0.154)
s87r (0.154)
s88s (0.152)
s95p (0.154)


(ε-3P)/T^4
6
5
4
3
2
1
0
100 200 300 400 500 600
T [MeV]
Budapest-Wuppertal
hotQCD, N_t = 12
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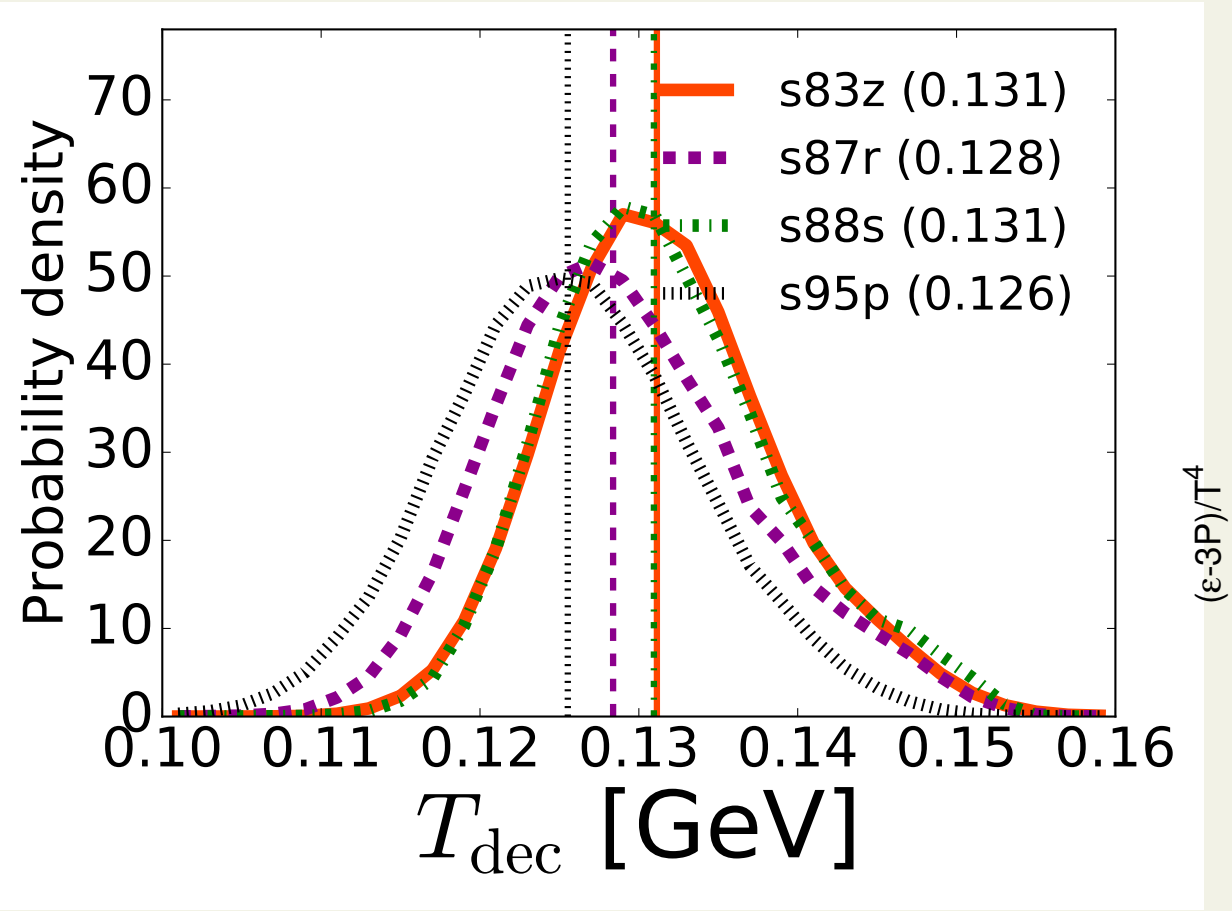
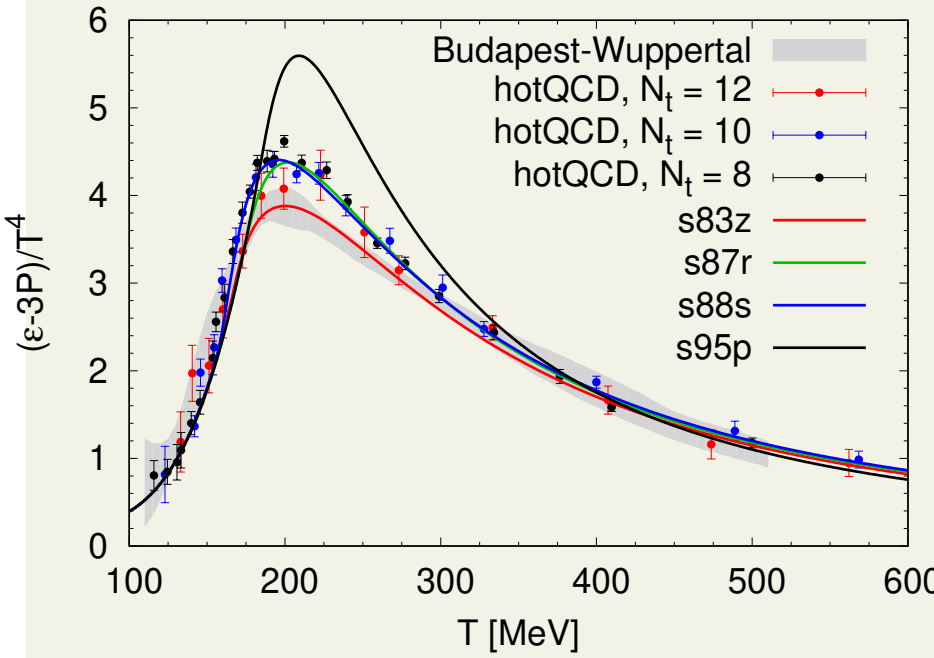
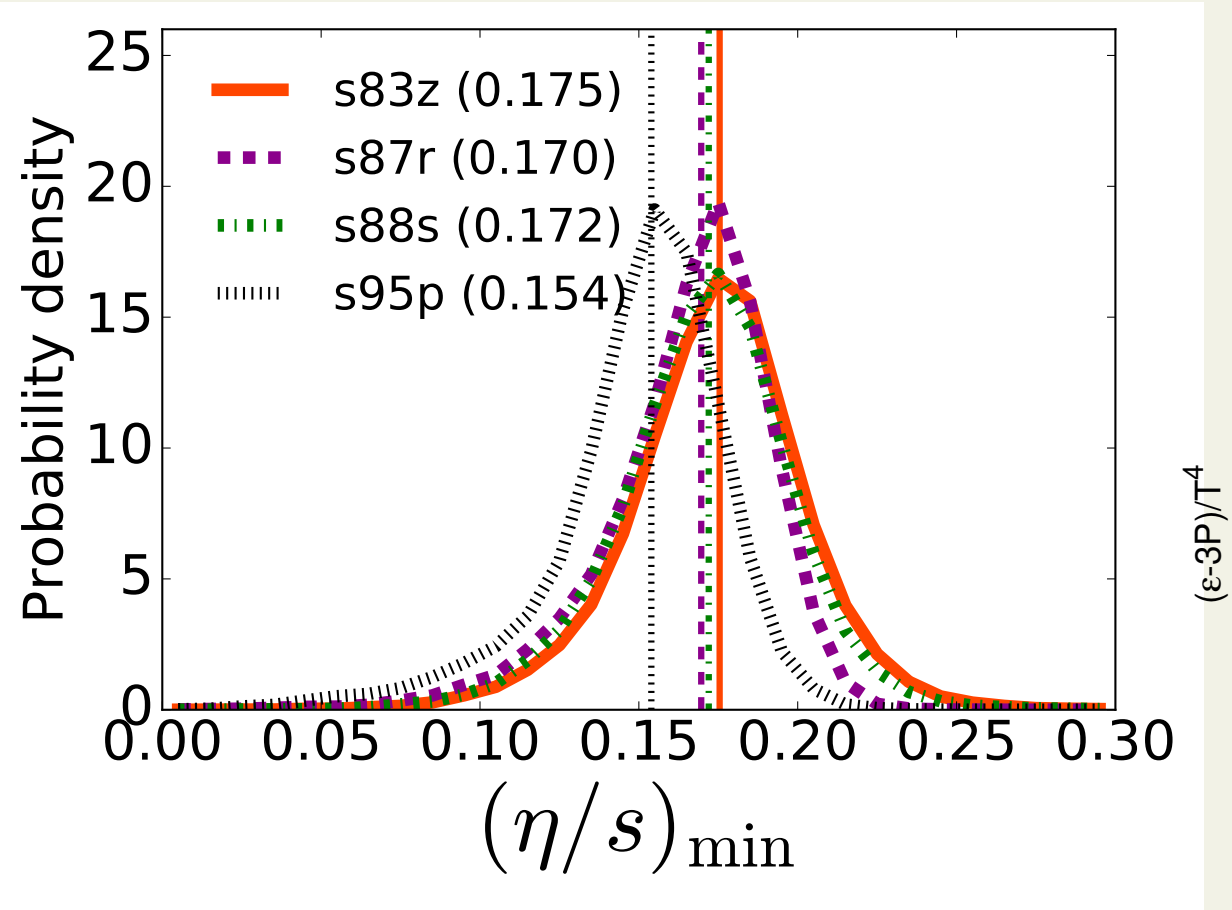
hotQCD, N_t = 8
s83z
s87r
s88s
s95p

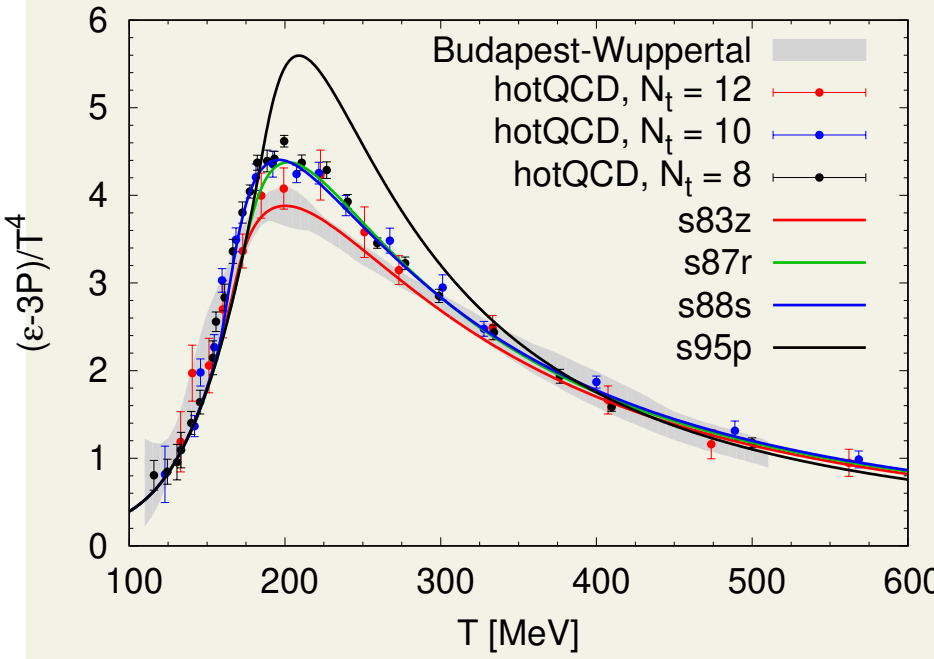
$T_{\rm dec}$


Probability density
0
10
20
30
40
50
60
70
0.10
0.11
0.12
0.13
0.14
0.15
0.16
$T_{\rm dec}$ [GeV]
s83z (0.131)
s87r (0.128)
s88s (0.131)
s95p (0.126)

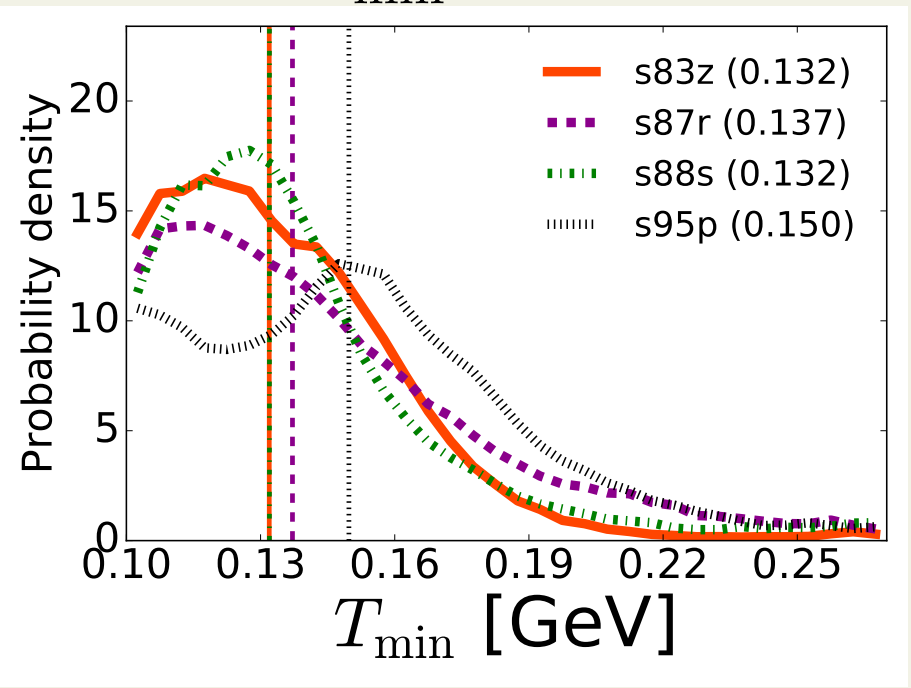

(ε-3P)/T^4
0
1
2
3
4
5
6
100
200
300
400
500
600
T [MeV]
Budapest-Wuppertal
hotQCD, N_t = 12
hotQCD, N_t = 10
hotQCD, N_t = 8
s83z
s87r
s88s
s95p

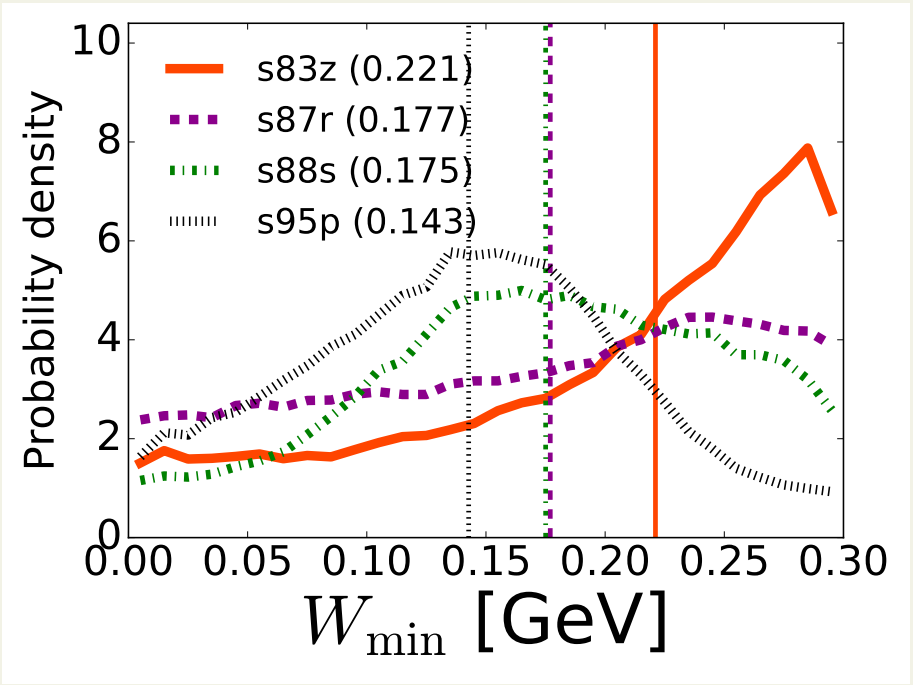
$(\eta/s)_{\mathrm{min}}$

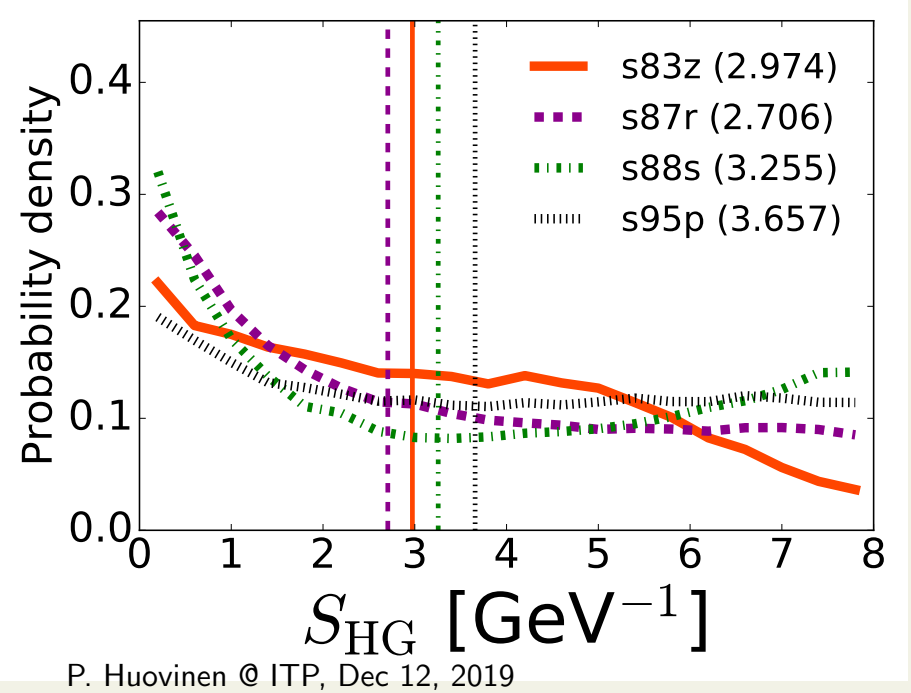


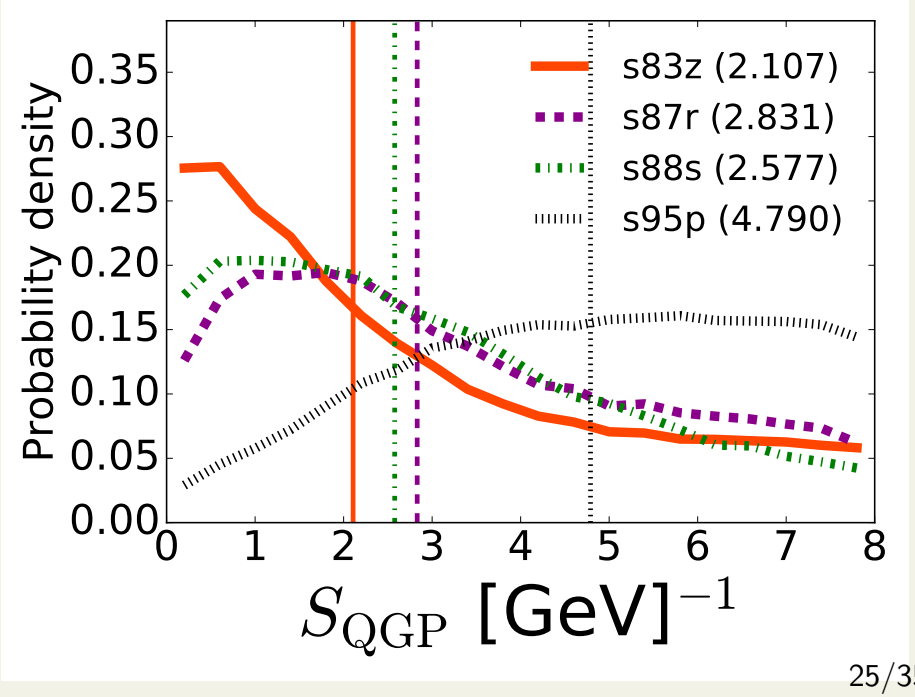


- **peak affected by EoS**
- **widths overlap**

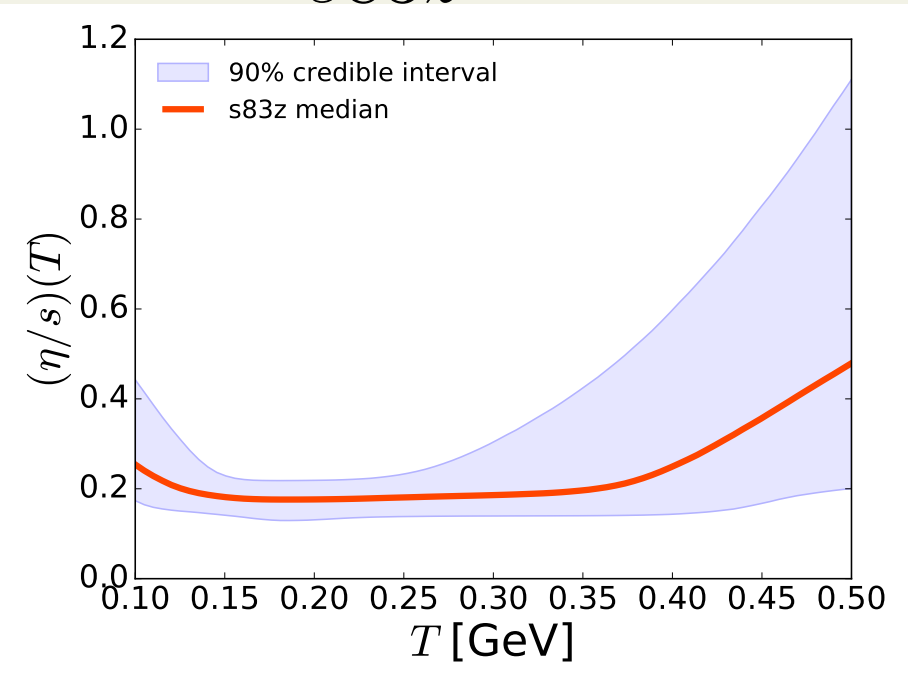

T_{min}
Probability density
s83z (0.132)
s87r (0.137)
s88s (0.132)
s95p (0.150)
T_{min} [GeV]


W
Probability density
s83z (0.221)
s87r (0.177)
s88s (0.175)
s95p (0.143)
W_{min} [GeV]

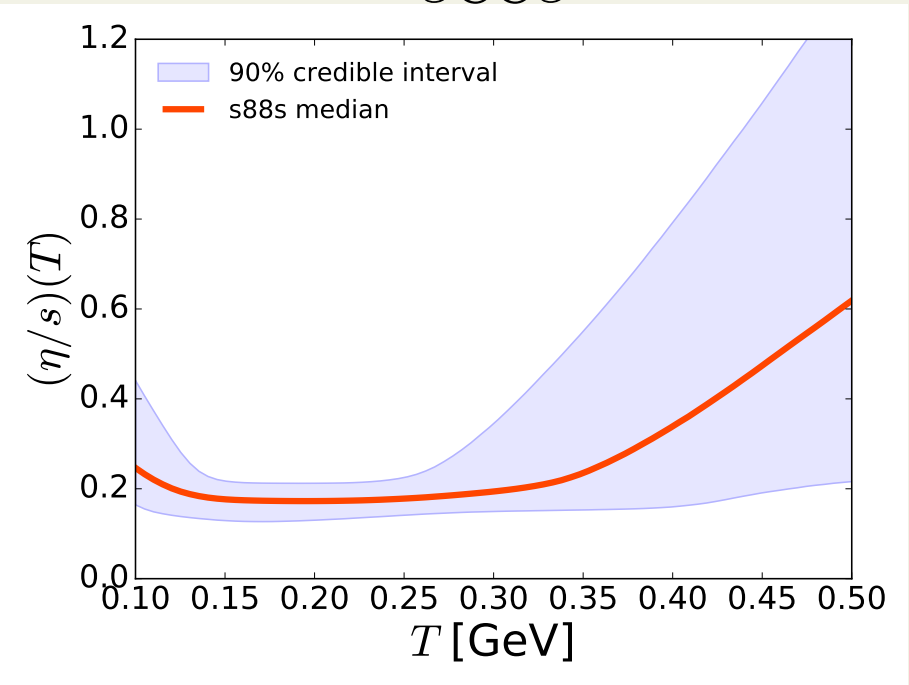

S_{HG}
Probability density
s83z (2.974)
s87r (2.706)
s88s (3.255)
s95p (3.657)
S_{HG} [GeV^{-1}]


S_{QGP}
Probability density
s83z (2.107)
s87r (2.831)
s88s (2.577)
s95p (4.790)
S_{QGP} [GeV]$^{-1}$

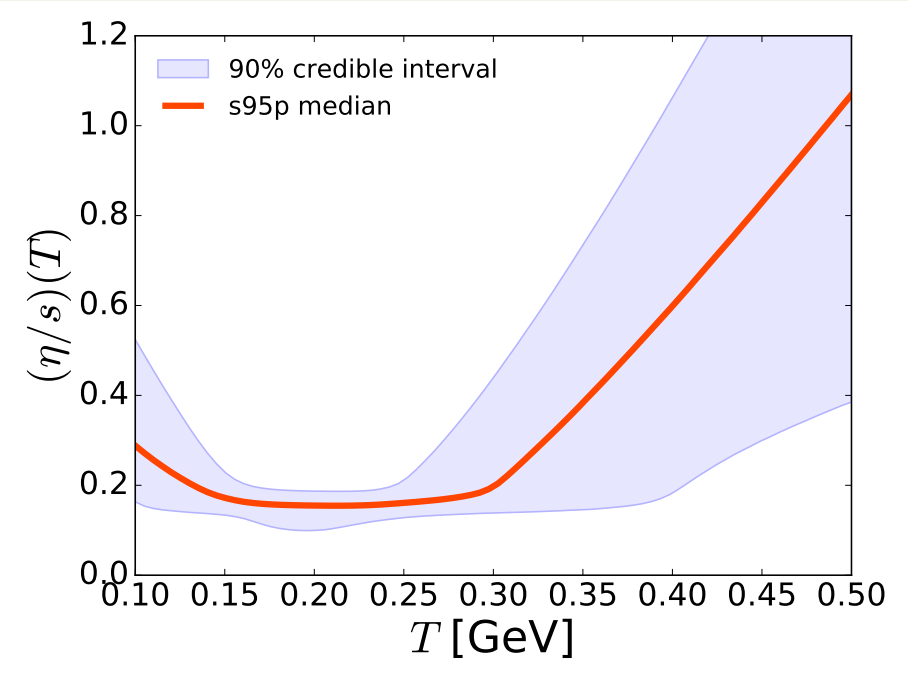
$s83z$



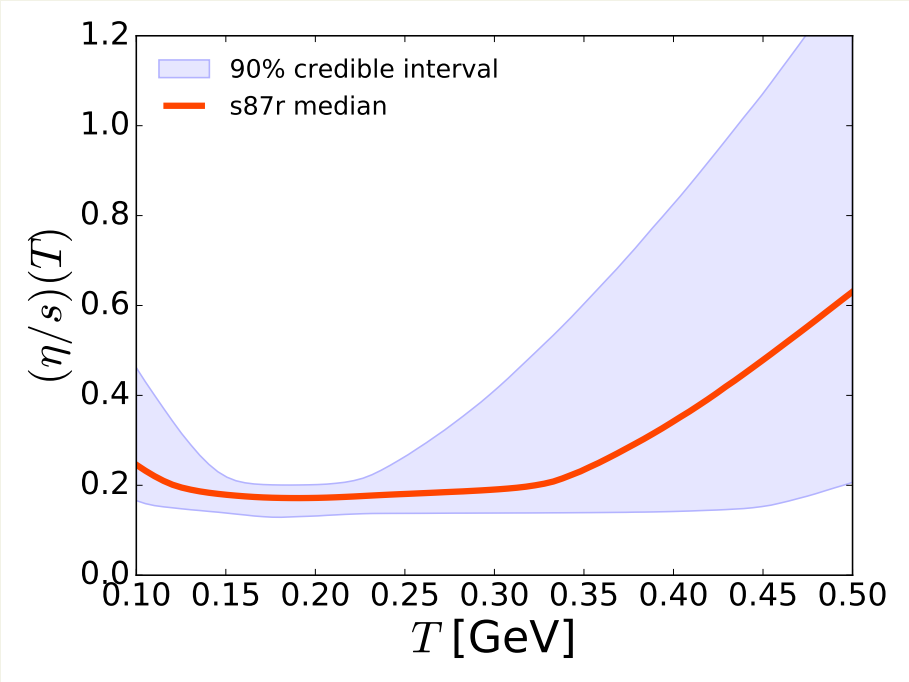
$s88s$

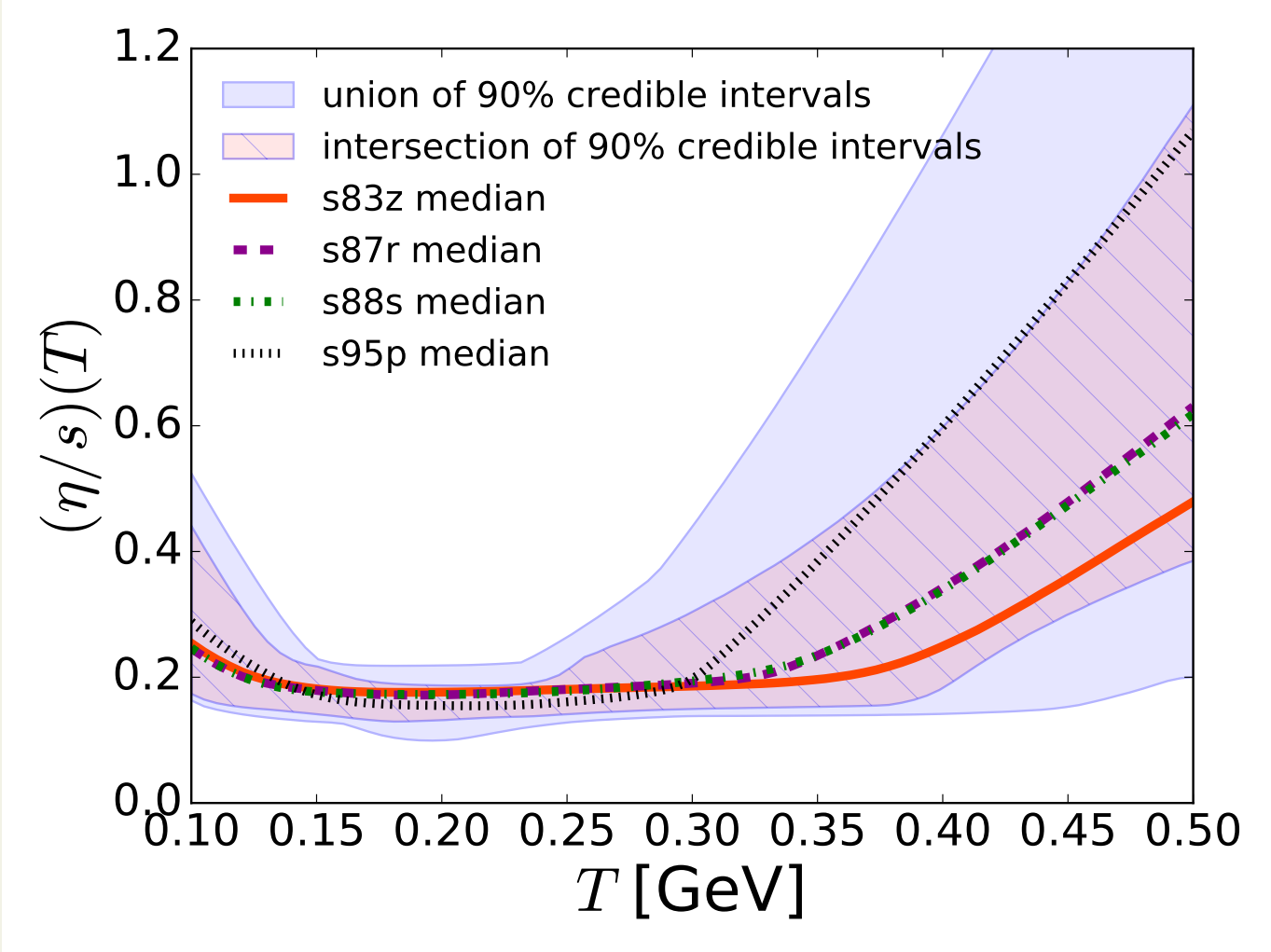


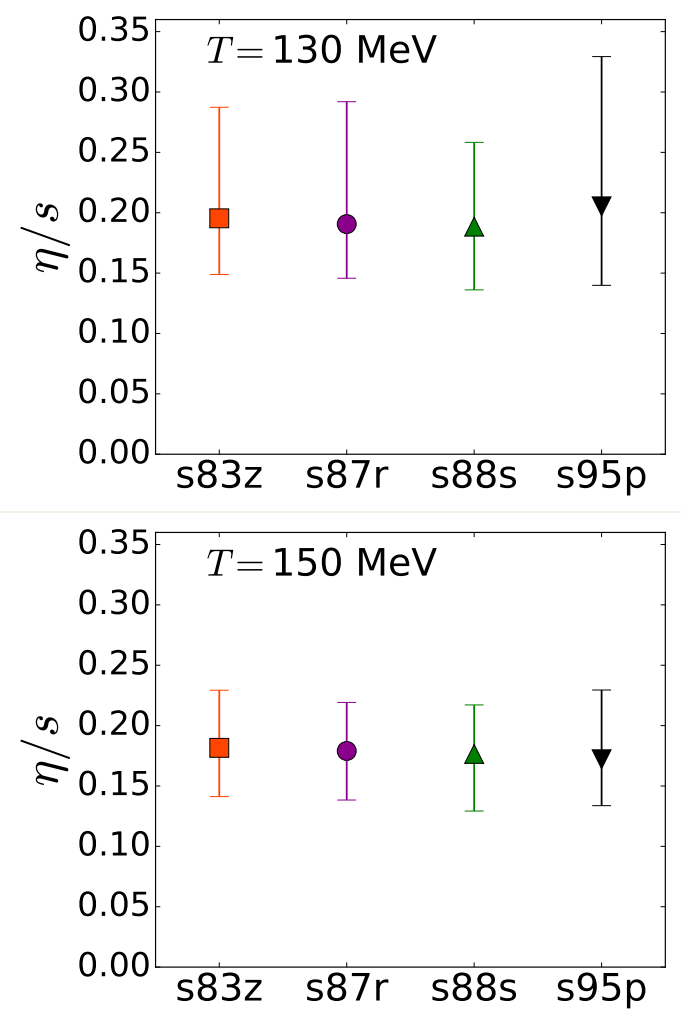
$s95p$



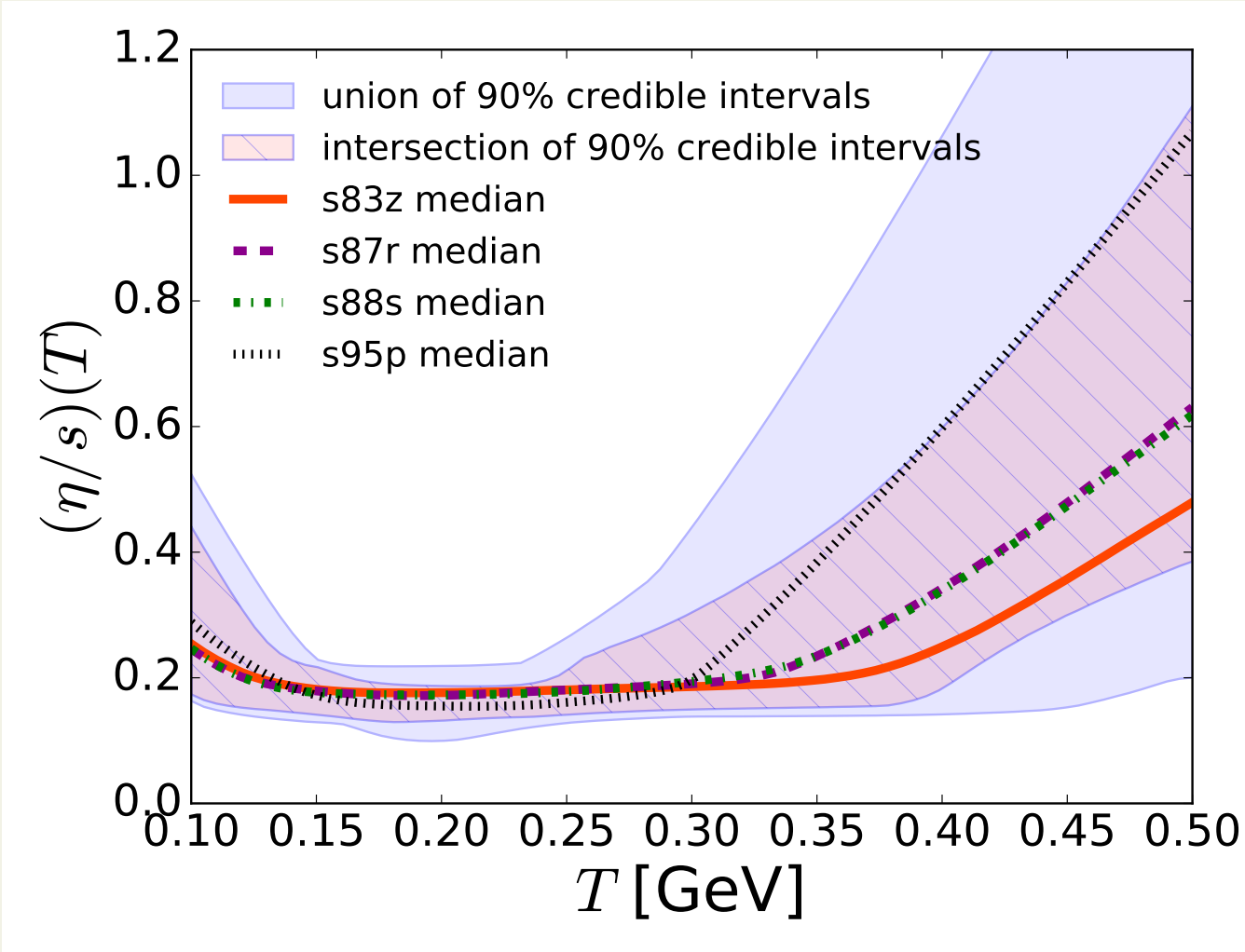
$s87r$

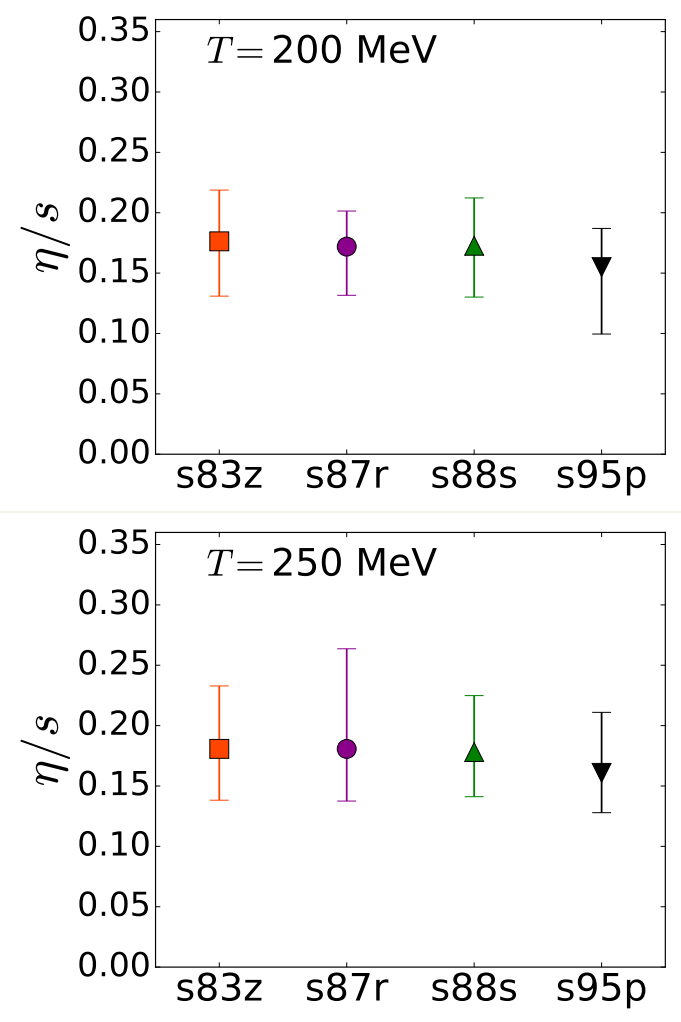



union of 90% credible intervals
intersection of 90% credible intervals
s83z median
s87r median
s88s median
s95p median
$(\eta/s)(T)$
T [GeV]
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50

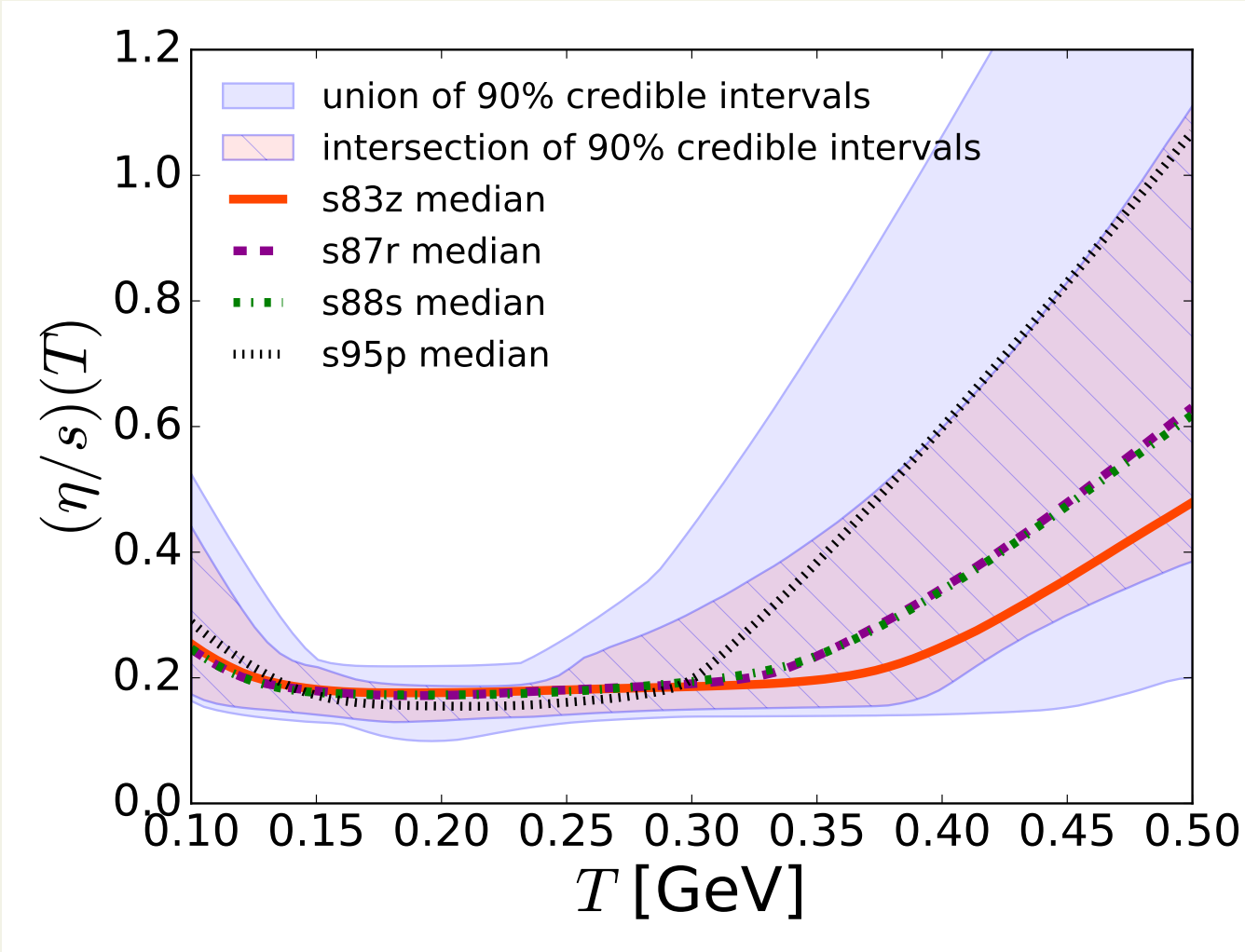

$T = 130$ MeV
η/s
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
s83z
s87r
s88s
s95p

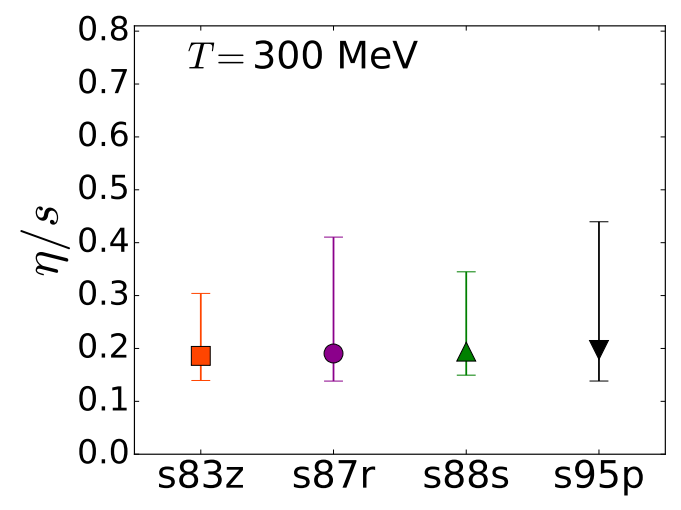
$T = 150$ MeV
η/s
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
s83z
s87r
s88s
s95p

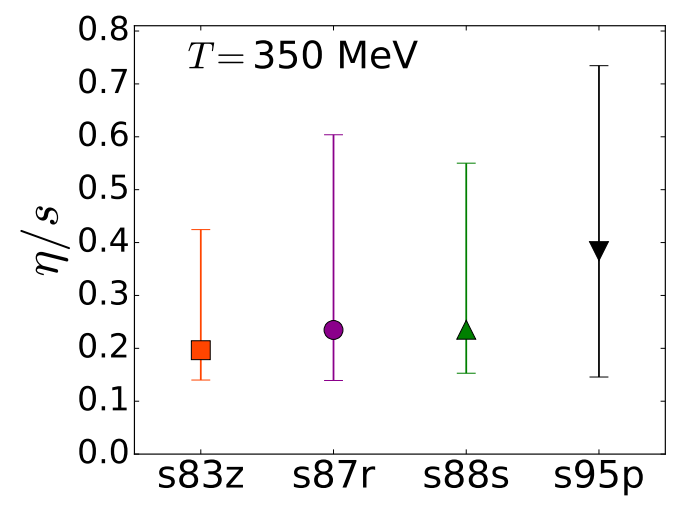

union of 90% credible intervals
intersection of 90% credible intervals
s83z median
s87r median
s88s median
s95p median
$(\eta/s)(T)$
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
T [GeV]

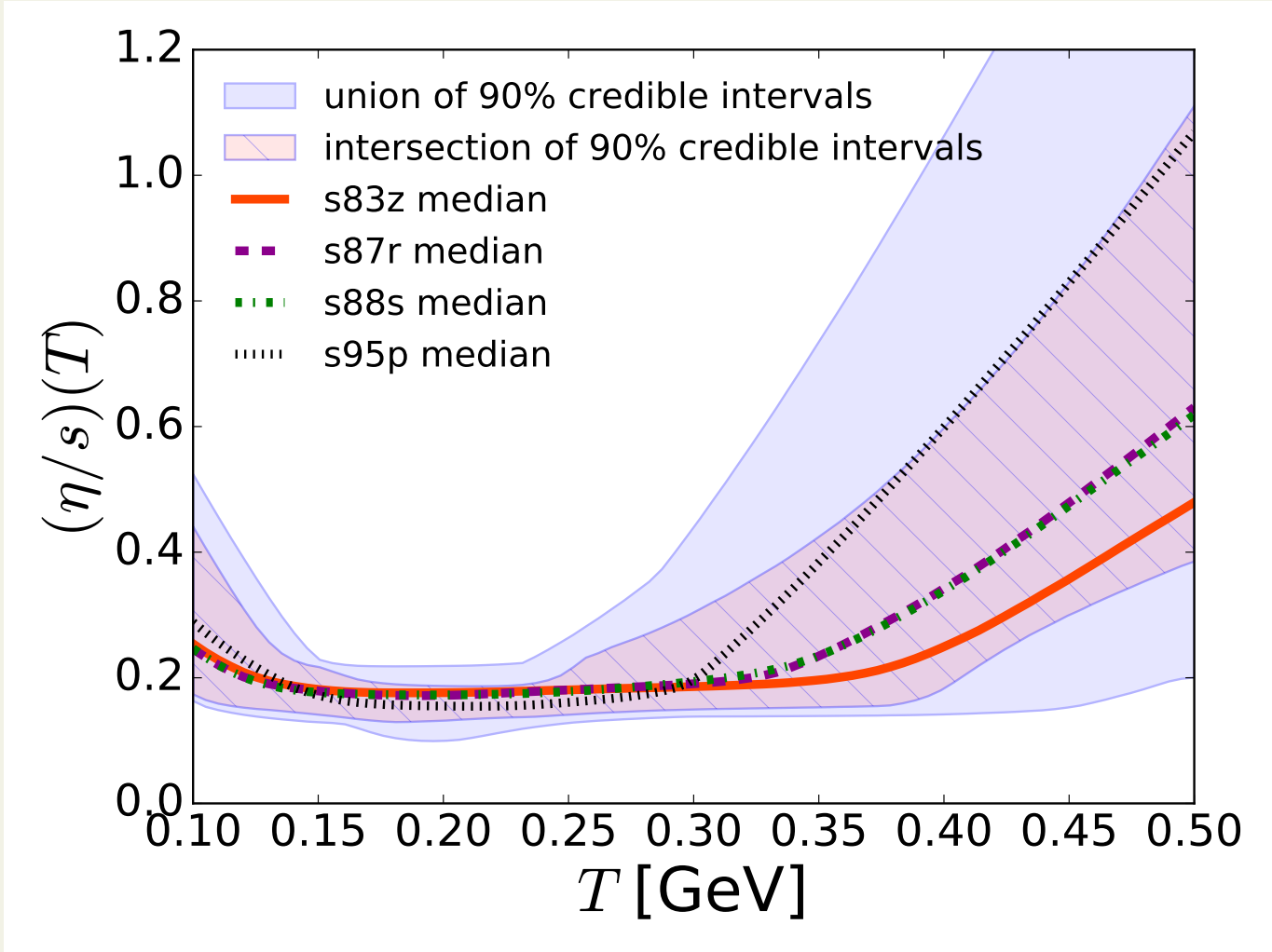

$T = 200$ MeV
η/s
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
s83z
s87r
s88s
s95p

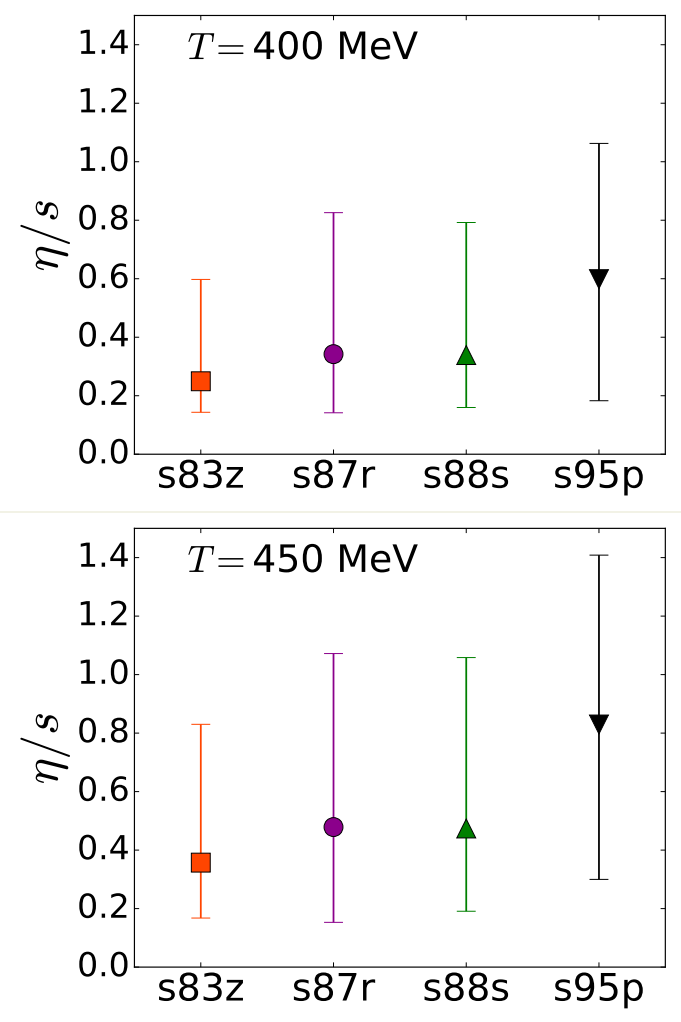
$T = 250$ MeV
η/s
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
s83z
s87r
s88s
s95p


union of 90% credible intervals
intersection of 90% credible intervals
s83z median
s87r median
s88s median
s95p median
$(\eta/s)(T)$
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
T [GeV]

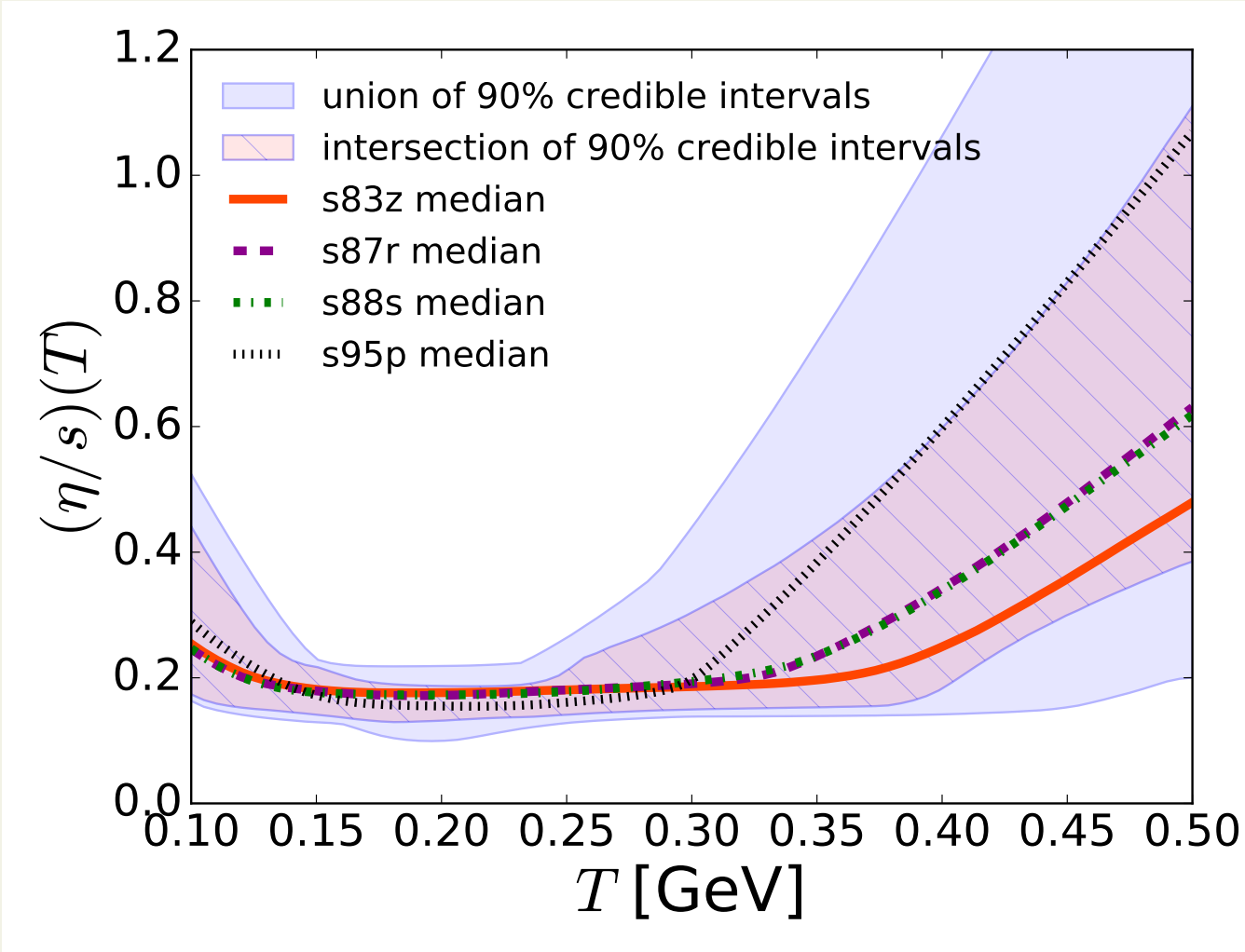

$T = 300$ MeV
η/s
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
s83z
s87r
s88s
s95p


$T = 350$ MeV
η/s
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
s83z
s87r
s88s
s95p


union of 90% credible intervals
intersection of 90% credible intervals
s83z median
s87r median
s88s median
s95p median
$(\eta/s)(T)$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
T [GeV]


$T = 400$ MeV
η/s
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
s83z
s87r
s88s
s95p

$T = 450$ MeV
η/s
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
s83z
s87r
s88s
s95p


union of 90% credible intervals
intersection of 90% credible intervals
s83z median
s87r median
s88s median
s95p median
$(\eta/s)(T)$
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
T [GeV]

- $\eta/s = 0.18^{+0.05}_{-0.05}$ **when** $150 \lesssim T/\mathrm{MeV} \lesssim 250$

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  $\eta/s = 0.07^{+0.05}_{-0.04}$ **at** $T = 154\,\mathrm{MeV}$

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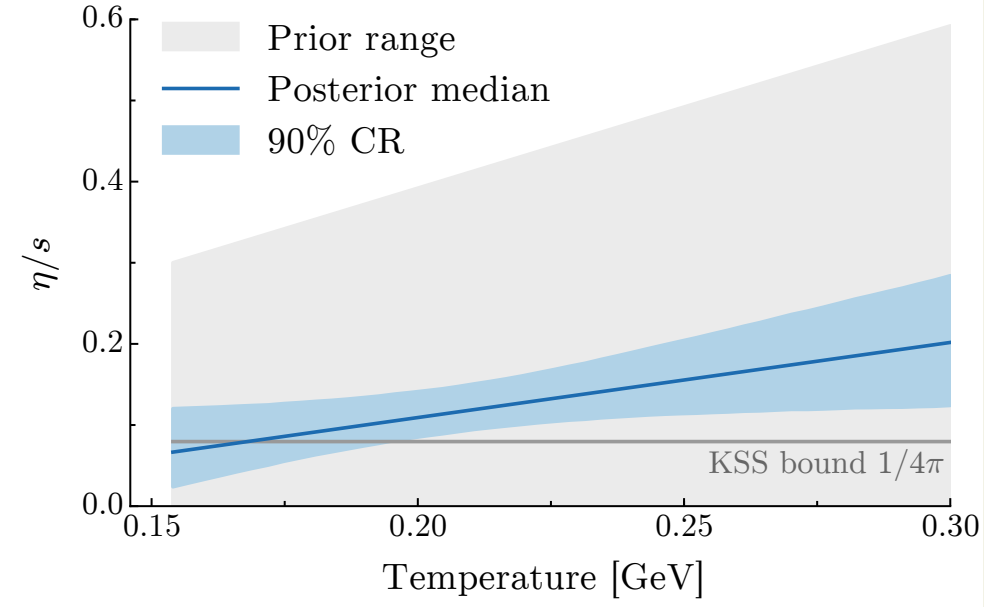
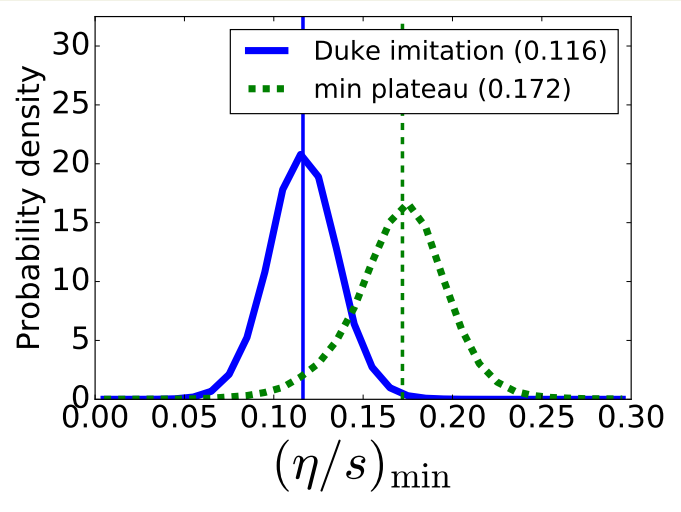
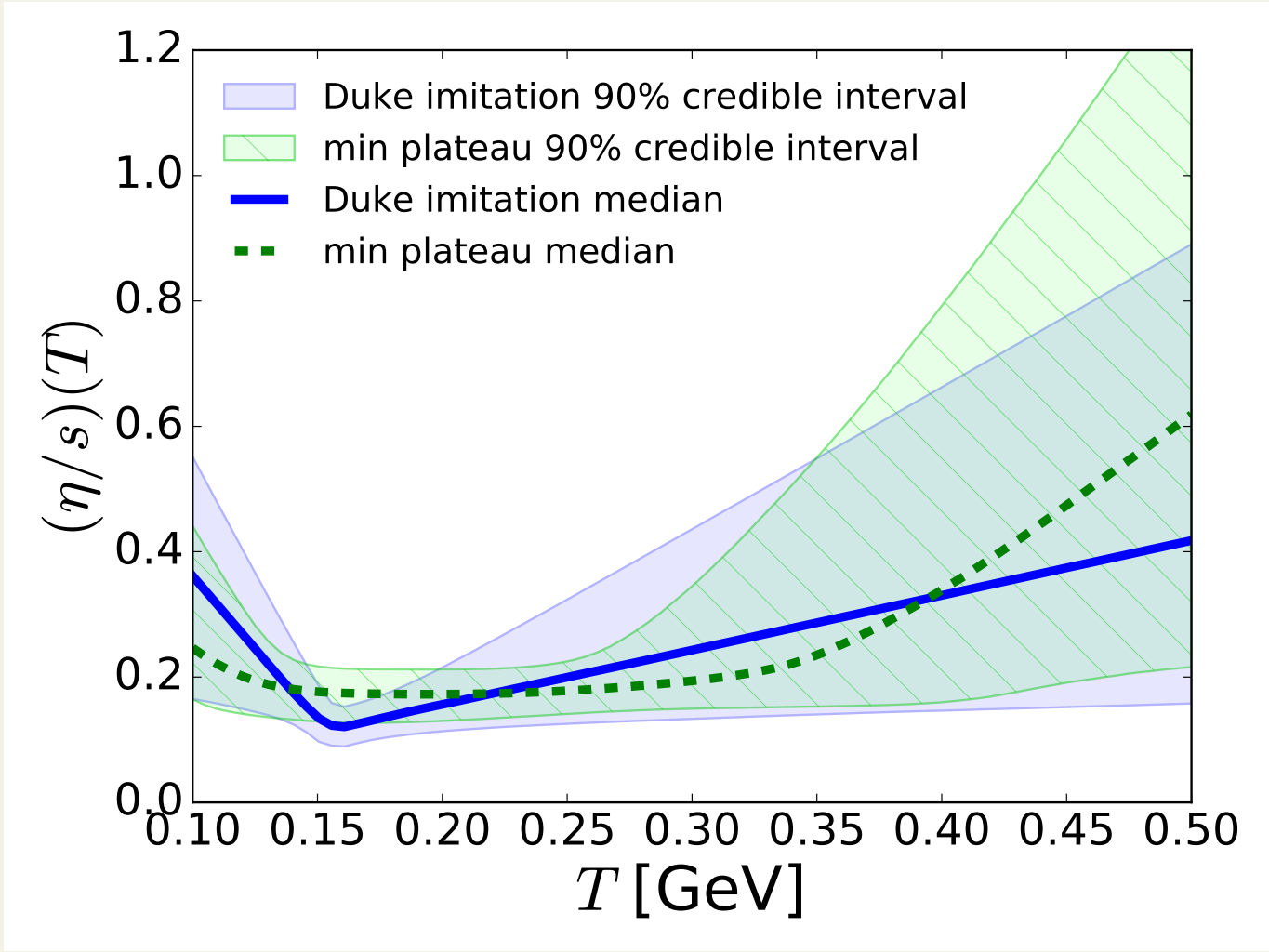
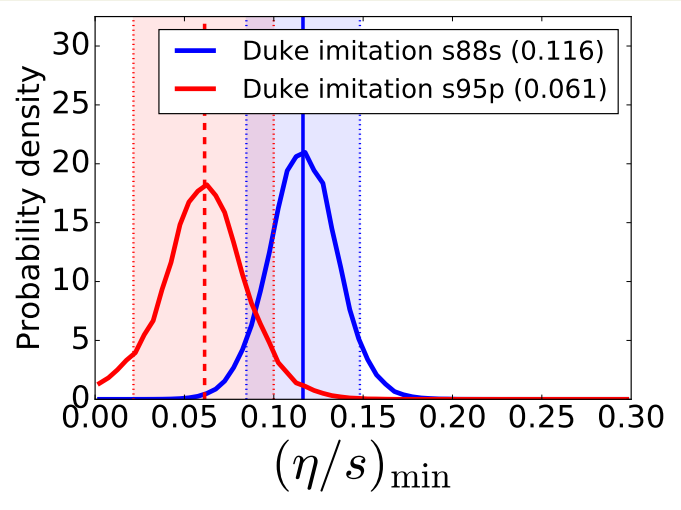


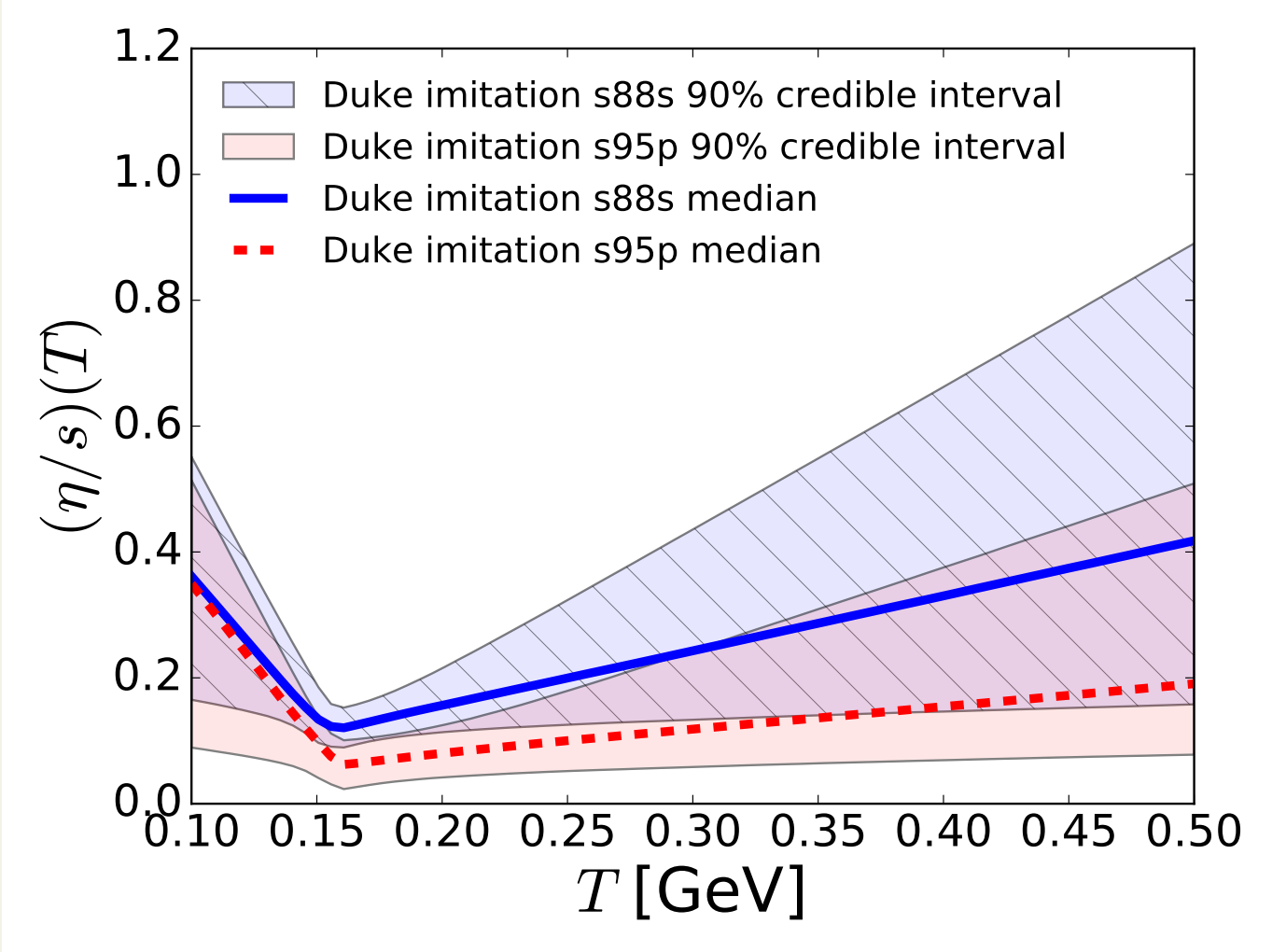
effect of $(\eta/s)(T)$ parametrisation





effect of $(\eta/s)(T)$ parametrisation





Does η/s depend on EoS?

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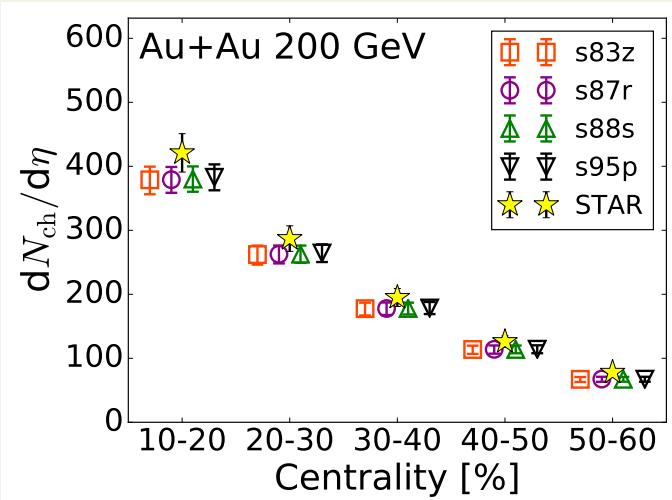
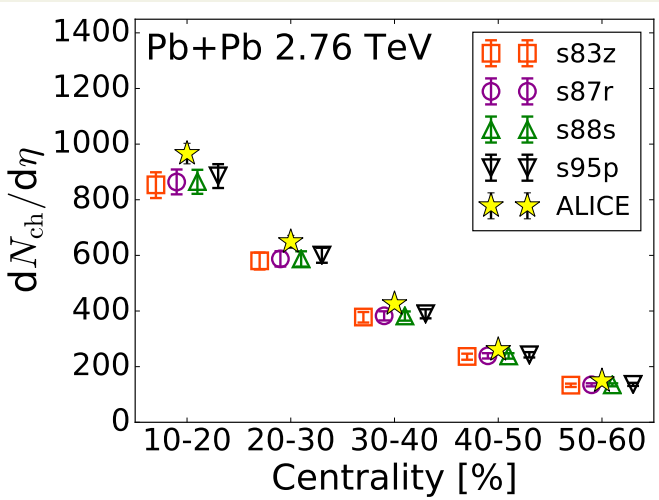
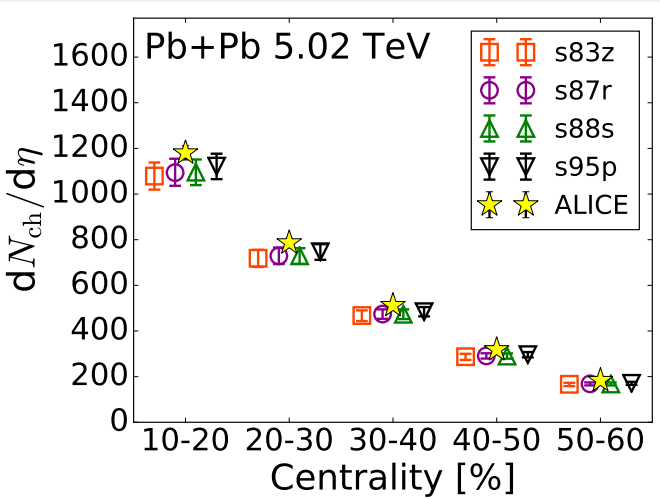
- η/s **badly constrained when** $T \lesssim 160\,\mathrm{MeV}$ **or** $T \gtrsim 250\,\mathrm{MeV}$
- **minimum value may depend on the parametrization**
  $\Rightarrow$ **take credibility limits seriously!**

This project has received funding from the European Research Council (ERC) under the European Unions Horizon 2020 research and innovation programme (grant agreement No 725741)

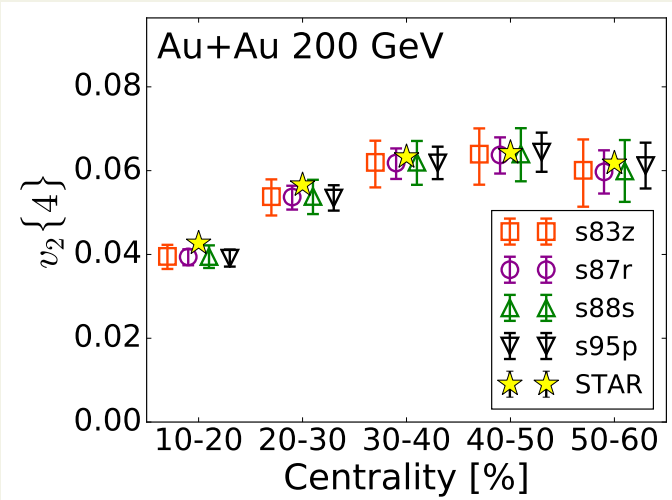
$\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta$

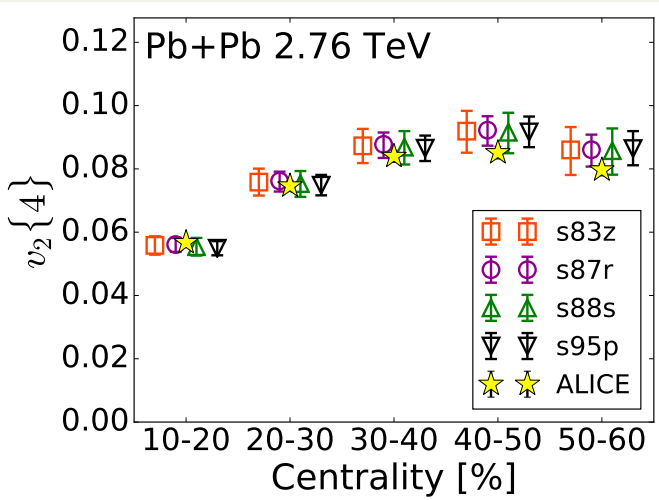


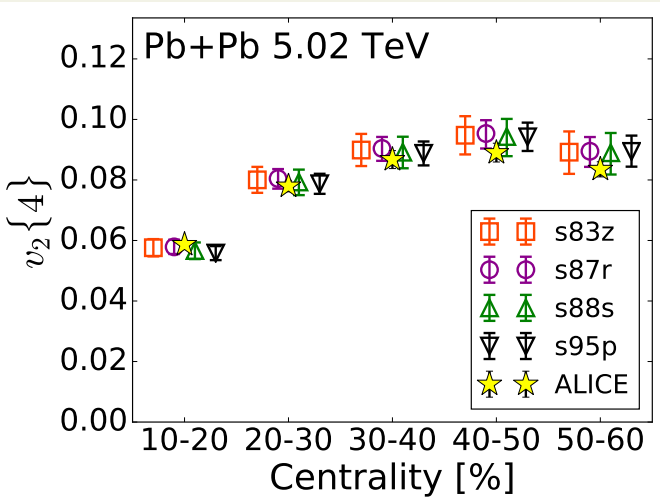




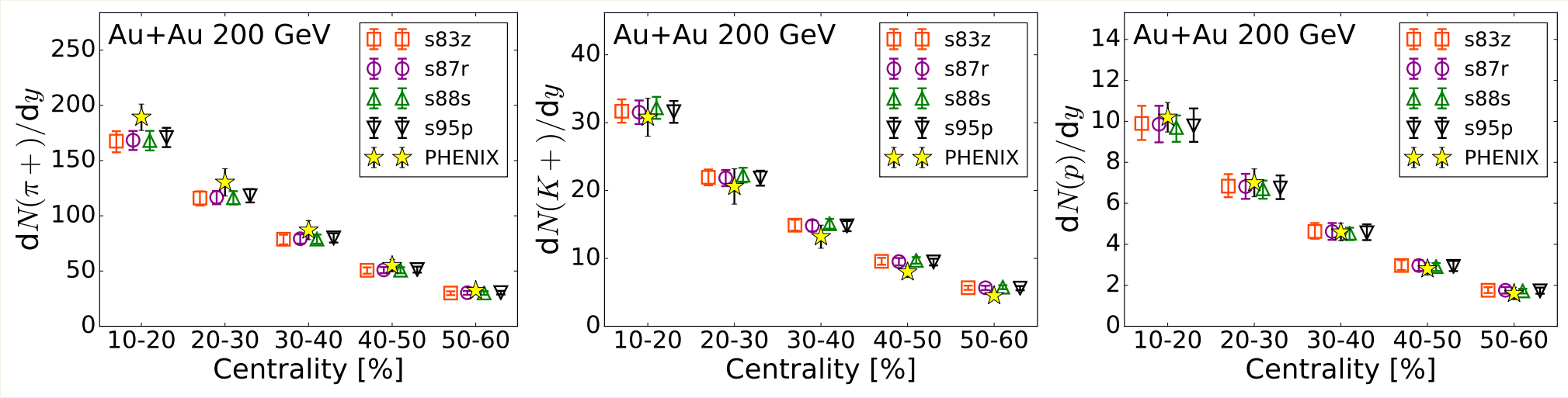
$v_2^{\mathrm{ch}}\{4\}$



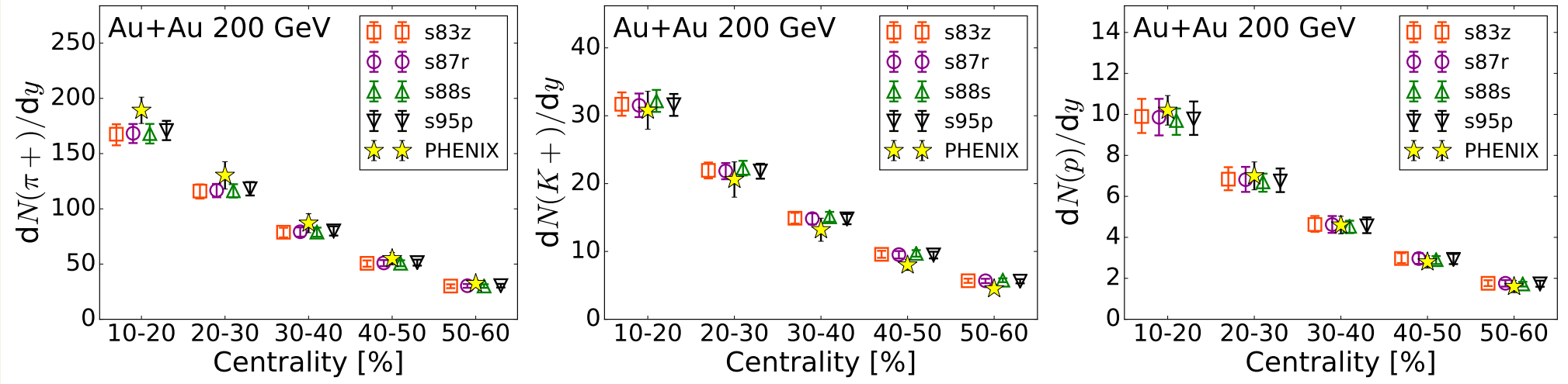




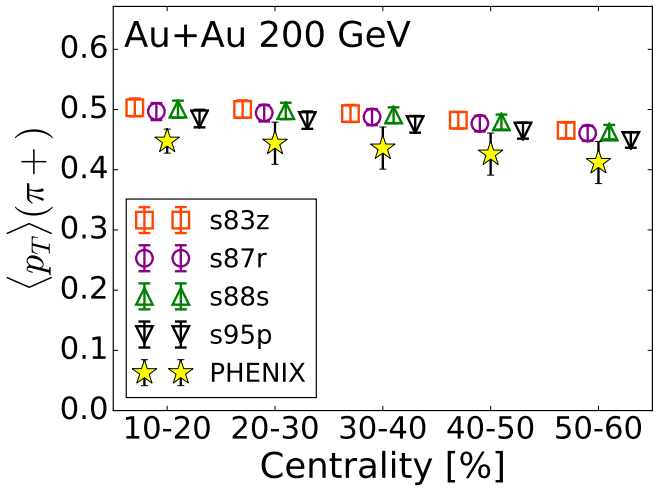
$\mathrm{d}N/\mathrm{d}y$, $\sqrt{s_{\mathrm{NN}}} = 200$ **GeV**

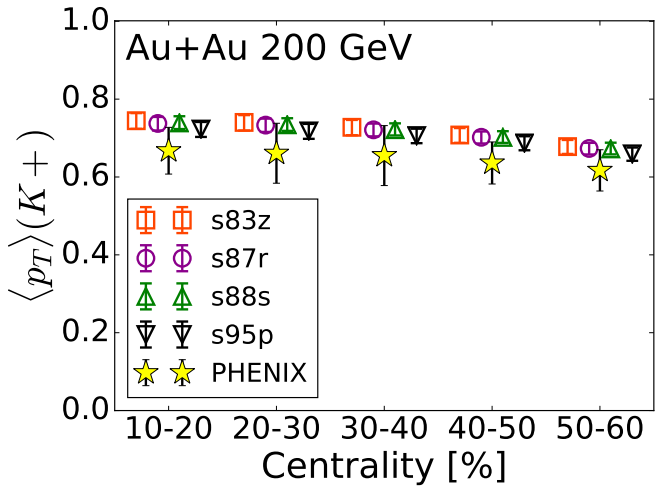


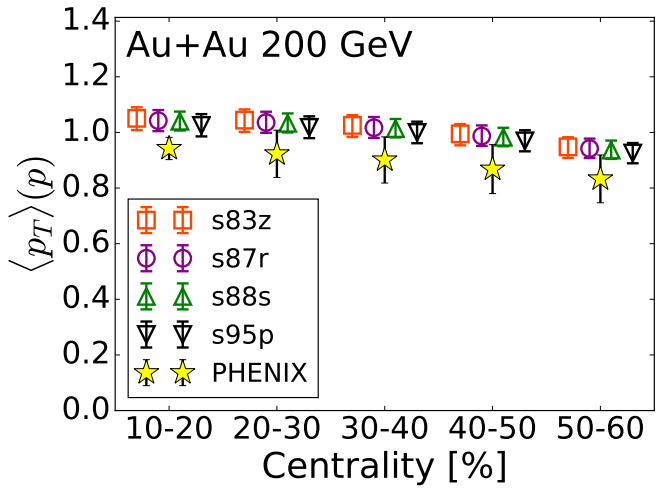
$\mathrm{d}N/\mathrm{d}y$, $\sqrt{s_{\mathrm{NN}}} = 2.76$ **TeV**



$\langle p_T \rangle$, $\sqrt{s_{\mathrm{NN}}} = 200$ **GeV**







$\langle p_T \rangle$, $\sqrt{s_{\mathrm{NN}}} = 2.76$ **TeV**