The Lifshitz regime in QCD

RDP, VV Skokov & A Tsvelik, 1801.08156

Chiral spirals and their fluctuations

1. Standard phase diagram in T & μ: critical end-point (CEP)

   Not seen from lattice at small μ

2. Quarkyonic phase at large N_c (analytic) and $N_c = 2$ (lattice)

3. Chiral Spirals in Quarkyonic matter: sigma models, SU(N) and U(1)

4. Phase diagram: *just* a 1st order line,
   with *large* fluctuations in the Lifshitz regime

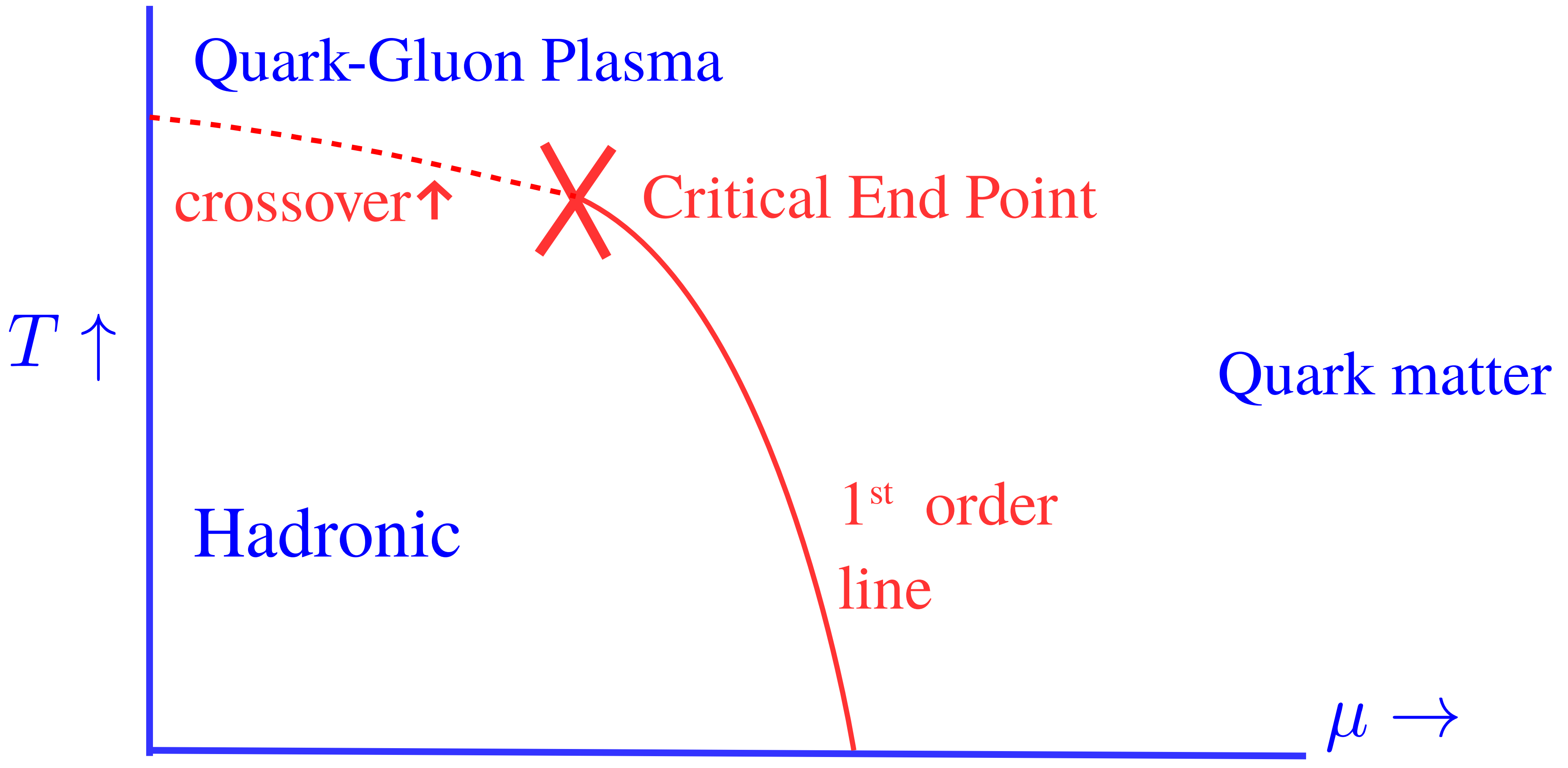
"Standard" phase diagram for QCD in T & μ: CEP?

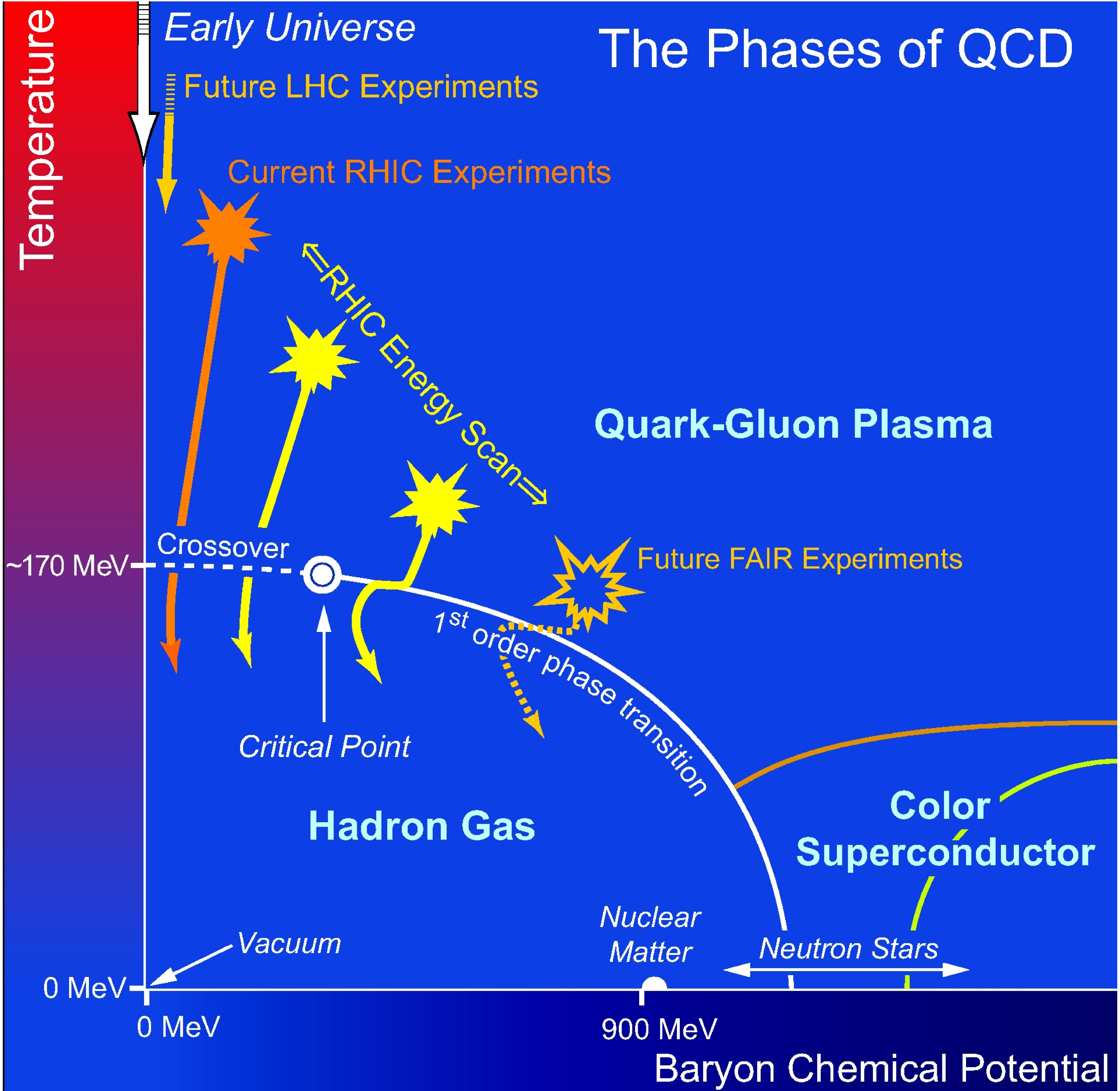
Lattice: at quark chemical potential $\mu = 0$, crossover at $T_{ch} \sim 154$ MeV

At $\mu \neq 0$, quarks *might* change scalar 4-pt coupling < 0, so transition 1st order

Must meet at a Critical End Point (CEP), *true* 2nd order phase transition

Asakawa & Yazaki '89, Stephanov, Rajagopal & Shuryak '98 & '99





The Phases of QCD
Temperature
Early Universe
Future LHC Experiments
Current RHIC Experiments
⇐RHIC Energy Scan⇒
Quark-Gluon Plasma
~170 MeV
Crossover
Future FAIR Experiments
1st order phase transition
Critical Point
Hadron Gas
Color Superconductor
Nuclear Matter
Neutron Stars
Vacuum
0 MeV
0 MeV
900 MeV
Baryon Chemical Potential

Lifshitz phase diagram for QCD

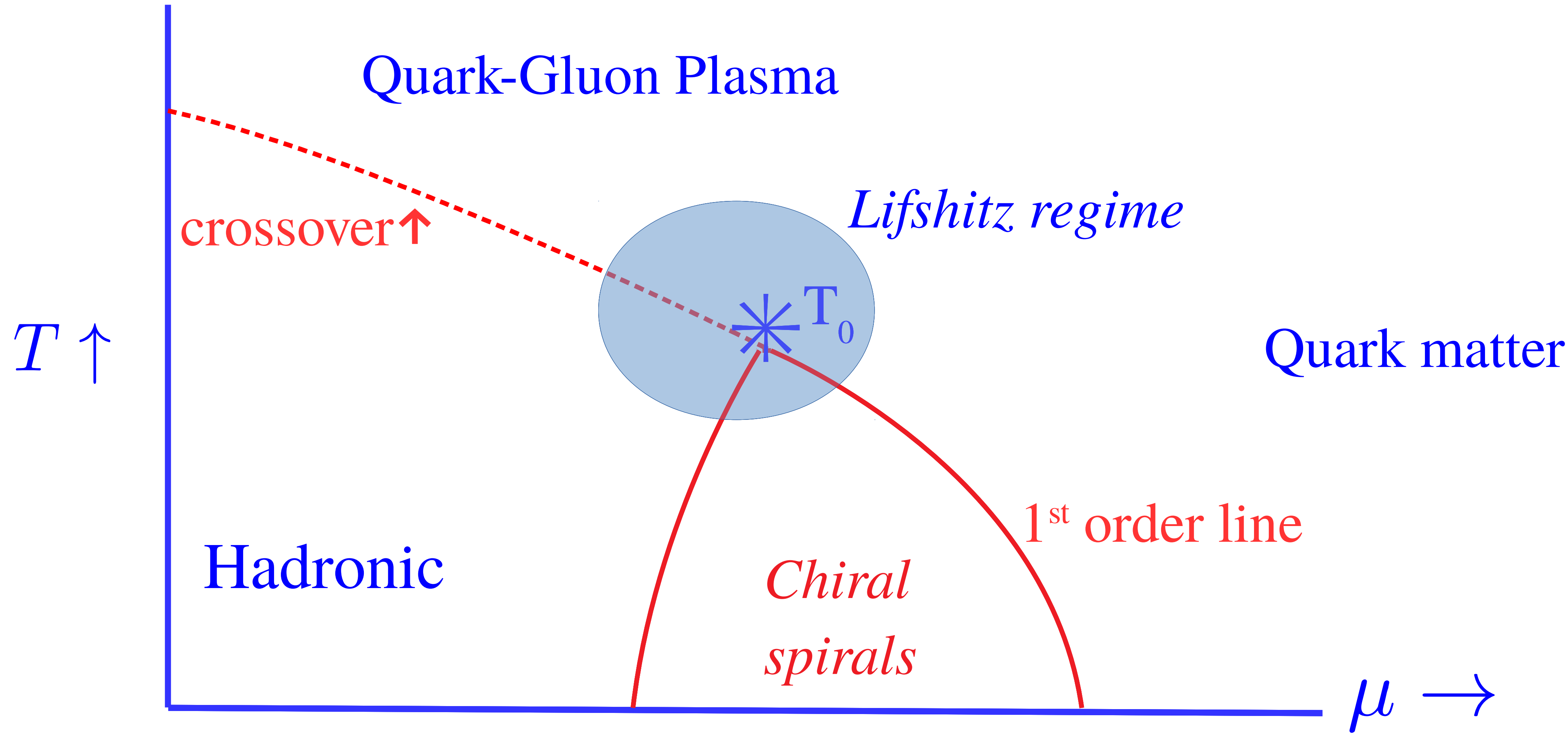
Instead: "Lifshitz regime": strongly coupled, large fluctuations

Unbroken 1st order line to spatially inhomogeneous phases = "chiral spirals"

Hints in heavy ion data?

Fundamental problem in field theory: analogies to phase diagram for polymers

Could be CEP as well...



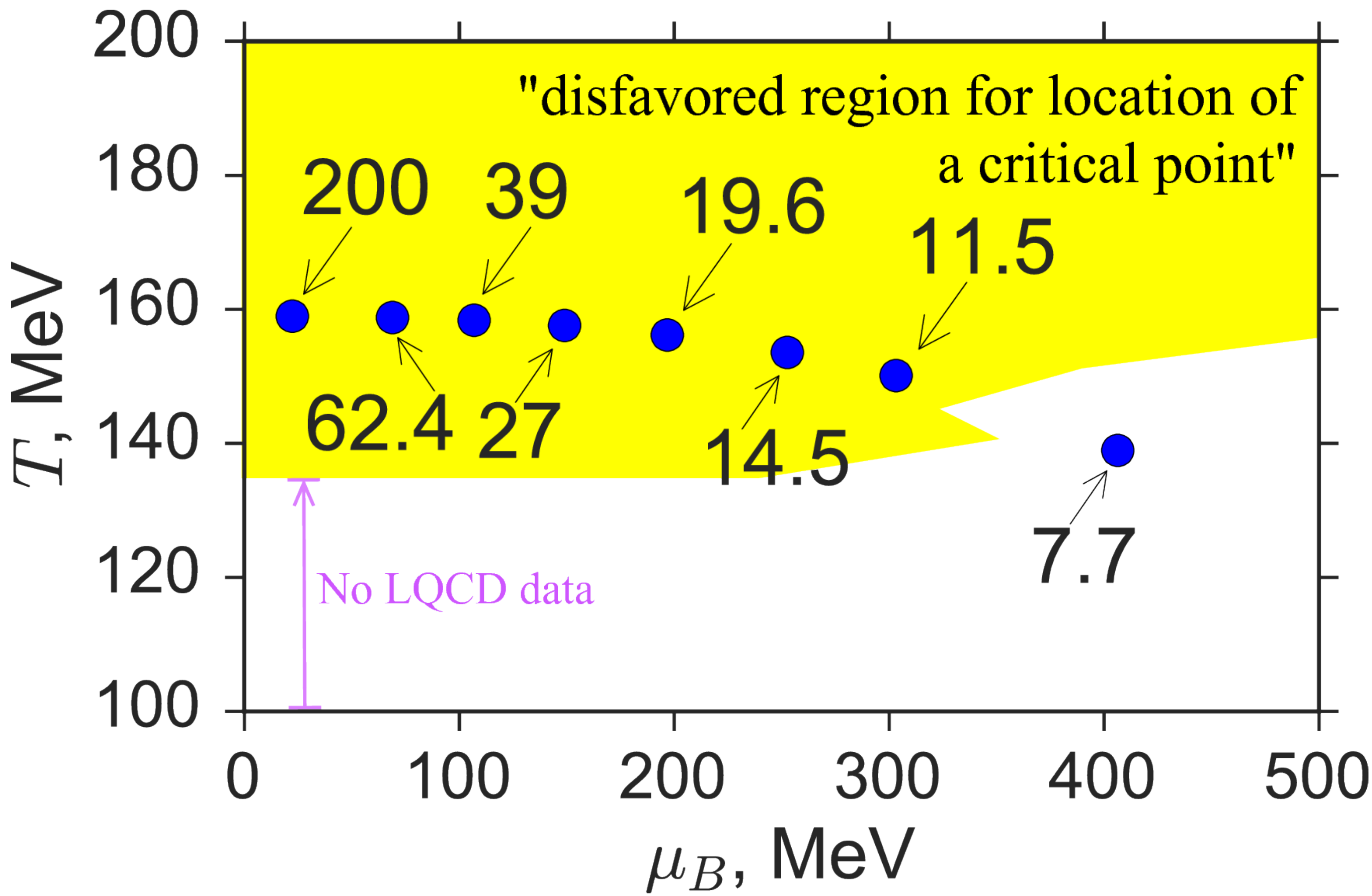
Lattice, hot QCD: no CEP at small μ

Lattice: Hot QCD, 1701.04325

Expand about μ = 0, power series in μ^{2n}, n = 1, 2, 3.

Estimate radius of convergence. *No* sign of CEP by μ_{qk} ~ T

$T \uparrow$



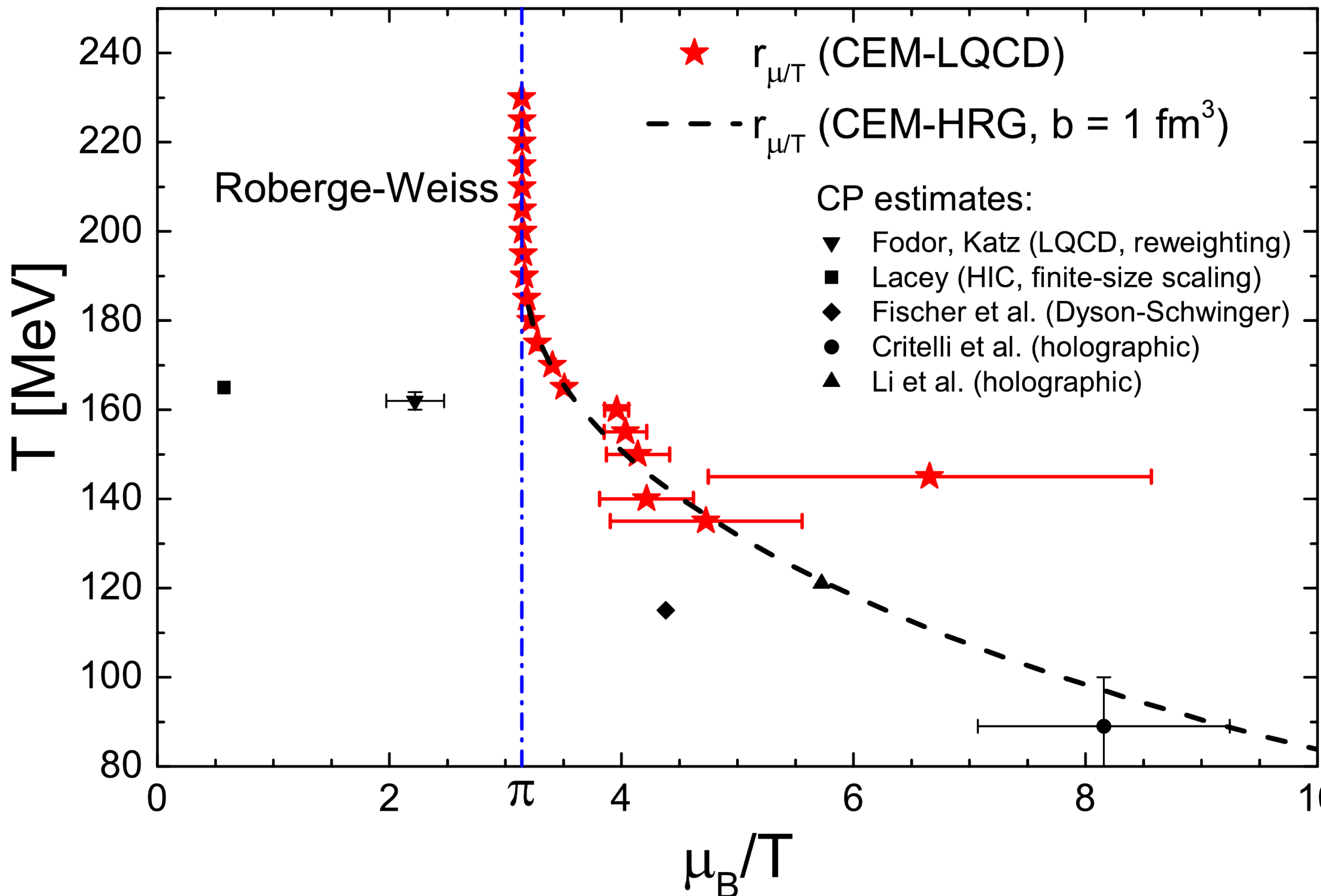
$3\mu_q \rightarrow$

Cluster expansion: no CEP at small μ

Lattice: Vovchenko, Steinheimer, Philipsen & Stoecker, 1701.04325

Use cluster expansion method, different way of estimating power series in μ

No sign of CEP by $\mu_{qk} \sim T$



$3\mu_{qk} \rightarrow$

So if there is no critical endpoint,

what could be going on?

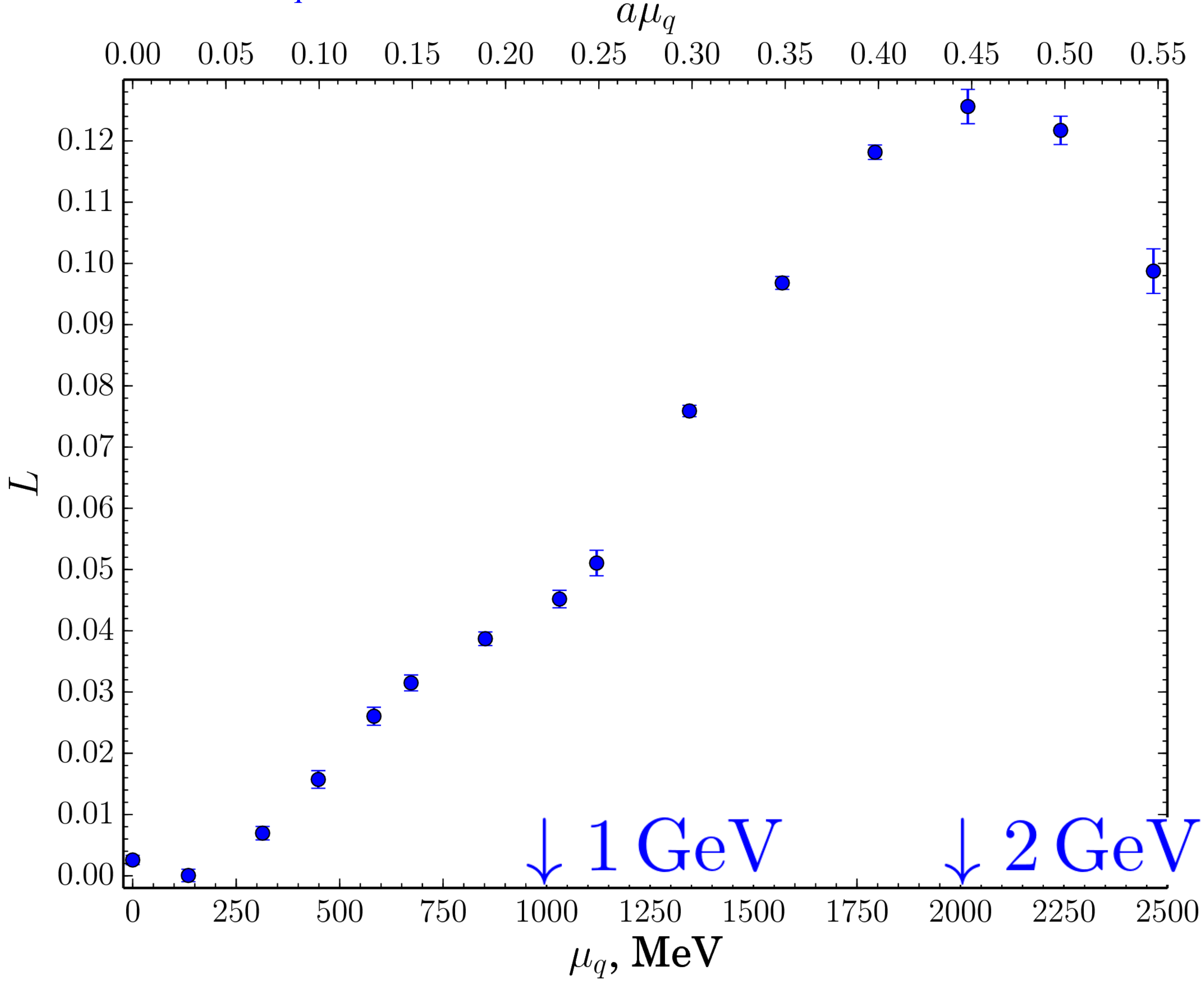
Lattice for T = 0, μ ≠ 0, *two* colors

Lattice: Bornyakov et al, 1711.01869. No sign problem for $N_c = 2$. Two flavors.

Heavy pions, $m_\pi \sim 740$ MeV. $\sqrt{\sigma} = 470$ MeV. 32^4 lattice, a ~ .04 fm

Confined until very high $\mu_{qk} \sim 1$ GeV. *Bare* Polyakov loop:

$\langle \text{loop} \rangle \uparrow$

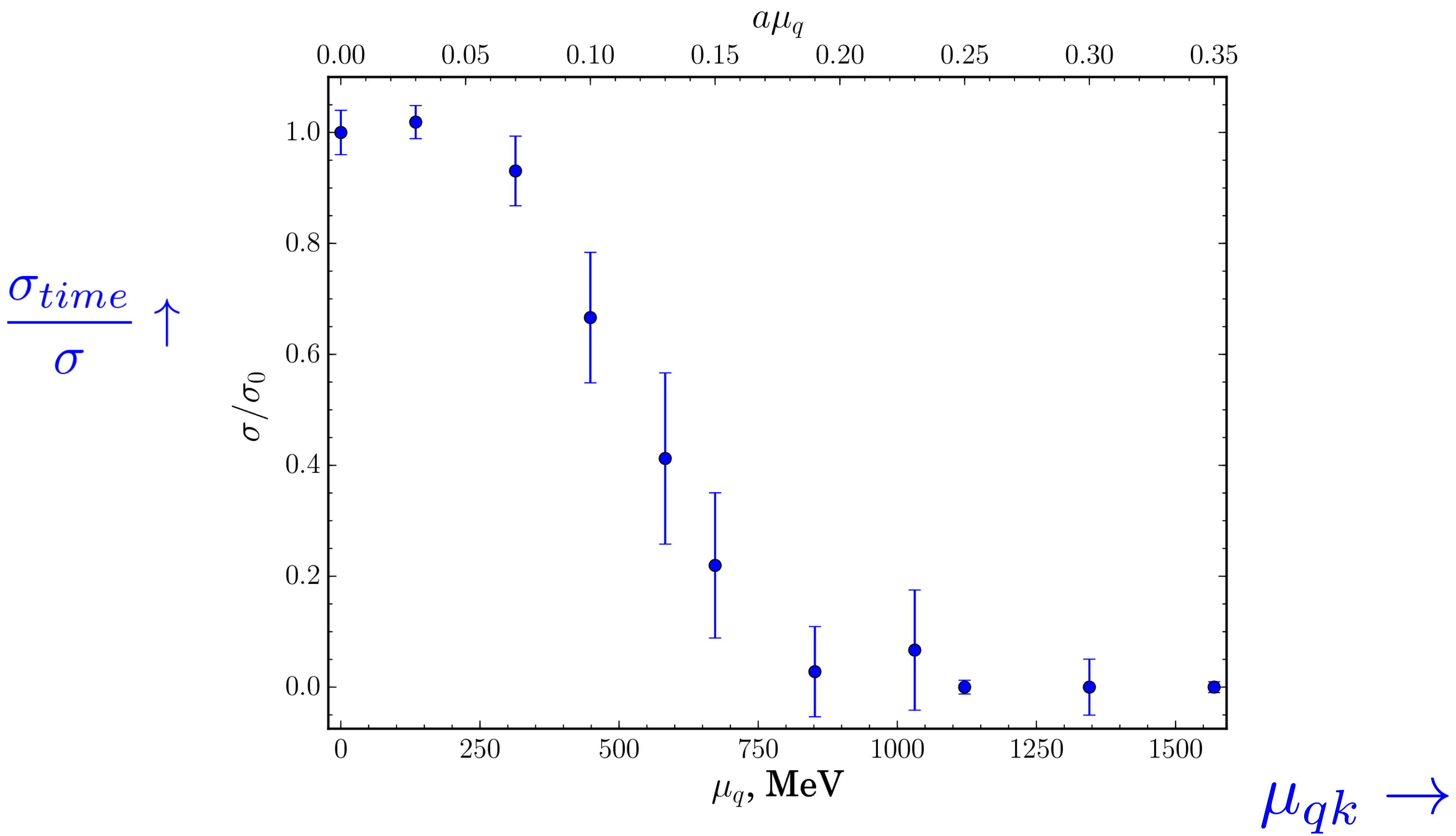


$\mu_{qk} \rightarrow$

Lattice for $T = 0$, $\mu \neq 0$, *two* colors

Lattice: Bornyakov et al, 1711.01869.

String tension in time: nonzero up to $\mu_{qk} \sim 750$ MeV



Phases for $N_c = 2$, $T \sim 0$, $\mu \neq 0$

Braguta, Ilgenfritz, Kotov, Molochkov, & Nikolaev, 1605.04090 (earlier: Hands, Skellerud + …)

Lattice: $N_c = 2$, $N_f = 2$. $m_\pi \sim 400$ MeV, fixed $T \sim 50$ MeV, vary μ_{qk}.

Hadronic phase: $0 \leq \mu_{qk} < m_\pi /2 \sim 200$ MeV. Confined, independent of μ

Dilute baryons: $200 < \mu_{qk} < 350$. Bose-Einstein condensate (BEC) of diquarks.

Dense Baryons: $350 < \mu_{qk} < 600$. Pressure *not* perturbative, BEC

Quarkyonic: $600 < \mu_{qk} < 1100$: pressure ~ perturbative, but excitations *confined*

(Wilson loop ~ area)

Perturbative: $1100 < \mu_{qk}$, but μa too large.

Quarkyonic matter

McLerran & RDP 0706.2191

At large N_c, $g^2 N_c \sim 1$, $g^2 N_f \sim 1/N_c$, so need to go to *large* $\mu \sim N_c^{1/2}$.

$$m^2_{Debye} = g^2((N_c + N_f/2)T^2/3 + N_f \mu^2/(2\pi^2))$$

Doubt large N_c applicable at $N_c = 2$.

When does perturbation theory work?

$T = \mu = 0$: scattering processes computable for momentum $p > 1$ GeV

$T \neq 0$: $p > 2 \pi T$, lowest Matsubara energy

$\mu \neq 0$, $T = 0$: μ is like a scattering scale, so *perhaps* $\mu_{pert} \sim 1$ GeV.

At least for the pressure. Excitations determined by region near Fermi surface

Possible phases of cold, dense quarks

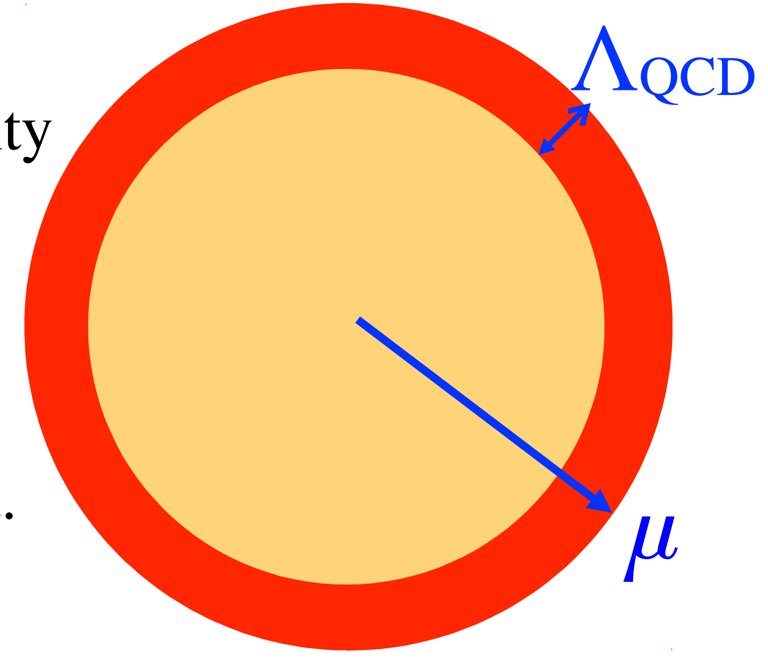
Confined: $0 \leq \mu_{qk} < m_{baryon}/3$. μ doesn't matter

Dilute baryons: $m_{baryon}\ 3 < \mu_{qk} < \mu_{dilute}$: . Effective models of baryons, pions

Dense baryons: $\mu_{dilute} < \mu_{qk} < \mu_{dense}$. Pion/kaon condensates.

Quarkyonic: $\mu_{dilute} < \mu_{qk} < \mu_{perturbative}$. 1-dim. chiral spirals.

Perturbative: $\mu_{perturbative} < \mu_{qk}$. Color superconductivity



$\mu_{perturbative}$ ~ 1 GeV?

Dense baryons and quarkyonic *continuously* related.

U(1) order parameter in both.

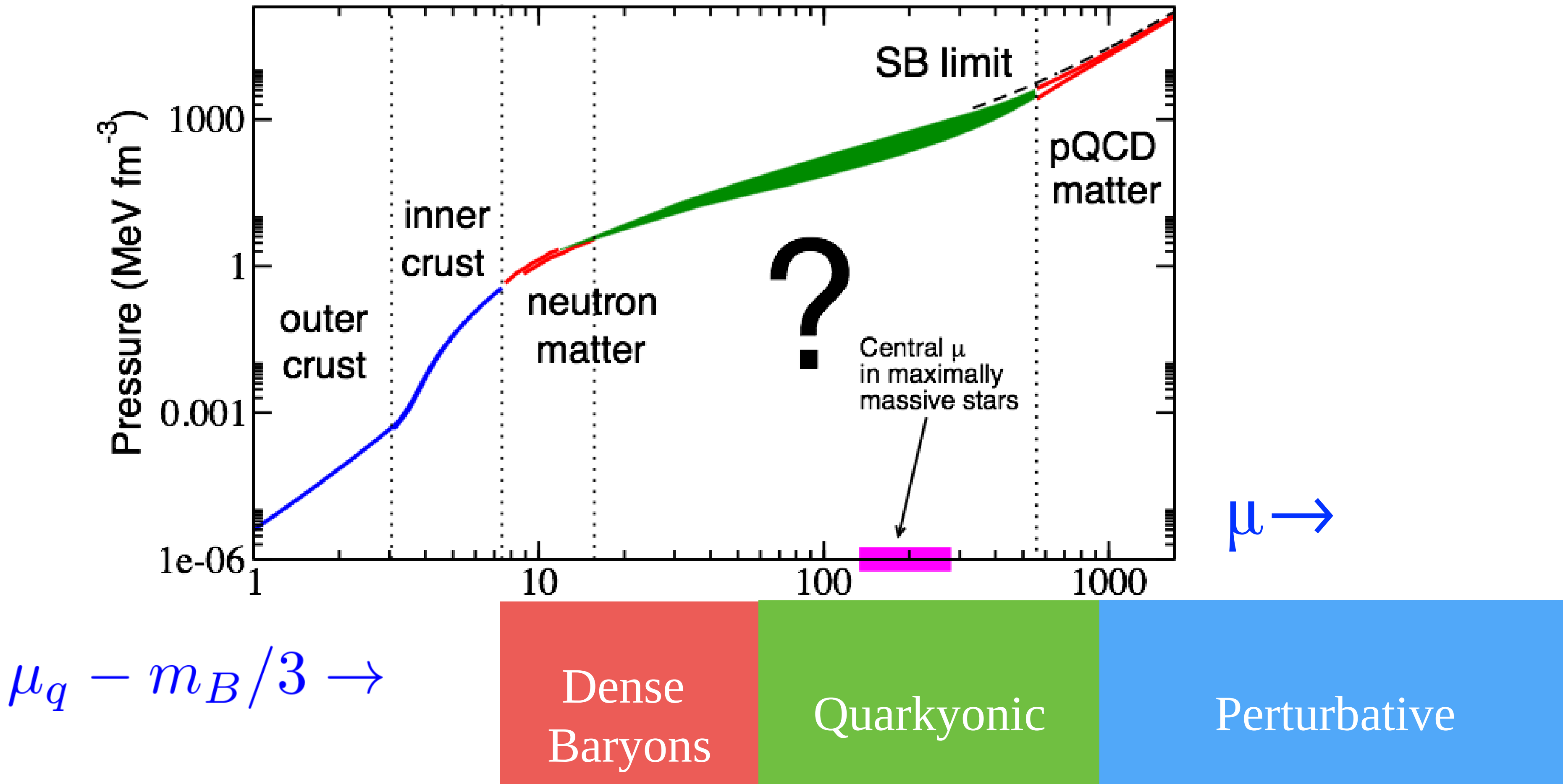
Relevance for neutron stars

Fraga, Kurkela, & Vuorinen 1402.6618.

Maximum μ_{qk} may reach quarkyonic (for pressure), but true perturbative?

Ghisoiu, Gorda, Kurkela, Romatschke, Säppi, & Vuorinen, 1609.04339: pressure(μ_{qk}) ~ g^6.

Will be able to compute $\Lambda_{pert} = \# \mu_{qk}$ # ~ 1?



Quarkyonic matter: 1-dim. reduction

Kojo, Hidaka, McLerran & RDP 0912.3800: as toy model, assume confining potential

$$\Delta_{00} = \frac{\sigma_0}{(\vec{p}^{\,2})^2} \ , \ \Delta_{ij} \sim \frac{1}{p^2}$$

Near the Fermi surface, reduces to effectively 1-dim. problem in patches.
For *either* massless or massive quarks, excitations have zero energy about Fermi surface; just Fermi velocity $v_F < 1$ if $m \neq 0$.

Spin in 4-dim. -> "flavor" in 1-dim., so *extended* $2N_f$ flavor symmetry, $SU(N_f)_L x SU(N_f)_R$ -> $SU(2\,N_f)_L x SU(2\,N_f)_R$. Similar to Glozman,1511.05857.

Extended $2\,N_f$ flavor sym. broken by transverse fluctuations, only approximate.

Number of patches $N_{patch} \sim \mu/\sigma_0$, so spherical Fermi surface recovered as σ_0 -> 0

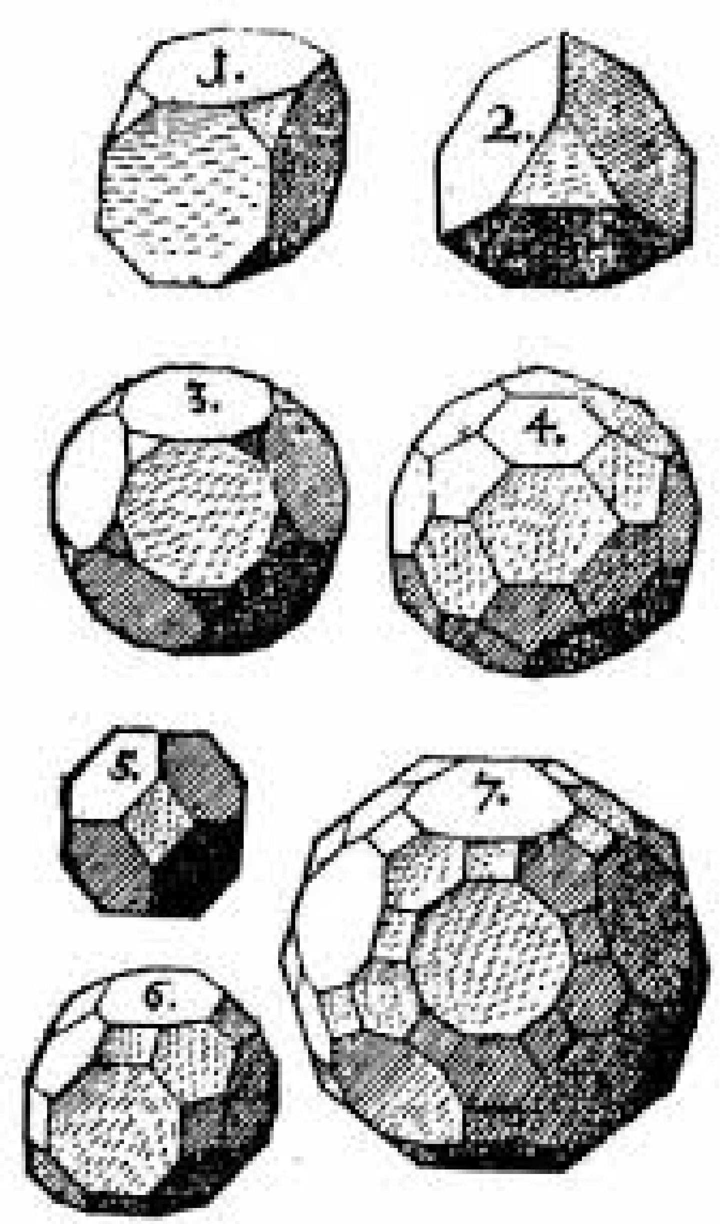
Transitions with # patches

Minimal number of patches = 6.

Probably occurs in dense baryonic phase.

In quarkyonic, presumably weak 1st order transitions as # patches changes.

Like Keplers....



Chiral spirals in 1+1 dimensions

In 1+1 dim., can eliminate μ by chiral rotation:

$$q' = \mathrm{e}^{i\mu z \Gamma_5} q \ , \ \overline{q}(\not{D} + i\mu\Gamma_0)q = \overline{q}' \not{D} q' \ , \ \Gamma_5 \Gamma_z = \Gamma_0$$

Thus a constant chiral condensate automatically becomes a chiral spiral:

$$\overline{q}' q' = \cos(2\mu z)\overline{q}q + i \sin(2\mu z)\overline{q}\gamma_5 q$$

Argument is only suggestive.

N.B.: anomaly ok, gives quark number: $\langle \overline{q}\Gamma_0 q \rangle = \mu/\pi$

Pairing is between quark & quark-hole,both at edge of Fermi sea.
Thus chiral condensate varies in z as ~ 2 μ.

Bosonization in 1+1 dimensions

Do not need detailed form of chiral spiral to determine excitations.

Use bosonization. For one fermion,

$$\overline{\psi}\ \partial\!\!\!/\ \psi \leftrightarrow (\partial_i \phi)^2$$

ϕ corresponds to U(1) of baryon number. In general, non-Abelian bosonization.

For flavor modes,

$$\mathcal{S}_{eff}^{flavor} = \int dt \int dz \; 3 \, \frac{1}{16\pi} \, \mathrm{tr}(\partial_\mu U^\dagger)(\partial_\mu U) + \ldots$$

where U is a SU(2 N_f) matrix.

Do not show Wess-Zumino-Witten terms for level 3 = # colors.

Also effects of transverse fluctuations, reduce SU(2 N_f) -> SU(N_f); quark mass

Lastly, SU(3) + level 2 N_f sigma model. Modes are gapped by confinement.

Pion/kaon condensates & U(1) phonon

Overhauser '60, Migdal '71....Kaplan & Nelson '86...

Pion/kaon condensate:

$$\langle \overline{q}_L q_R \rangle \sim \langle \Phi \rangle \sim \Phi_0 \; \exp(i(qz + \phi)t_3)$$

Condensate along σ and π^0 => t_3. Kaon condensate σ and K, etc.

Excitations are the SU(N_f) Goldstone bosons *and* a "phonon", φ.

Phases with pion/kaon condensates and quarkyonic Chiral Spirals both spontaneously break U(1), have associated massless field.

Continuously connected: SU(N_f) of π/K condensate => ~ SU(2 N_f) of CS's.

Fluctuations same in both.

Perhaps WZW terms for π/K condensates?

Anisotropic fluctuations in Chiral Spirals

Spontaneous breaking of global symmetry =>
Goldstone Bosons have derivative interactions, ~ ∂^2

π/K condensates and CS's break *both* global *and* rotational symmetries

Interactions along condensate direction usual quadratic, ~ $\partial_z{}^2$

Those quadratic in transverse momenta, ~ $\partial_\perp{}^2$, *cancel, leaving quartic*, ~ $\partial_\perp{}^4$.

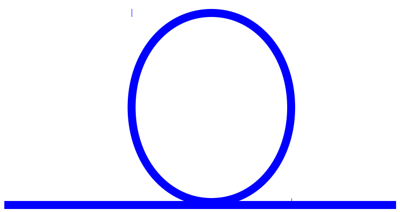
$$\mathcal{L}_{eff} = f_\pi^2 |(\partial_z - i k_0) U|^2 + \kappa |\partial_\perp^2 U|^2 + \ldots$$

Valid for *both* the U(1) phonon φ and Goldstone bosons U

Hidaka, Kamikado, Kanazawa & Noumi 1505.00848;
Lee, Nakano, Tsue, Tatsumi & Friman, 1504.03185; Nitta, Sasaki & Yokokura 1706.02938

No long range order in Chiral Spirals

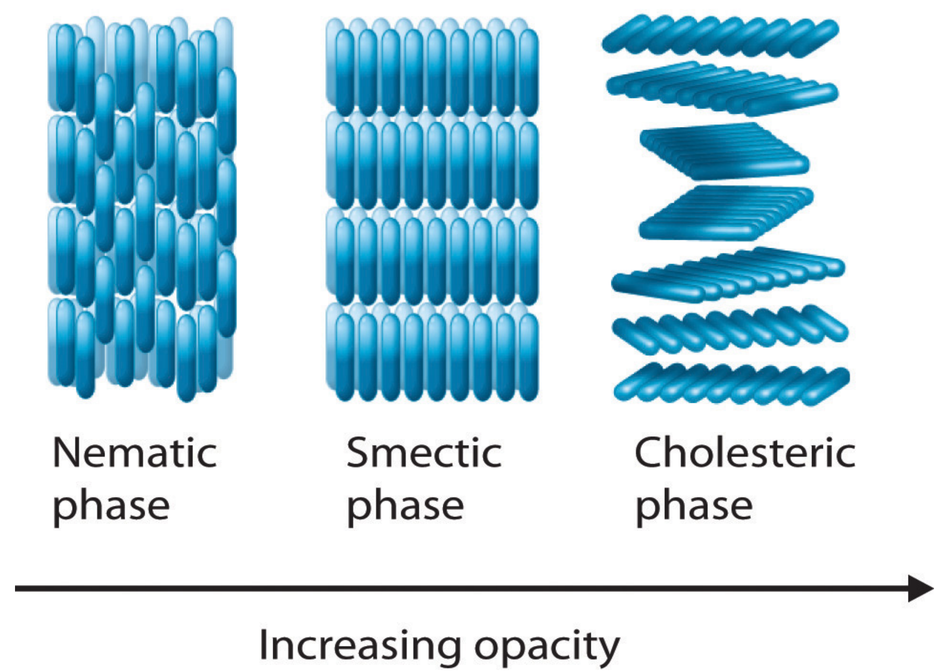
Consider tadpole diagram with anisotropic propagator



$$\int d^2k_\perp \, dk_z \, \frac{1}{(k_z - k_0)^2 + (k_\perp^2)^2} \sim \int d^2k_\perp \, \frac{1}{k_\perp^2} \sim \log \Lambda_{\rm IR}$$

Old story for π/K condensates: Kleinert ‘81; Baym, Friman, & Grinstein, ‘82 .

Similar to smectic-C liquid crystals:
ordering in one direction,
liquid in transverse.
Hence anisotropic propagator



Chiral Spirals in 1+1 dimensions

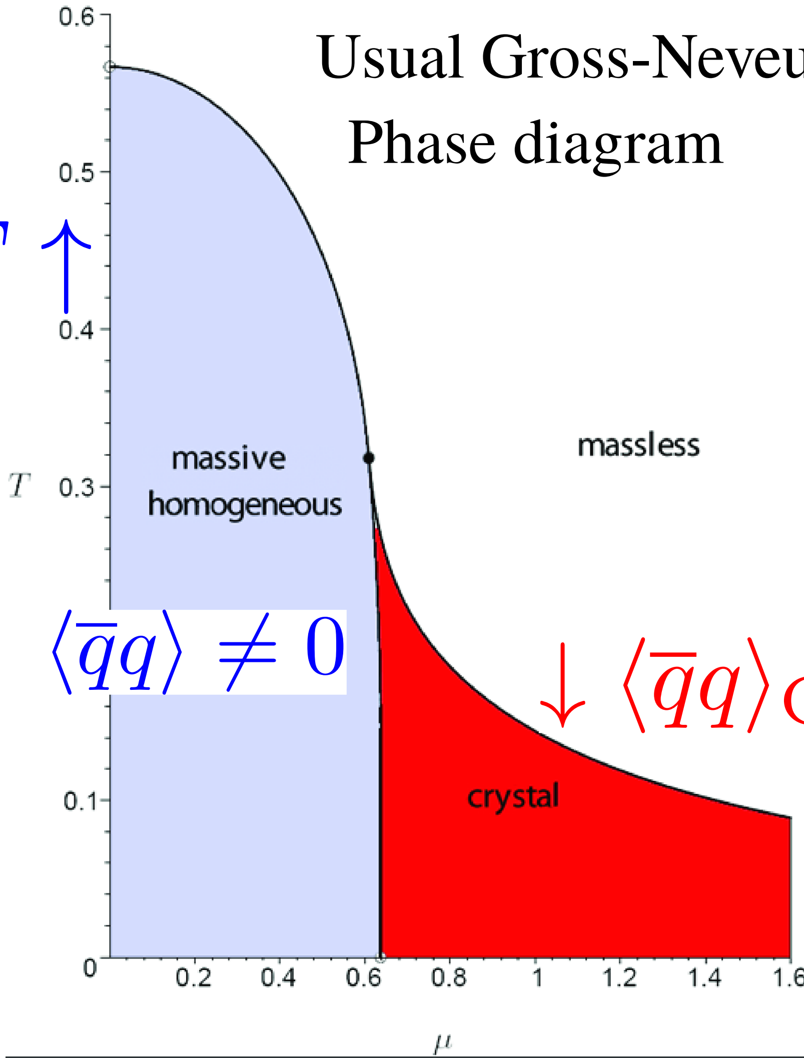
Overhauser/Migdal's pion condensate: $(\sigma, \pi^0) = f_\pi(\cos(k_0 z), \sin(k_0 z))$

Ubiquitous in 1+1 dimensions: Basar, Dunne & Thies, 0903.1868; Dunne & Thies 1309.2443+ ...

Wealth of exact solutions, phase diagrams at *infinite* N_f.

Usual Gross-Neveu model:

Phase diagram



$T \uparrow$

$\langle \overline{q} q \rangle \neq 0$

$\downarrow \langle \overline{q} q \rangle_{\rm CS} \neq 0$

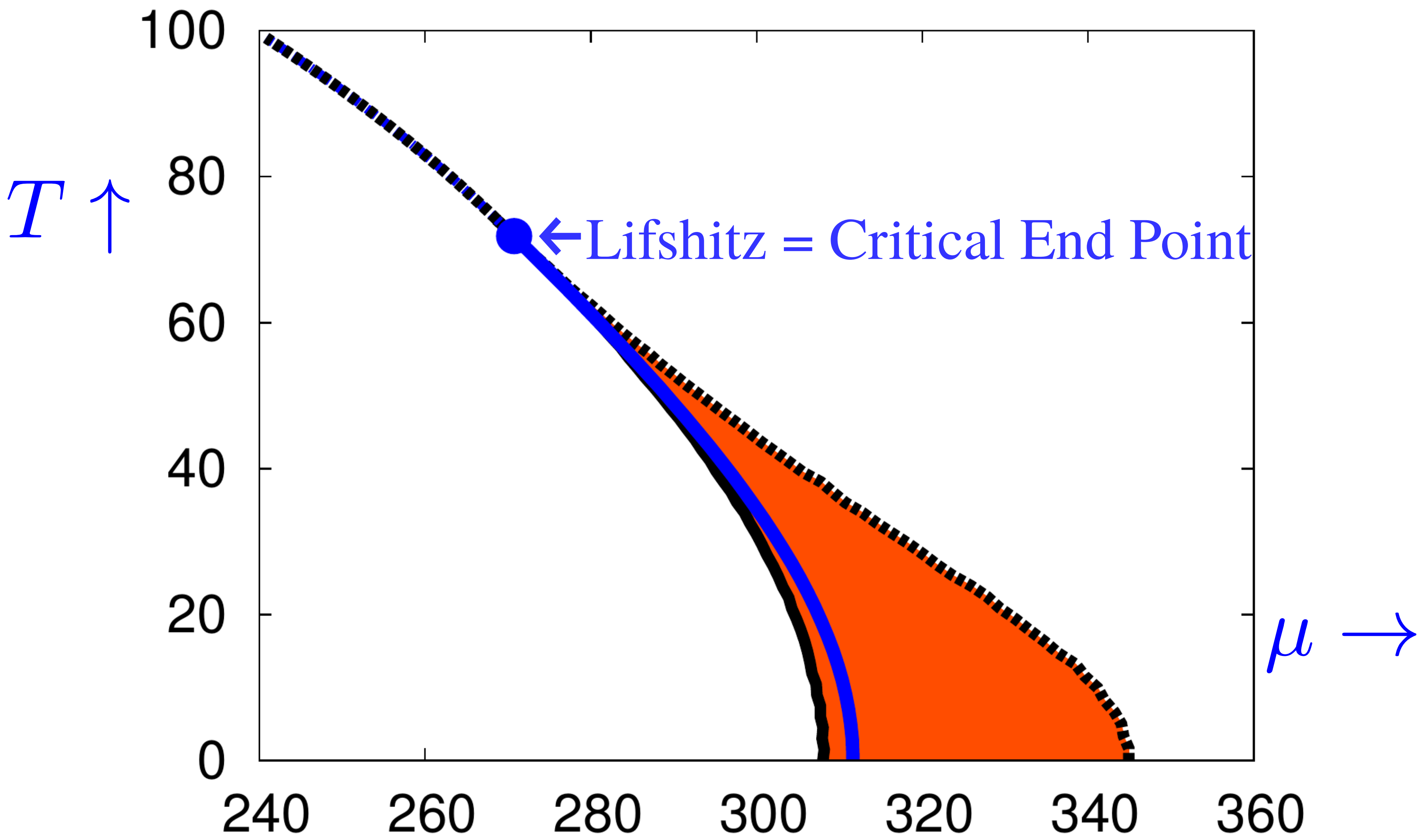
$\mu \longrightarrow$

Chiral spiral:

Chiral Spirals in 3+1 dimensions

In 3+1, *common* in NJL models: Nickel, 0902.1778 + ...Buballa & Carignano 1406.1367 + ...

In reduction to 1-dim, $\Gamma_5^{\text{1-dim}} = \gamma_0\gamma_z$, so chiral spiral between $\overline{q}q$ & $\overline{q}\gamma_0\gamma_z\gamma_5 q$



Both of these phase diagrams are
dramatically affected by fluctuations:

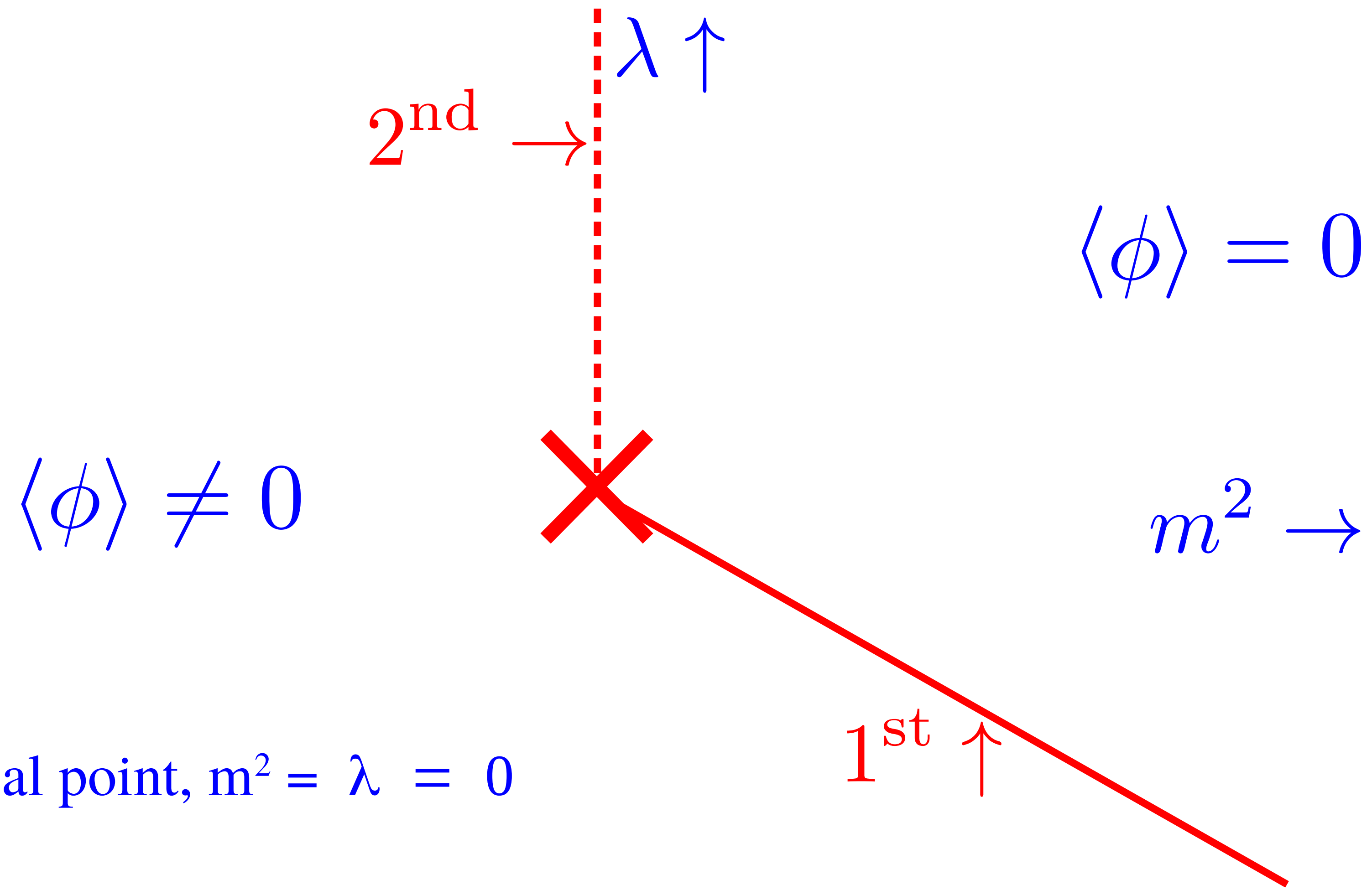
no Lifshitz point in 1+1 or 3+1 dimensions
at finite N

there *is* a *Lifshitz regime*

Standard phase diagram

$$\mathcal{L} = (\partial_\mu \phi)^2 + m^2\,\phi^2 + \lambda\,\phi^4 + \kappa\phi^6$$

Negative quartic coupling, λ, turns a 2nd order transition into 1st order.
Two phases.

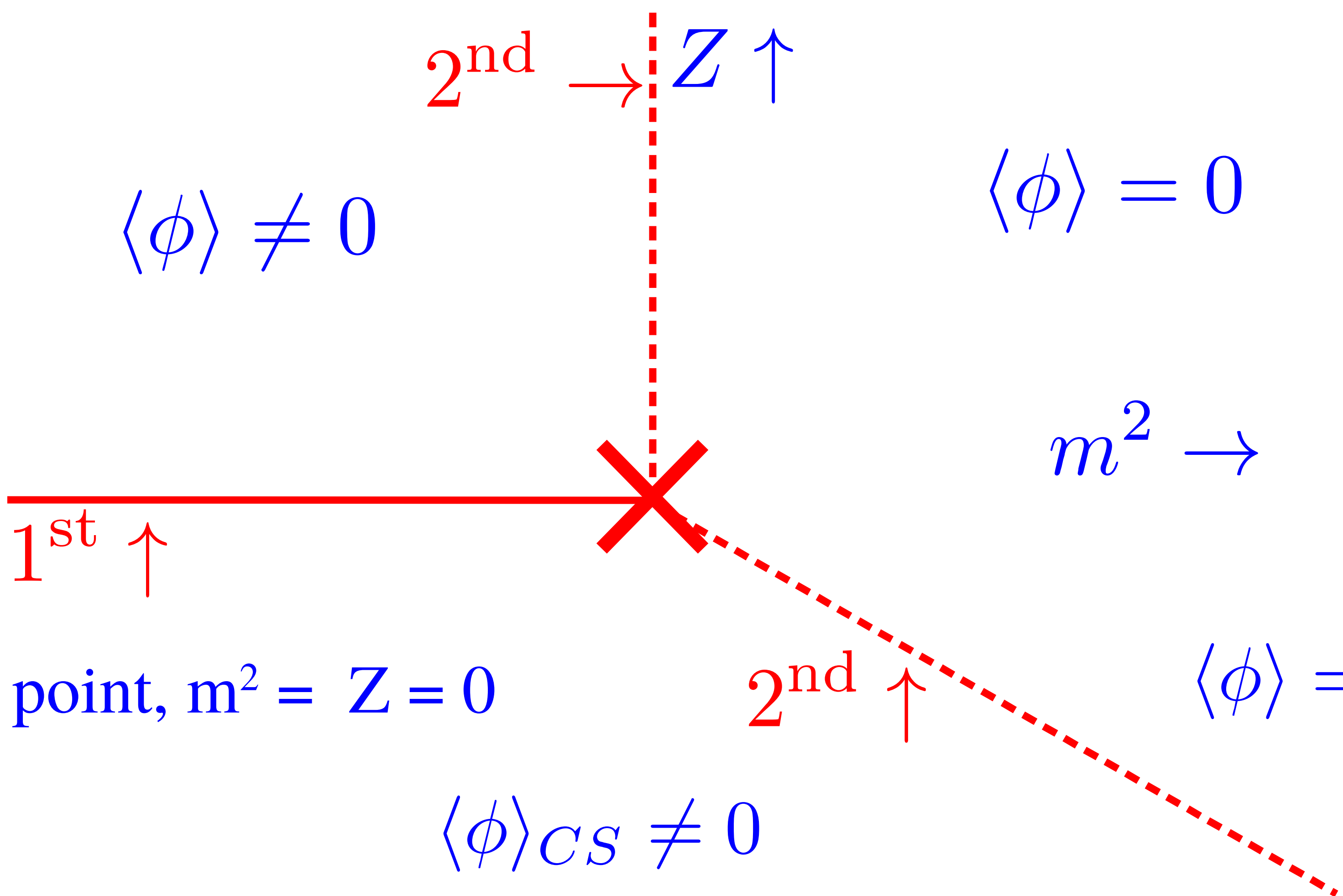


X = tri-critical point, $m^2 = \lambda = 0$

Lifshitz phase diagram (*in mean field theory*)

$$\mathcal{L}_{\text{Lifshitz}} = (\partial_0\phi)^2 + Z(\partial_i\phi)^2 + \frac{1}{M^2}(\partial_i^2\phi)^2 + m^2\phi^2 + \lambda\phi^4$$

Negative kinetic term, $Z < 0$, generates spatially inhomogeneous phase, CS.
Three phases.

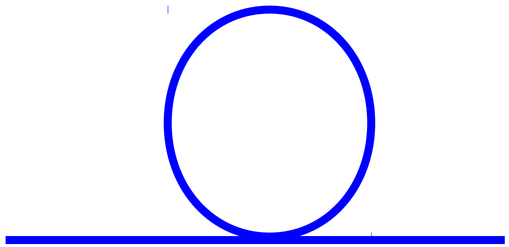


X = Lifshitz point, $m^2 = Z = 0$

No massless modes in too few dimensions

No massless modes in d ≤ 2 dimensions:

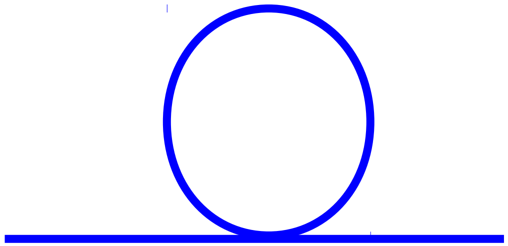
$$\int d^2k \, \frac{1}{k^2} \sim \log \Lambda_{\rm IR}$$



Cannot break a continuous symmetry in d ≤ 2 dimensions: instead of Goldstone bosons, generate a mass *non*-perturbatively.

Lifshitz point: Z = m^2 = 0, so propagator just ~ $1/k^4$:

$$\int d^4k \, \frac{1}{k^4} \sim \log \Lambda_{\rm IR}$$



Hence *no* Lifshitz point in d ≤ 4 (spatial) dimensions.

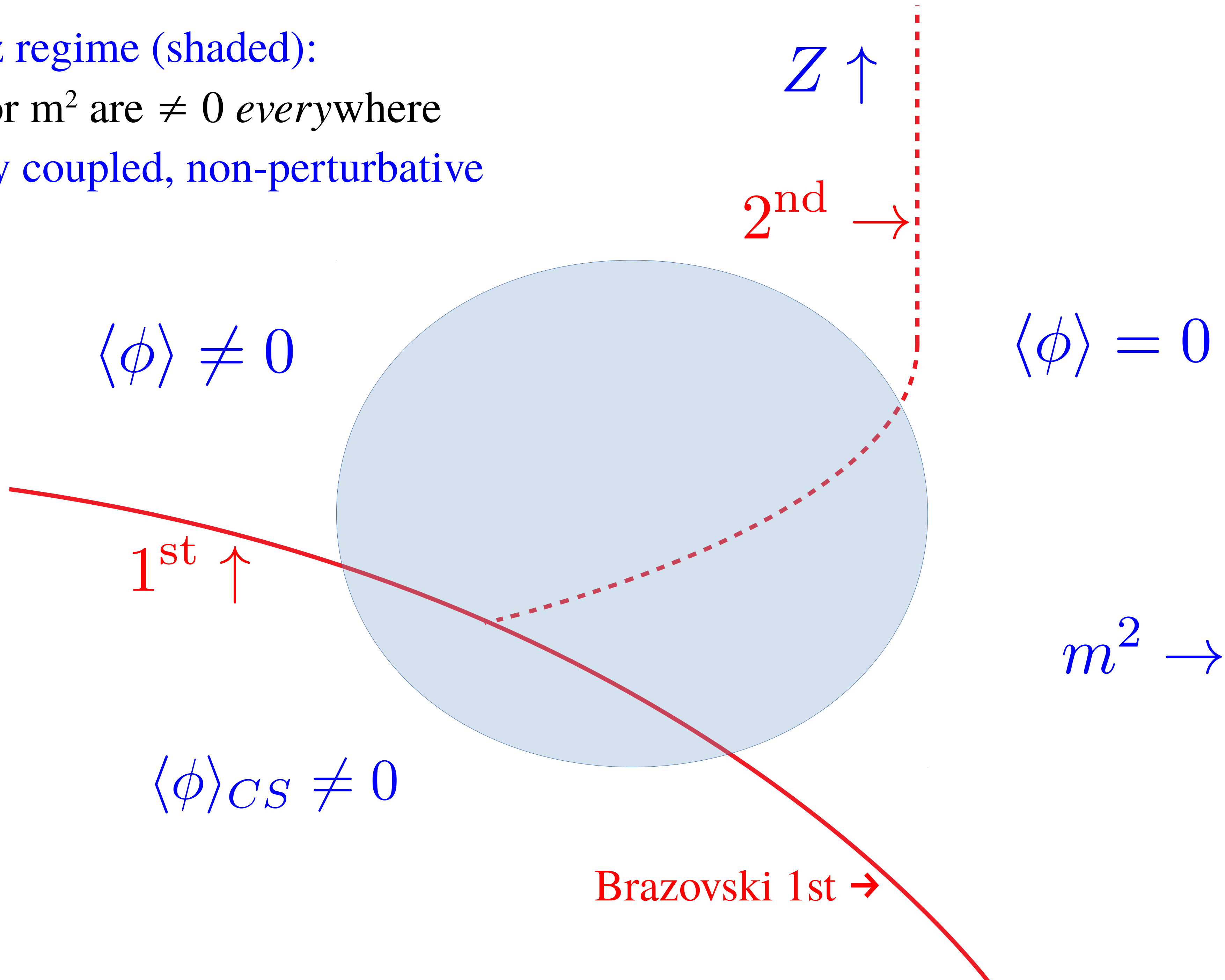
Must generate either a mass m^2, *or* term ~ Z $p^2 \neq 0$, *non*-perturbatively

Lifshitz regime

Lifshitz regime (shaded):

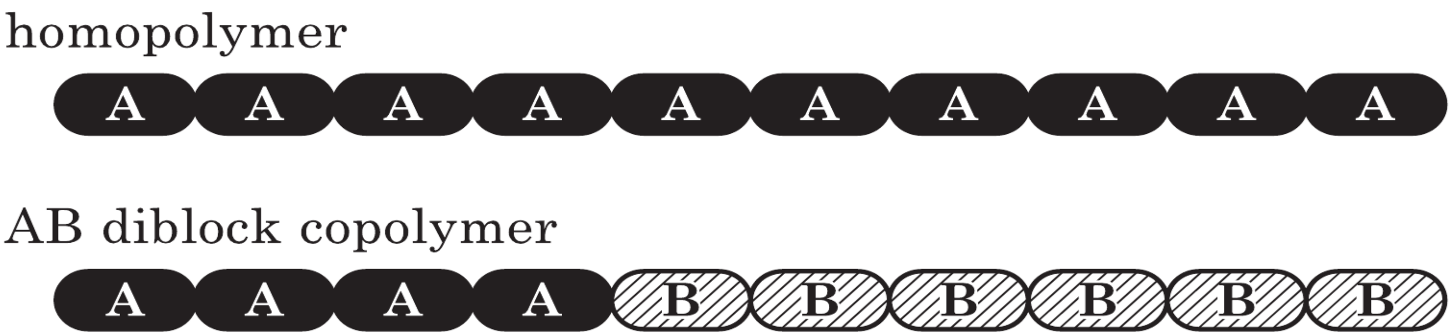
Z and/or m^2 are $\neq 0$ *every*where

strongly coupled, non-perturbative



Example: inhomogenous polymers

Like mixing oil & water: polymers A & B, with AB diblock copolymer (“co-AB”)



Three phases: high temperature, A & B mix, symmetric phase

low temperature, little co-AB: A & B seperate, broken phase

co-AB tends to decrease interface tension between A & B phases, can turn it negative. Like Z < 0

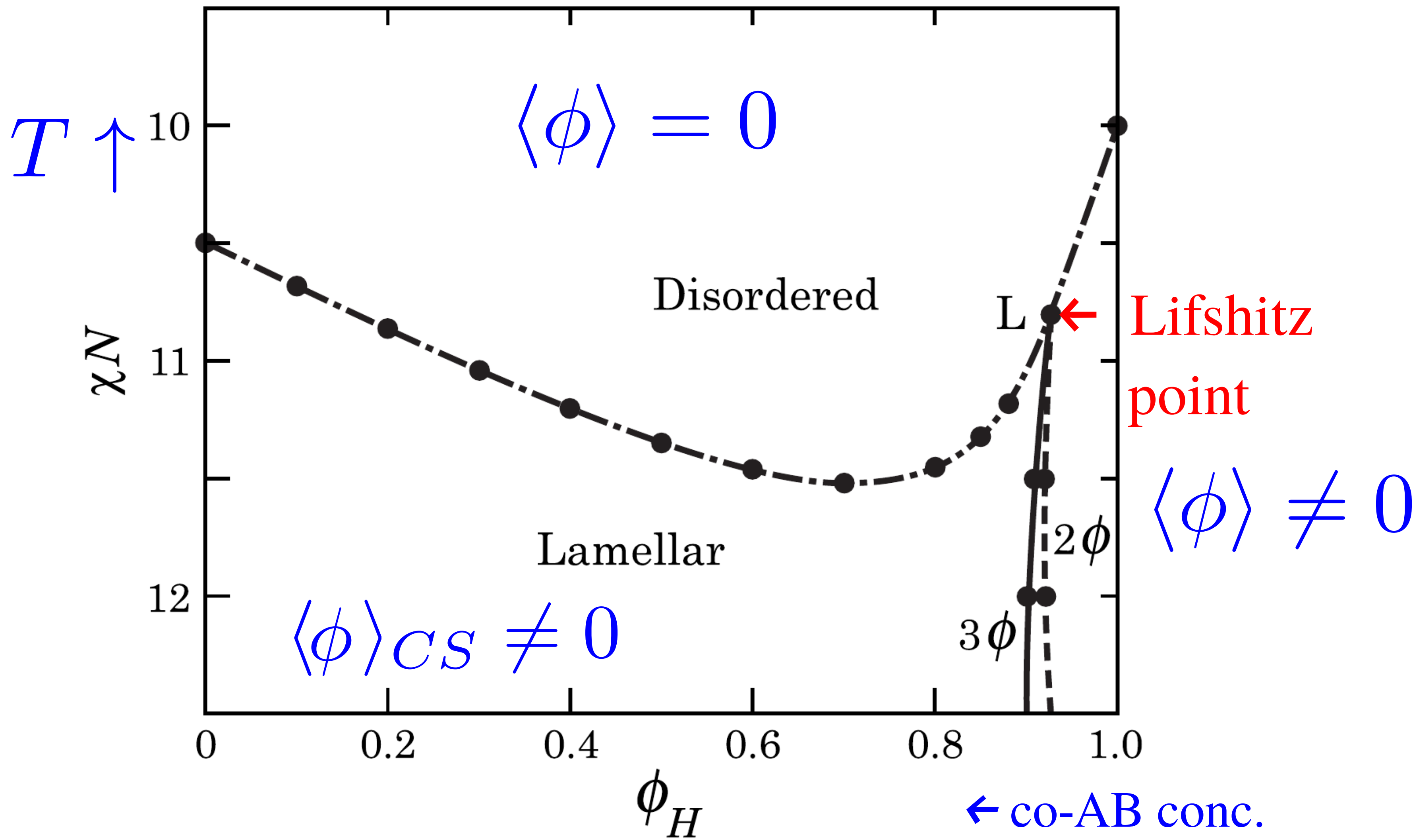
Low temperature, high concentration co-AB: “lamellar” phase, stripes of A & B. Like smectic.

Lifshitz point in inhomogenous polymers: mean field

Three phases, symmetric, broken, & spatially inhomogenous

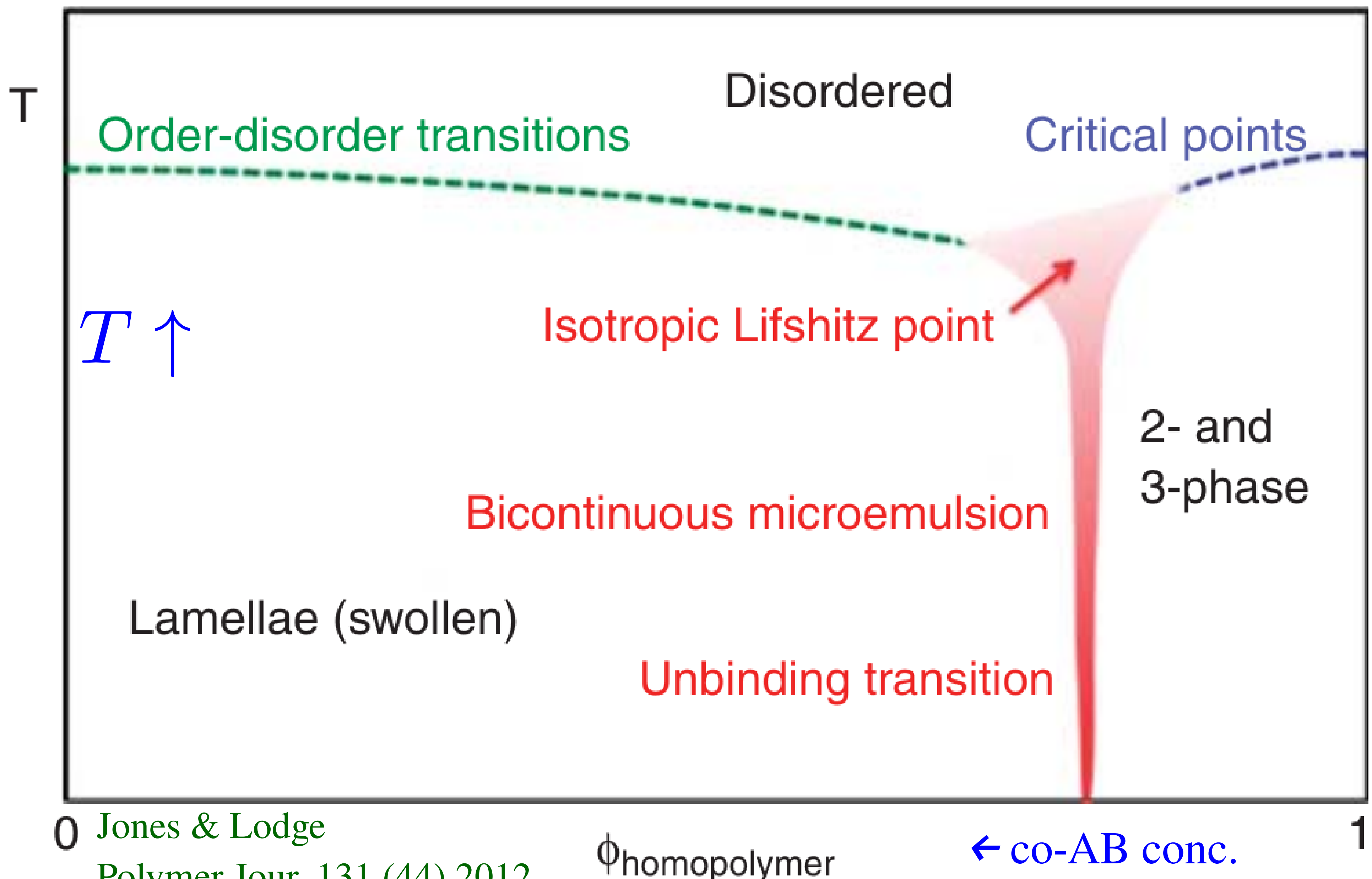
Mean field predicts Lifshitz point at given T & concentration of co-AB

Fredrickson & Bates, Jour. Polymer Sci. 35, 2775 (1997)



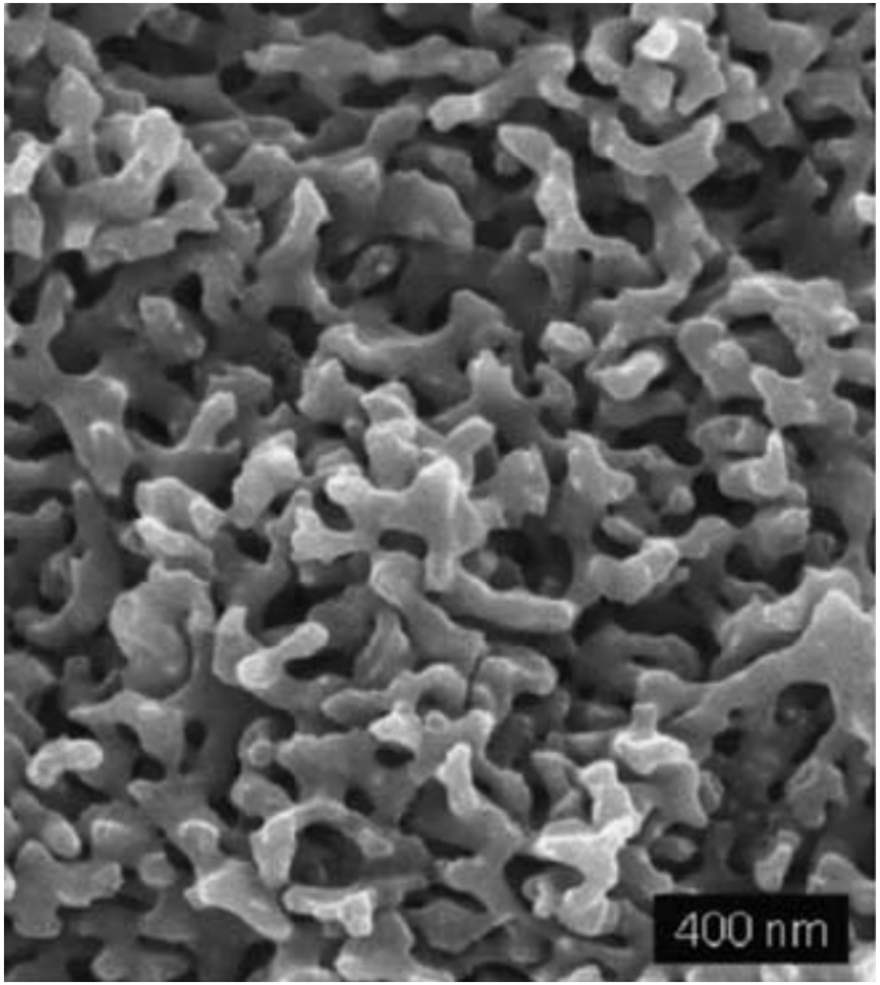
Lifshitz regime in inhomogenous polymers

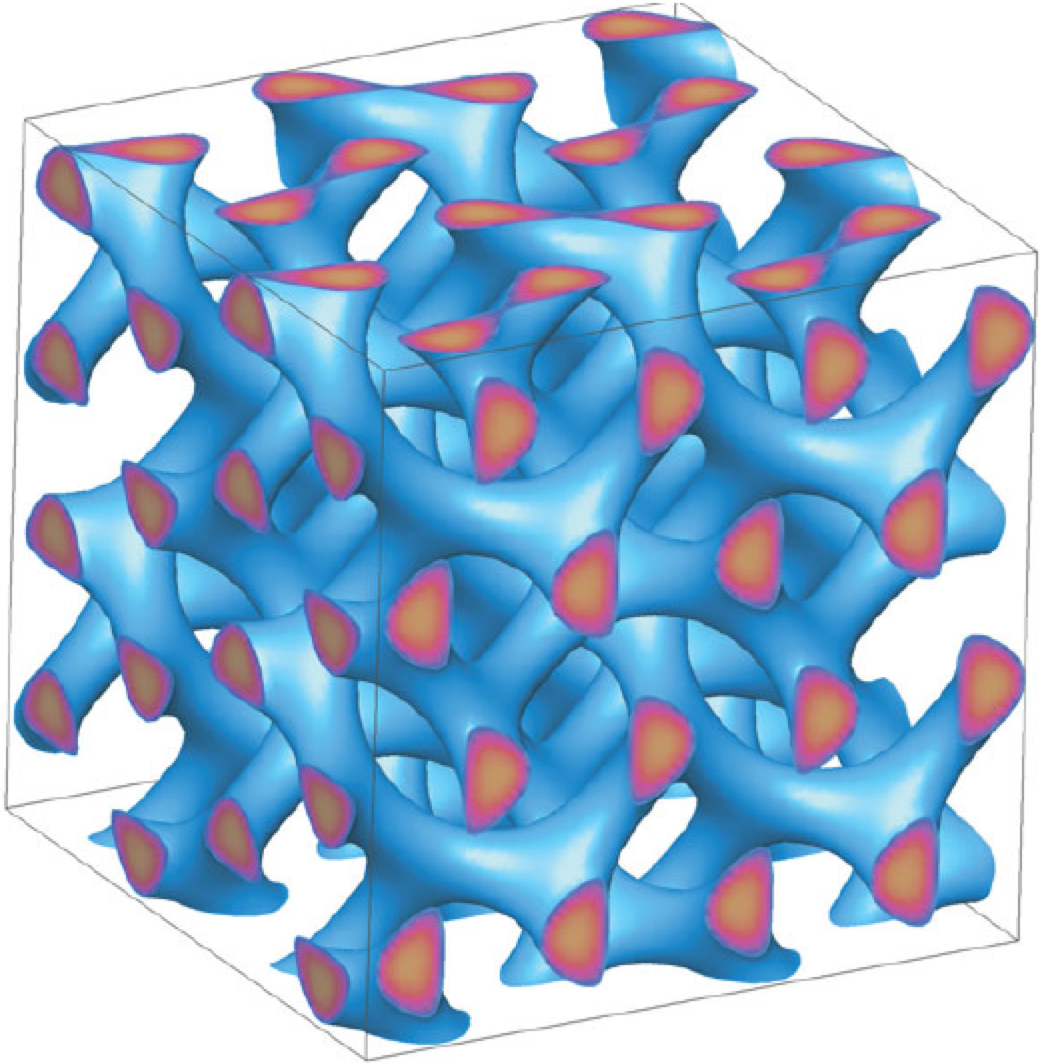
Instead of Lifshitz point predicted by mean field theory, find
Bicontinuous microemulsion: $Z \neq 0$, $m^2 = 0$: Lifshitz regime



Jones & Lodge
Polymer Jour. 131 (44) 2012

Bicontinuous microemulsion: $Z \approx 0$





Experiment

Jones & Lodge,
Polymer Jour. 131 (44) 2012

Self-consistent field theory

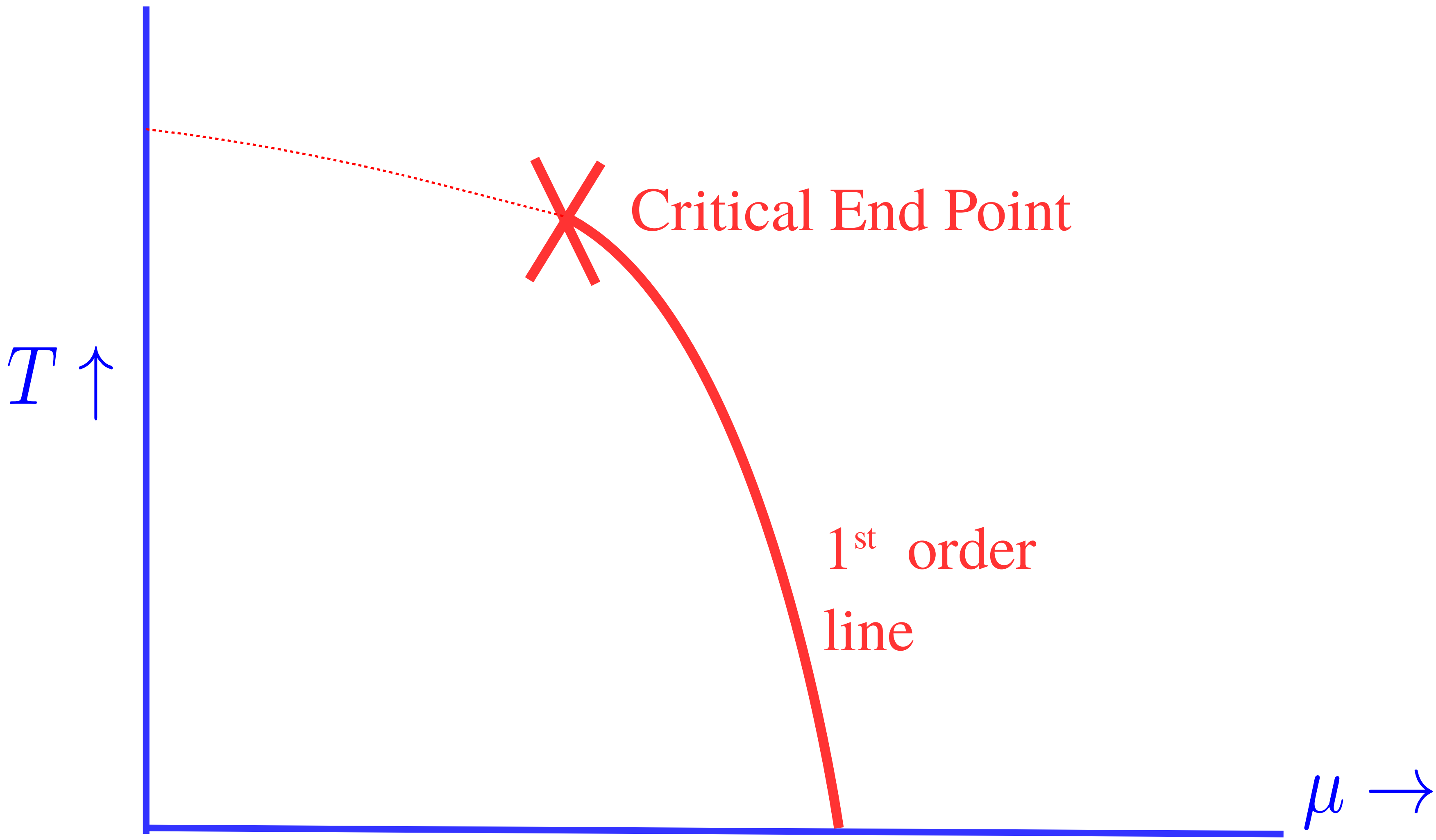
Fredrickson, "The equilibrium theory of
inhomogenous polymers"

Phase diagram for QCD in T & μ: usual picture

Two phases, one Critical End Point (CEP)

between crossover and line of 1st order transitions

Ising fixed point, dominated by *massless* fluctuations at CEP



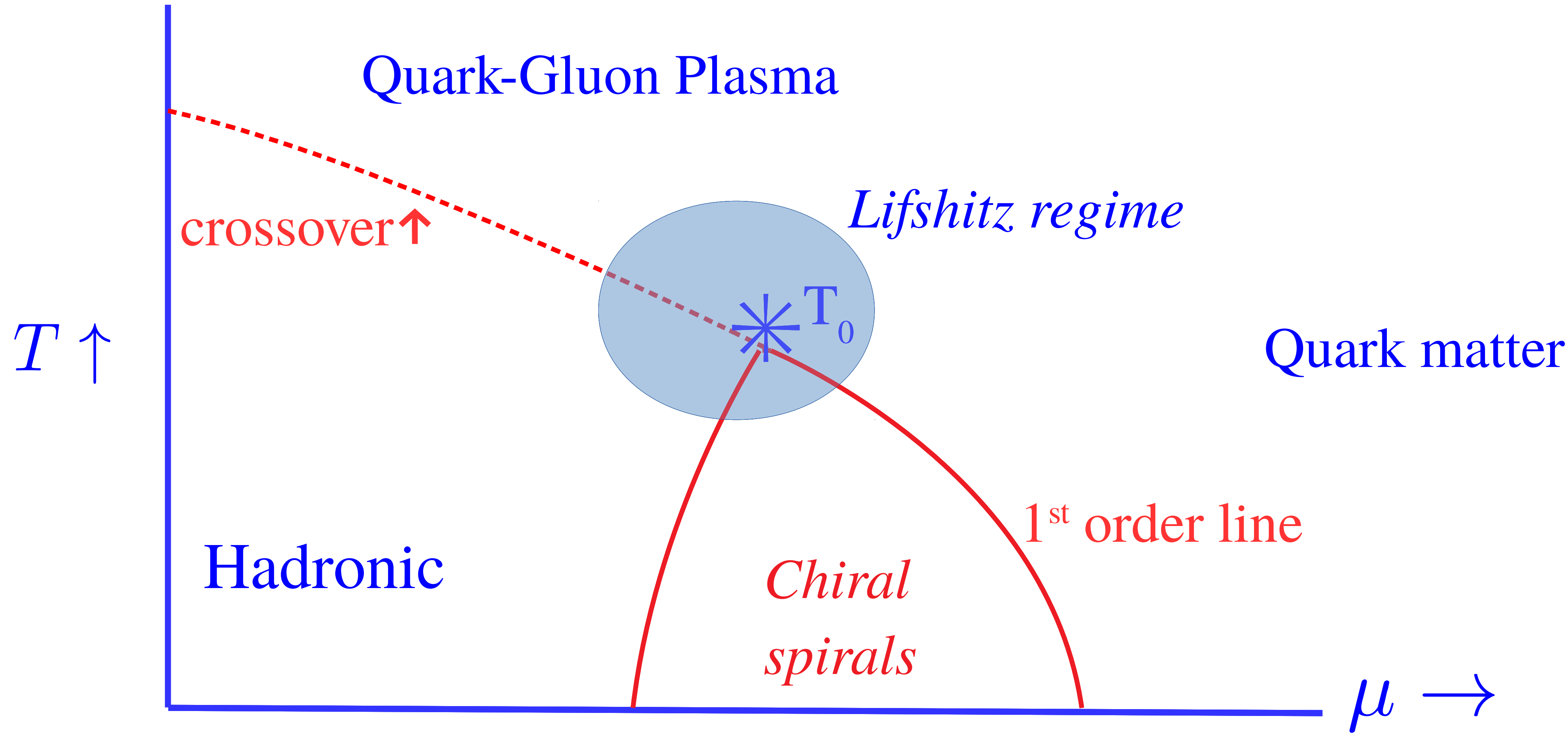
Lifshitz phase diagram for QCD

Lifshitz regime: strongly coupled, large fluctuations

Unbroken 1st order line to spatially inhomogeneous phases = "chiral spirals"

Heavy ions: could go through *two* 1st order transitions

T_0: maximum T, point of equal concentrations (unequal entropy)



Fluctuations at 7 GeV

Beam Energy Scan, down to 7 GeV.

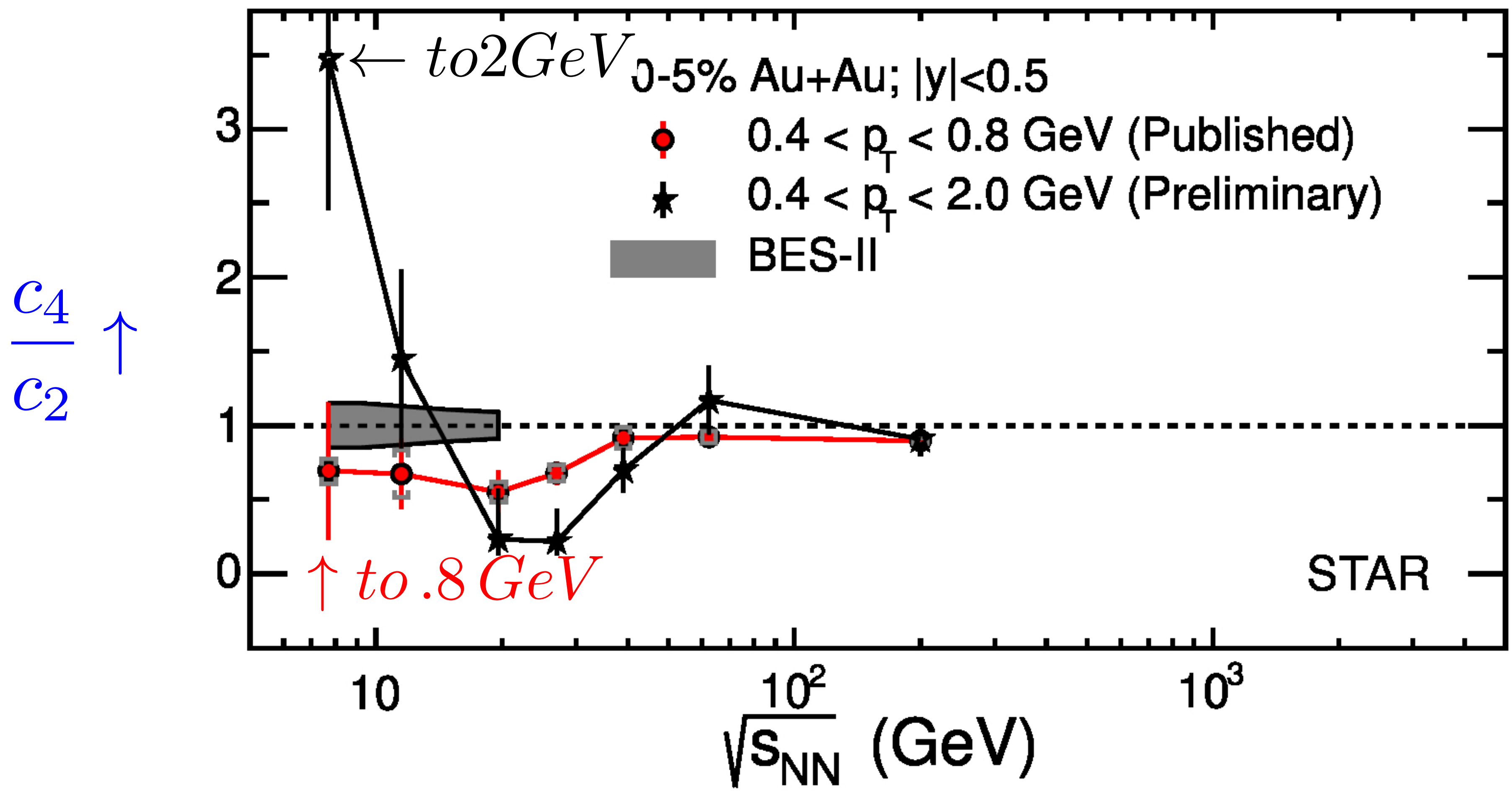
Fluctuations *MUCH* larger when up to 2 GeV than to 0.8 GeV

Trivial multiplicity scaling? ... or Chiral Spiral?

But fluctuations in nucleons, not pions.

$$c_n = \frac{\partial^n}{\partial \mu^n} p(T, \mu)$$

X. Luo & N. Xu, 1701.02105, fig. 37; Jowazee, 1708.03364



Experimentally

For *any* sort of periodic structure (1D, 2D, 3D...),

Fluctuations concentrated about some characteristic momentum k_0

So "slice and dice": bin in intervals, 0 to .5 GeV, .5 to 1., etc.

If peak in fluctuations in a bin not including zero,
may be evidence for $k_0 \neq 0$.

Signals for Lifshitz regime?

Must *measure fluctuations in pions, kaons...*

NJL models and Lifshitz points

Consider Nambu-Jona-Lasino models.

Nickel, 0902,1778 & 0906.5295 + ... + Buballa & Carignano 1406.1367

$$\mathcal{L}_{\rm NJL} = \overline{\psi}(\partial\!\!\!/ + g\sigma)\psi + \sigma^2$$

Integrating over ψ,

$$\mathrm{tr}\log(\partial\!\!\!/ + g\,\sigma) \sim \ldots + \kappa_1((\partial\sigma)^2 + \sigma^4) + \ldots$$

Due to scaling, ∂ -> $\lambda\partial$, σ -> $\lambda\sigma$.

Consequently, in NJL @ 1-loop, *tricritical = Lifshitz point*.

Special to including only σ at one loop.

Not generic: violated by the inclusion of more fields, to two loop order, etc.

Improved gradient expansion near critical point:

Carignano, Anzuni, Benhar, & Mannarelli, 1711.08607.

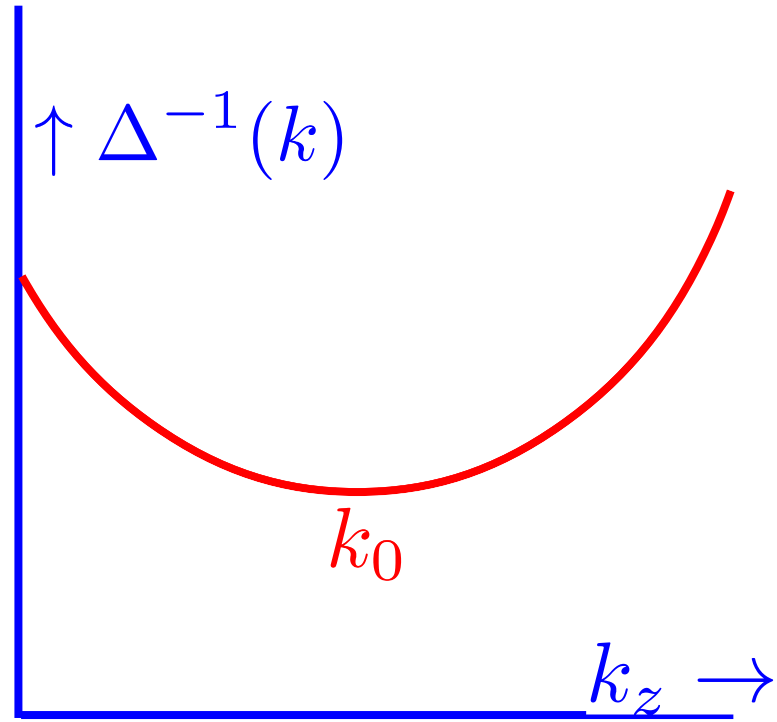
Symmetric to CS: 1D (Brazovski) fluctuations

Consider $m^2 > 0$, $Z < 0$: minimum in propagator at *non*zero momentum

Brazovski '75; Hohenberg & Swift '95 + ... ;

Lee, Nakano, Tsue, Tatsumi & Friman, 1504.03185; Yoshiike, Lee & Tatsumi 1702.01511

$$\Delta^{-1} = m^2 + Z\,k^2 + k^4/M^2$$
$$= m^2_{\rm eff} - 2\,Z\,k_z^2 + O(k_z^3, k_z k_\perp^2)$$



$k=(k_\perp,k_z-k_0)$: *no* terms in $k_\perp^2$, *only* $(k_\perp^2)^2$.

Due to spon. breaking of rotational sym.

1-loop tadpole diagram:

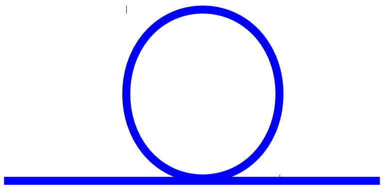
$$\int d^3k\, \frac{1}{k_z^2 + m^2_{\rm eff} + \ldots} \sim M^2 \int \frac{dk_z}{k_z^2 + m^2_{\rm eff}} \sim \frac{M^2}{m_{\rm eff}}$$

Effective reduction to 1-dim for any spatial dimension d, any global symmetry

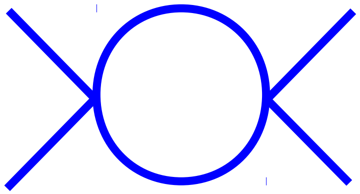
1st order transition in 1-dim.

Strong infrared fluctuations in 1-dim., both in the mass:

$$\Delta m^2 \sim \lambda \int d^3k \, \frac{1}{k_z^2 + m_{\rm eff}^2 + \ldots} \sim \lambda \, \frac{M}{m_{\rm eff}}$$



and for the coupling constant:



$$\Delta\lambda \sim -\lambda^2 \int \frac{d^3k}{(k_z^2 + m_{\rm eff}^2 + \ldots)^2} \sim -\lambda^2 M^3 \int_{m_{\rm eff}} \frac{dk_z}{k_z^4} \sim -\lambda \, \frac{M^3}{m_{\rm eff}^3}$$

Can*not* tune $m_{\rm eff}^2$ to 0: $\lambda_{\rm eff}$ goes negative, 1st order trans. induced by fluctuations

Not like other 1st order fluc-ind'd trans's: just that in 1-d, $m_{\rm eff}^2 \neq 0$ always