STRANGENESS NEUTRALITY AND THE QCD PHASE STRUCTURE

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[Fu, Pawlowski, FR, hep-ph/1808.00410]
[Fu, Pawlowski, FR, hep-ph/1809.01594]

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— NUCLEAR PHYSICS COLLOQUIUM —

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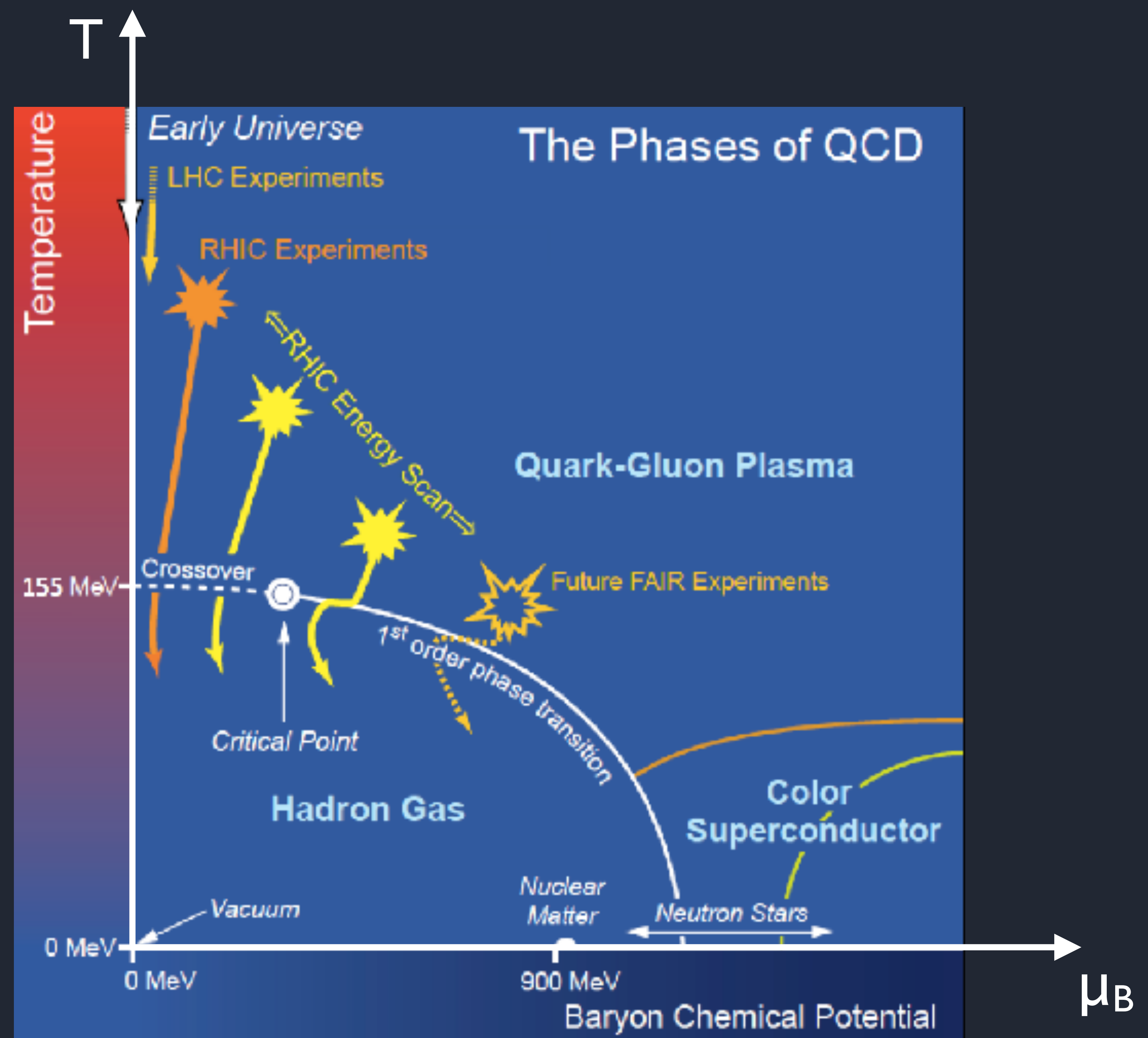
OUTLINE

- The QCD phase diagram & heavy-ion collisions
- Strangeness neutrality
- Low-energy QCD & functional RG
- Strangeness neutrality, correlations & phase structure

QCD PHASE DIAGRAM



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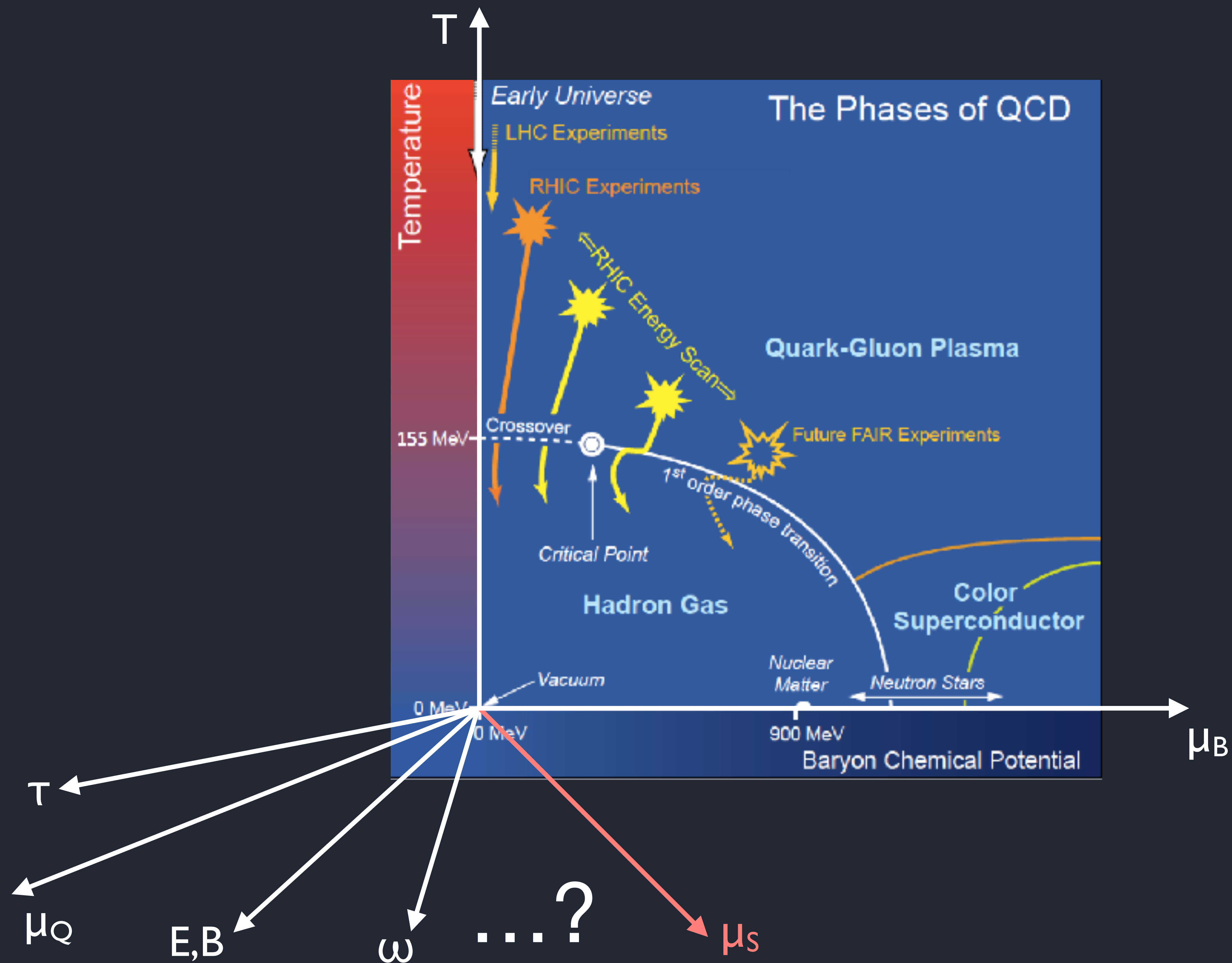
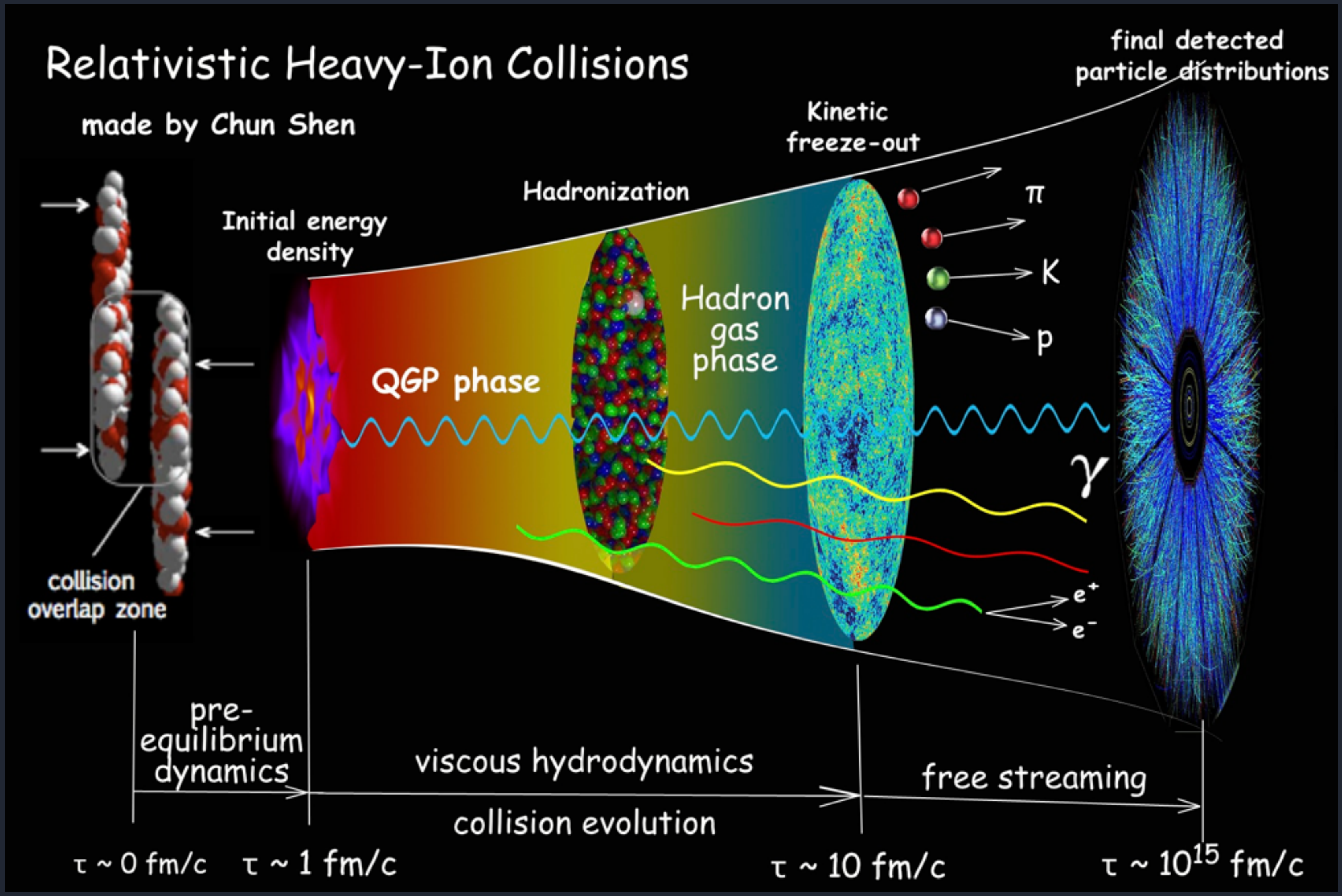


PROBING THE PHASE DIAGRAM



from strong and weak decays with the same final products [PDG]

- phase diagram probed at freeze-out; timescale ~10 fm/c
- typical timescales of strong and (flavor-changing) weak decays: ~1 fm/c vs ~10^{13} fm/c

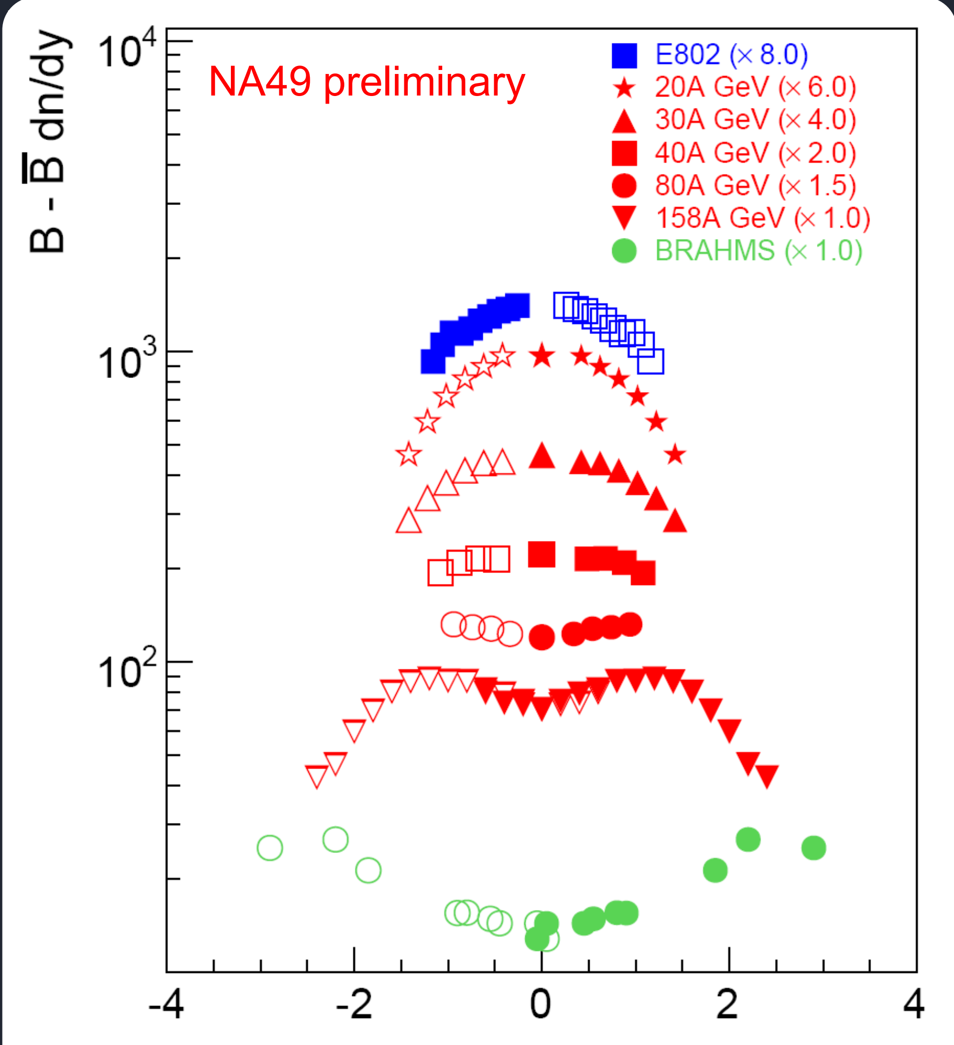
⟶ quark number conservation of the strong interactions at the freeze-out!

⟶ baryon number, strangeness & isospin are conserved

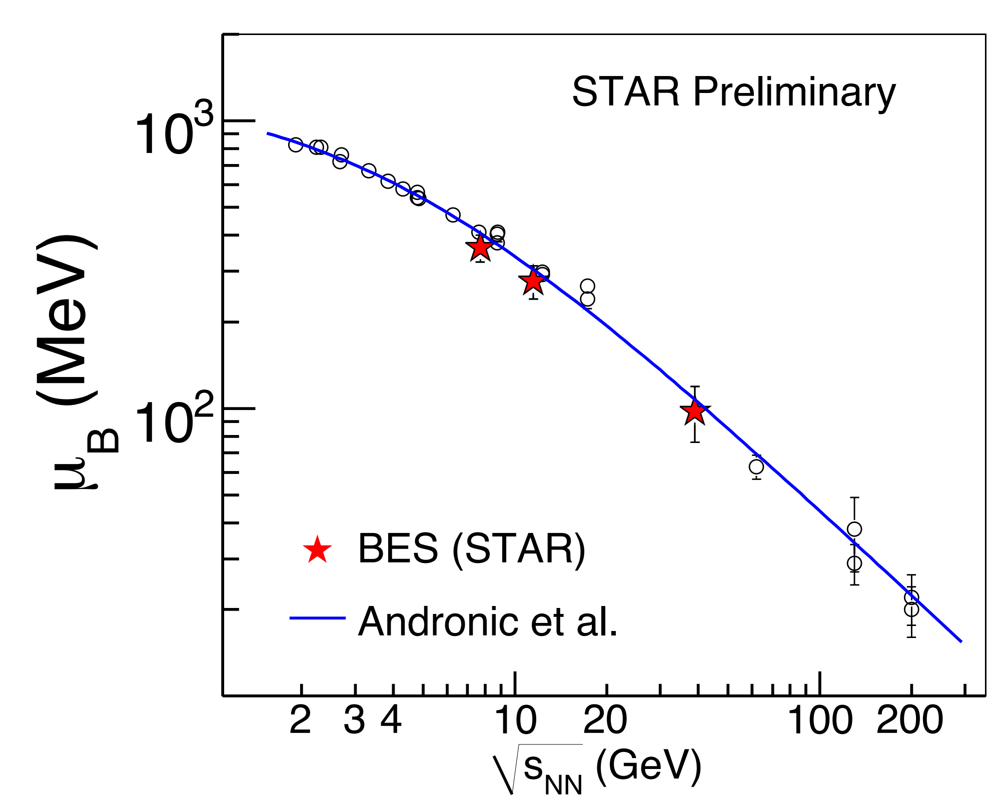
PROBING THE PHASE DIAGRAM

- net quark content determined by incident nuclei
- net strangeness has to be zero → fixes μ_S : strangeness neutrality
- net charged fixed → fixes μ_Q
- increasing baryon chemical potential with decreasing beam energy (at mid-rapidity)

→ μ_B is a `free´ parameter



[NA49, compiled by C. Blume]



[L. Kumar (STAR), J.Phys. G38 (2011)]

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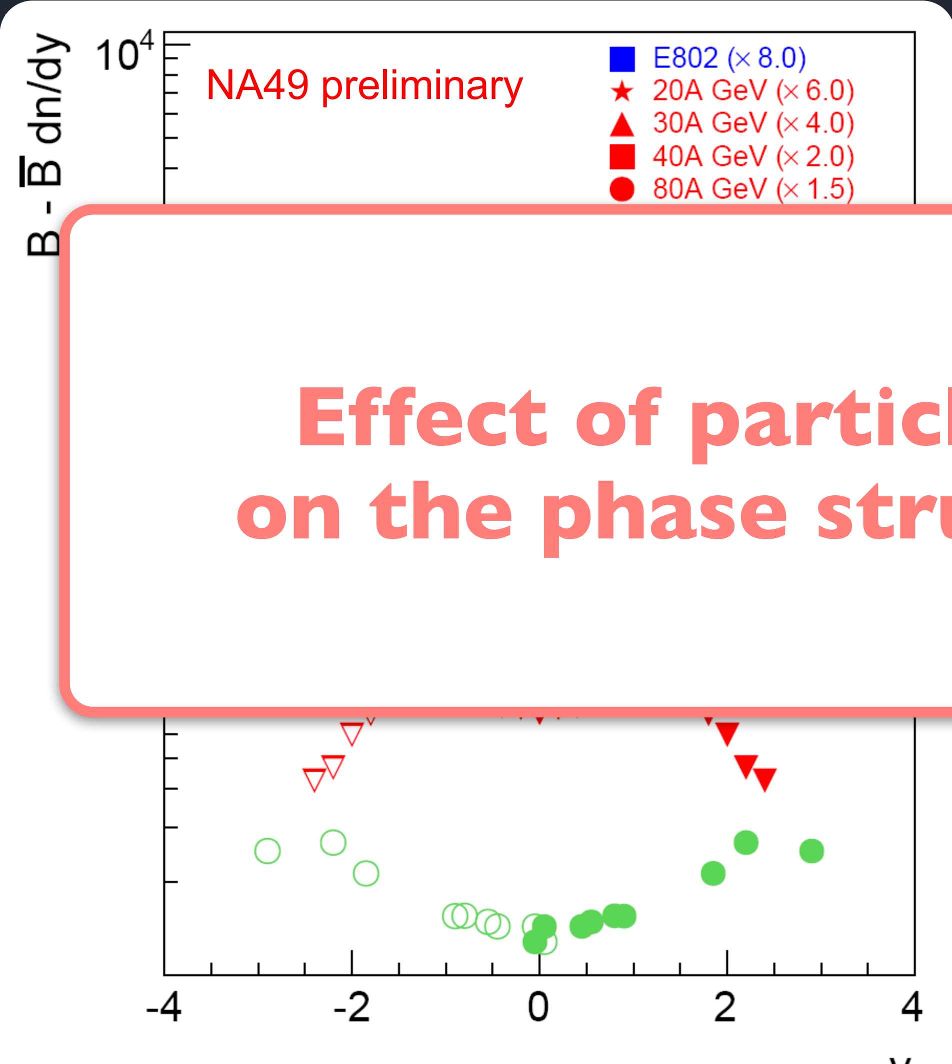

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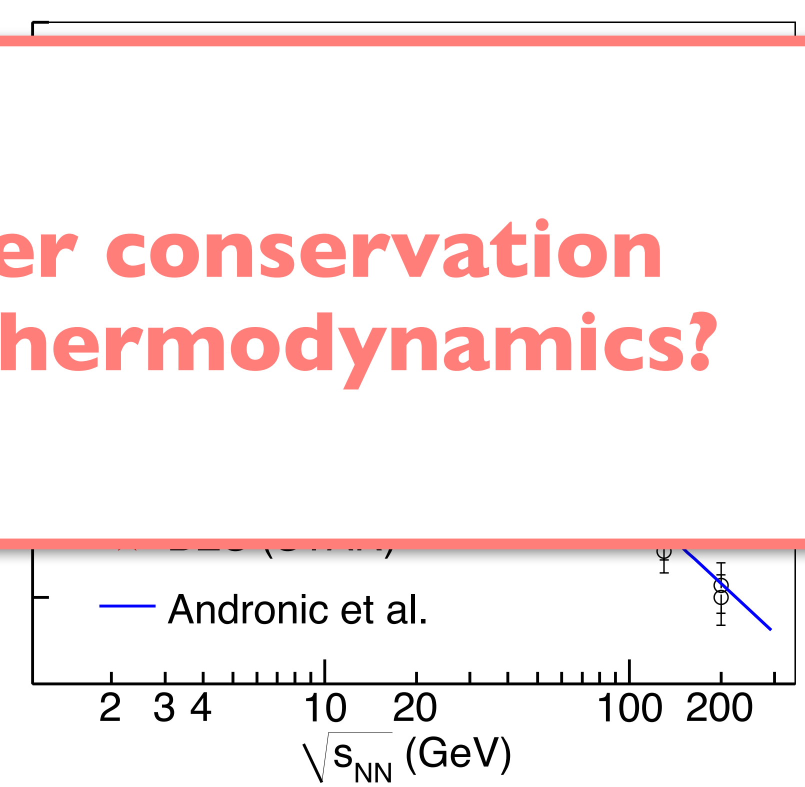

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Effect of particle number conservation on the phase structure/thermodynamics?

NOMENCLATURE

- chemical potentials:

charge

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \longrightarrow \mu = \begin{pmatrix} \mu_u \\ \mu_d \\ \mu_s \end{pmatrix} = \begin{pmatrix} \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \\ \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \\ \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S \end{pmatrix}$$

baryon strangeness

- here: $\mu_Q = 0$

- cumulants of conserved charges

pressure

$$\chi_{ij}^{BS} = T^{i+j-4} \frac{\partial^{i+j} p(T, \mu_B, \mu_S)}{\partial \mu_B^i \partial \mu_S^j}$$

net baryon number: $\langle B \rangle = \langle N_B - N_{\bar{B}} \rangle = \chi_{10}^{BS} V T^3$

net strangeness: $\langle S \rangle = \langle N_{\bar{S}} - N_S \rangle = \chi_{01}^{BS} V T^3$

BARYON-STRANGENESS CORRELATION

strangeness neutrality

$$C_{BS} \equiv -3\,\frac{\chi_{11}^{BS}}{\chi_{02}^{BS}} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2} = -3\frac{\langle BS\rangle}{\langle S^2\rangle}$$

[Koch, Majumder, Randrupp, nucl-th/0505052]

- **diagnostic tool for deconfinement:**

QGP

- all strangeness is carried by $s,\ \bar{s}$
- strict relation beween B and S: $B_s = -S_s/3$
- if all flavors are independent: $\chi_{11}^{BS} = -\chi_{02}^{BS}/3$

$\longrightarrow\ C_{BS} = 1$

hadronic phase

- mesons can carry only strangeness, baryons both

χ_{11}^{BS} : only strange baryons

χ_{02}^{BS} : strange baryons & mesons

$\longrightarrow\ C_{BS} \neq 1$

STRANGENESS NEUTRALITY

- HIC: colliding nuclei have zero strangeness $\longrightarrow$ $\langle S\rangle = 0$

- strangeness neutrality implicitly defines $\mu_{S0}(T,\mu_B) = \mu_S(T,\mu_B)\big|_{\langle S\rangle=0}$

$$\chi_{01}^{BS}(T,\mu_B,\mu_{S0}) = 0 \;\Rightarrow\; \frac{d\chi_{01}^{BS}}{d\mu_B} = 0 \;\Leftrightarrow\; \boxed{\frac{\partial\mu_{S0}}{\partial\mu_B} = \frac{1}{3}C_{BS}}$$

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particle number conservation ⟷ phases of QCD

⟶ access B-S correlation through strangeness neutrality!

LOW-ENERGY MODELING OF QCD

PQM MODEL

- Polyakov-loop enhanced quark-meson model

$$\Gamma_k = \int_0^{1/T} dx_0 \int d^3x \left\{ \bar{q}\big(\gamma_\nu D_\nu + \gamma_\nu C_\nu\big) q + \bar{q}\, h\cdot \Sigma_5 q + \mathrm{tr}\big(\bar{D}_\nu \Sigma \cdot \bar{D}_\nu \Sigma^\dagger\big) + \tilde{U}_k(\Sigma) + U_{\mathrm{glue}}(L, \bar{L}) \right\}$$

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- scalar and pseudoscalar meson nonets:

$$\Sigma = T^a(\sigma^a + i\pi^a) \ni \begin{array}{l} \{\sigma\,, f_0,\, a_0^0,\, a_0^+,\, a_0^-,\, \boxed{\kappa^0,\, \bar{\kappa}^0,\, \kappa^+,\, \kappa^-}\} \\ \{\eta\,, \eta',\, \pi^0,\, \pi^+,\, \pi^-,\, \boxed{K^0,\, \bar{K}^0,\, K^+,\, K^-}\} \end{array}$$

open strange mesons
$l\bar{s}\,,\; s\bar{l}$

$$\Sigma_5 = T^a(\sigma^a + i\gamma_5\pi^a)$$

- quarks (assume light isospin symmetry): $q = \begin{pmatrix} l \\ l \\ s \end{pmatrix}$

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- chemical potential matrix

$$\mu = \begin{pmatrix} \frac{1}{3}\mu_B & 0 & 0 \\ 0 & \frac{1}{3}\mu_B & 0 \\ 0 & 0 & \frac{1}{3}\mu_B - \mu_S \end{pmatrix}$$

- vector source for the chemical potential

$$C_\nu = \delta_{\nu 0}\, \mu$$

- covariant derivative couples chemical potentials to mesons

$$\bar{D}_\nu \Sigma = \partial_\nu \Sigma + [C_\nu, \Sigma]$$

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- effective meson potential:

$$\tilde{U}_k = U_k(\rho_1, \tilde{\rho}_2) - j_l \sigma_l - j_s \sigma_s - c_k\, \xi$$

anomalous $U(1)_A$ breaking via ’t Hooft determinant

$$\xi = \det \Sigma + \det \Sigma^\dagger$$

U(3) x U(3) symmetric potential (as a function of two chiral invariants)

$$\rho_i = \mathrm{tr}(\Sigma \cdot \Sigma^\dagger)^i$$

explicit chiral symmetry breaking: finite light & strange current quark masses

PQM MODEL

- Polyakov-loop enhanced quark-meson model

$$\Gamma_k = \int_0^{1/T} dx_0 \int d^3x \left\{ \bar{q}\left(\gamma_\nu D_\nu + \gamma_\nu C_\nu\right) q + \bar{q}\, h \cdot \Sigma_5 q + \mathrm{tr}\left(\bar{D}_\nu \Sigma \cdot \bar{D}_\nu \Sigma^\dagger\right) + \tilde{U}_k(\Sigma) + U_{\mathrm{glue}}(L, \bar{L}) \right\}$$

- temporal gluon background field in cov. derivative: $D_\nu = \partial_\nu - ig\delta_{\nu 0} A_0$
- Polyakov loop: $L = \dfrac{1}{N_c}\left\langle \mathrm{Tr}_f \mathcal{P}\, e^{ig\int_0^\beta d\tau A_0(\tau)} \right\rangle$
- Polyakov loop potential: [Lo et. al., hep-lat/1307.5958]

$$\frac{U_{\mathrm{glue}}(L, \bar{L})}{T^4} = -\frac{1}{2} a(T) \bar{L} L + b(T) \ln\left[M_H(L, \bar{L})\right] + \frac{1}{2} c(T)(L^3 + \bar{L}^3) + d(T)(\bar{L} L)^2$$

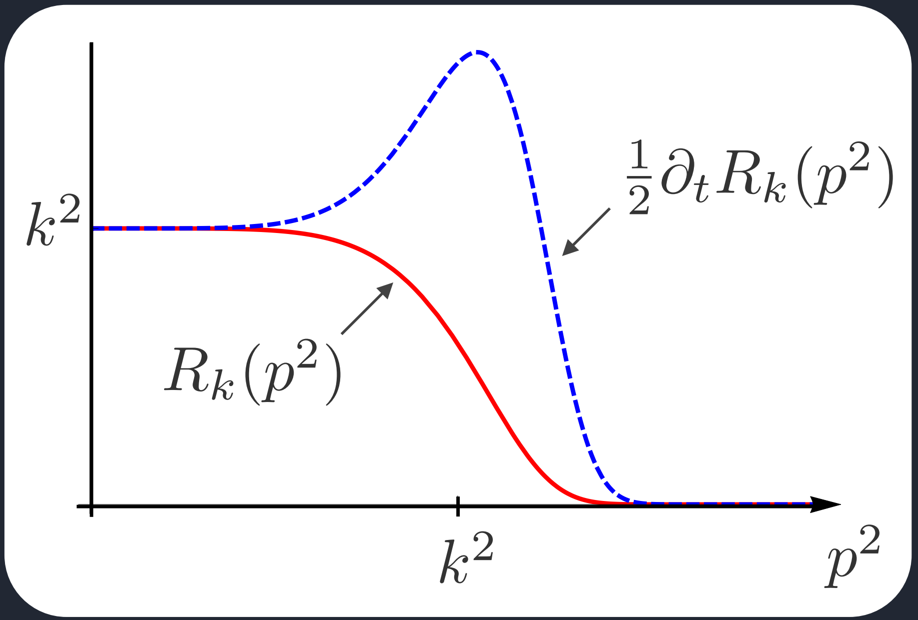
Haar measure $\quad M_H(L, \bar{L}) = 1 - 6\bar{L}L + 4(L^3 + \bar{L}^3) - 3(\bar{L}L)^2$

- parameters fitted to reproduce lattice pressure and Polyakov loop susceptibilities
- approximate N_f and μ dependence from QCD and HTL/HDL arguments

[Herbst et. al., hep-ph/1008.0081, 1302.1426]
[Haas et. al., hep-ph/1302.1993]

$\longrightarrow$ \`statistical confinement´

FUNCTIONAL RG



- introduce regulator to partition function to suppress momentum modes below energy scale k (Euclidean space):

$$Z_k[J] = \int \mathcal{D}\varphi \, e^{-S[\varphi]-\Delta S_k[\varphi]+\int_x J\varphi}$$

$$\Delta S_k[\varphi] = \frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\,\varphi(-q)R_k(q)\varphi(q)$$

- scale dependent effective action:

$$\Gamma_k[\phi] = \sup_J \left\{ \int_x J(x)\phi(x) - \ln Z_k[J] \right\} - \Delta S_k[J] \qquad \phi = \langle\varphi\rangle_J$$

- evolution equation for Γ_k: [Wetterich 1993]

$$\boxed{\partial_t \Gamma_k = \tfrac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1}\partial_t R_k\right]} \qquad \partial_t = k\frac{d}{dk}$$

successively integrate out fluctuations from UV to IR (Wilson RG)

full quantum effective action (generates 1PI correlators)

$\longrightarrow$ Γ_k is eff. action that incorporates all fluctuations down to scale k

$\longrightarrow$ lowering k: zooming out / coarse graining

PQM + FUNCTIONAL RG

- dynamical chiral symmetry breaking though U_k
- `statistical´ confinement through A_0 background (U_{glue})
- beyond mean-field (resummation of infinite class of diagrams through the FRG)
- thermal quark distributions modified through feedback from A_0

$$n_F(E) = \frac{1}{e^{(E-\mu)/T}+1} \qquad \xrightarrow{A_0 \neq 0} \qquad N_F(E;L,\bar{L}) = \frac{1+2\bar{L}e^{(E-\mu)/T}+Le^{2(E-\mu)/T}}{1+3\bar{L}e^{(E-\mu)/T}+3Le^{2(E-\mu)/T}+e^{3(E-\mu)/T}}$$

$$\longrightarrow \begin{cases} \frac{1}{e^{3(E-\mu)/T}+1}\,, & L\to 0 \text{ (confinement)} \\ \frac{1}{e^{(E-\mu)/T}+1}\,, & L\to 1 \text{ (deconfinement)} \end{cases}$$

→ `interpolation´ between baryon and quark d.o.f.

[Fukushima, hep-ph/0808.3382]
[Fu & Pawlowski, hep-ph/1508.06504]

→ correct `N_c - scaling´ of particle number fluctuations

[Ejiri et al., hep-ph/0509051]
[Skokov et al., hep-ph/1004.2665]

- thermodynamics from Euclidean (off-shell) formulation

→ simple hierarchy of relevant fluctuations: the lighter the particle, the more relevant it is

→ fluctuations of kaons and s-quarks (coupled to A_0) reasonable for qualitative description of the relevant strangeness effects (for moderate μ)

FLUCTUATIONS AND THE PHASE STRUCTURE AT STRANGENESS NEUTRALITY

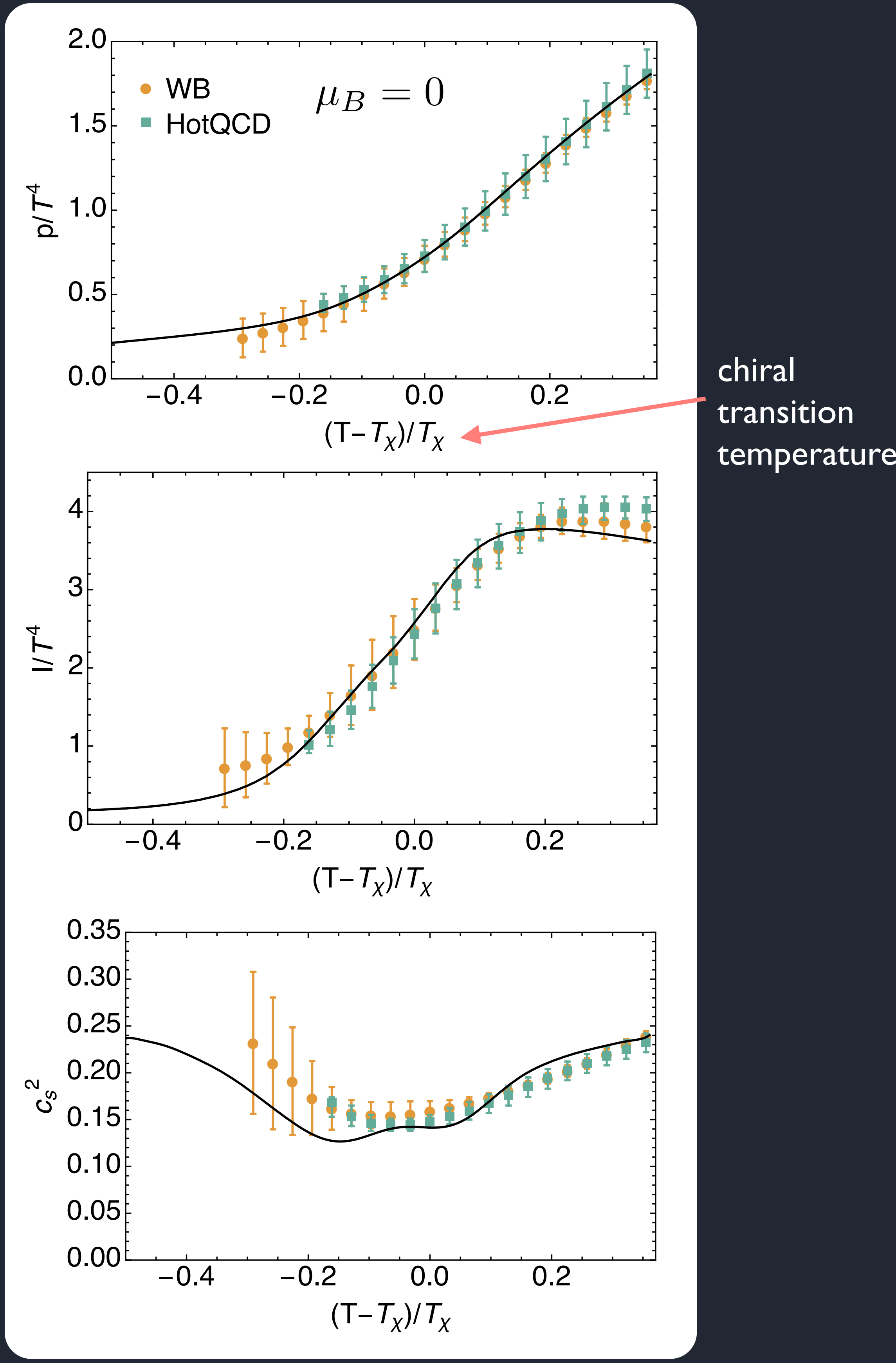
MODEL VS LATTICE EOS

- thermodynamic potential:

$$\Omega = \left(\widetilde{U}_0 + U_{\text{glue}}\right)\Big|_{\text{EoM}}$$

- thermodynamics:

$$p = -\Omega$$
$$s = \frac{\partial p}{\partial T}$$
$$\epsilon = -p + Ts + \mu_B n_B + \mu_S n_S$$
$$I = \epsilon - 3p$$
$$c_s^2 = \frac{s}{\partial \epsilon / \partial T}$$
$$n_B = \chi_{10}^{BS} T^3$$
$$n_S = \chi_{01}^{BS} T^3$$



[HotQCD, hep-lat/1407.6387 & 1701.04325]
[Wuppertal-Budapest, hep-lat/1309.5258]

MODEL VS LATTICE EOS

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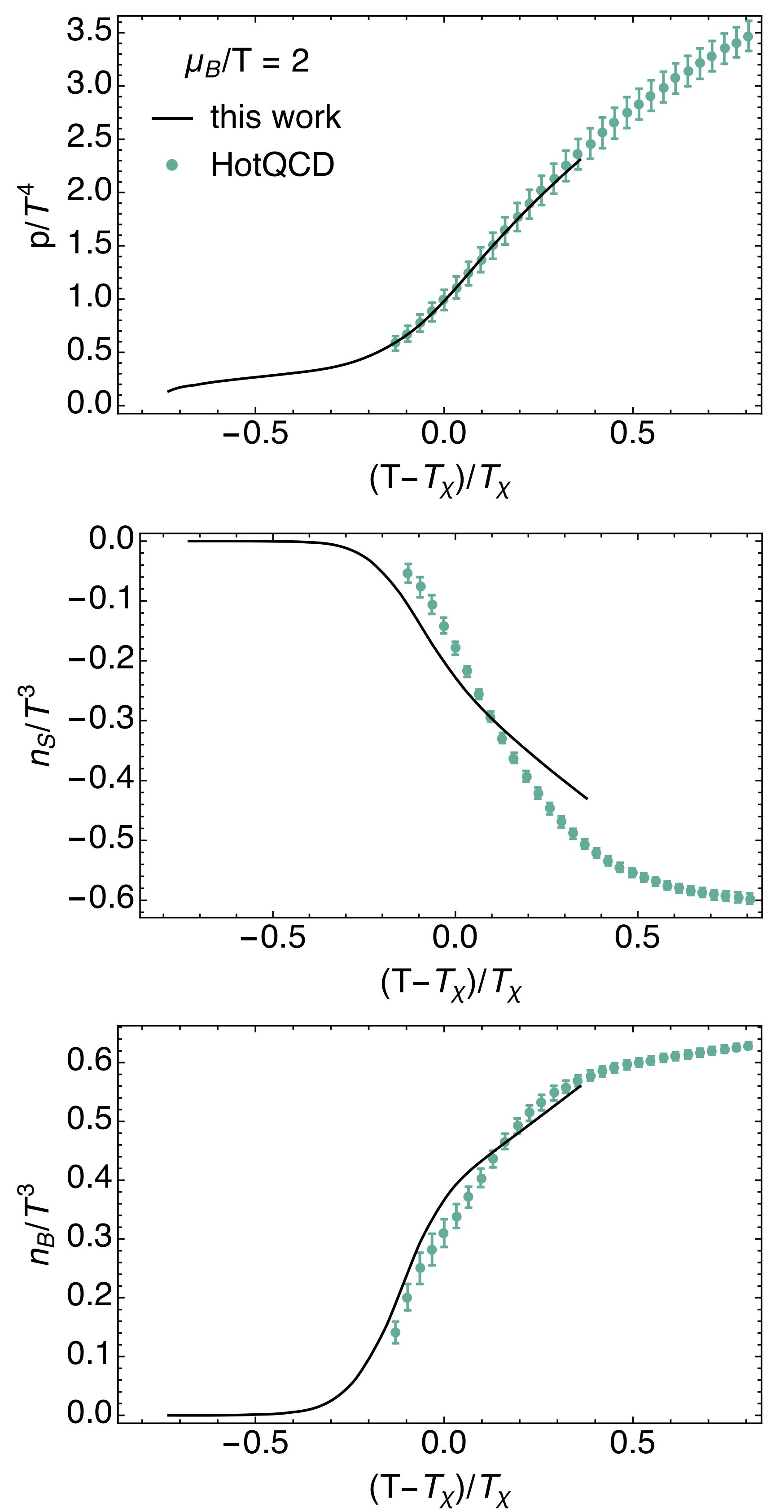

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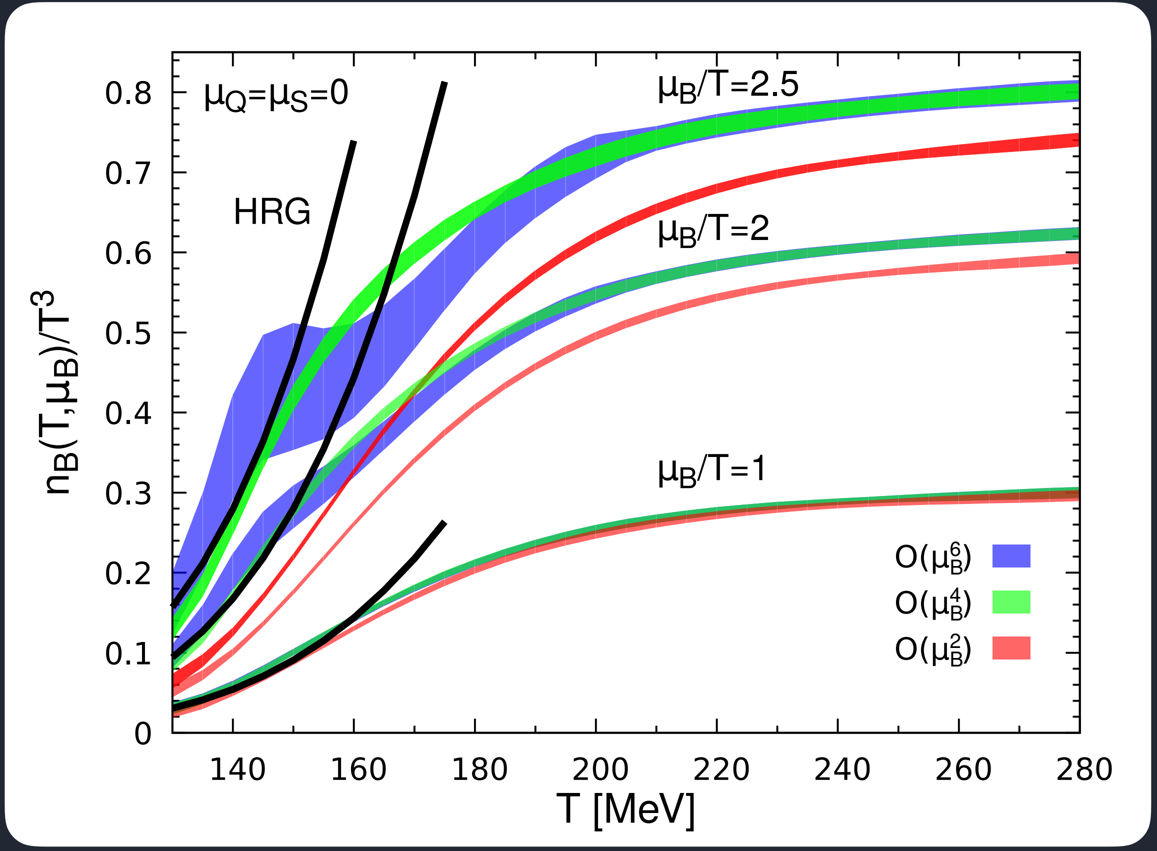

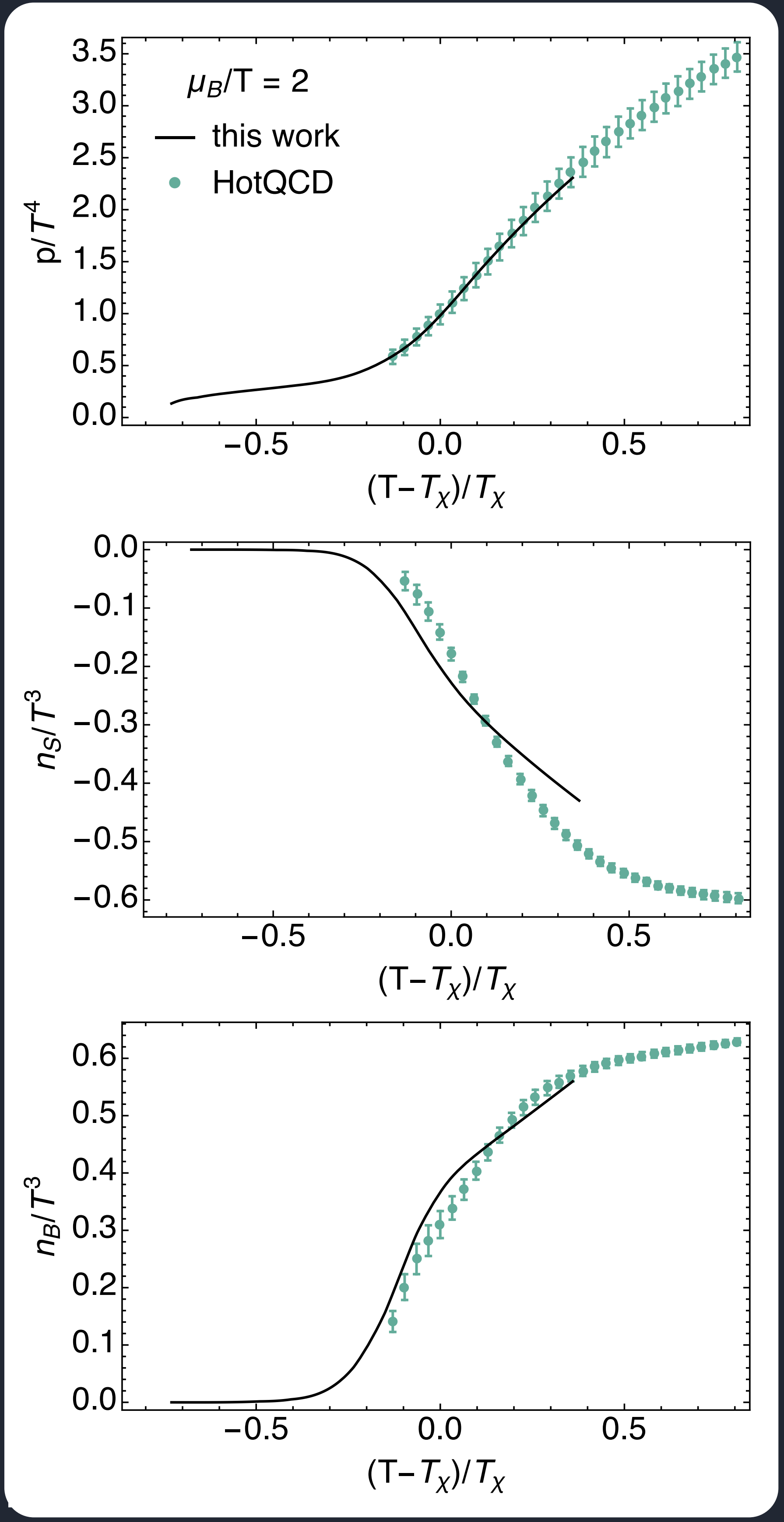

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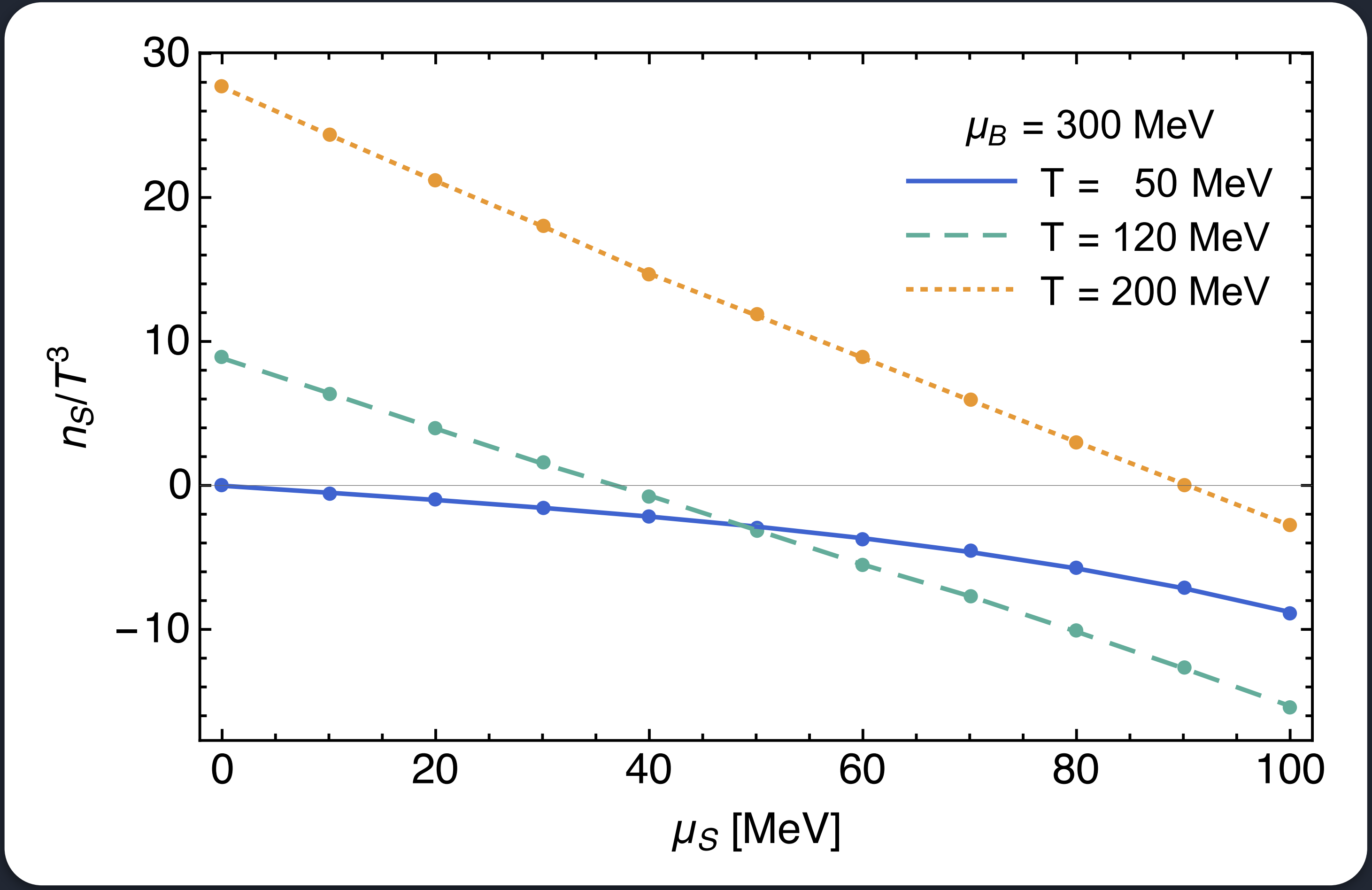
STRANGENESS DENSITY

as a function of μ_S



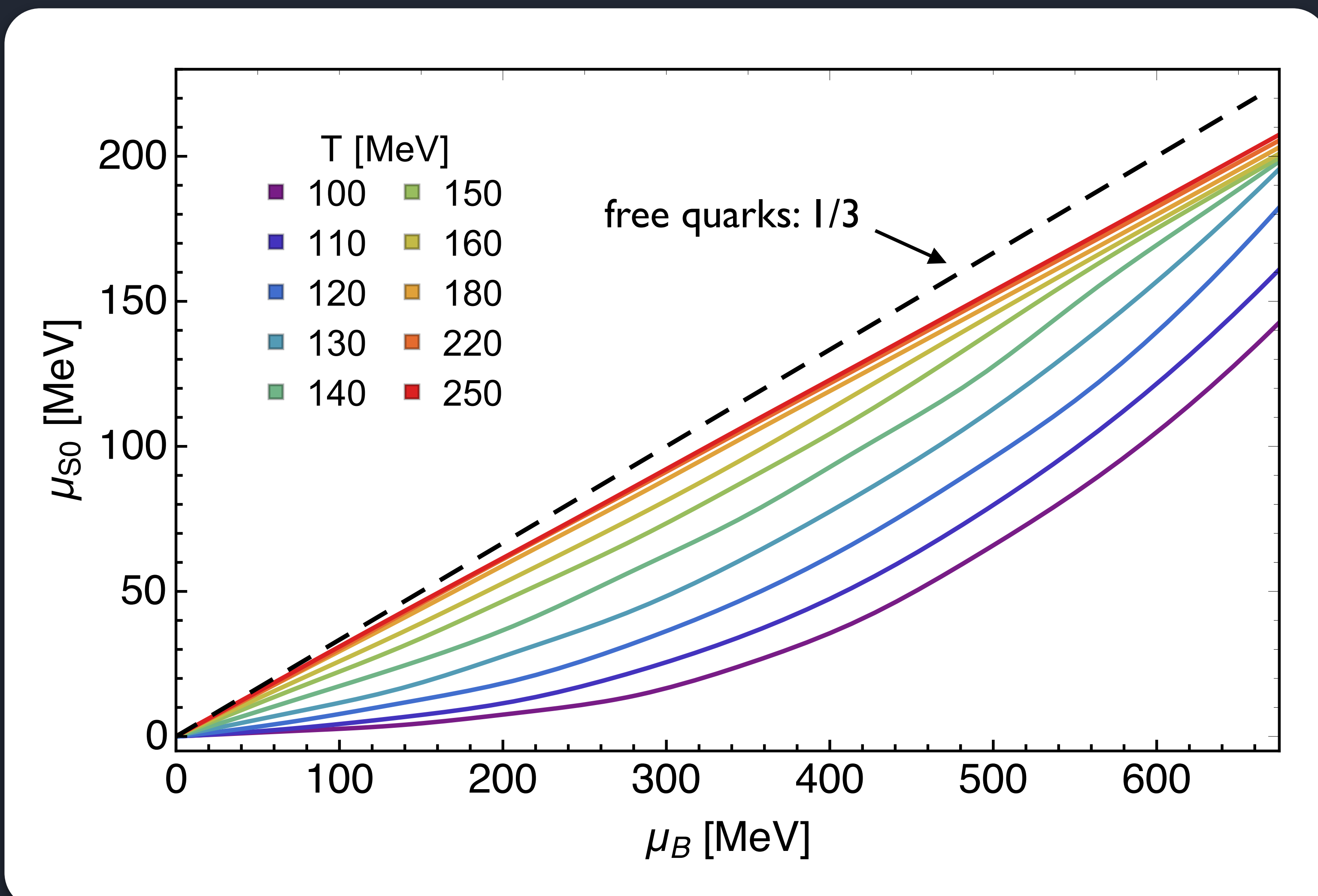
- strangeness number density decreases with increasing μ_S
- strangeness neutrality: zero crossing

$$\mu_{S0} \equiv \mu_S(T, \mu_B)\big|_{n_S=0}$$

- almost linear at larger T: higher strangeness cumulants suppressed?

STRANGENESS CHEMICAL POTENTIAL

as a function of μ_B at strangeness neutrality



- slope directly related to baryon-strangeness correlations:

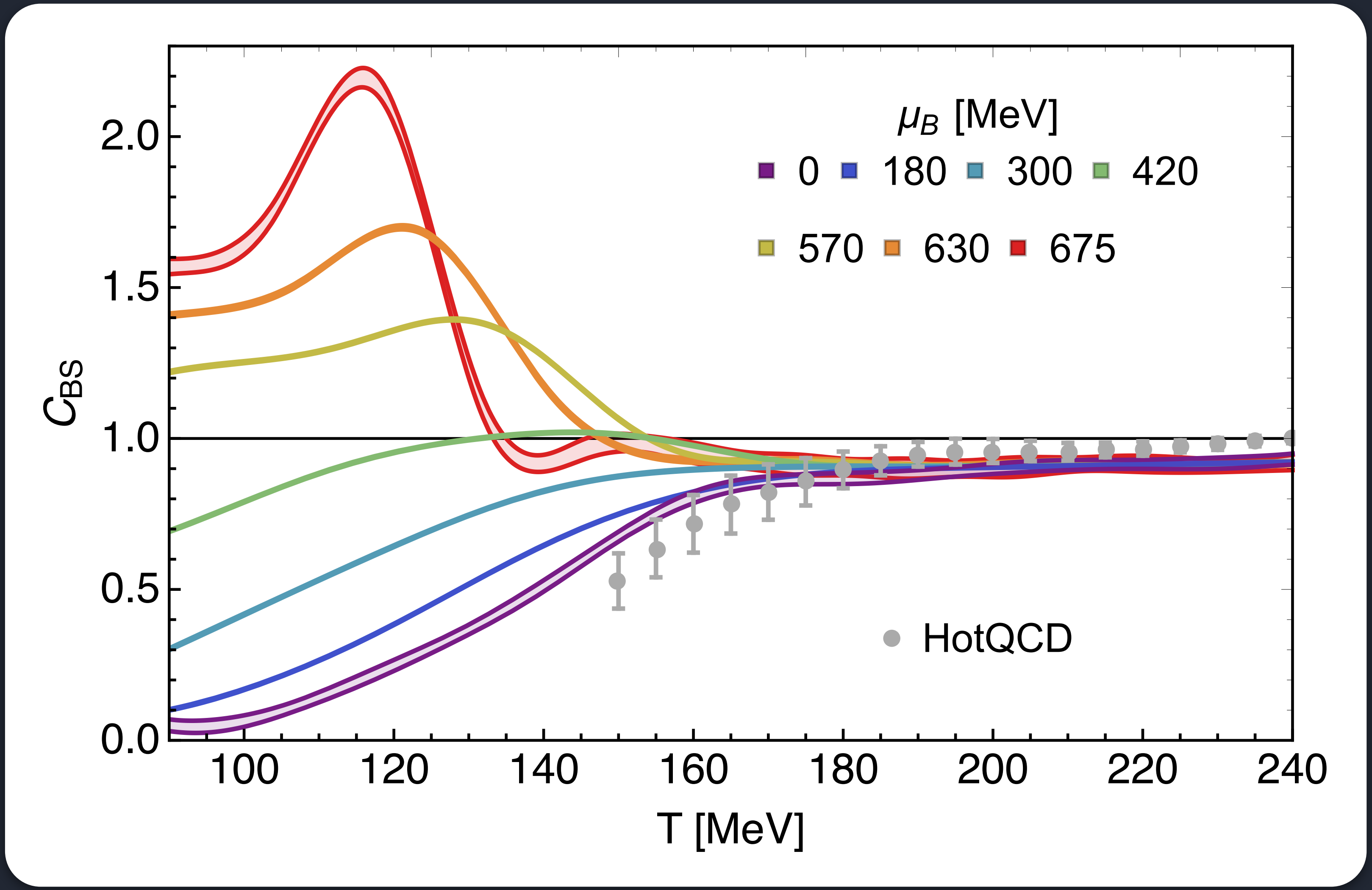
$$\frac{\partial \mu_{S0}}{\partial \mu_B} = \frac{1}{3} C_{BS}$$

→ C_{BS} for any T and μ

BARYON-STRANGENESS CORRELATION

at strangeness neutrality

$$3\frac{\partial\mu_{S0}}{\partial\mu_B} = C_{BS} \sim \frac{\langle\text{strange baryons}\rangle}{\langle\text{strange baryons \& mesons}\rangle} \sim \begin{cases} <1 & \text{mesons dominate} \\ =1 & \text{mesons \& baryons (or uncorrelated flavor)} \\ >1 & \text{baryons dominate} \end{cases}$$



lattice results: [HotQCD, hep-lat/1203.0784]

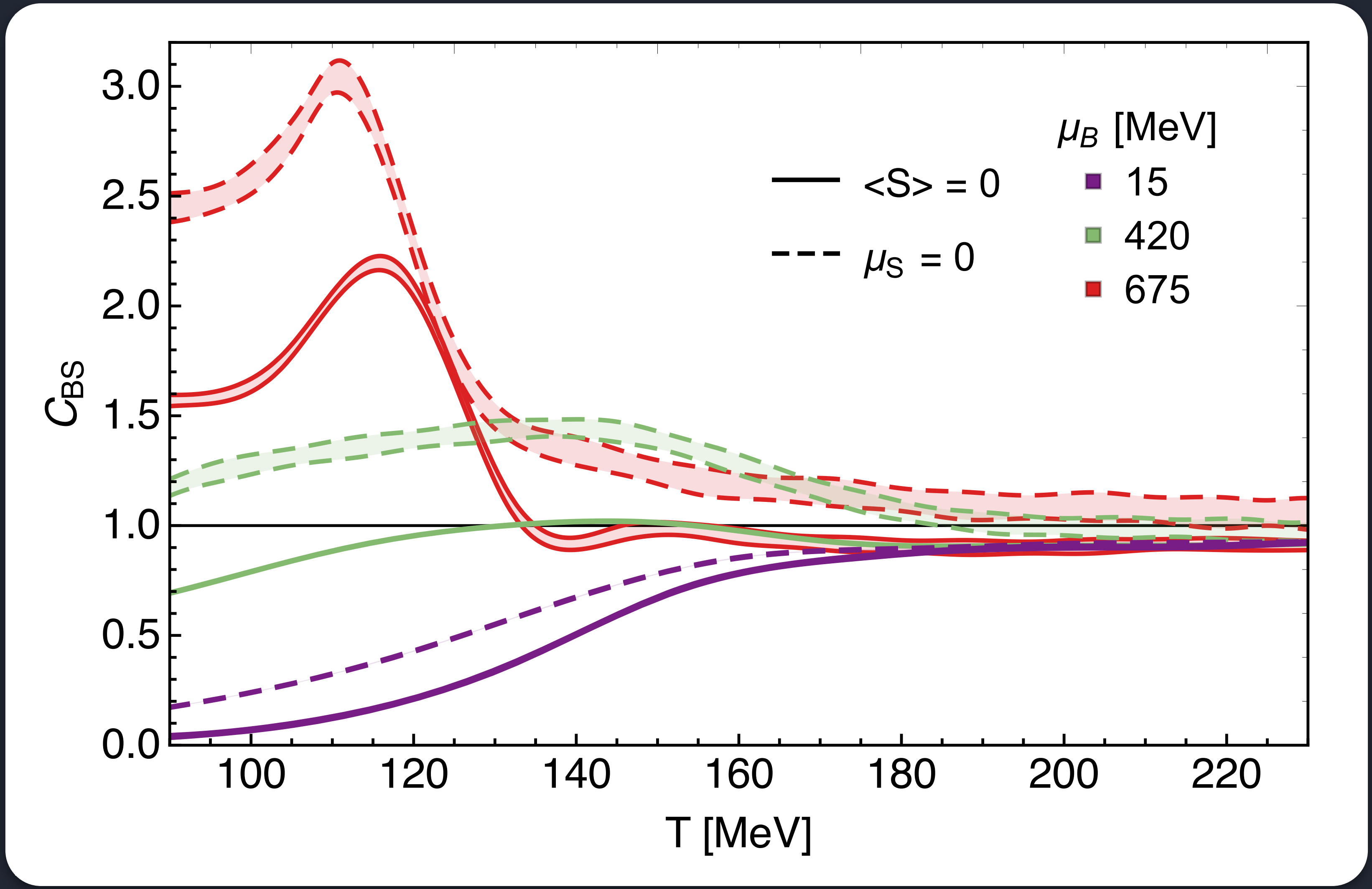
→ competition between baryonic and mesonic sources of strangeness!

maxima at the chiral transition!

→ direct sensitivity to the QCD phase transition

BARYON-STRANGENESS CORRELATION

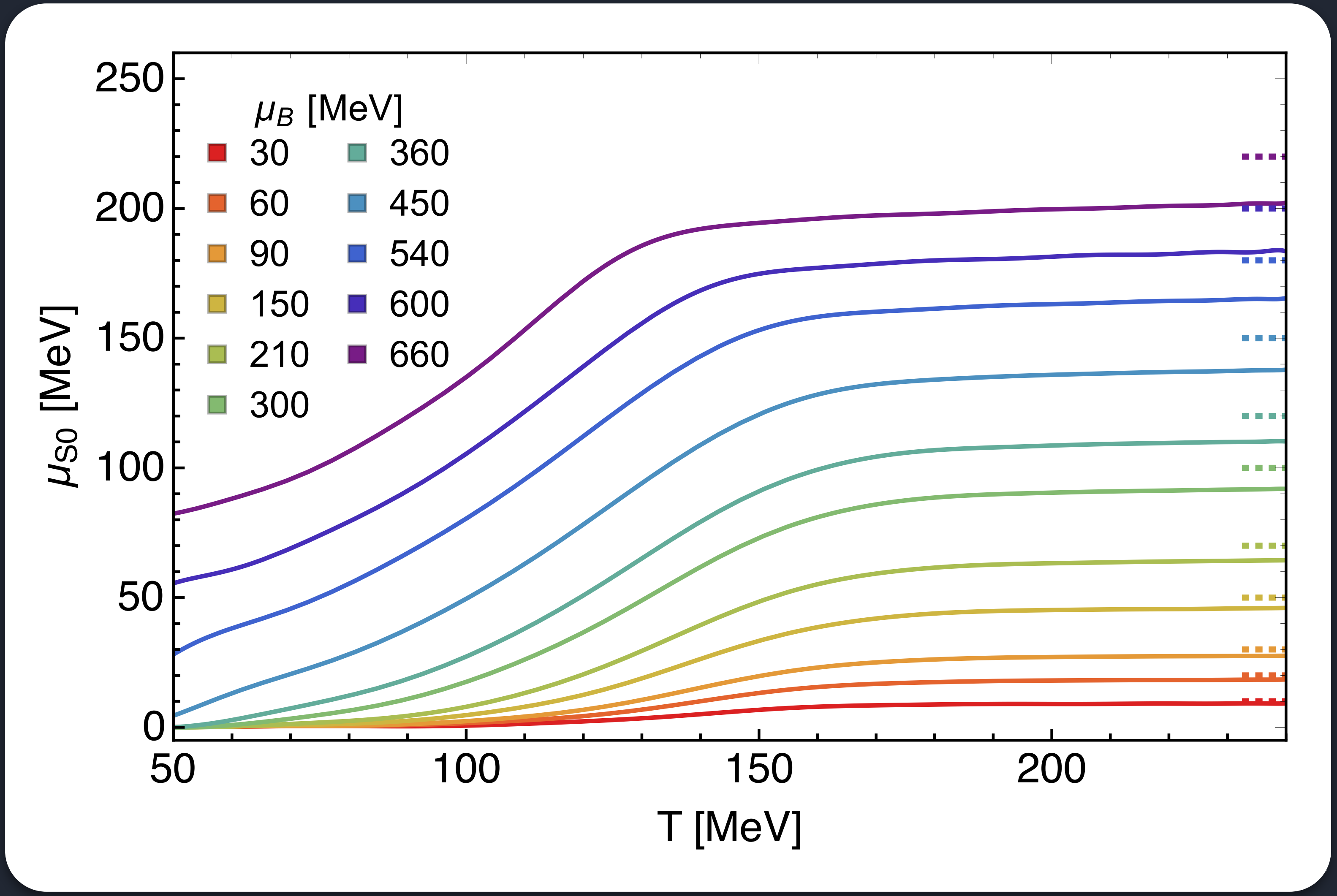
strangeness conservation vs non-conservation



→ sizable impact of strangeness neutrality!

STRANGENESS CHEMICAL POTENTIAL

as a function of T at strangeness neutrality



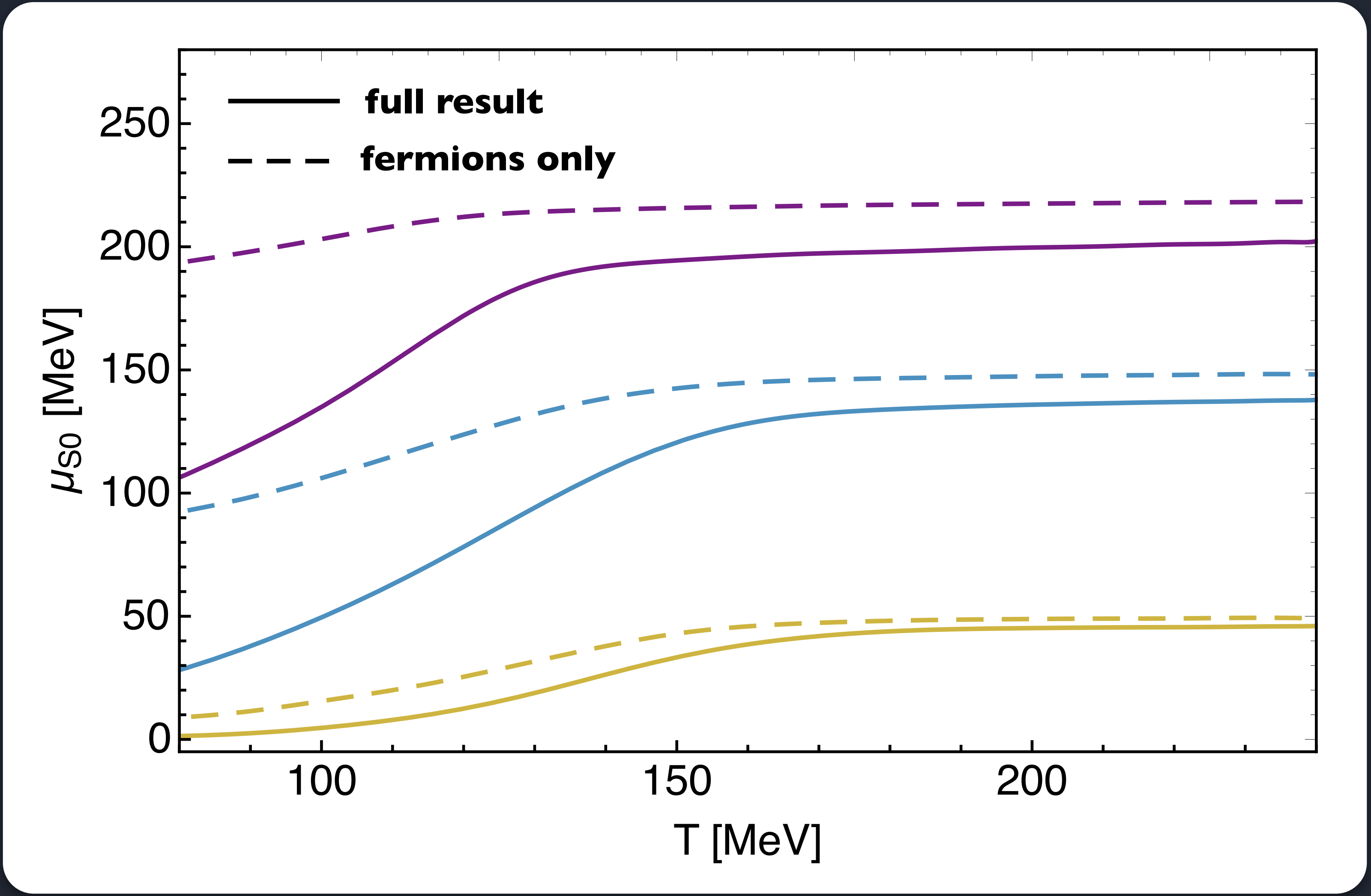
use as input for EoS at finite μ_B

STRANGENESS CHEMICAL POTENTIAL

role of open strange meson dynamics

- quark/baryon vs meson dynamics?
- equation for μ_{S0} from the fermion part of the RG flow [Fukushima, hep-ph/0901.0783]

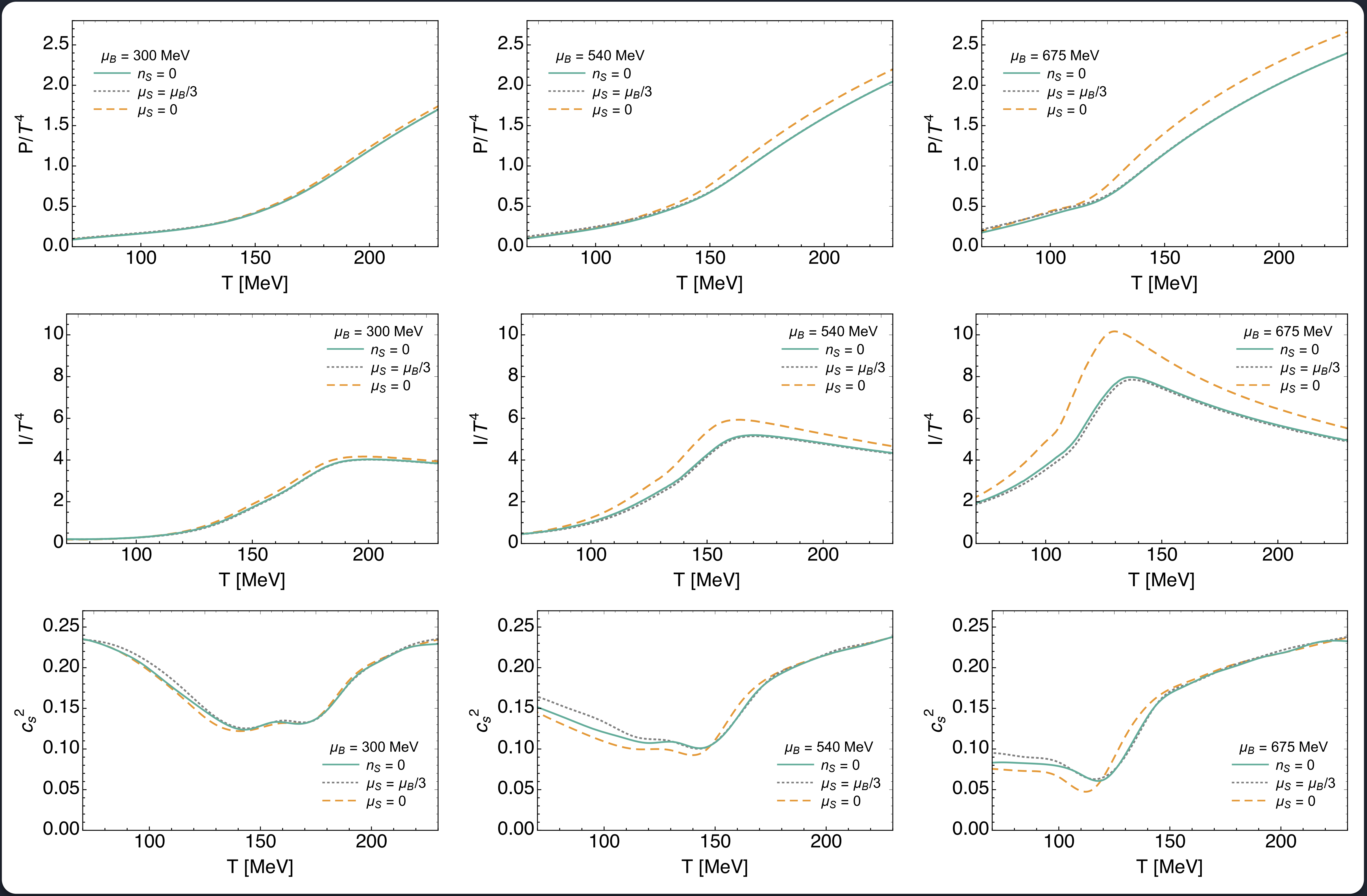
$$\mu_{S0}\big|_{\text{fermions}} \approx \frac{\mu_B}{2} - \frac{T}{2}\ln\left[\frac{\bar{L}(T,\mu_B)}{L(T,\mu_B)}\right]$$



open strange meson fluctuations crucial!

STRANGENESS NEUTRALITY

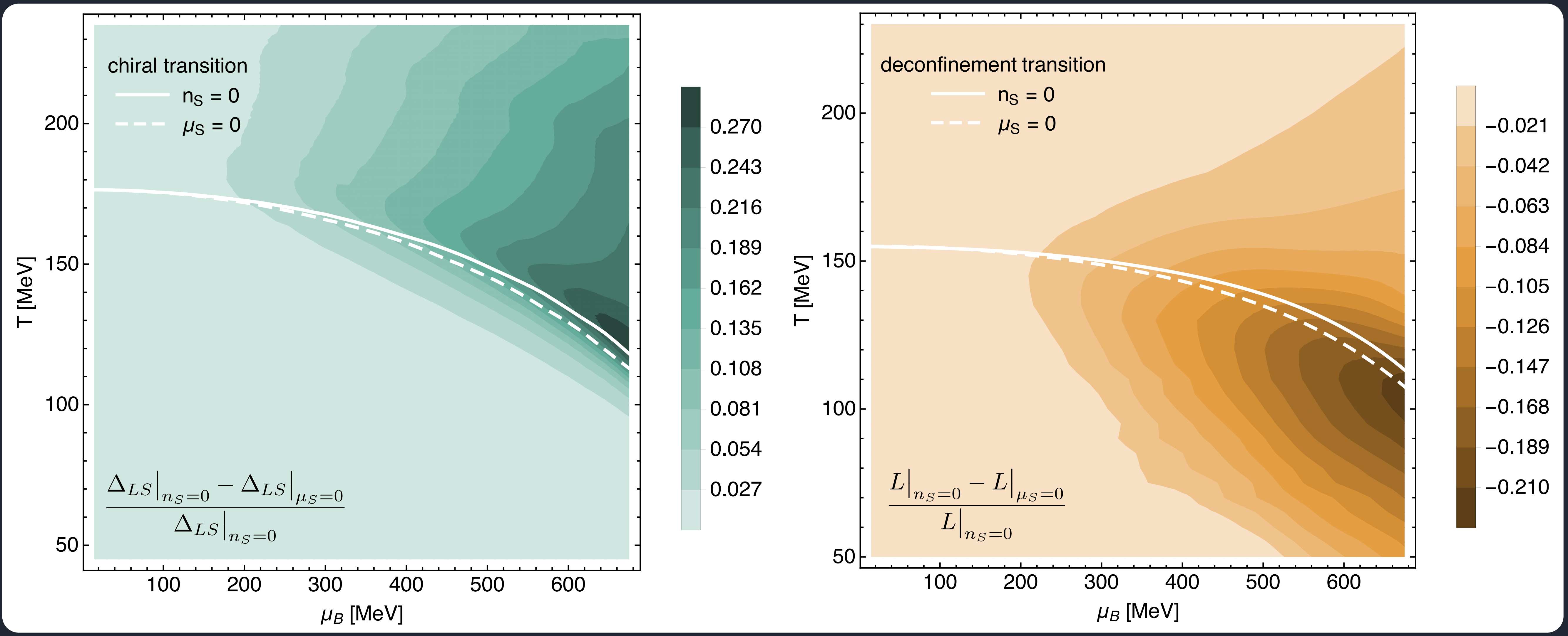
EoS at strangeness neutrality



STRANGENESS NEUTRALITY

phase structure at strangeness neutrality

$$\Delta_{LS} = \frac{\left(\sigma_L - \frac{j_L}{j_S}\sigma_S\right)\Big|_T}{\left(\sigma_L - \frac{j_L}{j_S}\sigma_S\right)\Big|_{T=0}} \qquad L = \frac{1}{N_c}\left\langle \mathrm{Tr}_f \mathcal{P}\, e^{ig\int_0^\beta d\tau A_0(\tau)} \right\rangle$$



→ transition to QGP at larger T (for fixed μ_B)

→ smaller curvature of the phase boundary

→ CEP at smaller μ_B & larger T (?)

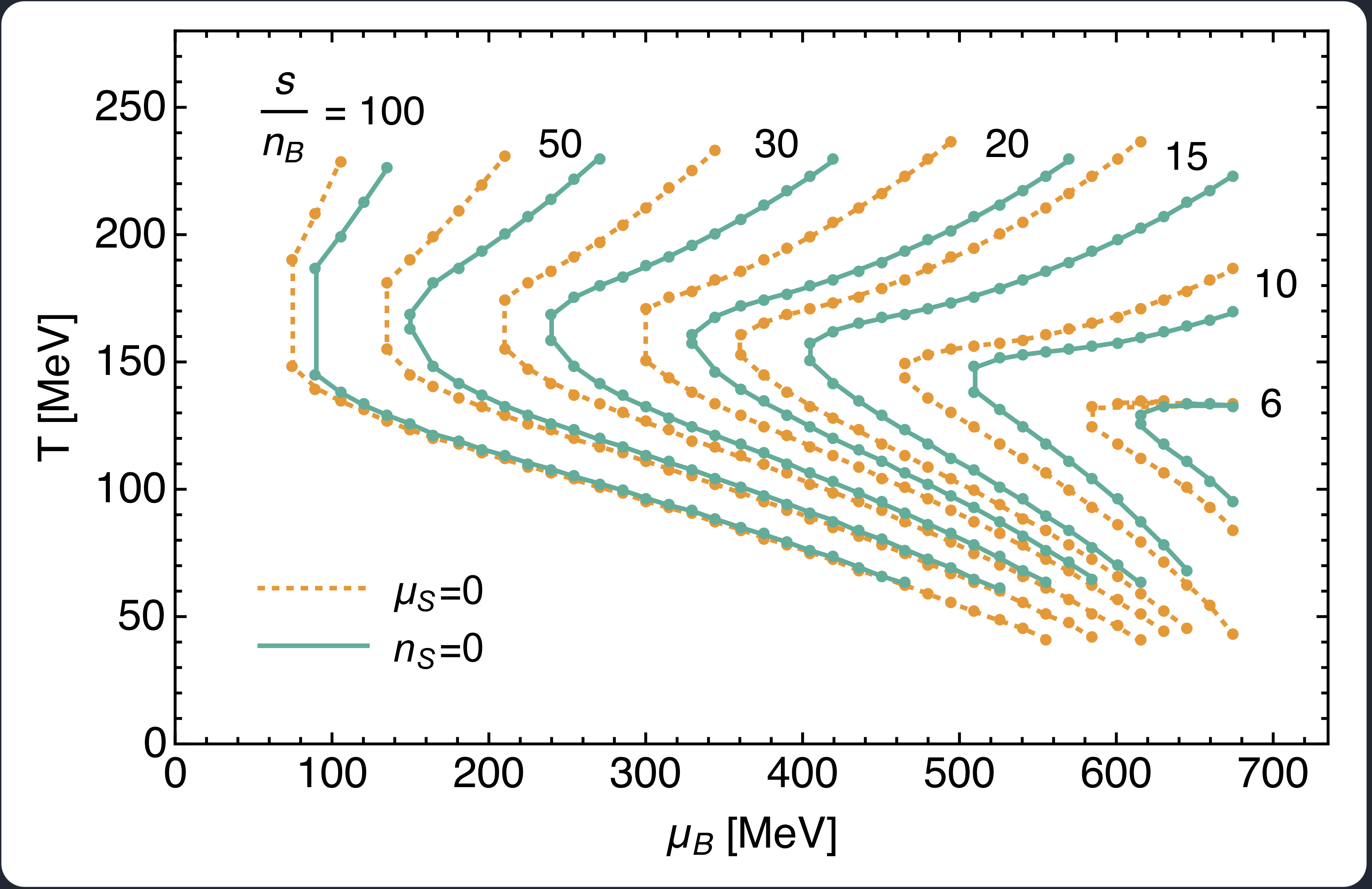
STRANGENESS NEUTRALITY

isentropes at strangeness neutrality

- QGP evolves hydrodynamically at late stages
- `almost´ perfect fluid: small viscosity over entropy density

→ QGP evolves close to isentropes in hydro regime

$$s/n_B = \mathrm{const.}$$



SUMMARY & OUTLOOK

- strangeness neutrality in heavy ion collisions
  - intimate relation between strangeness conservation and phases of QCD as probed in heavy-ion collisions
  - baryon-strangeness correlation via strangeness neutrality
  - finite μ_B requires finite μ_S: sensitive to interplay of QCD d.o.f.
- relevant for phase structure and thermodynamics at finite μ_B
  - ~30% effects already at moderate μ_B, C_{BS} is most sensitive
  - \`delayed´ transition to the QGP in the phase diagram

For the (near) future:

- study larger μ and the CEP
- repeat analysis also with charge chemical potential
- going beyond LPA
- including gluon fluctuations: dynamical hadronization
- self-consistent computation of the A_0 potential
- computation of off-diagonal cumulants

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thank you

BACKUP

STRANGENESS AND CHARGE CONSERVATION

- particle number conservation implicitly defines two functions:

$$\mu_{Q0}(T,\mu_B) = \mu_Q(T,\mu_B)\big|_{n_S=0,\, n_Q=rn_B}$$
$$\mu_{S0}(T,\mu_B) = \mu_S(T,\mu_B)\big|_{n_S=0,\, n_Q=rn_B}$$

$$r = \frac{Z}{A}$$

- this implies:

$$\frac{\partial \mu_{S0}}{\partial \mu_B} = \frac{1}{3} C_{BS} - \frac{\chi_{11}^{QS}}{\chi_2^S}\frac{\partial \mu_{Q0}}{\partial \mu_B}$$

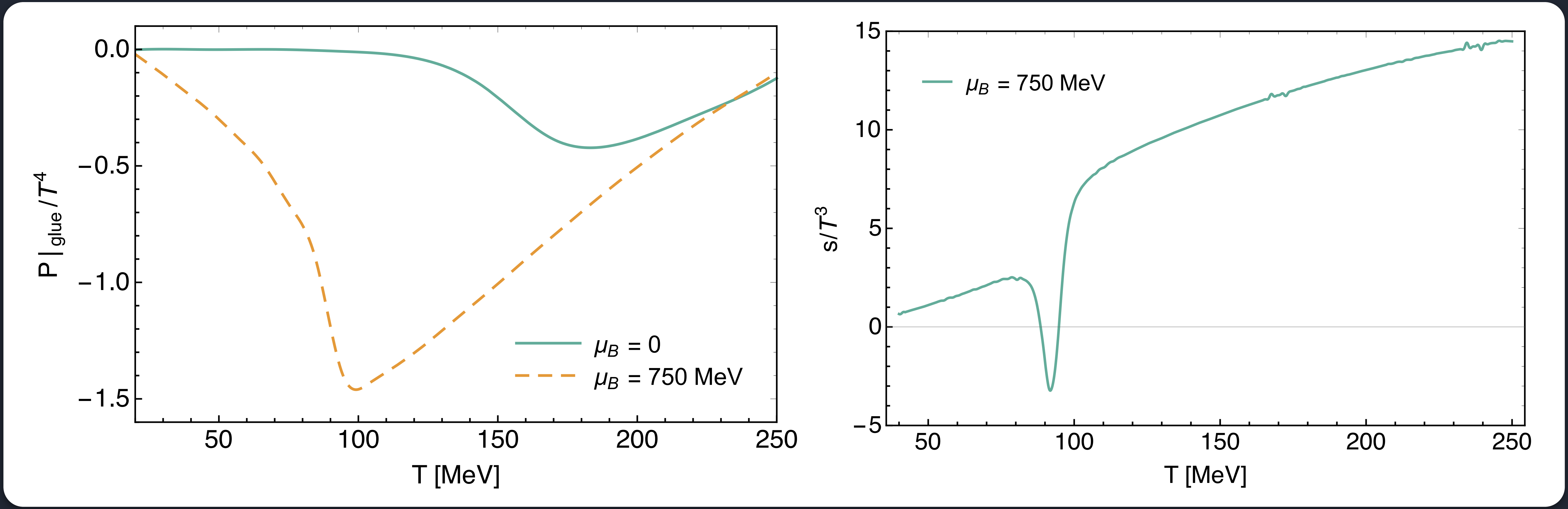
$$\frac{\partial \mu_{Q0}}{\partial \mu_B} = \frac{\chi_{11}^{BS}(\chi_{11}^{SQ} - r\chi_{11}^{BS}) - \chi_2^S(\chi_{11}^{BQ} - r\chi_2^B)}{\chi_2^S(\chi_2^Q - r\chi_{11}^{BQ}) - \chi_{11}^{SQ}(\chi_{11}^{SQ} - r\chi_{11}^{BS})}$$

$\longrightarrow$ generalization of \`freeze-out relations´ used on the lattice to any T and μ_B

2+1 FLAVOR PQM AT LARGE CHEMICAL POTENTIAL

- gluon contribution to the pressure at large chemical potential

$$p\big|_{\mathrm{glue}} = -U_{\mathrm{glue}}(L, \bar{L})$$



⟶ model becomes unphysical at large μ_B

likely due to missing feedback from the matter to the gauge sector / input potential not accurate at large $\bar{L} - L$