The Initial Correlations of the Glasma Energy-Momentum Tensor

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in collaboration with:
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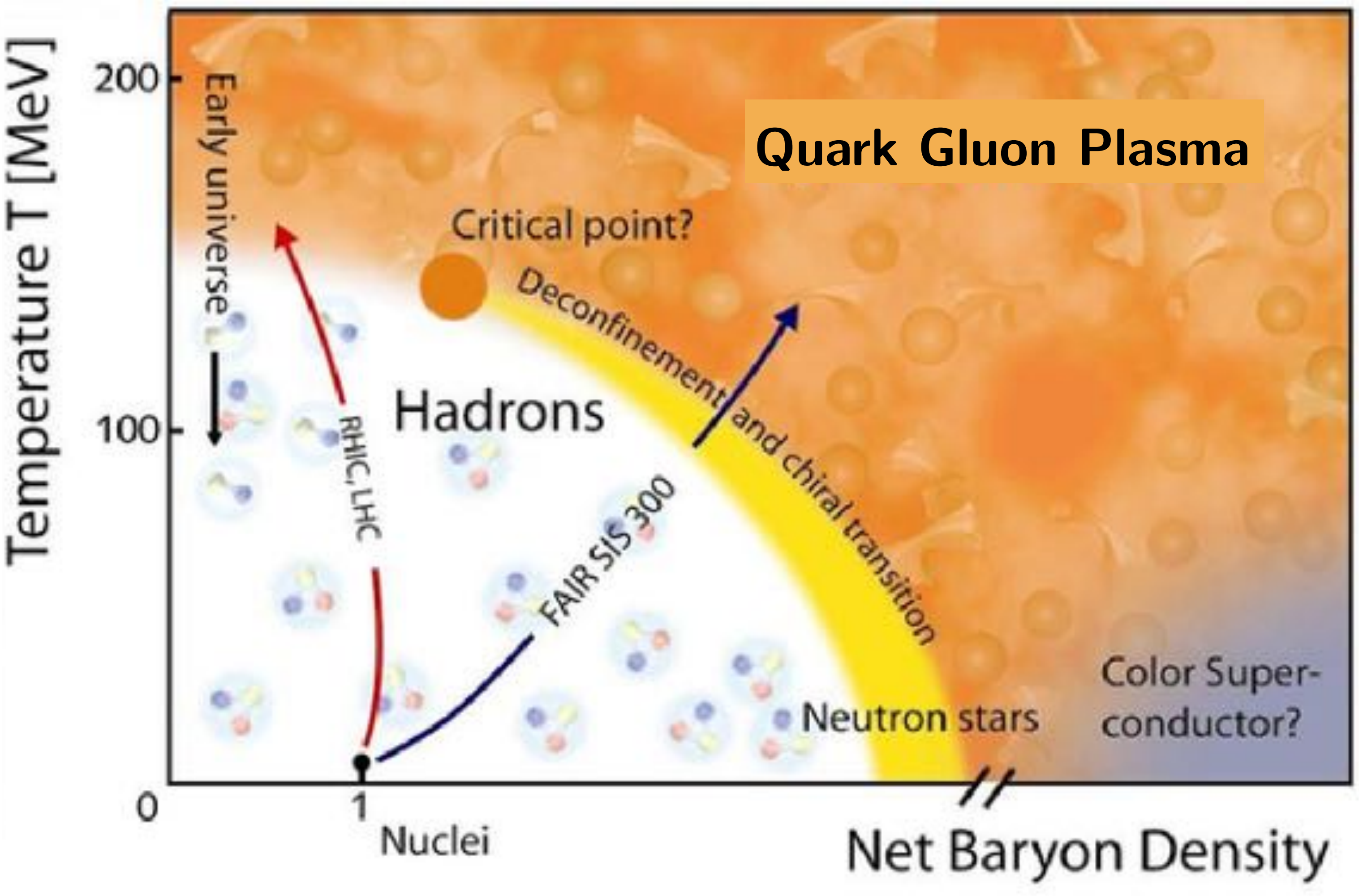
Outline

Introduction

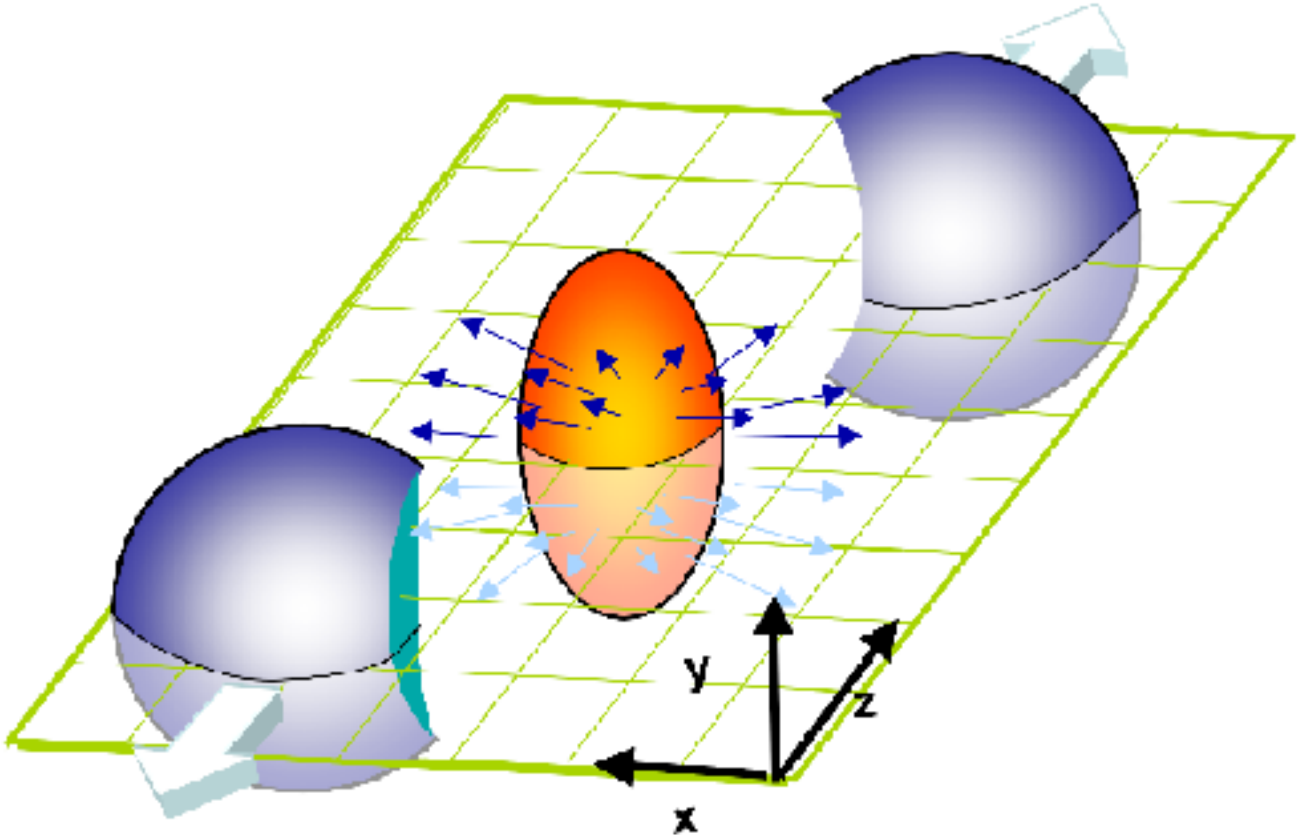
Introduction: The QCD phase space

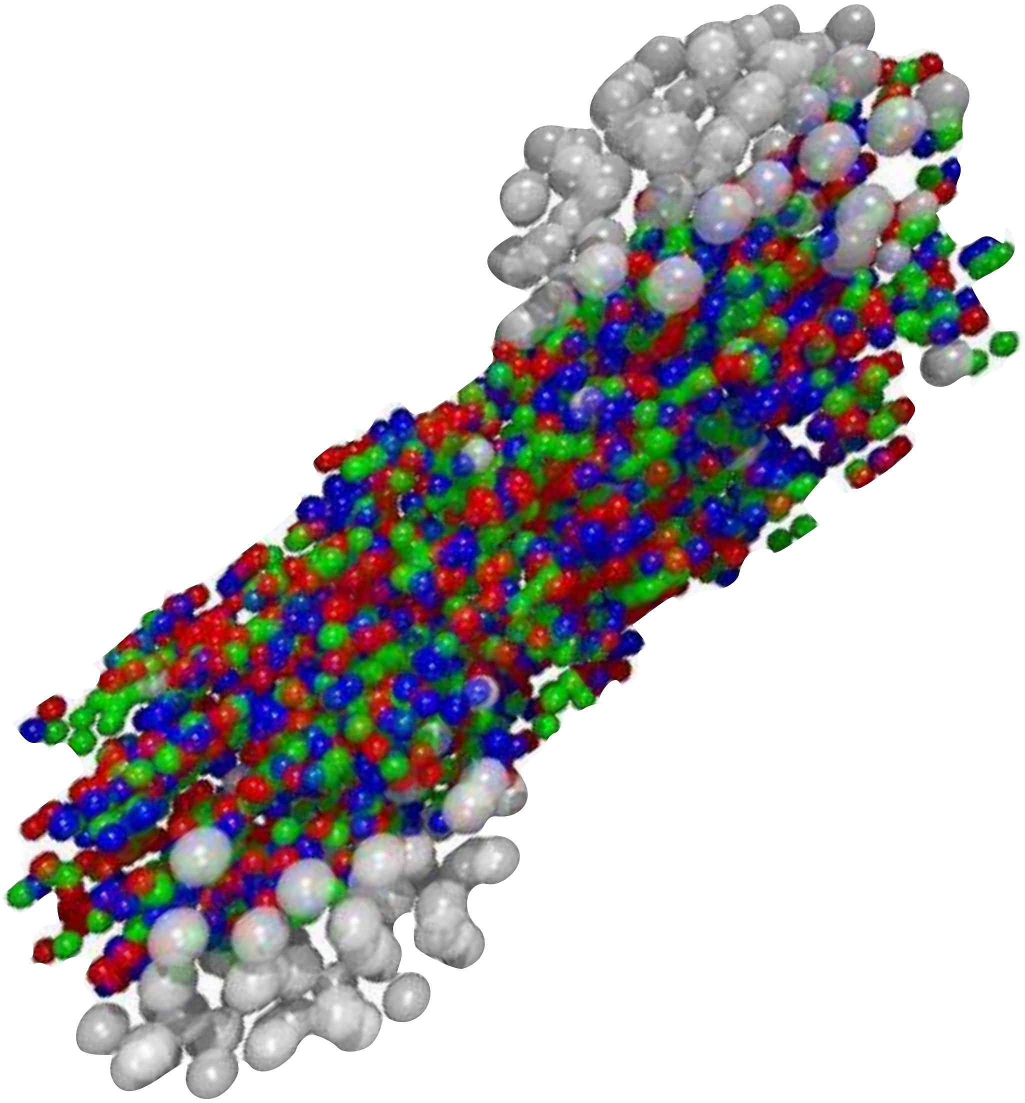


- Value of QCD's coupling constant depends on conditions of temperature and baryon density
- Low temperature and densities: **hadronic phase (confinement and spontaneously broken chiral symmetry)**
- Lattice simulations indicate a transition at high temperature to a **deconfined**, **chiral-symmetric phase:** The **QUARK-GLUON PLASMA**

Introduction: The QCD phase space

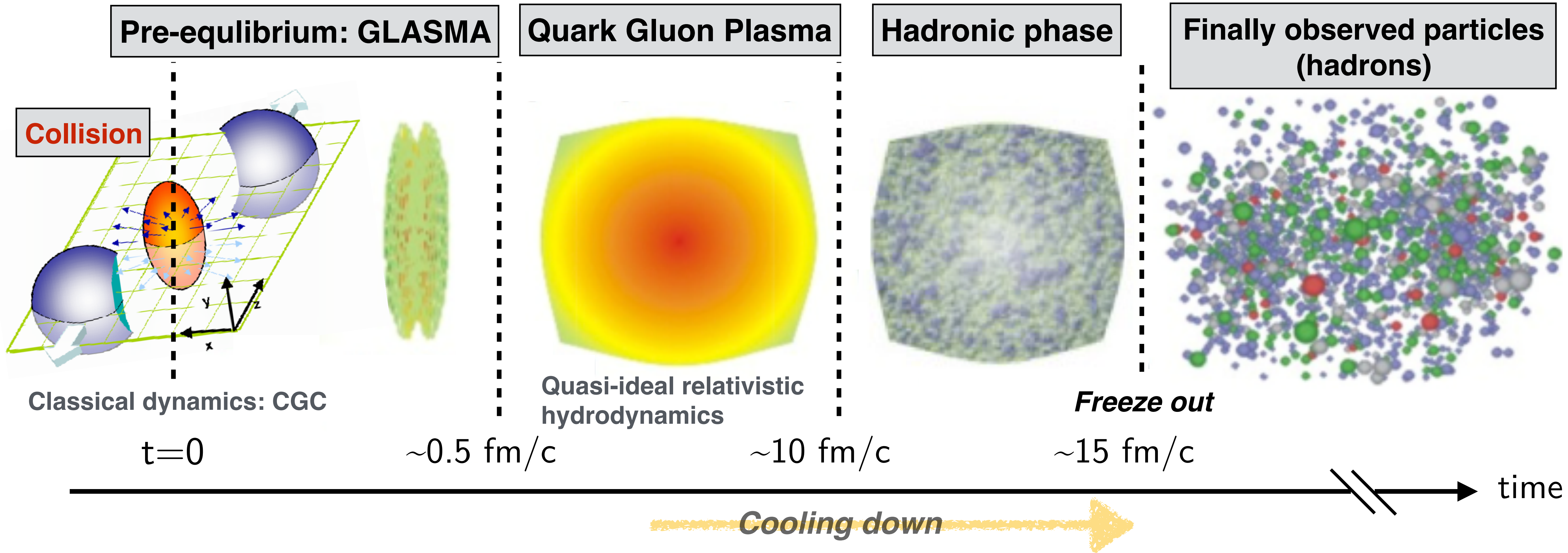
- This state of matter can be accessed in particle colliders through **Heavy Ion Collision** experiments





- Performed at Brookhaven National Laboratory's Relativistic Heavy Ion Collider (**RHIC**) and CERN's Large Hadron Collider (**ALICE** experiment)

Introduction: Stages of a heavy ion collision



Pre-equlibrium: GLASMA

Quark Gluon Plasma

Hadronic phase

Finally observed particles (hadrons)

Collision

Classical dynamics: CGC

Quasi-ideal relativistic hydrodynamics

Freeze out

t=0 ~0.5 fm/c ~10 fm/c ~15 fm/c time

Cooling down

- After the collision, matter goes through different phases as it cools down
- In the last part, it reaches the hadronic phase, and this is how it appears in the detectors

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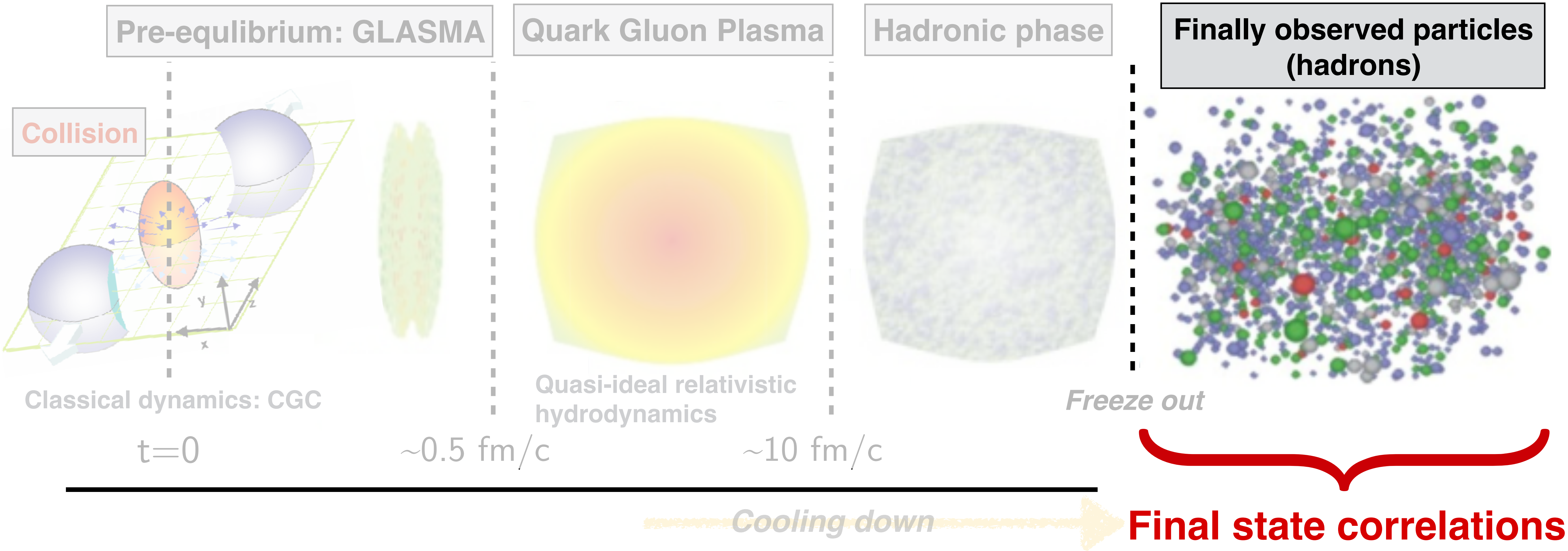

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- QGP can be studied through the **non-trivial correlations** between the measured particles

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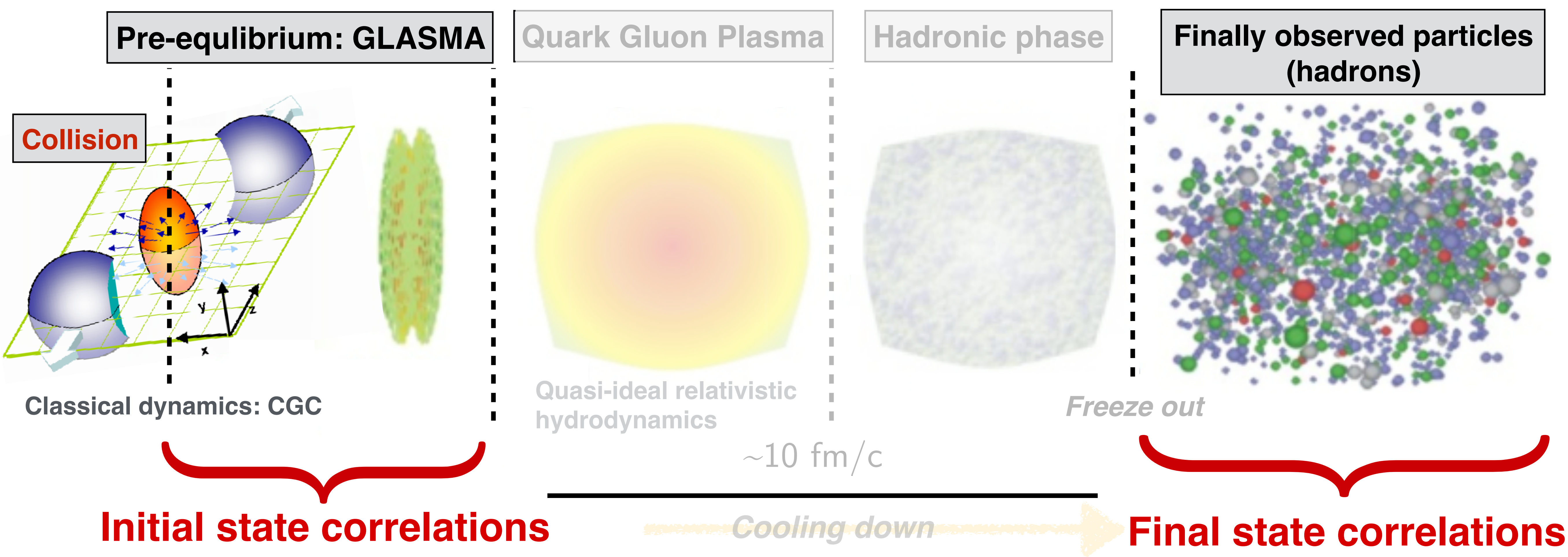

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Initial state correlations

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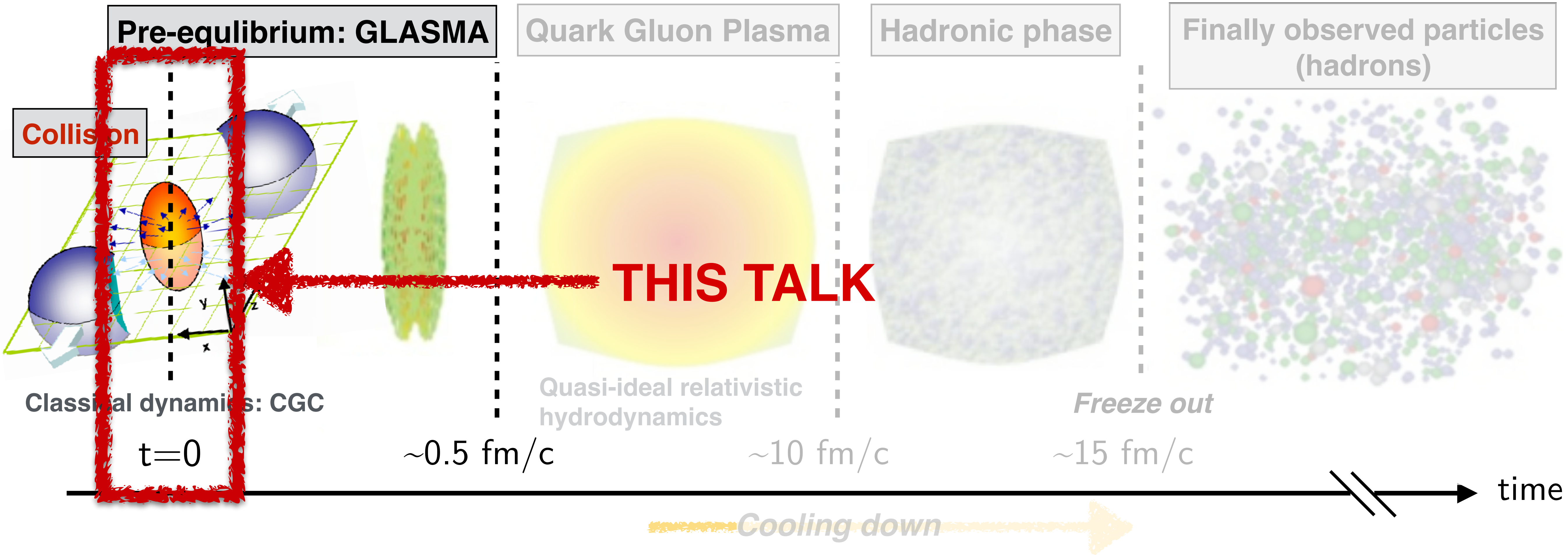
Final state correlations

- After the collision, matter goes through different phases as it cools down
- In the last part, it reaches the hadronic phase, and this is how it appears in the detectors
- QGP can be studied through the **non-trivial correlations** between the measured particles
- **BUT**: **Initial state fluctuations reflect in the final state correlations!**

→ We need robust theoretical description

Introduction: Stages of a heavy ion collision



- **No theoretical agreement** on the initial conditions of Glasma evolution
- Large degree of **phenomenological modeling**
- **Source of uncertainty** for parameters used in Hydro models

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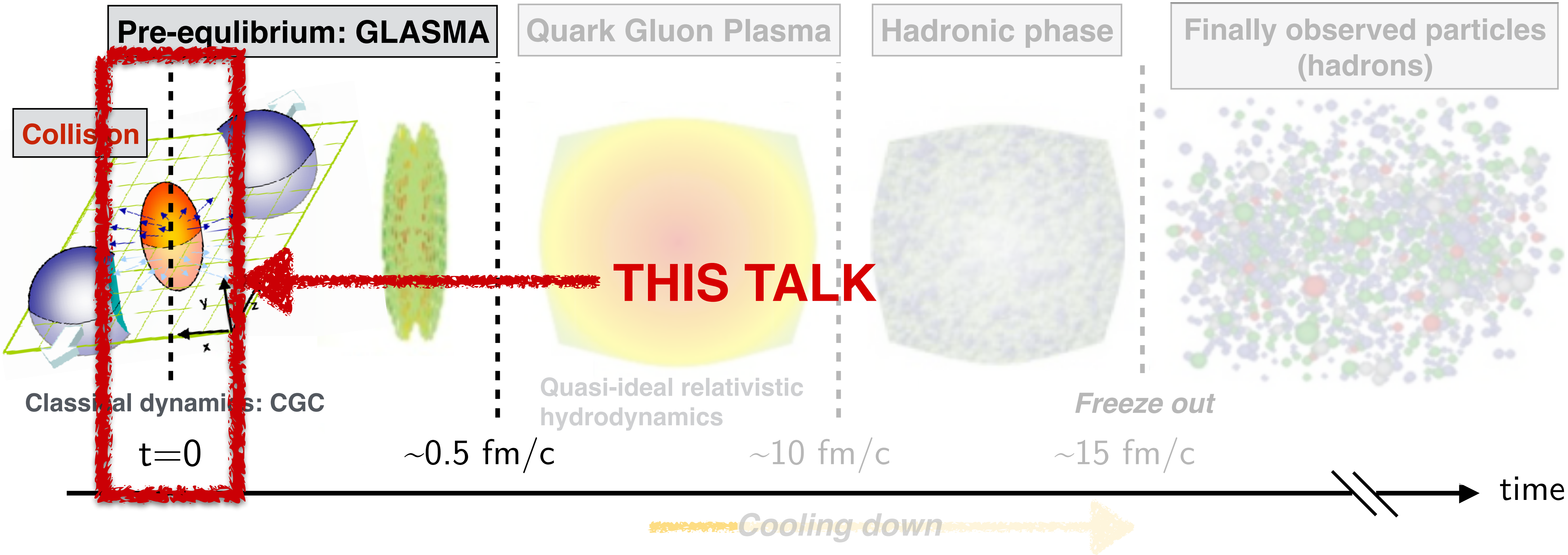

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Collision

THIS TALK

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Freeze out

t=0

~0.5 fm/c

~10 fm/c

~15 fm/c

time

Cooling down

- **No theoretical agreement** on the initial conditions of Glasma evolution
- Large degree of **phenomenological modeling**
- **Source of uncertainty** for parameters used in Hydro models
- We provide a **first-principles analytical calculation** of:

$$\langle T^{\mu\nu}(x_\perp)\rangle$$

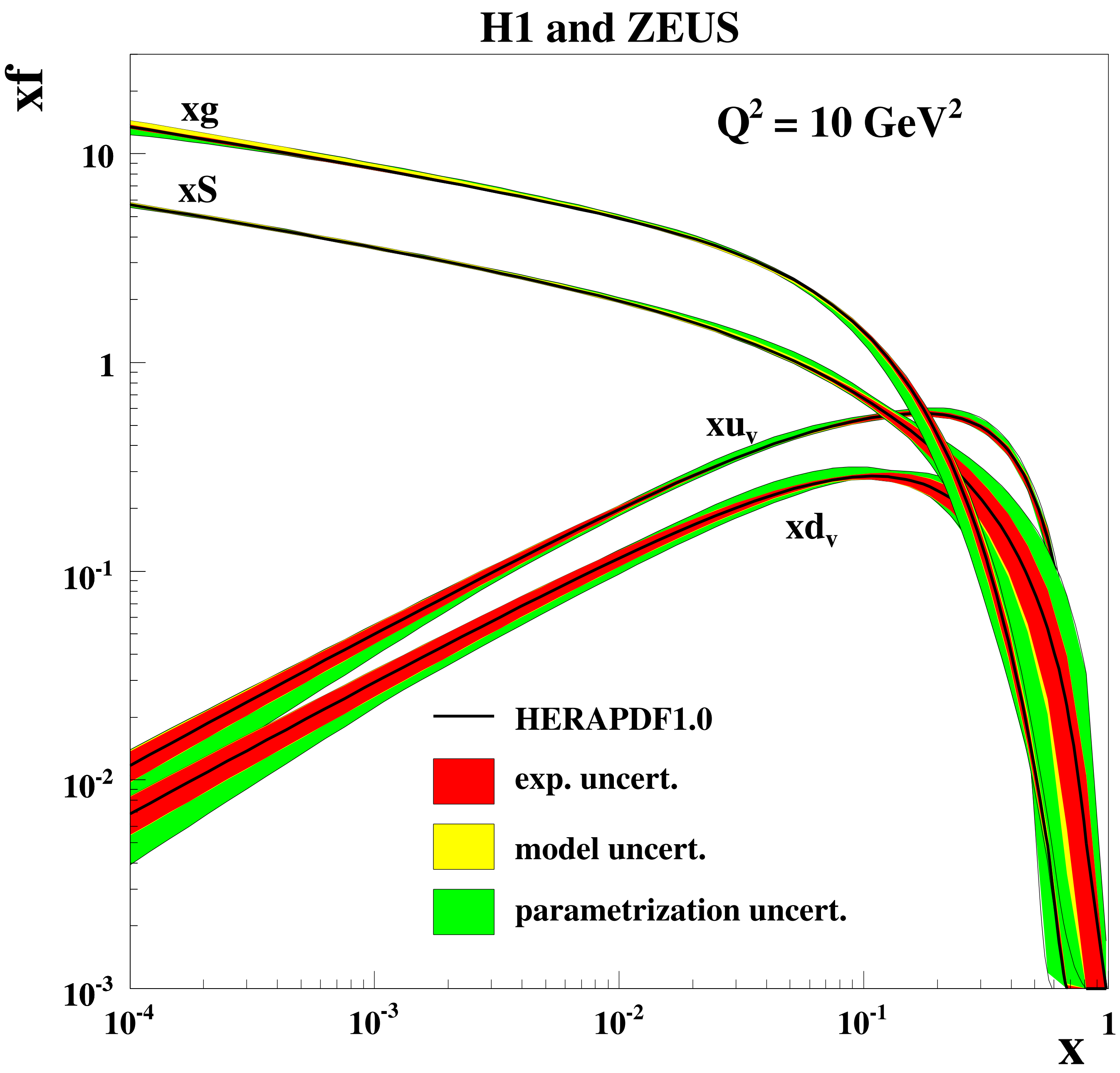
$$\langle T^{\mu\nu}(x_\perp)T^{\mu\nu}(y_\perp)\rangle$$

In the classical approximation (**Color Glass Condensate**)

Initial conditions: the Color-Glass Condensate

Highly Energetic Heavy Ion Collisions

- At high energies (or equivalently, low x) the partonic content of nucleons is **vastly dominated by a high density of gluons**



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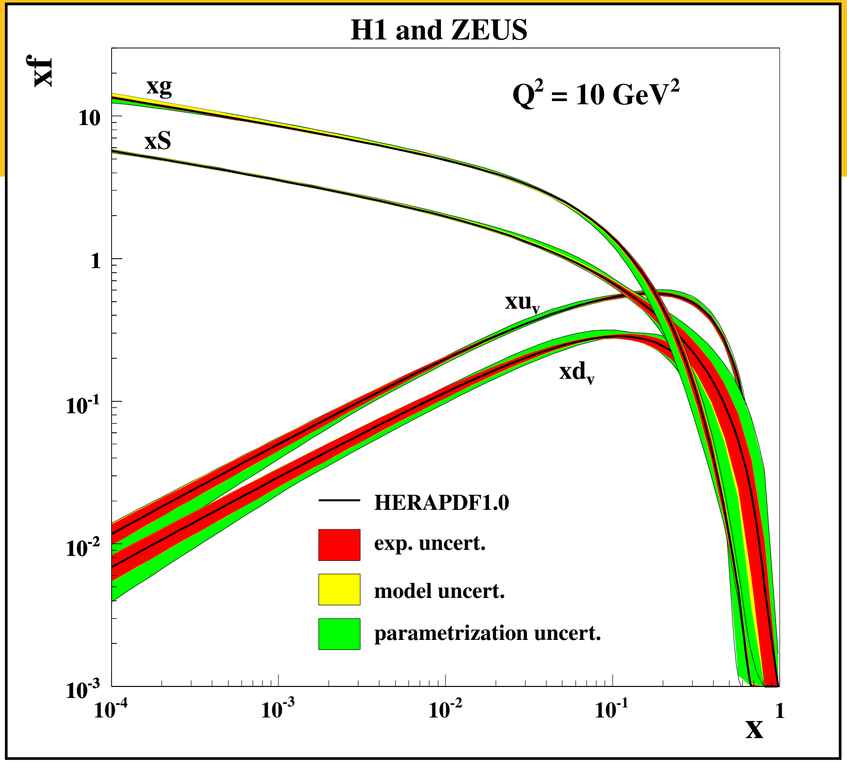
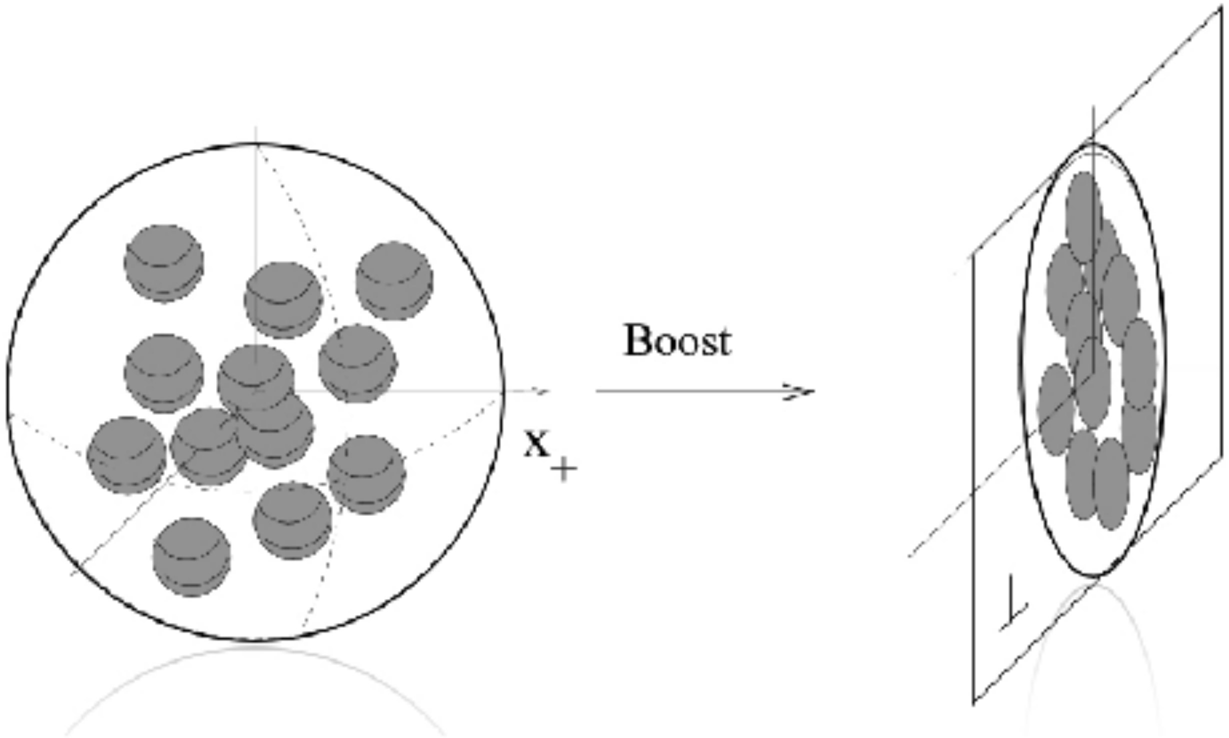


- At high energies (or equivalently, low x) the partonic content of nucleons is **vastly dominated by a high density of gluons**

- Relativistic kinematics: at high energies, the nuclei appear almost two-dimensional in the laboratory frame due to **Lorentz contraction**



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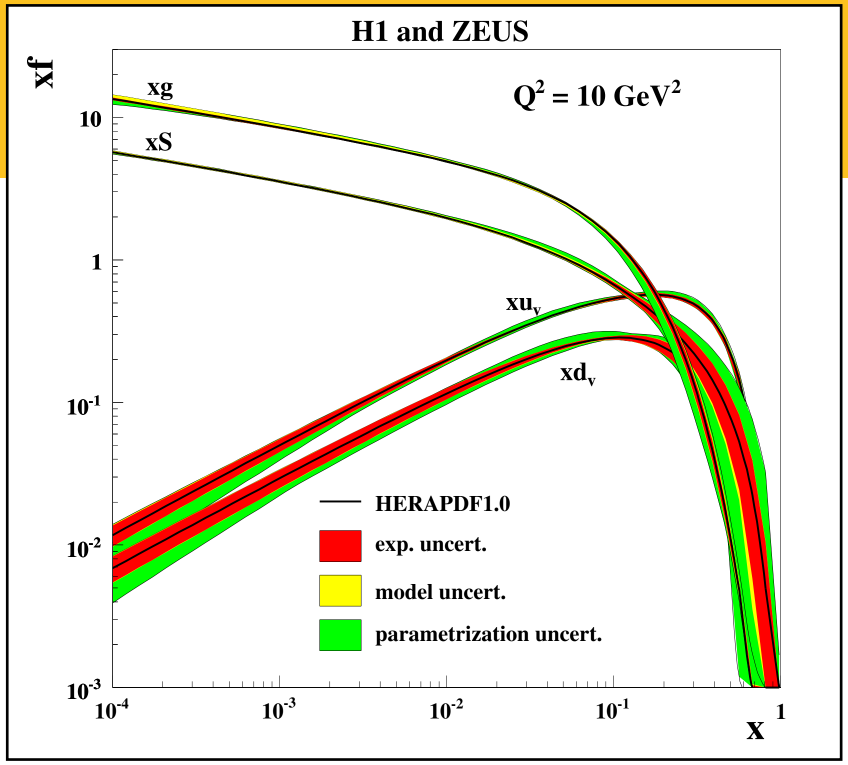

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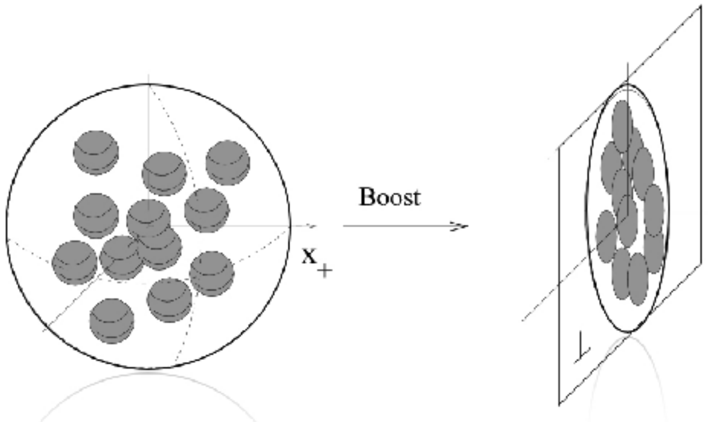

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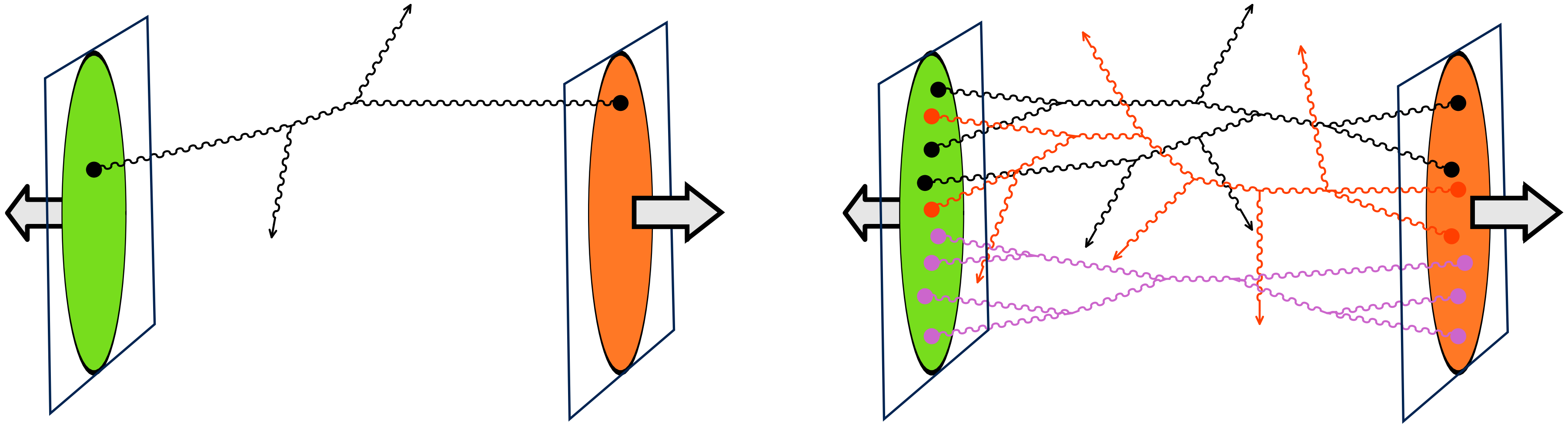
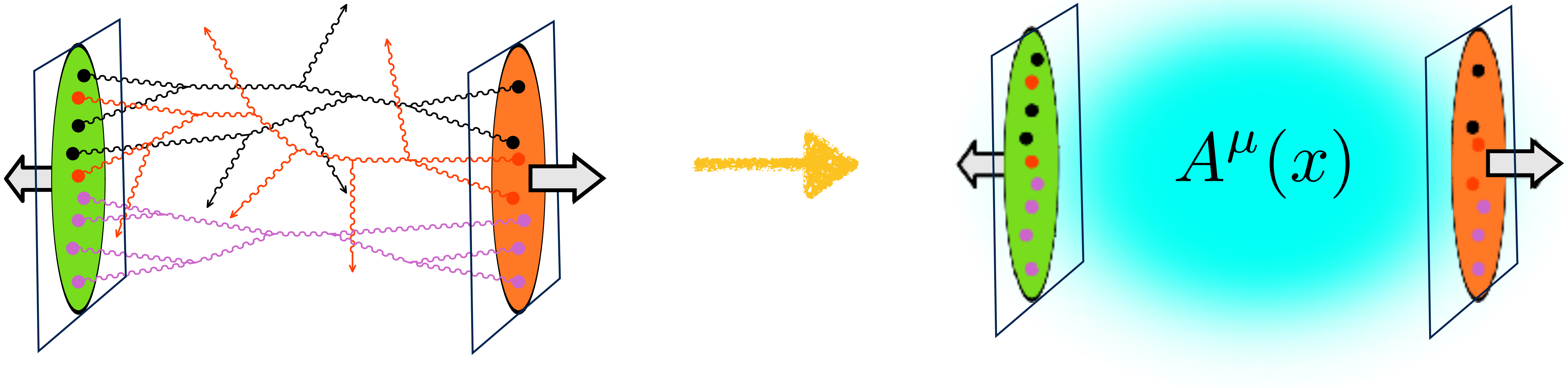
- QCD becomes **non-linear** and **non-perturbative**!



Color Glass Condensate: McLerran-Venugopalan model

- Perturbative techniques would require computing infinite diagrams
- We use an approximation of QCD for high gluon densities where we replace the gluons with a **classical field** generated by the valence quarks



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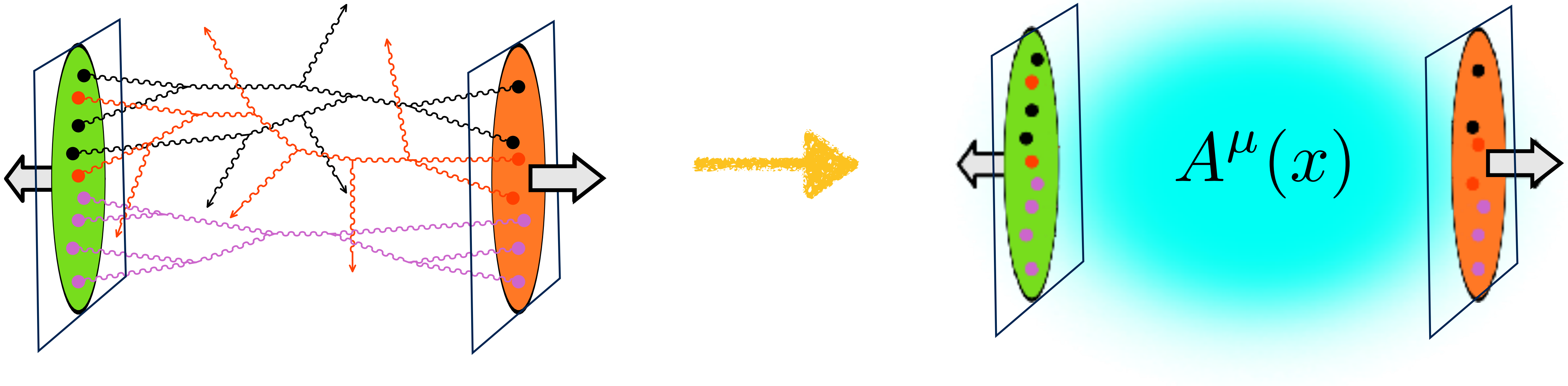
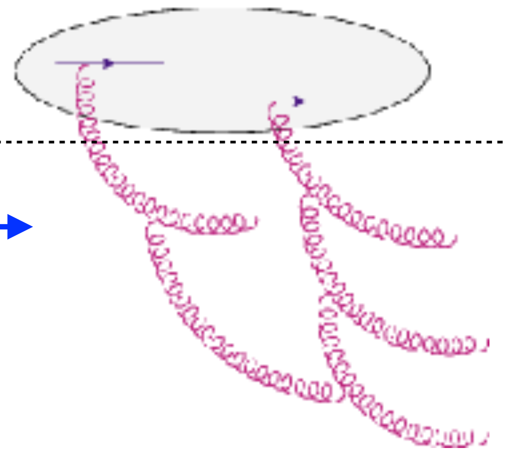


- Dynamics of the field described by **Yang-Mills** classical equations:

$$[D_\mu, F^{\mu\nu}] = J^\nu \propto \rho^a(x) t^a$$



$\longleftarrow J^{\nu,a} = \delta^{\nu+}\rho^a(x^-, x_\perp)$

$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig\,[A^\mu, A^\nu] \longrightarrow$

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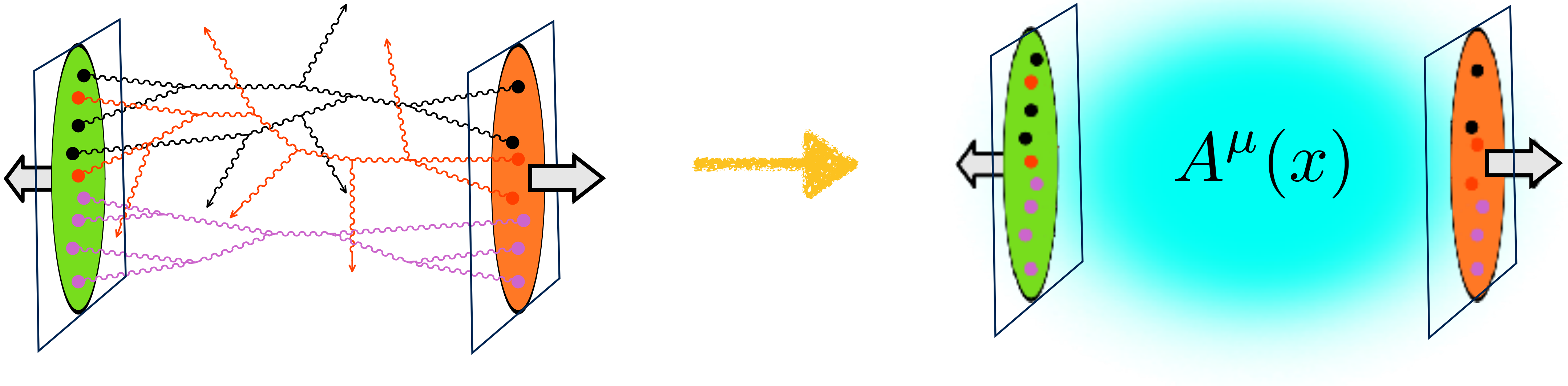

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- Calculation of observables: **average** over background classical fields

$$\langle \mathcal{O}[\rho] \rangle = \int [d\rho] \exp\left\{ - \int dx \, \mathrm{Tr}\left[\rho^2\right] \right\} \mathcal{O}[\rho]$$

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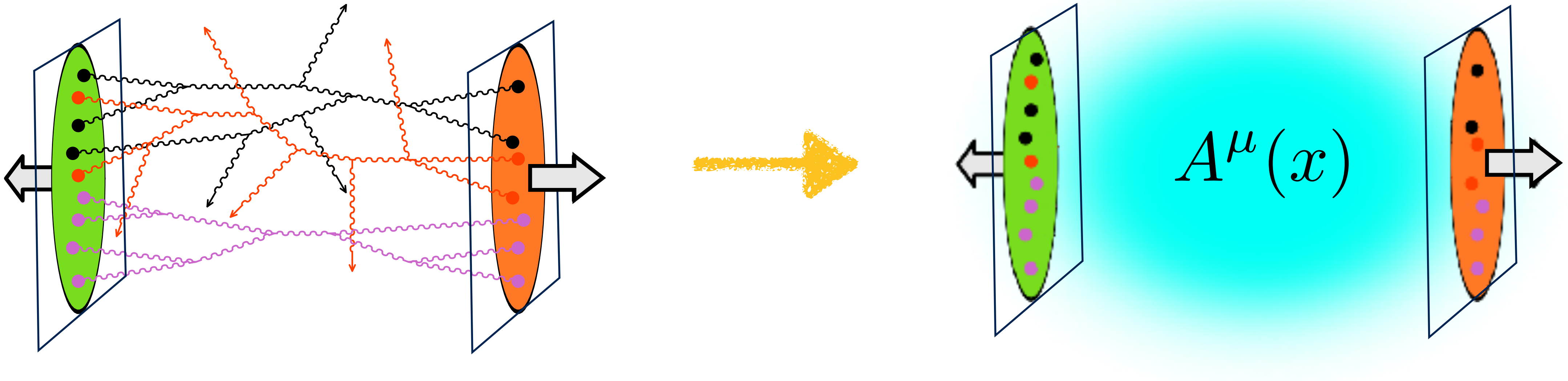

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- Calculation of observables: **average** over background classical fields
- Basic building block: **2-point correlator (McLerran-Venugopalan)**

$$\langle \rho^a(x^-, x_\perp)\rho^b(y^-, y_\perp)\rangle = \mu^2(x^-)\delta^{ab}\delta(x^- - y^-)\delta^{(2)}(x_\perp - y_\perp)$$

Color Glass Condensate: McLerran-Venugopalan model

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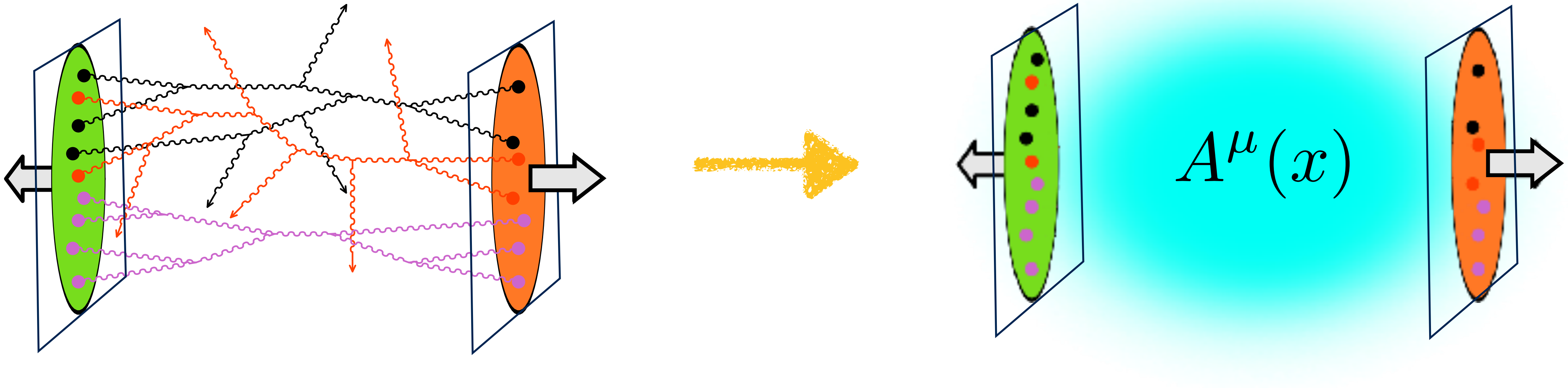

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$$[D_\mu, F^{\mu\nu}] = J^\nu \propto \rho^a(x)t^a$$

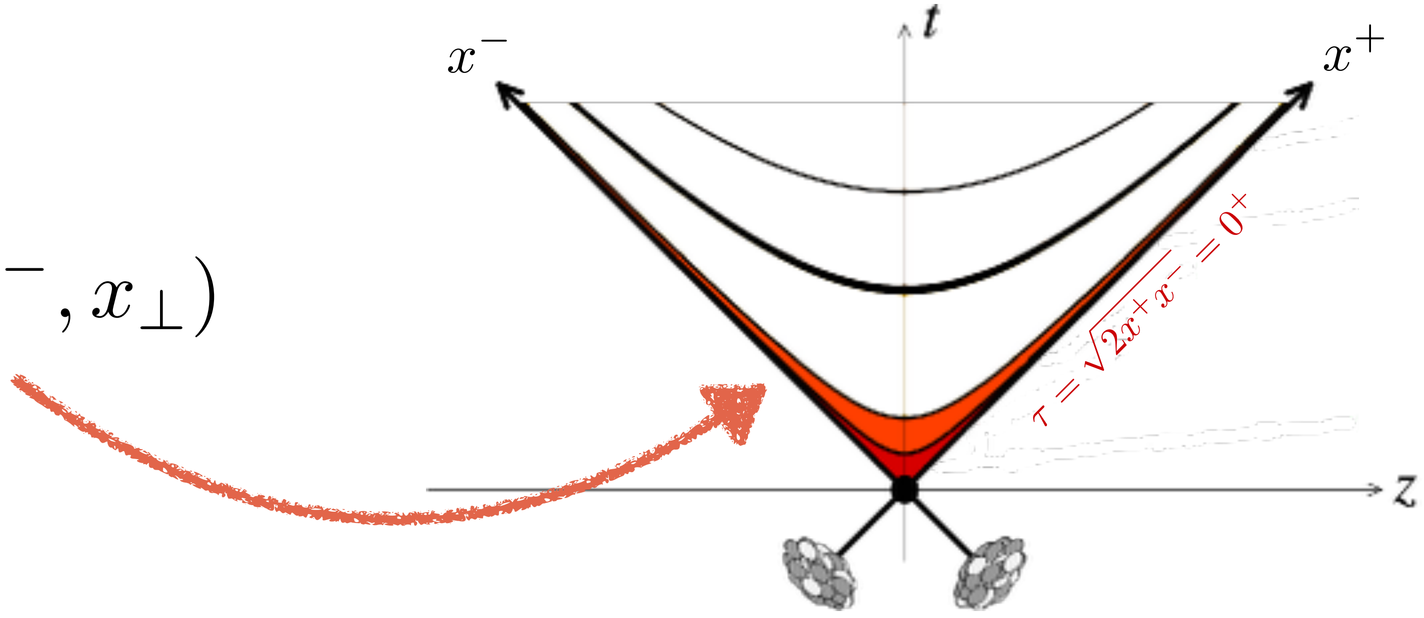
- Calculation of observables: **average** over background classical fields
- Basic building block: (**generalized**) **2-point correlator**

$$\langle \rho^a(x^-, x_\perp)\rho^b(y^-, y_\perp)\rangle = \mu^2(x^-)h(b_\perp)\delta^{ab}\delta(x^- - y^-)f(x_\perp - y_\perp)$$

Steps for the calculation

1) Calculate the gluon fields at early times in a HIC

$A_0^\mu(x^-, x_\perp)$



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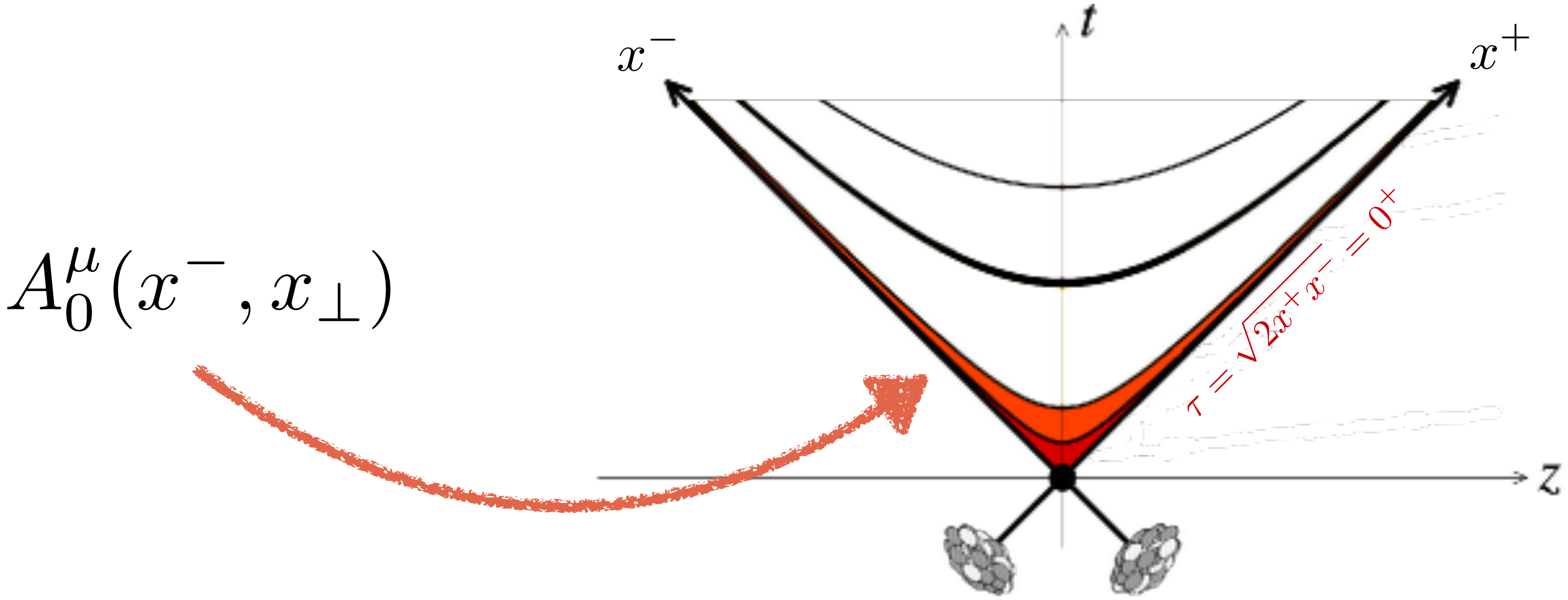

2) Build the energy-momentum tensor

$$T_0^{\mu\nu}(x_\perp) = 2\,\mathrm{Tr}\left\{\frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta} - F^{\mu\alpha}F^{\nu}_{\ \alpha}\right\}_0$$

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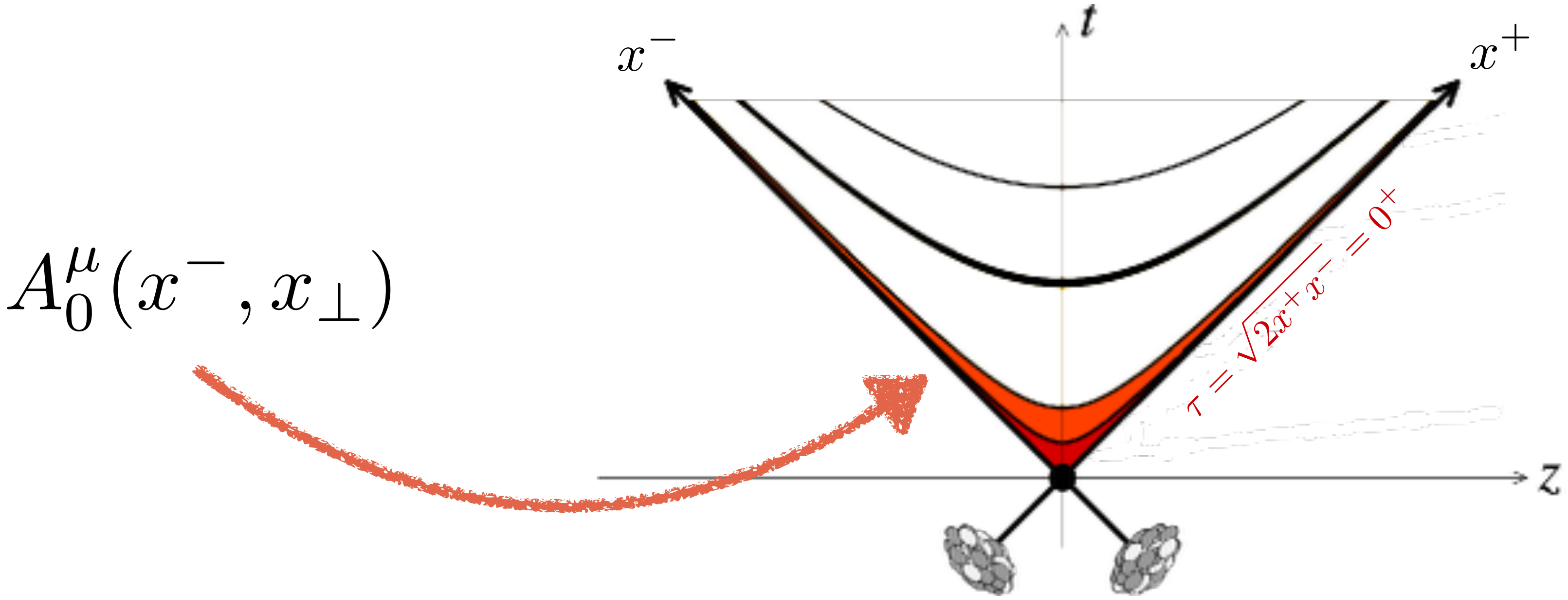

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3) Average over the color source distributions

$$\langle T_0^{\mu\nu}(x_\perp)\rangle = \int [d\rho_1]W_1[\rho_1][d\rho_2]W_2[\rho_2]T_0^{\mu\nu}(x_\perp)[\rho_1,\rho_2]$$

$$\langle T_0^{\mu\nu}(x_\perp)T_0^{\sigma\gamma}(y_\perp)\rangle = \int [d\rho_1]W_1[\rho_1][d\rho_2]W_2[\rho_2]\,T_0^{\mu\nu}(x_\perp)T_0^{\sigma\gamma}(y_\perp)[\rho_1,\rho_2]$$

Calculation of the gluon fields

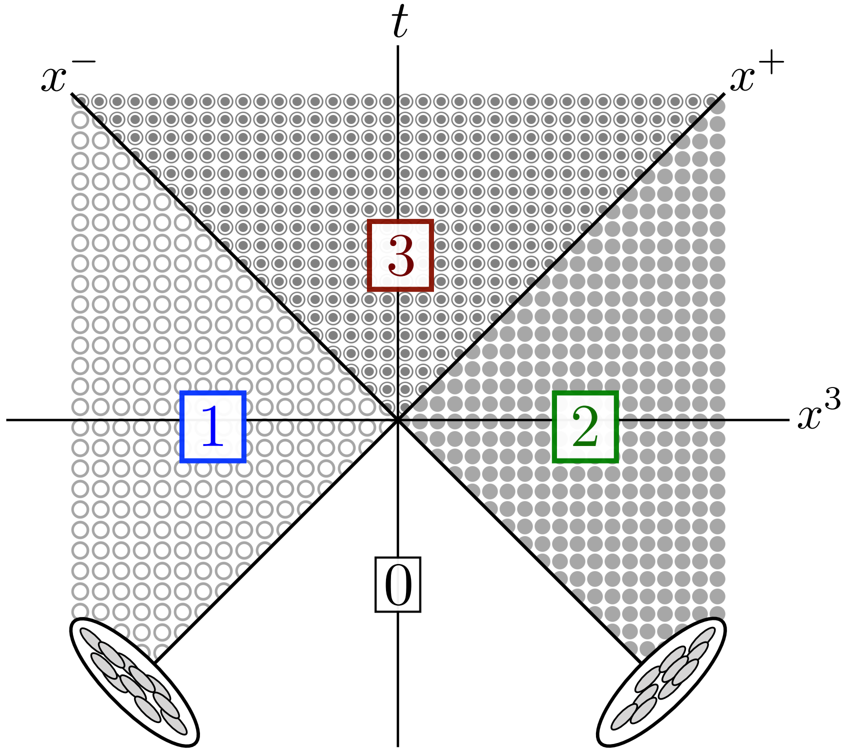
The gluon fields at $\tau=0^+$ in HICs

[Kovner, McLerran, Weigert 1998]

$$[D_\mu, F^{\mu\nu}] = J_1^\nu + J_2^\nu$$

$$\mathbf{J_1^\nu} = \rho_{\mathbf{1}}(\mathbf{x}_\perp)\delta(\mathbf{x}^-)\delta^{\nu+}$$

$$\mathbf{J_2^\nu} = \rho_{\mathbf{2}}(\mathbf{x}_\perp)\delta(\mathbf{x}^+)\delta^{\nu-}$$



The gluon fields at τ=0⁺ in HICs

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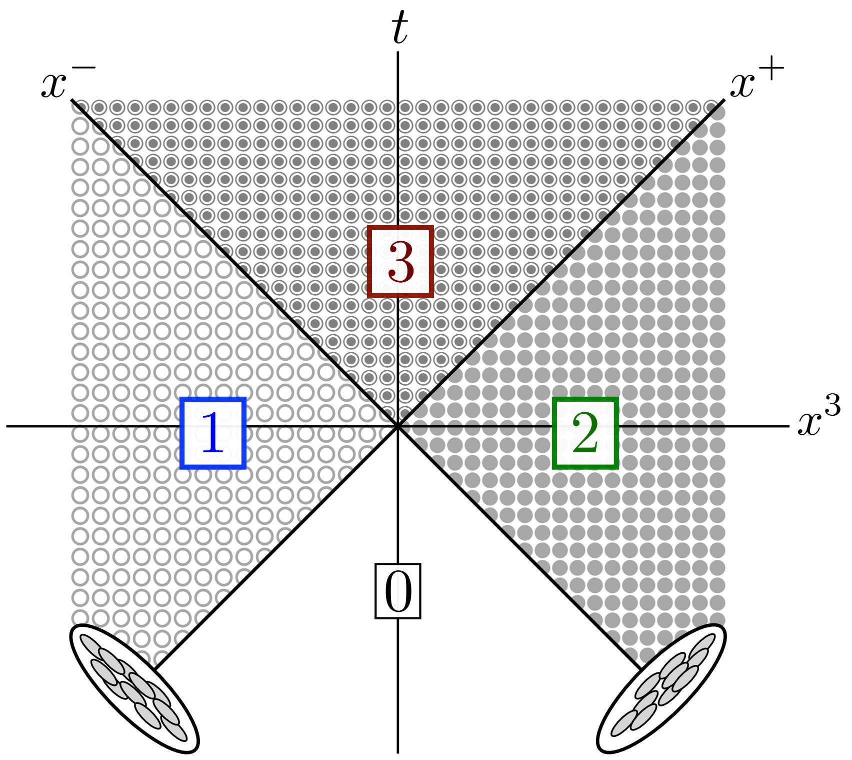

[1, 2] Single nucleus solution

$$A_1^{\pm} = 0$$

$$A_1^i = \theta(x^-)\int_{-\infty}^{\infty} dz^- \frac{\partial^i \tilde{\rho}_1^a(z^-, z_\perp)}{\nabla^2} U_1^{ab}(z^-, x_\perp)t^b \equiv \theta(x^-)\alpha_1^{i,b}(x_\perp)t^b$$

$$U_1^{ab}(x^-, x_\perp) = \mathrm{P}^- \exp\left\{-ig\int_{x_0^-}^{x^-} dz^- \frac{1}{\nabla^2}\tilde{\rho}_1(z^-, x_\perp)\right\}^{ab}$$

The gluon fields at $\tau=0^+$ in HICs

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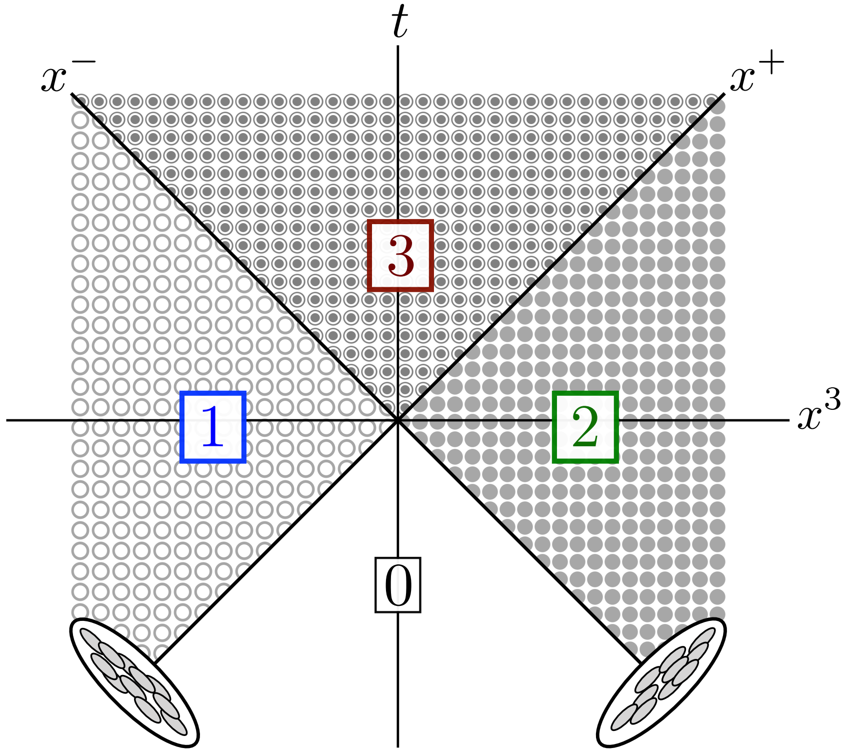

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[3] Forward light cone $\tau = 0^+$

$$A^\pm = \pm x^\pm \alpha(\tau = 0^+, x_\perp)$$

$$A^i = \alpha^i(\tau = 0^+, x_\perp)$$

$$\alpha^i(\tau = 0^+, x_\perp) = \alpha_1^i(x_\perp) + \alpha_2^i(x_\perp)$$

$$\alpha(\tau = 0^+, x_\perp) = \frac{ig}{2}\left[\alpha_1^i(x_\perp), \alpha_2^i(x_\perp)\right]$$

Calculation of the energy-momentum tensor $T^{\mu\nu}(\tau = 0^+)$

$$[D_\mu, F^{\mu\nu}] = J_1^\nu + J_2^\nu$$

$$\mathbf{J_1^\nu} = \rho_\mathbf{1}(\mathbf{x}_\perp)\delta(\mathbf{x}^-)\delta^{\nu +}$$

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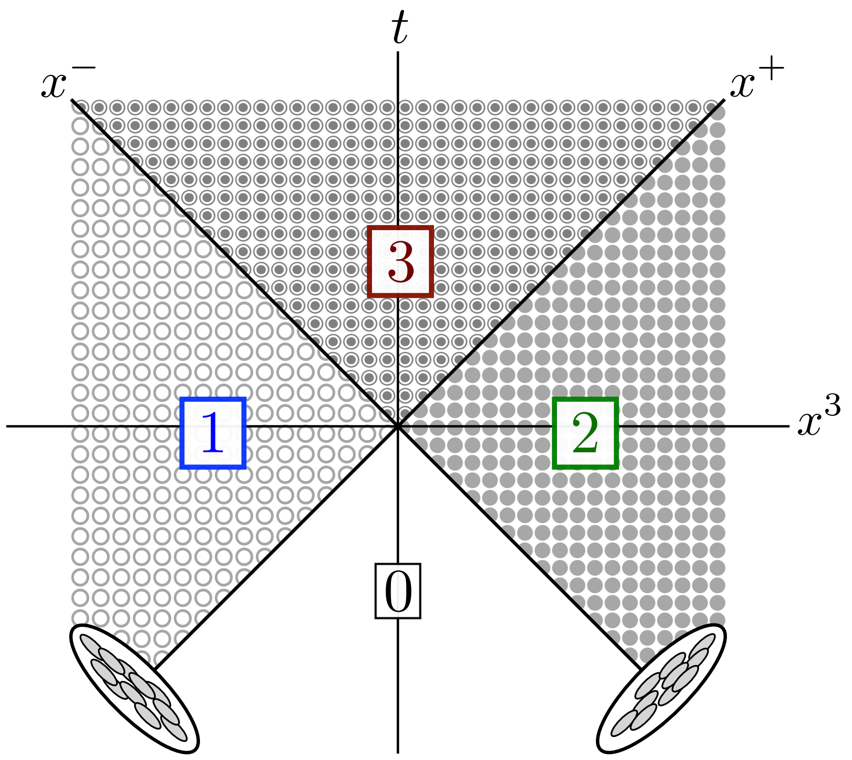

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- We can obtain the early-time energy-momentum tensor as:

$$T_0^{\mu\nu} = \frac{1}{4}g^{\mu\nu}F^{\alpha\beta,a}F_{\alpha\beta}^a - F^{\mu\alpha,a}F_\alpha^{\nu,a}$$

$$= -\frac{g^2}{2}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})\Big([\alpha_1^i, \alpha_2^j][\alpha_1^k, \alpha_2^l]\Big) \times \mathrm{diag}(1,1,1,-1)$$

$$\equiv \epsilon_0 \times \mathrm{diag}(1,1,1,-1) \equiv \epsilon_0 \times t^{\mu\nu}$$

Correlators of the energy-momentum tensor at $\tau = 0^+$

$\langle T^{\mu\nu}(x_\perp)\rangle = \langle \epsilon_0 \rangle\, t^{\mu\nu}$

REMINDER:

$$\boxed{\alpha_1^{i,b}(x_\perp) = \int_{-\infty}^{\infty} dz^- \frac{\partial^i \tilde{\rho}_1^a(z^-, z_\perp)}{\nabla^2} U_1^{ab}(z^-, x_\perp)}$$

- For the 1-point correlator of $T^{\mu\nu}$:

$$\begin{aligned}
\langle \epsilon_0 \rangle &= -g^2(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})\left\langle \mathrm{Tr}\left\{[\alpha_1^i, \alpha_2^j][\alpha_1^k, \alpha_2^l]\right\}\right\rangle \\
&= -g^2(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})\left\langle \alpha_1^{i,a}\alpha_2^{j,b}\alpha_1^{k,c}\alpha_2^{l,d}\right\rangle \mathrm{Tr}\left\{\left[t^a, t^b\right]\left[t^c, t^d\right]\right\} \\
&= \frac{g^2}{2}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl}) f^{abm} f^{cdm} \underbrace{\left\langle \alpha_1^{i,a}(x_\perp)\alpha_1^{k,c}(x_\perp)\right\rangle}_{\textbf{Building block of the calculation}} \left\langle \alpha_2^{j,b}(x_\perp)\alpha_2^{l,d}(x_\perp)\right\rangle
\end{aligned}$$

$\langle T^{\mu\nu}(x_\perp)\rangle = \langle \epsilon_0 \rangle \, t^{\mu\nu}$

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- We momentarily take two different transverse positions:

$$\langle \alpha^{i,a}(x_\perp)\alpha^{j,b}(y_\perp)\rangle = \int_{-\infty}^{\infty} dz^- dz^{-\prime} \left\langle \frac{\partial^i \tilde{\rho}^{a'}(z^-, x_\perp)}{\nabla^2} \underbrace{U^{a'a}(z^-, x_\perp)}_{\sim \exp\left\{ i \int^{z^-} dx^- \rho \right\}} \frac{\partial^j \tilde{\rho}^{b'}(z^{-\prime}, y_\perp)}{\nabla^2} \underbrace{U^{b'b}(z^{-\prime}, y_\perp)}_{\sim \exp\left\{ i \int^{z^{-\prime}} dx^- \rho \right\}} \right\rangle$$

$$\boxed{\langle \rho^a(x^-, x_\perp)\rho^b(y^-, y_\perp)\rangle = \mu^2(x^-)\delta^{ab}\delta(x^- - y^-)\delta^{(2)}(x_\perp - y_\perp)}$$

$\langle T^{\mu\nu}(x_\perp)\rangle = \langle \epsilon_0 \rangle \, t^{\mu\nu}$

REMINDER:

$$\alpha_1^{i,b}(x_\perp) = \int_{-\infty}^{\infty} dz^- \frac{\partial^i \tilde{\rho}_1^a(z^-, z_\perp)}{\nabla^2} U_1^{ab}(z^-, x_\perp)$$

- For the 1-point correlator of $T^{\mu\nu}$:

$$\langle \epsilon_0 \rangle = -g^2(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl}) \left\langle \mathrm{Tr}\left\{ [\alpha_1^i, \alpha_2^j][\alpha_1^k, \alpha_2^l] \right\} \right\rangle$$

$$= -g^2(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl}) \left\langle \alpha_1^{i,a}\alpha_2^{j,b}\alpha_1^{k,c}\alpha_2^{l,d} \right\rangle \mathrm{Tr}\left\{ \left[t^a, t^b\right] \left[t^c, t^d\right] \right\}$$

$$= \frac{g^2}{2}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl}) f^{abm} f^{cdm} \left\langle \alpha_1^{i,a}(x_\perp)\alpha_1^{k,c}(x_\perp) \right\rangle \left\langle \alpha_2^{j,b}(x_\perp)\alpha_2^{l,d}(x_\perp) \right\rangle$$

- We momentarily take two different transverse positions:

$$\langle \alpha^{i,a}(x_\perp)\alpha^{j,b}(y_\perp)\rangle = \int_{-\infty}^{\infty} dz^- dz^{-\prime} \left\langle \frac{\partial^i \tilde{\rho}^{a'}(z^-, x_\perp)}{\nabla_\perp^2} \frac{\partial^j \tilde{\rho}^{b'}(z^{-\prime} y_\perp)}{\nabla_\perp^2} \right\rangle \left\langle U^{a'a}(z^-, x_\perp) U^{b'b}(z^{-\prime}, y_\perp) \right\rangle$$

Luckily, in this case Wilson lines and (external) color source densities factorize

$$\boxed{\langle \rho^a(x^-, x_\perp)\rho^b(y^-, y_\perp)\rangle = \mu^2(x^-)\delta^{ab}\delta(x^- - y^-)\delta^{(2)}(x_\perp - y_\perp)}$$

$\langle T^{\mu\nu}(x_\perp)\rangle = \langle \epsilon_0\rangle \, t^{\mu\nu}$

REMINDER:

$$\alpha_1^{i,b}(x_\perp)=\int_{-\infty}^{\infty} dz^- \frac{\partial^i \tilde{\rho}_1^a(z^-,z_\perp)}{\nabla^2} U_1^{ab}(z^-,x_\perp)$$

- For the 1-point correlator of $T^{\mu\nu}$:

$$\langle\epsilon_0\rangle = -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \mathrm{Tr}\left\{[\alpha_1^i,\alpha_2^j][\alpha_1^k,\alpha_2^l]\right\}\right\rangle$$

$$= -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \alpha_1^{i,a}\alpha_2^{j,b}\alpha_1^{k,c}\alpha_2^{l,d}\right\rangle \mathrm{Tr}\left\{\left[t^a,t^b\right]\left[t^c,t^d\right]\right\}$$

$$= \frac{g^2}{2}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl}) f^{abm} f^{cdm} \left\langle \alpha_1^{i,a}(x_\perp)\alpha_1^{k,c}(x_\perp)\right\rangle \left\langle \alpha_2^{j,b}(x_\perp)\alpha_2^{l,d}(x_\perp)\right\rangle$$

- We momentarily take two different transverse positions:

$$\langle\alpha^{i,a}(x_\perp)\alpha^{j,b}(y_\perp)\rangle = \int_{-\infty}^{\infty} dz^- dz^- \boxed{\left\langle \frac{\partial^i\tilde{\rho}^{a'}(z^-,x_\perp)}{\nabla_\perp^2}\frac{\partial^j\tilde{\rho}^{b'}(z^{-\prime}y_\perp)}{\nabla_\perp^2}\right\rangle} \left\langle U^{a'a}(z^-,x_\perp)U^{b'b}(z^{-\prime},y_\perp)\right\rangle$$

$$\delta^{a'b'}\mu^2(x^-)\delta(x^- - y^-)\partial_x^i\partial_y^j L(x_\perp - y_\perp)$$

Where:

$$L(x_\perp - y_\perp) = \int d^2 z_\perp G(x_\perp - z_\perp) G(y_\perp - z_\perp).$$

$\langle T^{\mu\nu}(x_\perp)\rangle = \langle \epsilon_0\rangle\, t^{\mu\nu}$

REMINDER:

$$\alpha_1^{i,b}(x_\perp) = \int_{-\infty}^{\infty} dz^- \frac{\partial^i \tilde{\rho}_1^a(z^-, z_\perp)}{\nabla^2} U_1^{ab}(z^-, x_\perp)$$

- For the 1-point correlator of $T^{\mu\nu}$:

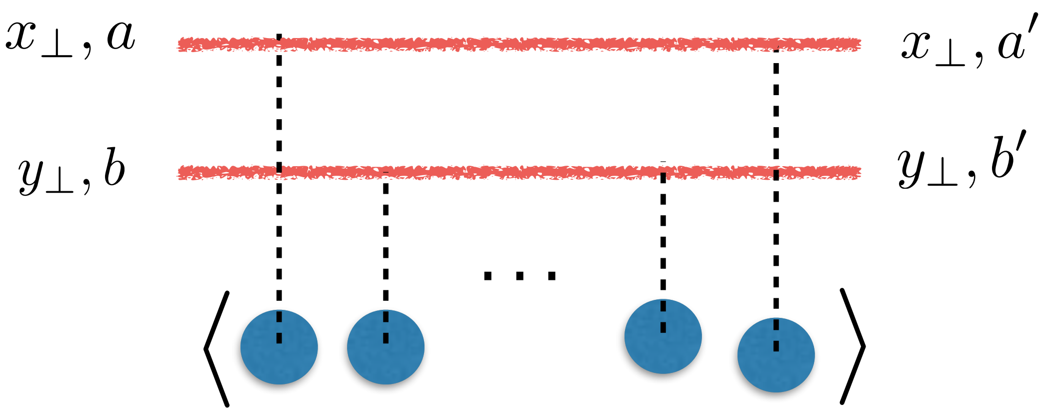
$$\langle \epsilon_0\rangle = -g^2(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})\Big\langle \mathrm{Tr}\left\{[\alpha_1^i, \alpha_2^j][\alpha_1^k, \alpha_2^l]\right\}\Big\rangle$$

$$= -g^2(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})\left\langle \alpha_1^{i,a}\alpha_2^{j,b}\alpha_1^{k,c}\alpha_2^{l,d}\right\rangle \mathrm{Tr}\left\{\left[t^a, t^b\right]\left[t^c, t^d\right]\right\}$$

$$= \frac{g^2}{2}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl}) f^{abm} f^{cdm} \left\langle \alpha_1^{i,a}(x_\perp)\alpha_1^{k,c}(x_\perp)\right\rangle \left\langle \alpha_2^{j,b}(x_\perp)\alpha_2^{l,d}(x_\perp)\right\rangle$$

- We momentarily take two different transverse positions:

$$\langle \alpha^{i,a}(x_\perp)\alpha^{j,b}(y_\perp)\rangle = \int_{-\infty}^{\infty} dz^- dz^{-\prime} \left\langle \frac{\partial^i \tilde{\rho}^{a'}(z^-, x_\perp)}{\nabla_\perp^2} \frac{\partial^j \tilde{\rho}^{b'}(z^{-\prime} y_\perp)}{\nabla_\perp^2} \right\rangle \left\langle U^{a'a}(z^-, x_\perp) U^{b'b}(z^{-\prime}, y_\perp)\right\rangle$$



$$\frac{\delta^{ab}\delta^{a'b'}}{N} \exp\left[-g^2 \frac{N}{2} \Gamma(x_\perp, y_\perp)\bar{\mu}^2(x^-)\right]$$

$$\equiv \frac{\delta^{ab}\delta^{a'b'}}{N} C_{\mathrm{adj}}^{(2)}(x^-; x_\perp, y_\perp).$$

Where:

$$\Gamma(x_\perp, y_\perp) = 2(L(0_\perp) - L(x_\perp - y_\perp)).$$

$\langle T^{\mu\nu}(x_\perp)\rangle = \langle \epsilon_0 \rangle \, t^{\mu\nu}$

REMINDER:

$$\alpha_1^{i,b}(x_\perp) = \int_{-\infty}^{\infty} dz^- \frac{\partial^i \tilde{\rho}_1^a(z^-, z_\perp)}{\nabla^2} U_1^{ab}(z^-, x_\perp)$$

- For the 1-point correlator of $T^{\mu\nu}$:

$$\begin{aligned}
\langle \epsilon_0 \rangle &= -g^2(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})\left\langle \mathrm{Tr}\left\{[\alpha_1^i, \alpha_2^j][\alpha_1^k, \alpha_2^l]\right\}\right\rangle \\
&= -g^2(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})\left\langle \alpha_1^{i,a}\alpha_2^{j,b}\alpha_1^{k,c}\alpha_2^{l,d}\right\rangle \mathrm{Tr}\left\{\left[t^a, t^b\right]\left[t^c, t^d\right]\right\} \\
&= \frac{g^2}{2}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl}) f^{abm} f^{cdm} \left\langle \alpha_1^{i,a}(x_\perp)\alpha_1^{k,c}(x_\perp)\right\rangle \left\langle \alpha_2^{j,b}(x_\perp)\alpha_2^{l,d}(x_\perp)\right\rangle \\
&= \frac{g^2}{8} f^{abm} f^{cdm}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})\delta^{ac}\delta^{ik}\delta^{bd}\delta^{jl}\bar{\mu}_1^2\bar{\mu}_2^2(\partial^2 L(0_\perp))^2 \\
&= g^2 C_A^2 C_F \bar{\mu}_1^2\bar{\mu}_2^2(\partial^2 L(0_\perp))^2 \\
&= \boxed{\frac{C_F}{g^2}\bar{Q}_{s1}^2(x_\perp)\bar{Q}_{s2}^2(x_\perp)\left(4\pi\,\partial^2 L(0_\perp)\right)^2}
\end{aligned}$$

Notation:

$$\bar{\mu}^2 = \int_{-\infty}^{\infty} dz^- \mu^2(z^-)$$

- Here we have introduced a **momentum scale** characterizing each nucleus:

$$\bar{Q}_s^2 = \alpha_s N_c \, \bar{\mu}^2(x_\perp)$$

- In the MV model the factor $\partial^2 L(0_\perp)$ yields a **logarithmic UV divergence:**

$$\partial_\perp^2 L(0_\perp)_{\mathrm{MV}} = \frac{1}{4\pi}\lim_{r\to 0}\left[\ln\left(\frac{m^2 r^2}{4}\right)\right]$$

$\langle T^{\mu\nu}(x_\perp)T^{\sigma\rho}(y_\perp)\rangle = \langle\epsilon(x_\perp)\epsilon(y_\perp)\rangle\, t^{\mu\nu}\, t^{\sigma\rho}$

- For the 2-point correlator of $T^{\mu\nu}$: prepare for trouble and make it double

$$\langle\epsilon(x_\perp)\epsilon(y_\perp)\rangle = \frac{g^4}{4}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'}+\epsilon^{i'j'}\epsilon^{k'l'})f^{abn}f^{cdn}f^{a'b'm}f^{c'd'm}$$

$$\times\, \underbrace{\langle\alpha^{i\,a}_{1\,x}\alpha^{k\,c}_{1\,x}\alpha^{i'a'}_{1\,y}\alpha^{k'c'}_{1\,y}\rangle}_{\textbf{Building block of the calculation}}\langle\alpha^{j\,b}_{2\,x}\alpha^{l\,d}_{2\,x}\alpha^{j'b'}_{2\,y}\alpha^{l'd'}_{2\,y}\rangle$$

REMINDER:

$$\alpha_1^{i,b}(x_\perp)=\int_{-\infty}^{\infty}dz^-\frac{\partial^i\tilde{\rho}_1^a(z^-,z_\perp)}{\nabla^2}U_1^{ab}(z^-,x_\perp)$$

$\langle T^{\mu\nu}(x_\perp) T^{\sigma\rho}(y_\perp)\rangle = \langle \epsilon(x_\perp)\epsilon(y_\perp)\rangle\, t^{\mu\nu}\, t^{\sigma\rho}$

- For the 2-point correlator of $T^{\mu\nu}$: prepare for trouble and make it double

$$\langle \epsilon(x_\perp)\epsilon(y_\perp)\rangle = \frac{g^4}{4}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'} + \epsilon^{i'j'}\epsilon^{k'l'}) f^{abn} f^{cdn} f^{a'b'm} f^{c'd'm}$$
$$\times \langle \alpha^{i\,a}_{1\,x}\alpha^{k\,c}_{1\,x}\alpha^{i'a'}_{1\,y}\,\alpha^{k'c'}_{1\,y}\rangle \langle \alpha^{j\,b}_{2\,x}\alpha^{l\,d}_{2\,x}\alpha^{j'b'}_{2\,y}\,\alpha^{l'd'}_{2\,y}\rangle$$

- The building block:

$$\langle \alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle = \int_{-\infty}^{\infty} dz^- dw^- dz^{-\prime} dw^{-\prime} \Big\langle \frac{\partial^i \tilde{\rho}^e(z^-, x_\perp)}{\nabla^2} U^{ea}(z^-, x_\perp)$$
$$\frac{\partial^k \tilde{\rho}^f(w^-, x_\perp)}{\nabla^2} U^{fc}(w^-, x_\perp) \frac{\partial^{i'} \tilde{\rho}^{e'}(z^{-\prime}, y_\perp)}{\nabla^2} U^{e'a'}(z^{-\prime}, y_\perp) \frac{\partial^{k'} \tilde{\rho}^{f'}(w^{-\prime}, y_\perp)}{\nabla^2} U^{f'c'}(w^{-\prime}, y_\perp) \Big\rangle .$$

$$\neq \int_{-\infty}^{\infty} dz^- dw^- dz^{-\prime} dw^{-\prime} \Big\langle \frac{\partial^i \tilde{\rho}^e(z^-, x_\perp)}{\nabla^2} \frac{\partial^k \tilde{\rho}^f(w^-, x_\perp)}{\nabla^2} \frac{\partial^{i'} \tilde{\rho}^{e'}(z^{-\prime}, y_\perp)}{\nabla^2} \frac{\partial^{k'} \tilde{\rho}^{f'}(w^{-\prime}, y_\perp)}{\nabla^2} \Big\rangle$$
$$\times \Big\langle U^{ea}(z^-, x_\perp) U^{fc}(w^-, x_\perp) U^{e'a'}(z^{-\prime}, y_\perp) U^{f'c'}(w^{-\prime}, y_\perp) \Big\rangle$$

!!!

REMINDER:

$$\boxed{\alpha_1^{i,b}(x_\perp) = \int_{-\infty}^{\infty} dz^- \frac{\partial^i \tilde{\rho}_1^a(z^-, z_\perp)}{\nabla^2} U_1^{ab}(z^-, x_\perp)}$$

Correlator of *n* Wilson lines and *m* external sources

Based on *[Fillion-Gourdeau & Jeon '09]*

$$F^{m,n}(b^-,a^-) = G^m H^{0,n} + \sum_{i,j,i<j} G^{m-2}_{(1,\ldots,i-1,\{i\},i+1,\ldots,j-1,\{j\},j+1,\ldots,m)} H^{2,n}_{(\{1,\ldots,i-1\},i,\{i+1,\ldots,j-1\},j,\{j+1,\ldots,m\})}$$

$$+ \sum_{i,j,k,l,i<j<k<l} G^{m-4}_{(1,\ldots i-1,\{i\},i+1,\ldots,j-1,\{j\},j+1,\ldots,k-1,\{k\},k+1,\ldots,l-1,\{l\},l+1,\ldots,m)}$$

$$\times H^{4,n}_{(\{1,\ldots,i-1\},i,\{i+1,\ldots,j-1\},j,\{j+1,\ldots,k-1\},k,\{k+1,\ldots,l-1\},l,\{l+1,\ldots,m\})}$$

$$+\ldots + \sum_{i,j,i<j} G^{2}_{(\{1,\ldots,i-1\},i,\{i+1,\ldots,j-1\},j,\{j+1,\ldots,m\})} H^{2,n}_{(1,\ldots,i-1,\{i\},i+1,\ldots,j-1,\{j\},j+1,\ldots,m)} + H^{m,n}$$

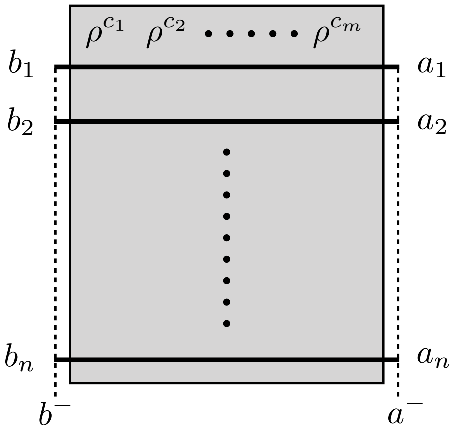
Where: $G^{m-1}_{(1,\ldots,j-1,\{j\},j+1,\ldots,m)} \equiv \langle \rho_1 \ldots \rho_{j-1}\rho_{j+1}\ldots\rho_m \rangle$

$$H^{j,n}_{(\{1,\ldots,J_1-1\},J_1,\{J_1+1,\ldots,J_2-1\},J_2,\{J_2+1,\ldots\}\ldots\{J_j-1\},J_j,\{J_{j+1},\ldots,m\})} \equiv \langle \rho_{J_1}\rho_{J_2}\ldots\rho_{J_j}U_1\ldots U_n\rangle_c$$

✗

✓

Correlator of *n* Wilson lines and *m* external sources



$$= G^m H^{0,n} + \sum_{i,j,i<j} G^{m-2}_{(1,\dots,i-1,\{i\},i+1,\dots,j-1,\{j\},j+1,\dots,m)} H^{2,n}_{(\{1,\dots,i-1\},i,\{i+1,\dots,j-1\},j,\{j+1,\dots,m\})}$$

$$+ \sum_{i,j,k,l,i<j<k<l} G^{m-4}_{(1,\dots i-1,\{i\},i+1,\dots,j-1,\{j\},j+1,\dots,k-1,\{k\},k+1,\dots,l-1,\{l\},l+1,\dots,m)}$$
$$\times H^{4,n}_{(\{1,\dots,i-1\},i,\{i+1,\dots,j-1\},j,\{j+1,\dots,k-1\},k,\{k+1,\dots,l-1\},l,\{l+1,\dots,m\})}$$

$$+ \dots + \sum_{i,j,i<j} G^{2}_{(\{1,\dots,i-1\},i,\{i+1,\dots,j-1\},j,\{j+1,\dots,m\})} H^{2,n}_{(1,\dots,i-1,\{i\},i+1,\dots,j-1,\{j\},j+1,\dots,m)} + H^{m,n}$$

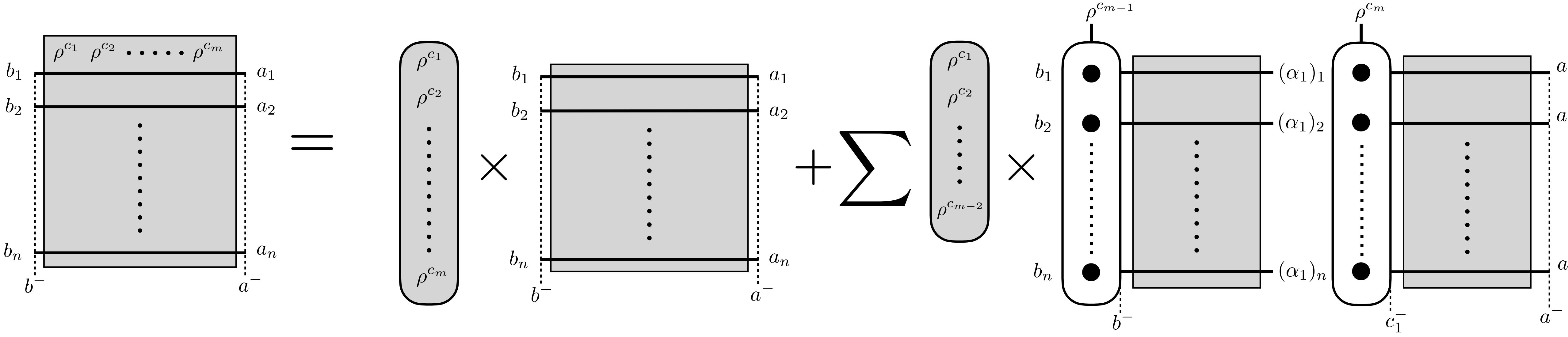
Where: $G^{m-1}_{(1,\dots,j-1,\{j\},j+1,\dots,m)} \equiv \langle \rho_1 \dots \rho_{j-1} \rho_{j+1} \dots \rho_m \rangle$

$$H^{j,n}_{(\{1,\dots,J_1-1\},J_1,\{J_1+1,\dots,J_2-1\},J_2,\{J_2+1,\dots\}\dots\{J_j-1\},J_j,\{J_{j+1},\dots,m\})} \equiv \langle \rho_{J_1} \rho_{J_2} \dots \rho_{J_j} U_1 \dots U_n \rangle_c$$

✗

✓

Correlator of n Wilson lines and m external sources



$$+\sum_{i,j,k,l,i<j<k<l} G^{m-4}_{(1,\dots i-1,\{i\},i+1,\dots,j-1,\{j\},j+1,\dots,k-1,\{k\},k+1,\dots,l-1,\{l\},l+1,\dots,m)}$$

$$\times H^{4,n}_{(\{1,\dots,i-1\},i,\{i+1,\dots,j-1\},j,\{j+1,\dots,k-1\},k,\{k+1,\dots,l-1\},l,\{l+1,\dots,m\})}$$

$$+\dots+\sum_{i,j,i<j} G^{2}_{(\{1,\dots,i-1\},i,\{i+1,\dots,j-1\},j,\{j+1,\dots,m\})} H^{2,n}_{(1,\dots,i-1,\{i\},i+1,\dots,j-1,\{j\},j+1,\dots,m)} + H^{m,n}$$

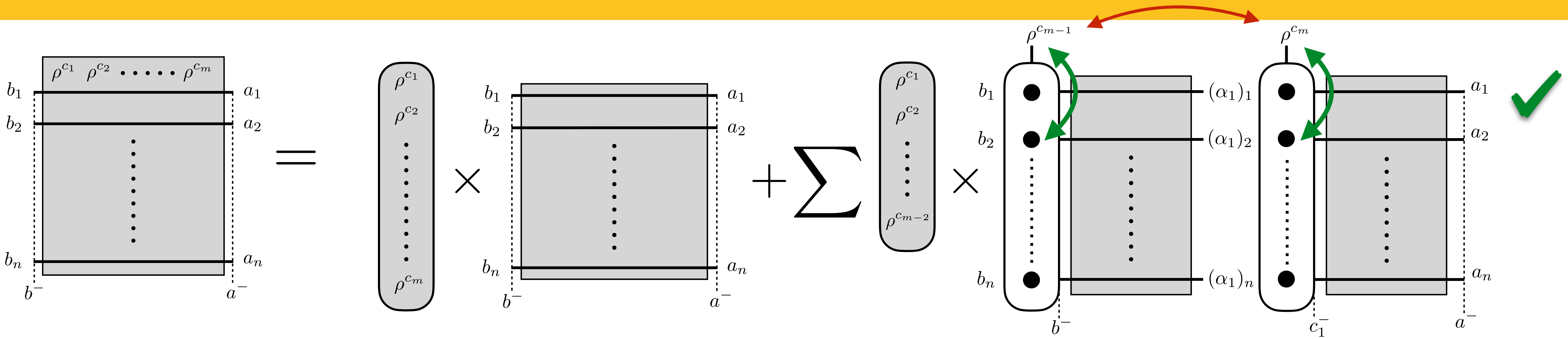
Where: $G^{m-1}_{(1,\dots,j-1,\{j\},j+1,\dots,m)} \equiv \langle \rho_1 \dots \rho_{j-1} \rho_{j+1} \dots \rho_m \rangle$

$$H^{j,n}_{(\{1,\dots,J_1-1\},J_1,\{J_1+1,\dots,J_2-1\},J_2,\{J_2+1,\dots\}\dots\{J_j-1\},J_j,\{J_{j+1},\dots,m\})} \equiv \langle \rho_{J_1} \rho_{J_2} \dots \rho_{J_j} U_1 \dots U_n \rangle_c$$

✗

✓

Correlator of *n* Wilson lines and *m* external sources ✗



$$+\sum_{i,j,k,l,i<j<k<l} G^{m-4}_{(1,\dots i-1,\{i\},i+1,\dots,j-1,\{j\},j+1,\dots,k-1,\{k\},k+1,\dots,l-1,\{l\},l+1,\dots,m)}$$

$$\times H^{4,n}_{(\{1,\dots,i-1\},i,\{i+1,\dots,j-1\},j,\{j+1,\dots,k-1\},k,\{k+1,\dots,l-1\},l,\{l+1,\dots,m\})}$$

$$+\dots+\sum_{i,j,i<j} G^{2}_{(\{1,\dots,i-1\},i,\{i+1,\dots,j-1\},j,\{j+1,\dots,m\})} H^{2,n}_{(1,\dots,i-1,\{i\},i+1,\dots,j-1,\{j\},j+1,\dots,m)} + H^{m,n}$$

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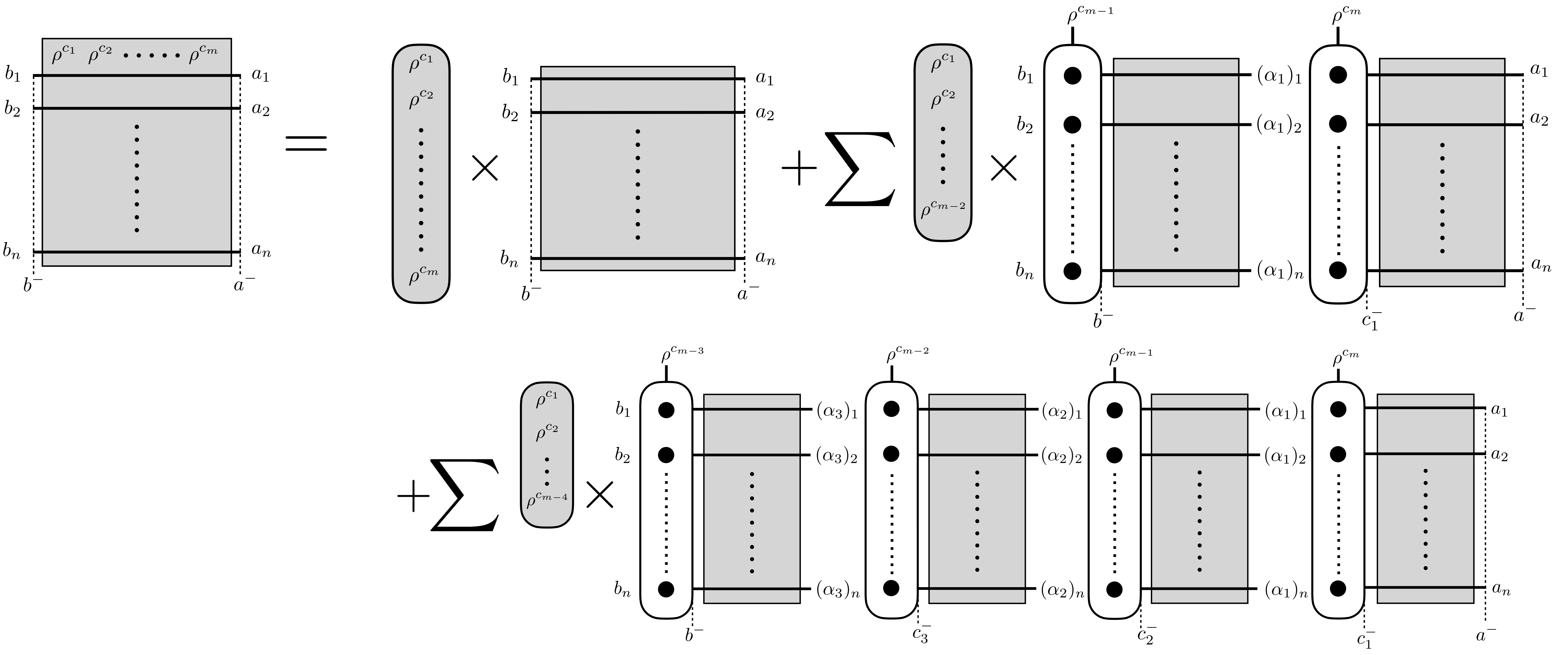
$H^{j,n}_{(\{1,\dots,J_1-1\},J_1,\{J_1+1,\dots,J_2-1\},J_2,\{J_2+1,\dots\}\dots\{J_j-1\},J_j,\{J_{j+1},\dots,m\})} \equiv \langle \rho_{J_1}\rho_{J_2}\dots\rho_{J_j}U_1\dots U_n \rangle_c$ ✗ ✓

Correlator of *n* Wilson lines and *m* external sources



$$+\ldots+\sum_{i,j,i<j} G^{2}_{(\{1,\ldots,i-1\},i,\{i+1,\ldots,j-1\},j,\{j+1,\ldots,m\})} H^{2,n}_{(1,\ldots,i-1,\{i\},i+1,\ldots,j-1,\{j\},j+1,\ldots,m)} + H^{m,n}$$

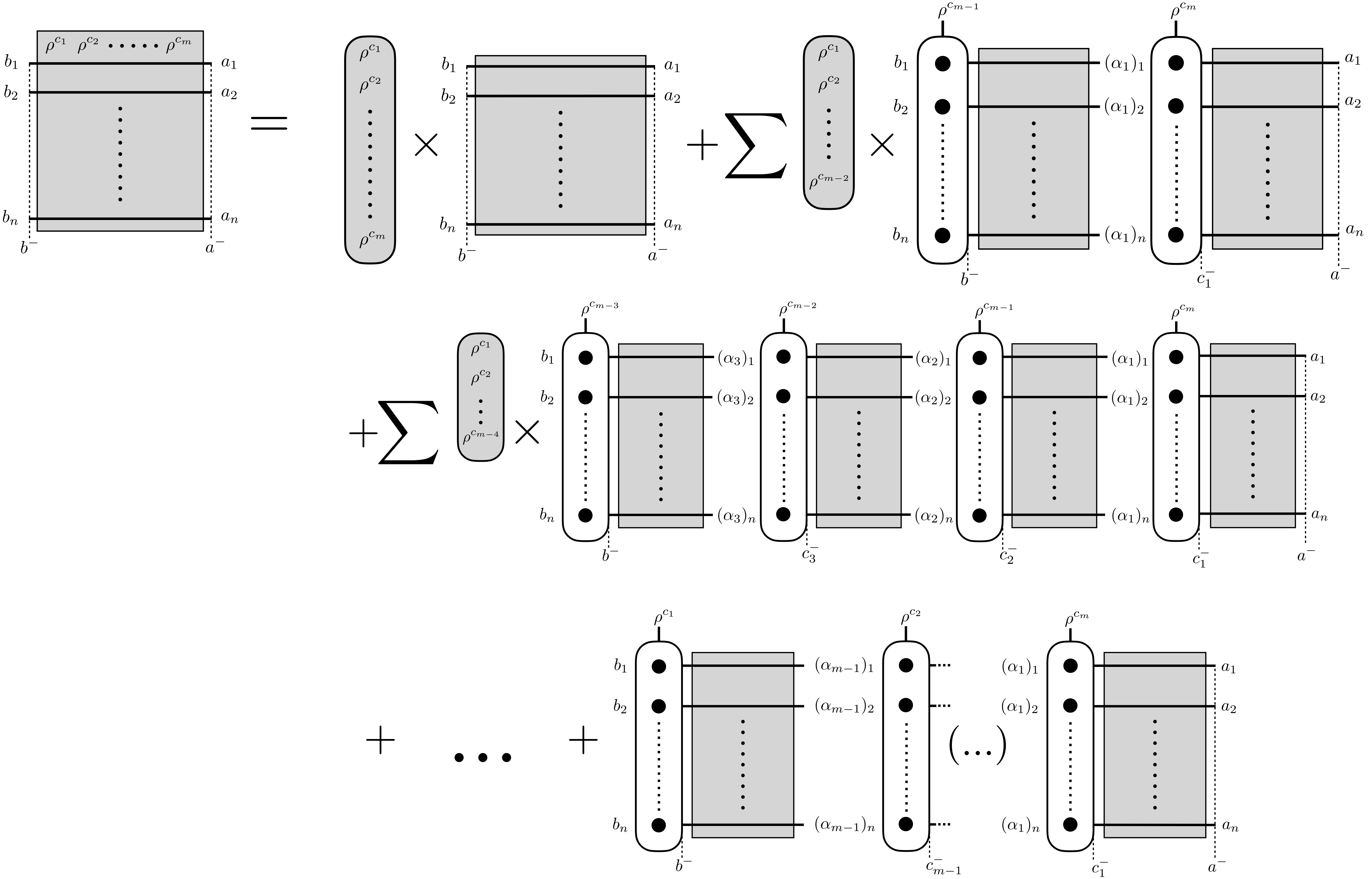
Where: $G^{m-1}_{(1,\ldots,j-1,\{j\},j+1,\ldots,m)} \equiv \langle \rho_1 \ldots \rho_{j-1}\rho_{j+1}\ldots\rho_m \rangle$

$$H^{j,n}_{(\{1,\ldots,J_1-1\},J_1,\{J_1+1,\ldots,J_2-1\},J_2,\{J_2+1,\ldots\}\ldots\{J_j-1\},J_j,\{J_{j+1},\ldots,m\})} \equiv \langle \rho_{J_1}\rho_{J_2}\ldots\rho_{J_j}U_1\ldots U_n\rangle_c$$

✗

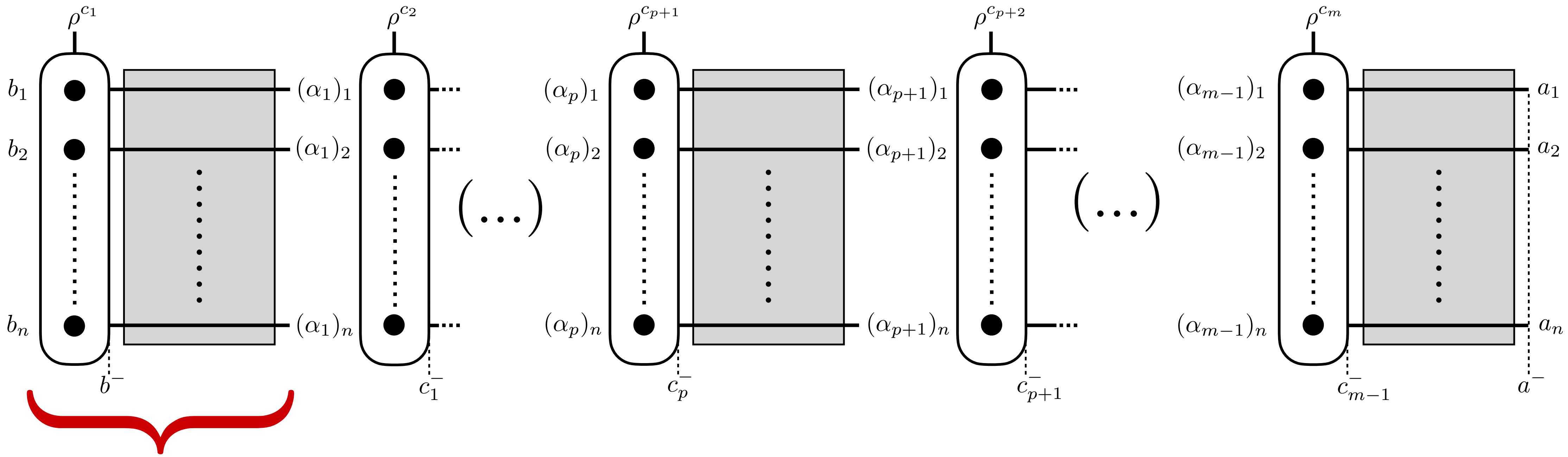
✓

Correlator of *n* Wilson lines and *m* external sources



Detail on "connected" correlators

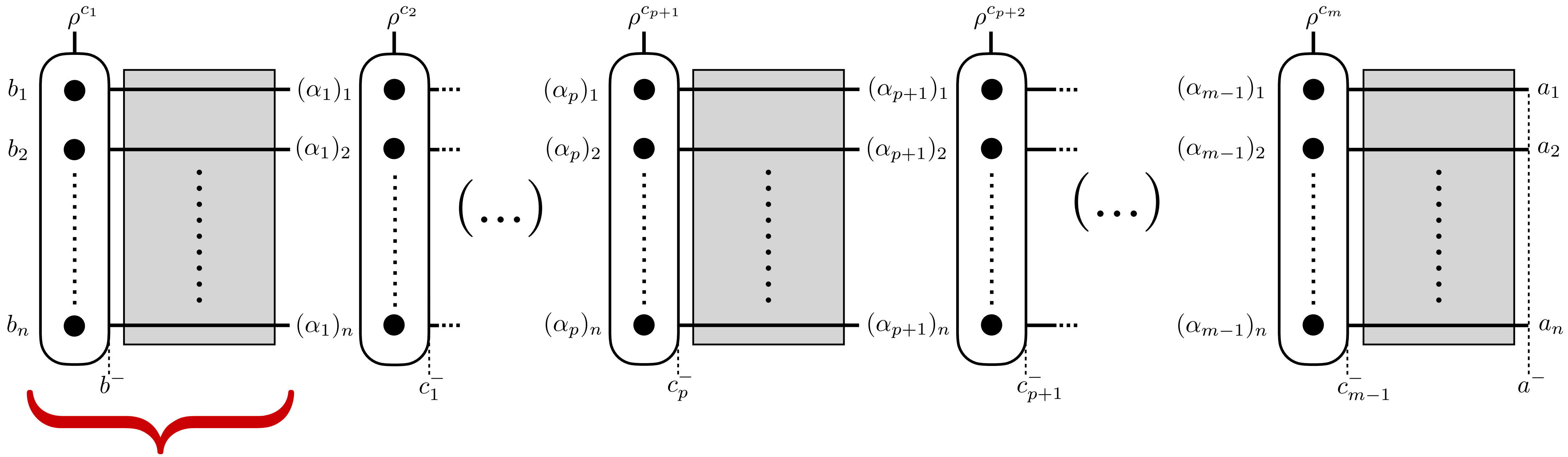
$$H^{m,n}(b^-,a^-|\{b\},\{a\}) = H^{1,n}(b^-,c_1^-|\{b\},\{\alpha_1\})\left[\prod_{p=1}^{m-2} H^{1,n}(c_p^-,c_{p+1}^-|\{\alpha_p\},\{\alpha_{p+1}\})\right]H^{1,n}(c_{m-1}^-,a^-|\{\alpha_{m-1}\},\{a\})$$



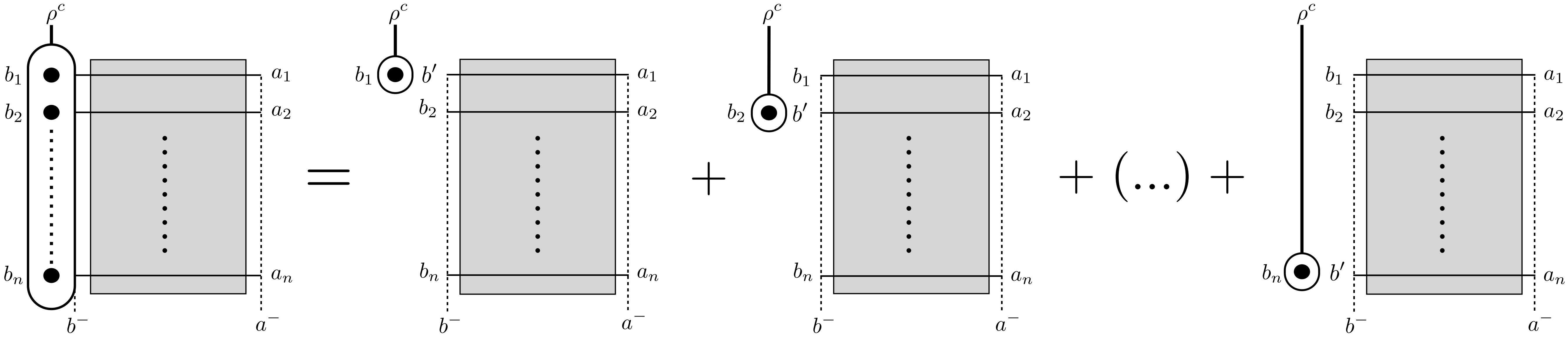
Building block of connected correlators

Detail on “connected” correlators

$$H^{m,n}(b^-,a^-|\{b\},\{a\}) = H^{1,n}(b^-,c_1^-|\{b\},\{\alpha_1\})\left[\prod_{p=1}^{m-2} H^{1,n}(c_p^-,c_{p+1}^-|\{\alpha_p\},\{\alpha_{p+1}\})\right] H^{1,n}(c_{m-1}^-,a^-|\{\alpha_{m-1}\},\{a\})$$

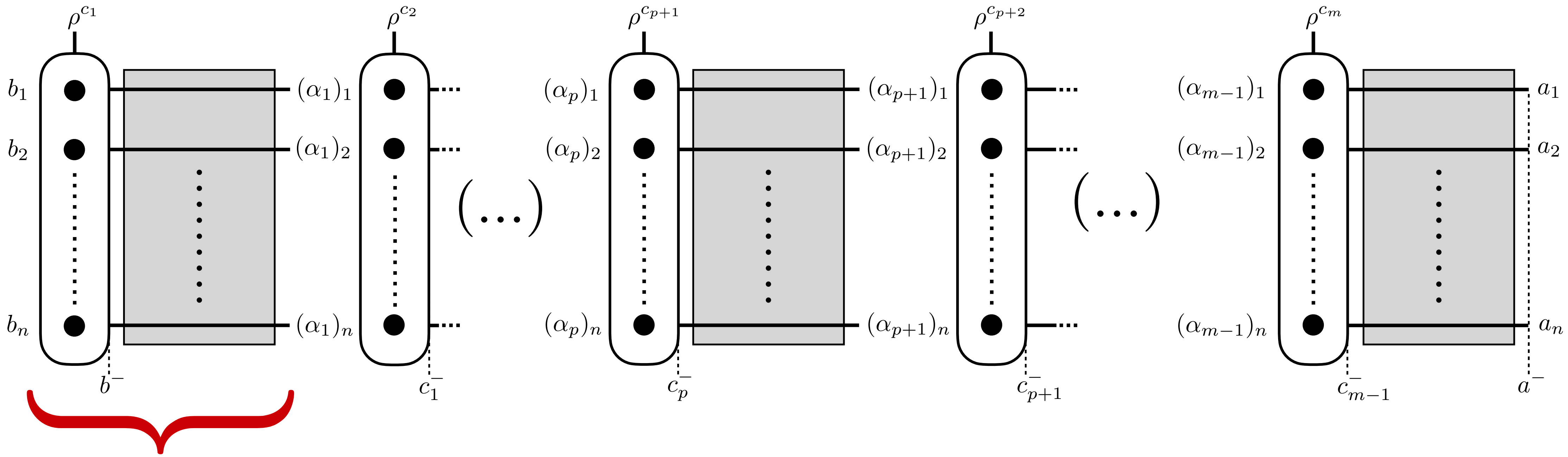


$$H^{1,n}(b^-,a^-|\{b\},\{a\})^i = g\mu^2(b^-)\sum_{j=1}^{n} \partial_y^i L(x_{j\perp}-y_\perp) f^{c\,b_j b'} F^n(b^-,a^-|\{\beta\}\{a\})|_{\beta_j=b'}$$

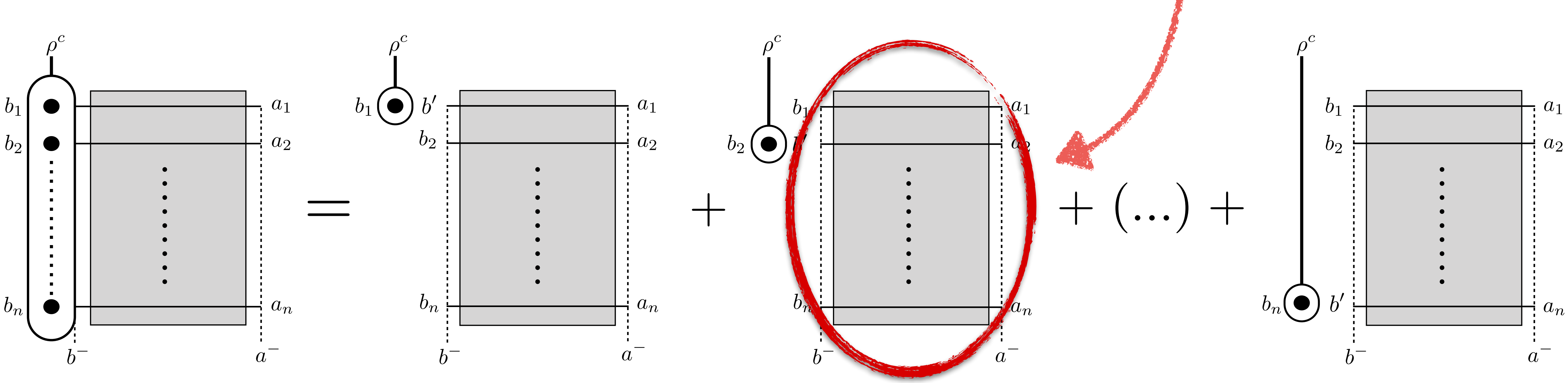


Detail on "connected" correlators

$$H^{m,n}(b^-,a^-|\{b\},\{a\}) = H^{1,n}(b^-,c_1^-|\{b\},\{\alpha_1\})\left[\prod_{p=1}^{m-2} H^{1,n}(c_p^-,c_{p+1}^-|\{\alpha_p\},\{\alpha_{p+1}\})\right] H^{1,n}(c_{m-1}^-,a^-|\{\alpha_{m-1}\},\{a\})$$



$$H^{1,n}(b^-,a^-|\{b\},\{a\})^i = g\mu^2(b^-)\sum_{j=1}^{n} \partial_y^i L(x_{j\perp}-y_\perp) f^{c\,b_j b'} \underline{F^n(b^-,a^-|\{\beta\}\{a\})|_{\beta_j=b'}}$$



Correlator of 4 Wilson lines and 4 external sources

NOTATION:

$$\boxed{\rho \equiv \frac{\partial\tilde{\rho}}{\nabla_\perp^2}}$$

- Applying the previous rules we can decompose this correlator as:

$$\langle\rho^4 U^4\rangle = \langle\rho^4\rangle\langle U^4\rangle + \langle\rho^2\rangle\langle\rho^2 U^4\rangle_c + \langle\rho^4 U^4\rangle_c$$

Correlator of 4 Wilson lines and 4 external sources

- Applying the previous rules we can decompose this correlator as:

$$\langle \rho^4 U^4 \rangle = \langle \rho^4 \rangle \langle U^4 \rangle + \langle \rho^2 \rangle \langle \rho^2 U^4 \rangle_c + \langle \rho^4 U^4 \rangle_c$$

Disconnected terms:

$$\int_{-\infty}^{\infty} dz^- dw^- dz^{-\prime} dw^{-\prime} \left\langle \tilde{\rho}_x^{i,e}\, \tilde{\rho}_x^{k,f}\, \tilde{\rho}_y^{i',e'}\, \tilde{\rho}_y^{k',f'} \right\rangle \left\langle U^{ea}(z^-, x_\perp) U^{fc}(w^-, x_\perp) U^{e'a'}(z^{-\prime}, y_\perp) U^{f'c'}(w^{-\prime}, y_\perp) \right\rangle$$

REMINDER:

$$\langle \rho^a(x^-, x_\perp) \rho^b(y^-, y_\perp) \rangle = \mu^2(x^-) \delta^{ab} \delta(x^- - y^-) \delta^{(2)}(x_\perp - y_\perp)$$

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$$= \int_{-\infty}^{\infty} dz^- dw^- dz^{-\prime} dw^{-\prime} \left(\langle \tilde{\rho}_x^{i,e}\, \tilde{\rho}_x^{k,f} \rangle \langle \tilde{\rho}_y^{i',e'}\, \tilde{\rho}_y^{k',f'} \rangle + \langle \tilde{\rho}_x^{i,e}\, \tilde{\rho}_y^{i',e'} \rangle \langle \tilde{\rho}_x^{k,f}\, \tilde{\rho}_y^{k',f'} \rangle + \langle \tilde{\rho}_x^{i,e}\, \tilde{\rho}_y^{k',f'} \rangle \langle \tilde{\rho}_x^{k,f}\, \tilde{\rho}_y^{i',e'} \rangle \right)$$
$$\times \left\langle U^{ea}(z^-, x_\perp) U^{fc}(w^-, x_\perp) U^{e'a'}(z^{-\prime}, y_\perp) U^{f'c'}(w^{-\prime}, y_\perp) \right\rangle$$

(Wick's theorem)

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$$= \int_{-\infty}^{\infty} dz^- dw^- dz^{-\prime} dw^{-\prime} \left(\underline{\langle \tilde{\rho}_x^{i,e}\, \tilde{\rho}_x^{k,f} \rangle \langle \tilde{\rho}_y^{i',e'}\, \tilde{\rho}_y^{k',f'} \rangle}^{\mathbf{1}} + \underline{\langle \tilde{\rho}_x^{i,e}\, \tilde{\rho}_y^{i',e'} \rangle \langle \tilde{\rho}_x^{k,f}\, \tilde{\rho}_y^{k',f'} \rangle}^{\mathbf{2}} + \underline{\langle \tilde{\rho}_x^{i,e}\, \tilde{\rho}_y^{k',f'} \rangle \langle \tilde{\rho}_x^{k,f}\, \tilde{\rho}_y^{i',e'} \rangle}^{\mathbf{3}} \right)$$

$$\times \left\langle U^{ea}(z^-, x_\perp) U^{fc}(w^-, x_\perp) U^{e'a'}(z^{-\prime}, y_\perp) U^{f'c'}(w^{-\prime}, y_\perp) \right\rangle$$

With:

1- $\propto \delta^{ef} \delta^{e'f'} \left\langle U^{ea}(z^-, x_\perp) U^{fc}(w^-, x_\perp) U^{e'a'}(z^{-\prime}, y_\perp) U^{f'c'}(w^{-\prime}, y_\perp) \right\rangle = \delta^{ac} \delta^{a'c'}$

2- $\propto \delta^{ee'} \delta^{ff'} \left\langle U^{ea}(z^-, x_\perp) U^{fc}(w^-, x_\perp) U^{e'a'}(z^{-\prime}, y_\perp) U^{f'c'}(w^{-\prime}, y_\perp) \right\rangle$

3- $\propto \delta^{ef'} \delta^{fe'} \left\langle U^{ea}(z^-, x_\perp) U^{fc}(w^-, x_\perp) U^{e'a'}(z^{-\prime}, y_\perp) U^{f'c'}(w^{-\prime}, y_\perp) \right\rangle$

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3 terms

Disconnected terms:

$$\propto \delta^{ee'} \delta^{ff'} \langle U^{ea}(x_\perp) U^{fc}(y_\perp) U^{e'a'}(x_\perp) U^{f'c'}(y_\perp) \rangle$$

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3 terms

Disconnected terms:

$$\propto \delta^{ee'} \delta^{ff'} \langle U^{ea}(x_\perp) U^{fc}(y_\perp) U^{e'a'}(x_\perp) U^{f'c'}(y_\perp) \rangle$$

Connected terms:

$$\int_{-\infty}^{\infty} dz^- dz^{-\prime} dw^- dw^{-\prime} \left\langle \tilde{\rho}_u^{i,a'} \tilde{\rho}_{u'}^{j,b'} \right\rangle \left\langle \tilde{\rho}_v^{k,c'} \tilde{\rho}_{v'}^{l,d'} U^{a'a}(z^-, u_\perp) U^{b'b}(z^{-\prime}, u'_\perp) U^{c'c}(w^-, v_\perp) U^{d'd}(w^{-\prime}, v'_\perp) \right\rangle_{\mathrm{c}}$$

$$= g^2 \partial_u^i \partial_{u'}^j L(u_\perp - u'_\perp) \int_{-\infty}^{\infty} dz^- \int_{-\infty}^{z^-} dw^- \int_{-\infty}^{w^-} dw^{-\prime} \mu^2(z^-) \mu^2(w^-) \mu^2(w^{-\prime})$$

$$\times C^{(2)}_{\mathrm{adj}}(z^-, w^-; u_\perp, u'_\perp) \Big(\Big[\partial_v^k (L(v_\perp - u'_\perp) - L(v_\perp - u_\perp)) C^{(3)}_{\mathrm{adj}}(w^-, w^{-\prime}; u_\perp, u'_\perp, v_\perp)$$

$$\times \partial_{v'}^l \left(f^{AeD} f^{CBe} L(v'_\perp - u_\perp) + f^{ACe} f^{DBe} L(v'_\perp - u'_\perp) + f^{ABe} f^{eCD} L(v'_\perp - v_\perp) \right)$$

$$\times Q^{ABCD}_{abcd}(w^{-\prime}; u_\perp, u'_\perp, v_\perp, v'_\perp) \Big] + \left[\begin{array}{c} l \longleftrightarrow k \\ c \longleftrightarrow d \\ v_\perp \longleftrightarrow v'_\perp \end{array} \right] \Big)$$

With $u_\perp = v_\perp \equiv x_\perp$ **and** $u'_\perp = v'_\perp \equiv y_\perp$ **:**

$$= 2 g^2 \partial_x^i \partial_y^j L(x_\perp - y_\perp) \int_{-\infty}^{\infty} dz^- \int_{-\infty}^{z^-} dw^- \int_{-\infty}^{w^-} dw^{-\prime} \mu^2(z^-) \mu^2(w^-) \mu^2(w^{-\prime})$$

$$\times C^{(2)}_{\mathrm{adj}}(z^-, w^-; x_\perp, y_\perp) \partial_x^k (L(x_\perp - x_\perp) - L(x_\perp - y_\perp)) C^{(3)}_{\mathrm{adj}}(w^-, w^{-\prime}; x_\perp, y_\perp, x_\perp)$$

$$\times \partial_y^l (L(y_\perp - y_\perp) - L(y_\perp - x_\perp)) f^{ACe} f^{BDe} Q^{ABCD}_{abcd}(w^{-\prime}; x_\perp, y_\perp, x_\perp, y_\perp)$$

!!!

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3 terms **4 terms**

Disconnected terms:

$$\propto \delta^{ee'} \delta^{ff'} \langle U^{ea}(x_\perp) U^{fc}(y_\perp) U^{e'a'}(x_\perp) U^{f'c'}(y_\perp) \rangle$$

Connected terms:

$$\propto f^{ACe} f^{BDe} \langle U^{Aa}(x_\perp) U^{Bc}(y_\perp) U^{Ca'}(x_\perp) U^{Dc'}(y_\perp) \rangle$$

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Fully connected terms:

$$\int_{-\infty}^{\infty} dz^- dw^- dz^{-\prime} dw^{-\prime} \left\langle \tilde{\rho}_x^{i,e} \tilde{\rho}_x^{k,f} \tilde{\rho}_y^{i',e'} \tilde{\rho}_y^{k',f'} U^{ea}(z^-, x_\perp) U^{fc}(w^-, x_\perp) U^{e'a'}(z^{-\prime}, y_\perp) U^{f'c'}(w^{-\prime}, y_\perp) \right\rangle_{\mathrm{c}}$$

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Fully connected terms:

$$\int_{-\infty}^{\infty} dz^- dw^- dz^{-\prime} dw^{-\prime} \underline{\left\langle \tilde{\rho}_x^{i,e} \tilde{\rho}_x^{k,f} \tilde{\rho}_y^{i',e'} \tilde{\rho}_y^{k',f'} U^{ea}(z^-, x_\perp) U^{fc}(w^-, x_\perp) U^{e'a'}(z^{-\prime}, y_\perp) U^{f'c'}(w^{-\prime}, y_\perp) \right\rangle_{\mathrm{c}}}$$

$$H^{4,4}(z^-, -\infty | e, f, e', f' \,;\, a, c, a', c') = \underline{H^{1,1}(z^-, w^- | e \,;\, \alpha_1)^i} H^{1,2}(w^-, z^{-\prime} | \alpha_1, f \,;\, \alpha_2, \beta_1)^k$$
$$\times H^{1,3}(z^{-\prime}, w^{-\prime} | \alpha_2, \beta_1, e' \,;\, \alpha_3, \beta_2, \gamma_1)^{i'}$$
$$\times H^{1,4}(w^{-\prime}, -\infty | \alpha_3, \beta_2, \gamma_1, f' \,;\, a, c, a', c')^{k'}$$

$$H^{1,1}(z^-, w^- | e \,;\, \alpha_1)^i = g\, \lambda(z^-, b_\perp) \partial_x^i L(0_\perp) f^{ee\alpha} \langle U^{\alpha\alpha_1}(z^-, w^- ; x_\perp) \rangle = 0$$

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3 terms **4 terms** **0 terms**

Disconnected terms:

$$\propto \delta^{ee'} \delta^{ff'} \langle U^{ea}(x_\perp) U^{fc}(y_\perp) U^{e'a'}(x_\perp) U^{f'c'}(y_\perp) \rangle$$

Connected terms:

$$\propto f^{ACe} f^{BDe} \langle U^{Aa}(x_\perp) U^{Bc}(y_\perp) U^{Ca'}(x_\perp) U^{Dc'}(y_\perp) \rangle$$

Fully connected terms:

$$\propto f^{ee\alpha} = 0$$

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3 terms **4 terms** **0 terms**

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$$\propto \delta^{ee'}\delta^{ff'}\langle U^{ea}(x_\perp)U^{fc}(y_\perp)U^{e'a'}(x_\perp)U^{f'c'}(y_\perp)\rangle$$

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Fully connected terms:

$$\propto f^{ee\alpha} = 0$$

- These terms contain specific **projections in color space of the correlator of four Wilson lines in the adjoint representation**.
- We will calculate them via a discretization of space in the x^--direction:

$$\int dx^- \longrightarrow \sum_{i=0}^{n}\Delta x^-$$

Wilson line correlators

Based on *[Kovner & Wiedemann '01]*

- Discretization of Wilson line: $\boxed{U(x^-, x_\perp)_{ij} = (U^n(x_n^-, x_\perp)U^{n-1}(x_{n-1}^-, x_\perp)\ldots U^1(x_1^-, x_\perp))_{ij}}$

Wilson line correlators

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- Discretization of two-point correlator: $\langle \tilde{A}^{+a}(x^-,x_\perp)\tilde{A}^{+b}(y^-,y_\perp)\rangle = \delta_{x^-y^-}\delta^{ab}B_{xy}(x^-,b_\perp)$

  with $B_{xy}(x^-,b_\perp) \equiv g^2\Delta x^-\lambda(x^-,b_\perp)L(x_\perp - y_\perp)$.
- We expand one of the n factors to order g^2 :

$$\boxed{U(x^-,x_\perp)_{ij} \approx \left(\delta_{ik} + ig\tilde{A}^{+a}(x_n^-,x_\perp)t^a_{ik}\Delta x^- - \frac{C_F}{2}\delta_{ik}B_{xx}(x_n^-,x_\perp)\right)U^{(n-1)}_{kj}}$$

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Correlator of 2 Wilson lines in the fundamental representation

$$\left\langle \mathrm{Tr}\left\{ U(x_\perp) U^\dagger(y_\perp) \right\} \right\rangle$$
$$= \left\langle \mathrm{Tr}\left\{ U(x_\perp) U^\dagger(y_\perp) \right\} \right\rangle^{(n-1)} \left(1 - \frac{g^2}{2} C_F \Delta x^- \lambda(x_n^-, b_\perp) \Gamma(x_\perp - y_\perp) \right)$$

We will neglect terms of order $(\Delta x^-)^2$ or higher

Wilson line correlators

- Discretization of Wilson line: $\boxed{U(x^-,x_\perp)_{ij}=(U^n(x_n^-,x_\perp)U^{n-1}(x_{n-1}^-,x_\perp)\ldots U^1(x_1^-,x_\perp))_{ij}}$
- Discretization of two-point correlator: $\langle\tilde{A}^{+a}(x^-,x_\perp)\tilde{A}^{+b}(y^-,y_\perp)\rangle=\delta_{x^-y^-}\delta^{ab}B_{xy}(x^-,b_\perp)$

  with $B_{xy}(x^-,b_\perp)\equiv g^2\Delta x^-\lambda(x^-,b_\perp)L(x_\perp-y_\perp)$.
- We expand one of the n factors to order g^2 :

$$\boxed{U(x^-,x_\perp)_{ij}\approx\left(\delta_{ik}+ig\tilde{A}^{+a}(x_n^-,x_\perp)t^a_{ik}\Delta x^--\frac{C_F}{2}\delta_{ik}B_{xx}(x_n^-,x_\perp)\right)U^{(n-1)}_{kj}}$$

Correlator of 2 Wilson lines in the fundamental representation

$$\left\langle\mathrm{Tr}\left\{U(x_\perp)U^\dagger(y_\perp)\right\}\right\rangle$$

$$=\left\langle\mathrm{Tr}\left\{U(x_\perp)U^\dagger(y_\perp)\right\}\right\rangle^{(n-1)}\left(1-\frac{g^2}{2}C_F\Delta x^-\lambda(x_n^-,b_\perp)\Gamma(x_\perp-y_\perp)\right)$$

- We iterate the process:

$$=\left(1-\frac{g^2}{2}C_F\,\Gamma(x_\perp-y_\perp)h(b_\perp)\sum_{i=1}^{n}\Delta x^-\mu^2(x_i^-)\right)=\left(1-\frac{g^2}{2}C_F\,\Gamma(x_\perp-y_\perp)\bar{\lambda}(x^-,b_\perp)\right)$$

Wilson line correlators

- Discretization of Wilson line: $\boxed{U(x^-,x_\perp)_{ij} = (U^n(x_n^-,x_\perp)U^{n-1}(x_{n-1}^-,x_\perp)\dots U^1(x_1^-,x_\perp))_{ij}}$
- Discretization of two-point correlator: $\langle \tilde{A}^{+a}(x^-,x_\perp)\tilde{A}^{+b}(y^-,y_\perp)\rangle = \delta_{x^-y^-}\delta^{ab}B_{xy}(x^-,b_\perp)$

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- We expand one of the n factors to order g^2 :

$$\boxed{U(x^-,x_\perp)_{ij} \approx \left(\delta_{ik} + ig\tilde{A}^{+a}(x_n^-,x_\perp)t^a_{ik}\Delta x^- - \frac{C_F}{2}\delta_{ik}B_{xx}(x_n^-,x_\perp)\right)U^{(n-1)}_{kj}}$$

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- We iterate the process:

$$= \left(1 - \frac{g^2}{2}C_F\,\Gamma(x_\perp - y_\perp)h(b_\perp)\sum_{i=1}^{n}\Delta x^-\mu^2(x_i^-)\right) = \left(1 - \frac{g^2}{2}C_F\,\Gamma(x_\perp - y_\perp)\bar{\lambda}(x^-,b_\perp)\right)$$

- **Reexponentiation:**
  We assume that the neglected higher order terms add up to an exponential expression:

$$= \boxed{\exp\left\{-\frac{g^2}{2}C_F\,\Gamma(x_\perp - y_\perp)\bar{\lambda}(x^-,b_\perp)\right\}.}$$

Correlator of 4 Wilson lines in the adjoint representation

$$\left\langle U^{ab}(z_\perp)U^{cd}(z_\perp)U^{ef}(x_\perp)U^{gh}(y_\perp)\right\rangle$$

Correlator of 4 Wilson lines in the adjoint representation

$$\begin{aligned}
\left\langle U^{ab}(z_\perp)U^{cd}(z_\perp)U^{ef}(x_\perp)U^{gh}(y_\perp)\right\rangle &= \left\langle U^{aa'}(z_\perp)U^{cc'}(z_\perp)U^{ee'}(x_\perp)U^{gg'}(y_\perp)\right\rangle^{(n-1)} \\
&\times\Bigg(\delta^{a'b}\delta^{c'd}\delta^{e'f}\delta^{g'h}\left(1-\frac{N_c}{2}\left(2B_z+B_x+B_y\right)\right)+\delta^{a'b}\delta^{c'd}f^{e'mf}f^{g'mh}B_{xy} \\
&+\delta^{a'b}\delta^{e'f}f^{c'md}f^{g'mh}B_{zy}+\delta^{a'b}\delta^{g'h}f^{e'mf}f^{c'md}B_{zx}+\delta^{e'f}\delta^{c'd}f^{a'mb}f^{g'mh}B_{zy} \\
&+\delta^{g'h}\delta^{c'd}f^{e'mf}f^{a'mb}B_{zx}+\delta^{e'f}\delta^{g'h}f^{a'mb}f^{c'md}B_z\Bigg)
\end{aligned}$$

Correlator of 4 Wilson lines in the adjoint representation

$$\left\langle U^{ab}(z_\perp)U^{cd}(z_\perp)U^{ef}(x_\perp)U^{gh}(y_\perp)\right\rangle = \left\langle U^{aa'}(z_\perp)U^{cc'}(z_\perp)U^{ee'}(x_\perp)U^{gg'}(y_\perp)\right\rangle^{(n-1)}$$

$$\times\Bigg(\delta^{a'b}\delta^{c'd}\delta^{e'f}\delta^{g'h}\left(1-\frac{N_c}{2}(2B_z+B_x+B_y)\right)+\delta^{a'b}\delta^{c'd}f^{e'mf}f^{g'mh}B_{xy}$$

$$+\delta^{a'b}\delta^{e'f}f^{c'md}f^{g'mh}B_{zy}+\delta^{a'b}\delta^{g'h}f^{e'mf}f^{c'md}B_{zx}+\delta^{e'f}\delta^{c'd}f^{a'mb}f^{g'mh}B_{zy}$$

$$+\delta^{g'h}\delta^{c'd}f^{e'mf}f^{a'mb}B_{zx}+\delta^{e'f}\delta^{g'h}f^{a'mb}f^{c'md}B_z\Bigg)$$

- We express the previous lines as a **matrix equation:**

$$U^{aceg}_{bdfh}=(U^{aceg}_{a'c'e'g'})^{(n-1)}T^{a'c'e'g'}_{bdfh}$$

using the following color vector basis:

$$u_1=\delta^{ea}\delta^{gc}\qquad u_2=\delta^{ca}\delta^{ge}\qquad u_3=\delta^{ga}\delta^{ec}$$
$$w_1=d^{eam}d^{gcm}\quad w_2=d^{cam}d^{gem}\quad w_3=d^{gam}d^{ecm}$$

- In this base, $T^{a'c'e'g'}_{bdfh}$ can be written as $(1+M(x^-_n))^{a'c'e'g'}_{bdfh}$ with $M(x^-_n)^{a'c'e'g'}_{bdfh}$ of order Δx^-.

Correlator of 4 Wilson lines in the adjoint representation

$$\left\langle U^{ab}(z_\perp)U^{cd}(z_\perp)U^{ef}(x_\perp)U^{gh}(y_\perp)\right\rangle = \left\langle U^{aa'}(z_\perp)U^{cc'}(z_\perp)U^{ee'}(x_\perp)U^{gg'}(y_\perp)\right\rangle^{(n-1)}$$
$$\times\Bigg(\delta^{a'b}\delta^{c'd}\delta^{e'f}\delta^{g'h}\left(1-\frac{N_c}{2}\left(2B_z+B_x+B_y\right)\right)+\delta^{a'b}\delta^{c'd}f^{e'mf}f^{g'mh}B_{xy}$$
$$+\delta^{a'b}\delta^{e'f}f^{c'md}f^{g'mh}B_{zy}+\delta^{a'b}\delta^{g'h}f^{e'mf}f^{c'md}B_{zx}+\delta^{e'f}\delta^{c'd}f^{a'mb}f^{g'mh}B_{zy}$$
$$+\delta^{g'h}\delta^{c'd}f^{e'mf}f^{a'mb}B_{zx}+\delta^{e'f}\delta^{g'h}f^{a'mb}f^{c'md}B_z\Bigg)$$

- We express the previous lines as a **matrix equation:**

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- In this base, $T^{a'c'e'g'}_{bdfh}$ can be written as $(1+M(x^-_n))^{a'c'e'g'}_{bdfh}$ with $M(x^-_n)^{a'c'e'g'}_{bdfh}$ of order Δx^-.
- Iterating the expansion process we get:

$$U^{aceg}_{bdfh}=1+\sum_{i=1}^{n}M^{a'c'e'g'}_{bdfh}(x^-_i)=1+\int^{x^-}dz'^-M^{a'c'e'g'}_{bdfh}(z'^-)=1+\bar{M}(x^-)$$

Correlator of 4 Wilson lines in the adjoint representation

- **Reexponentiation:** we need to diagonalize $\bar{M}$. We get (using Mathematica):

$$\bar{M}_d = \begin{bmatrix} N_c R_a & 0 & 0 & 0 & 0 & 0 \\ 0 & N_c R_b & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}(R_a + R_b)N_c & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(R_a + R_b)N_c & 0 & 0 \\ 0 & 0 & 0 & 0 & N_c R_a - R_d & 0 \\ 0 & 0 & 0 & 0 & 0 & N_c R_a + R_d \end{bmatrix} .$$

with: $R_a = -\frac{g^2}{2}\bar{\lambda}(x^-, b_\perp)\left(\Gamma(z_\perp - x_\perp) - \Gamma(z_\perp - y_\perp)\right)$, $R_b = -\frac{g^2}{2}\bar{\lambda}(x^-, b_\perp)\left(\Gamma(x_\perp - y_\perp)\right)$

and: $R_d = R_b - R_a$.

and thus: $$\boxed{U^{aceg}_{bdfh} \doteq (1 + \bar{M}_d)^{aceg}_{bdfh} \longrightarrow U^{aceg}_{bdfh} \doteq (e^{\bar{M}_d})^{aceg}_{bdfh}}$$

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with: $R_a = -\frac{g^2}{2} \bar{\lambda}(x^-, b_\perp) \left(\Gamma(z_\perp - x_\perp) - \Gamma(z_\perp - y_\perp) \right) \quad , \quad R_b = -\frac{g^2}{2} \bar{\lambda}(x^-, b_\perp) \left(\Gamma(x_\perp - y_\perp) \right)$

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$$\left. \begin{array}{r} z_\perp \equiv x_\perp \\ x_\perp = y_\perp \equiv y_\perp \end{array} \right\} \longrightarrow \quad \bar{M}_d = \begin{bmatrix} N_c R_a & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} N_c R_a & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} N_c R_a & 0 & 0 \\ 0 & 0 & 0 & 0 & (N_c + 1) R_a & 0 \\ 0 & 0 & 0 & 0 & 0 & (N_c - 1) R_a \end{bmatrix} .$$

Correlator of 4 Wilson lines in the adjoint representation

$$\langle U^{Aa}(x_\perp)U^{Bb}(x_\perp)U^{Cc}(y_\perp)U^{Dd}(y_\perp)\rangle \doteq \exp\{\bar{M}_d\} = \begin{bmatrix} e^{N_c R_a} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{\frac{1}{2}N_c R_a} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\frac{1}{2}N_c R_a} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{(N_c+1)R_a} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{(N_c-1)R_a} \end{bmatrix}.$$

- We need to calculate the following projections:

$$\boxed{\begin{array}{c} f^{ABe}f^{DCe}\,\langle U^{Aa}(x_\perp)U^{Bb}(x_\perp)U^{Cc}(y_\perp)U^{Dd}(y_\perp)\rangle \\ \delta^{AC}\delta^{BD}\,\langle U^{Aa}(x_\perp)U^{Bb}(x_\perp)U^{Cc}(y_\perp)U^{Dd}(y_\perp)\rangle \end{array}}$$

Correlator of 4 Wilson lines in the adjoint representation

$$\langle U^{Aa}(x_\perp)U^{Bb}(x_\perp)U^{Cc}(y_\perp)U^{Dd}(y_\perp)\rangle \doteq \exp\{\bar{M}_d\} = \begin{bmatrix} e^{N_cR_a} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{\frac{1}{2}N_cR_a} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\frac{1}{2}N_cR_a} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{(N_c+1)R_a} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{(N_c-1)R_a} \end{bmatrix}.$$

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- The first projection corresponds to the trivial propagation of an eigenvector by $\exp\{\bar{M}_d\}$:

$$f^{ABe}f^{DCe} = \begin{pmatrix} \frac{2}{N_c} \\ 0 \\ -\frac{2}{N_c} \\ 1 \\ 0 \\ -1 \end{pmatrix} \doteq \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \longrightarrow$$

$$f^{ABe}f^{DCe}\exp\{\bar{M}_d\} = f^{abe}f^{dce}\exp\left\{\frac{1}{2}N_cR_a\right\}$$

$$= f^{abe}f^{dce}\exp\left\{-g^2\frac{N_c}{2}\Gamma(x_\perp - y_\perp)\bar{\lambda}(x^-,b_\perp)\right\}$$

Correlator of 4 Wilson lines in the adjoint representation

$$\langle U^{Aa}(x_\perp)U^{Bb}(x_\perp)U^{Cc}(y_\perp)U^{Dd}(y_\perp)\rangle \doteq \exp\{\bar{M}_d\} = \begin{bmatrix} e^{N_c R_a} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{\frac{1}{2}N_c R_a} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\frac{1}{2}N_c R_a} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{(N_c+1)R_a} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{(N_c-1)R_a} \end{bmatrix}.$$

- We need to calculate the following projections:

$$\boxed{\begin{aligned} & f^{ABe} f^{DCe} \left\langle U^{Aa}(x_\perp)U^{Bb}(x_\perp)U^{Cc}(y_\perp)U^{Dd}(y_\perp)\right\rangle \\ & \delta^{AC}\delta^{BD} \left\langle U^{Aa}(x_\perp)U^{Bb}(x_\perp)U^{Cc}(y_\perp)U^{Dd}(y_\perp)\right\rangle \end{aligned}}$$

- The second projection is remarkably more difficult:

$$\delta^{AC}\delta^{BD} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \doteq \begin{pmatrix} 1/N_c \\ 1/(N_c^2-1) \\ -1/N_c \\ N_c/(N_c^2-4) \\ 1/4 \\ -1/4 \end{pmatrix}$$

Correlator of 4 Wilson lines in the adjoint representation

- After propagation we obtain:

$$
\begin{aligned}
&\delta^{AC}\delta^{BD}\langle U^{Aa}(x_\perp)U^{Bb}(x_\perp)U^{Cc}(y_\perp)U^{Dd}(y_\perp)\rangle \doteq \delta^{AC}\delta^{BD}\exp\{\bar{M}_d\} = \\
&\delta^{ac}\delta^{bd}\left(\frac{N_c^2-4}{2N_c^2}e^{-g^2N_c\Gamma\bar{\lambda}} + \frac{2}{N_c^2}e^{-g^2\frac{N_c}{2}\Gamma\bar{\lambda}} + \frac{N_c+2}{4N_c}e^{-g^2(N_c+1)\Gamma\bar{\lambda}} + \frac{N_c-2}{4N_c}e^{-g^2(N_c-1)\Gamma\bar{\lambda}}\right) \\
&+\delta^{ab}\delta^{cd}\left(\frac{1}{N_c^2-1} - \frac{N_c+2}{2N_c(N_c+1)}e^{-g^2(N_c+1)\Gamma\bar{\lambda}} + \frac{N_c-2}{2N_c(N_c-1)}e^{-g^2(N_c-1)\Gamma\bar{\lambda}}\right) \\
&+\delta^{ad}\delta^{bc}\left(-\frac{N_c^2-4}{2N_c^2}e^{-g^2N_c\Gamma\bar{\lambda}} - \frac{2}{N_c^2}e^{-g^2\frac{N_c}{2}\Gamma\bar{\lambda}} + \frac{N_c+2}{4N_c}e^{-g^2(N_c+1)\Gamma\bar{\lambda}} + \frac{N_c-2}{4N_c}e^{-g^2(N_c-1)\Gamma\bar{\lambda}}\right) \\
&+d^{acn}d^{bdn}\left(-\frac{1}{N_c}e^{-g^2N_c\Gamma\bar{\lambda}} + \frac{1}{N_c}e^{-g^2\frac{N_c}{2}\Gamma\bar{\lambda}} + \frac{1}{4}e^{-g^2(N_c+1)\Gamma\bar{\lambda}} - \frac{1}{4}e^{-g^2(N_c-1)\Gamma\bar{\lambda}}\right) \\
&+d^{abn}d^{cdn}\left(\frac{N_c}{N_c^2-4}e^{-g^2\frac{N_c}{2}\Gamma\bar{\lambda}} - \frac{N_c+4}{4(N_c+2)}e^{-g^2(N_c+1)\Gamma\bar{\lambda}} + \frac{N_c-4}{4(N_c-2)}e^{-g^2(N_c-1)\Gamma\bar{\lambda}}\right) \\
&+d^{adn}d^{bcn}\left(\frac{1}{N_c}e^{-g^2N_c\Gamma\bar{\lambda}} - \frac{1}{N_c}e^{-g^2\frac{N_c}{2}\Gamma\bar{\lambda}} + \frac{1}{4}e^{-g^2(N_c+1)\Gamma\bar{\lambda}} - \frac{1}{4}e^{-g^2(N_c-1)\Gamma\bar{\lambda}}\right)
\end{aligned}
$$

A remarkably complicated contribution.

Correlator of 4 Wilson lines and 4 external sources

- Applying the previous rules we can decompose this correlator as:

$$\langle\alpha^4\rangle = \langle\rho^4 U^4\rangle = \underbrace{\langle\rho^4\rangle\langle U^4\rangle}_{\textbf{3 terms}} + \underbrace{\langle\rho^2\rangle\langle\rho^2 U^4\rangle_c}_{\textbf{4 terms}}$$

- This expression can be written in terms of the following functions:

Disconnected terms:

$$D^{ik;i'k'}_{ac;a'c'}(x_\perp,x_\perp,y_\perp,y_\perp) = \frac{1}{4}\delta^{ik}\delta^{i'k'}\big(\partial^2 L(0_\perp)\big)^2\delta^{ac}\delta^{a'c'}\bar{\lambda}^2(b_\perp)$$

$$D^{ij;kl}_{ab;cd}(x_\perp,y_\perp,x_\perp,y_\perp) = 2\,\partial^i_x\partial^j_y L(x_\perp-y_\perp)\partial^k_x\partial^l_y L(x_\perp-y_\perp)\int_{-\infty}^{\infty}dz^-\int_{-\infty}^{z^-}dw^-\lambda(z^-,b_\perp)\lambda(w^-,b_\perp)$$
$$\times\delta^{AC}\delta^{BD}\,\langle U^{Aa}(x_\perp)U^{Bc}(x_\perp)U^{Cb}(y_\perp)U^{Dd}(y_\perp)\rangle$$

Connected terms:

$$C^{ij;kl}_{ab;cd}(x_\perp,y_\perp,x_\perp,y_\perp) = f^{ace}f^{bde}\partial^i_x\partial^j_y L(x_\perp-y_\perp)\partial^k_x\Gamma(x_\perp-y_\perp)\partial^l_y\Gamma(y_\perp-x_\perp)$$
$$\times\left(\frac{4}{\Gamma^3 g^4 N_c^3}-\left(\frac{\bar{\lambda}^2(b_\perp)}{2\Gamma N_c}+\frac{4}{\Gamma^3 g^4 N_c^3}+\frac{2\bar{\lambda}(b_\perp)}{\Gamma^2 g^2 N_c^2}\right)C^{(2)}_{\mathrm{adj}}(x_\perp,y_\perp)\right)$$

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3 terms **4 terms**

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$$\times\delta^{AC}\delta^{BD}\left\langle U^{Aa}(x_\perp)U^{Bc}(x_\perp)U^{Cb}(y_\perp)U^{Dd}(y_\perp)\right\rangle$$

Connected terms:

$$C^{ij;kl}_{ab;cd}(x_\perp, y_\perp, x_\perp, y_\perp) = f^{ace}f^{bde}\partial^i_x\partial^j_y L(x_\perp - y_\perp)\partial^k_x\Gamma(x_\perp - y_\perp)\partial^l_y\Gamma(y_\perp - x_\perp)$$
$$\times\left(\frac{4}{\Gamma^3 g^4 N_c^3} - \left(\frac{\bar{\lambda}^2(b_\perp)}{2\Gamma N_c} + \frac{4}{\Gamma^3 g^4 N_c^3} + \frac{2\bar{\lambda}(b_\perp)}{\Gamma^2 g^2 N_c^2}\right) C^{(2)}_{\text{adj}}(x_\perp, y_\perp)\right)$$

$$\begin{aligned}\langle\alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle &= D^{ik;i'k'}_{ac;a'c'}(x_\perp, x_\perp, y_\perp, y_\perp) + D^{ii';kk'}_{aa';cc'}(x_\perp, y_\perp, x_\perp, y_\perp)\\ &+ D^{ik';ki'}_{ac';ca'}(x_\perp, y_\perp, x_\perp, y_\perp) + C^{ii';kk'}_{aa';cc'}(x_\perp, y_\perp, x_\perp, y_\perp) + C^{ik';ki'}_{ac';ca'}(x_\perp, y_\perp, x_\perp, y_\perp)\\ &+ C^{kk';ii'}_{cc';aa'}(x_\perp, y_\perp, x_\perp, y_\perp) + C^{ki';ik'}_{ca';ac'}(x_\perp, y_\perp, x_\perp, y_\perp)\end{aligned}$$

$\langle T^{\mu\nu}(x_\perp) T^{\sigma\rho}(y_\perp)\rangle = \langle \epsilon(x_\perp)\epsilon(y_\perp)\rangle\, t^{\mu\nu}\, t^{\sigma\rho}$

- For the 2-point correlator of $T^{\mu\nu}$: prepare for trouble and make it double

$$\langle \epsilon(x_\perp)\epsilon(y_\perp)\rangle = \frac{g^4}{4}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'} + \epsilon^{i'j'}\epsilon^{k'l'}) f^{abn} f^{cdn} f^{a'b'm} f^{c'}$$

$$\times \langle \alpha_{1\,x}^{i\,a}\alpha_{1\,x}^{k\,c}\alpha_{1\,y}^{i'a'}\alpha_{1\,y}^{k'c'}\rangle\langle \alpha_{2\,x}^{j\,b}\alpha_{2\,x}^{l\,d}\alpha_{2\,y}^{j'b'}\alpha_{2\,y}^{l'd'}\rangle$$

- **And finally:**

The **color structure** of this object is frustratingly complex. Even with all parts analytically calculated, the contraction of the color indices demands a computational treatment (via FeynCalc or FORM)

$$\begin{aligned}\langle \alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle &= D^{ik;i'k'}_{ac;a'c'}(x_\perp,x_\perp,y_\perp,y_\perp) + D^{ii';kk'}_{aa';cc'}(x_\perp,y_\perp,x_\perp,y_\perp)\\ &+ D^{ik';ki'}_{ac';ca'}(x_\perp,y_\perp,x_\perp,y_\perp) + C^{ii';kk'}_{aa';cc'}(x_\perp,y_\perp,x_\perp,y_\perp) + C^{ik';ki'}_{ac';ca'}(x_\perp,y_\perp,x_\perp,y_\perp)\\ &+ C^{kk';ii'}_{cc';aa'}(x_\perp,y_\perp,x_\perp,y_\perp) + C^{ki';ik'}_{ca';ac'}(x_\perp,y_\perp,x_\perp,y_\perp)\end{aligned}$$

$\mathrm{Cov}[\epsilon_0](x_\perp, y_\perp) = \langle \epsilon_0(x_\perp)\epsilon_0(y_\perp)\rangle - \langle \epsilon_0(x_\perp)\rangle\langle \epsilon_0(y_\perp)\rangle$

$$
\begin{aligned}
\mathrm{Cov}[\epsilon](\tau = 0^+; x_\perp, y_\perp) = {} & \frac{\partial_x^i \Gamma \partial_y^i \Gamma (N_c^2 - 1) A (4A^2 - B^2)}{16 N_c^2 \Gamma^5 g^4} (p_1 q_2 + p_2 q_1) \\
& + \frac{(N_c^2 - 1)(16A^4 + B^4)}{2N_c^2 \Gamma^4 g^4} p_1 p_2 + \frac{(\partial_x^i \Gamma \partial_y^i \Gamma)^2 (N_c^2 - 1) A^2}{64 N_c^2 \Gamma^6 g^4} q_1 q_2 \\
& + \frac{(N_c^2 - 1)(4A^2 + B^2)}{2N_c^2 \Gamma^2 g^4} \left(4\pi\, \partial^2 L(0_\perp)\right)^2 \left(\left[\bar{Q}_{s1}^4 (Q_{s2}^2 r^2 - 4 + 4e^{-\frac{Q_{s2}^2 r^2}{4}}) \right] + [1 \leftrightarrow 2] \right) \\
& + \frac{(4A^2 + B^2)^2}{g^4 \Gamma^4 N_c^2} \left(\left[\frac{N_c^6 + 2N_c^4 - 19N_c^2 + 8}{(N_c^2 - 1)^2} - 4 \frac{N_c^6 - 3N_c^4 - 26N_c^2 + 16}{(N_c^2 - 1)(N_c^2 - 4)} e^{-\frac{Q_{s1}^2 r^2}{4}} \right. \right. \\
& + \frac{(N_c - 1)(N_c + 3) N_c^3}{(N_c + 1)^2 (N_c + 2)^2} \left(\frac{N_c}{2} e^{-\frac{(N_c+1) r^2 Q_{s2}^2}{2N_c}} + (N_c + 2) - 2(N_c + 1) e^{-\frac{Q_{s2}^2 r^2}{4}} \right) e^{-\frac{(N_c+1) r^2 Q_{s1}^2}{2N_c}} \\
& + \frac{(N_c + 1)(N_c - 3) N_c^3}{(N_c - 1)^2 (N_c - 2)^2} \left(\frac{N_c}{2} e^{-\frac{(N_c-1) r^2 Q_{s2}^2}{2N_c}} + (N_c - 2) - 2(N_c - 1) e^{-\frac{Q_{s2}^2 r^2}{4}} \right) e^{-\frac{(N_c-1) r^2 Q_{s1}^2}{2N_c}} \\
& \left. + \frac{r^4}{2} Q_{s1}^2 Q_{s2}^2 - 4r^2 Q_{s1}^2 \left(1 - e^{-\frac{Q_{s2}^2 r^2}{4}} \right) + 4 \frac{(N_c^2 - 8)(N_c^2 - 1)(N_c^2 + 4)}{(N_c^2 - 4)^2} e^{-\frac{(Q_{s1}^2 + Q_{s2}^2) r^2}{4}} \right]
\end{aligned}
$$

With: $p_{1,2} \equiv e^{-\frac{Q_{s1,2}^2 r^2}{4}} (Q_{s1,2}^2 r^2 + 4) - 4$, $q_{1,2} \equiv e^{-\frac{Q_{s1,2}^2 r^2}{4}} \left(Q_{s1,2}^4 r^4 + 8Q_{s1,2}^2 r^2 + 32\right) - 32$.

And some model-dependent parameters: $\Gamma(r_\perp)_{\mathrm{MV}} \approx \frac{r^2}{8\pi} \ln\left(\frac{4}{m^2 r^2}\right)$

$$A(r_\perp)_{\mathrm{MV}} \approx \frac{1}{8\pi} \ln\left(\frac{4}{m^2 r^2}\right)$$

$$B(r_\perp)_{\mathrm{MV}} = \frac{1}{4\pi}$$

And the saturation scale:

$$\frac{r^2 Q_s^2}{4} = g^2 \frac{N_c}{2} \Gamma(r_\perp) \bar{\lambda}(b_\perp)$$

Pocket formulae

- Omitting (for the moment) the issues with logarithmic divergencies (GBW model):

$$r \to 0$$

$$\lim_{r\to 0} \mathrm{Cov}[\epsilon](0^+; x_\perp, y_\perp) = \frac{3C_F}{g^4 2N_c} Q_{s1}^4 Q_{s2}^4$$

$$\lim_{r\to 0} \frac{\mathrm{Cov}[\epsilon](0^+; x_\perp), y_\perp)}{\langle \epsilon_0(x_\perp)\rangle\langle \epsilon_0(y_\perp)\rangle} = \frac{3}{(N_c^2 - 1)}$$

Usual suppression factor characteristic of non-trivial color correlators

Pocket formulae

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$$rQ_s \to \infty$$

$$\lim_{rQ_s \gg 1} \mathrm{Cov}[\epsilon](0^+; x_\perp, y_\perp) = \frac{2\left(N_c^2-1\right)\left(Q_{s1}^4 Q_{s2}^2 + Q_{s2}^4 Q_{s1}^2\right)}{g^4 N_c^2 r^2}.$$

Power law:
Remarkably slow decay!

$$\lim_{rQ_s \gg 1} \frac{\mathrm{Cov}[\epsilon](0^+; x_\perp, y_\perp)}{\langle \epsilon_0(x_\perp)\rangle\langle \epsilon_0(y_\perp)\rangle} = \frac{8}{(N_c^2-1)\, r^2}\left(\frac{1}{Q_{s1}^2} + \frac{1}{Q_{s2}^2}\right)$$

Comparison with the ‘Glasma Graph’ approximation

- Glasma Graph approximation *[Lappi & Schlichting 2018, Muller & Schaefer 2012].* Assume Gaussian distribution of the produced gluon fields:

$$\begin{aligned}\langle\alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle_{\text{GG}} = &\langle\alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\rangle\langle\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle\\ &+\langle\alpha^{i\,a}(x_\perp)\alpha^{i'a'}(y_\perp)\rangle\langle\alpha^{k\,c}(x_\perp)\alpha^{k'c'}(y_\perp)\rangle\\ &+\langle\alpha^{i\,a}(x_\perp)\alpha^{k'c'}(y_\perp)\rangle\langle\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\rangle.\end{aligned}$$

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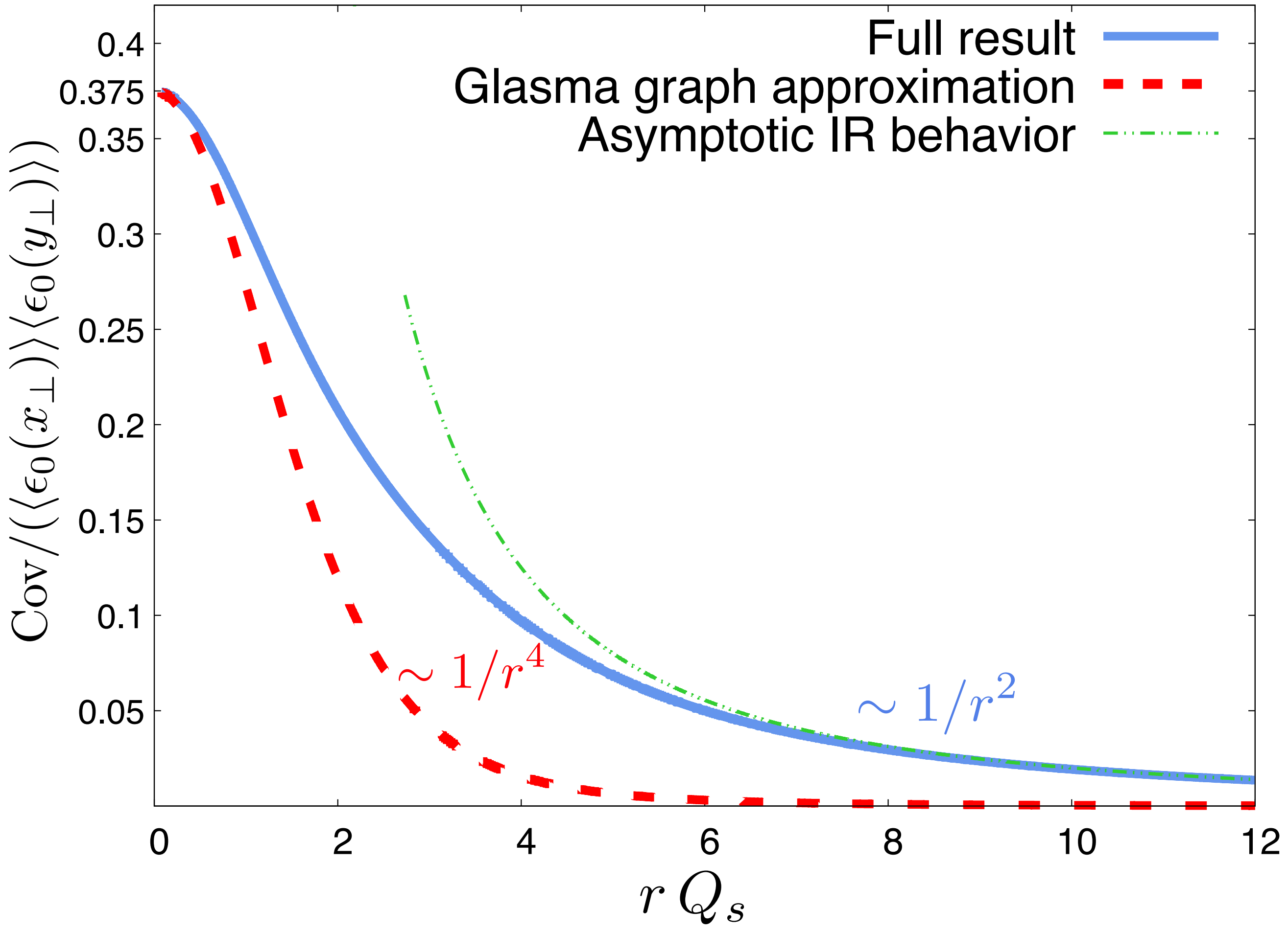
$$\begin{aligned}\langle\alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle_{\text{GG}} = &\langle\alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\rangle\langle\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle\\ &+\langle\alpha^{i\,a}(x_\perp)\alpha^{i'a'}(y_\perp)\rangle\langle\alpha^{k\,c}(x_\perp)\alpha^{k'c'}(y_\perp)\rangle\\ &+\langle\alpha^{i\,a}(x_\perp)\alpha^{k'c'}(y_\perp)\rangle\langle\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\rangle.\end{aligned}$$

- Agreement with full result in the r->0 limit. Strong discrepancies in the r-> ∞ limit



- This slowly decaying behavior could potentially have an impact in both physical interpretations and numerical results for any observable built from this quantity.

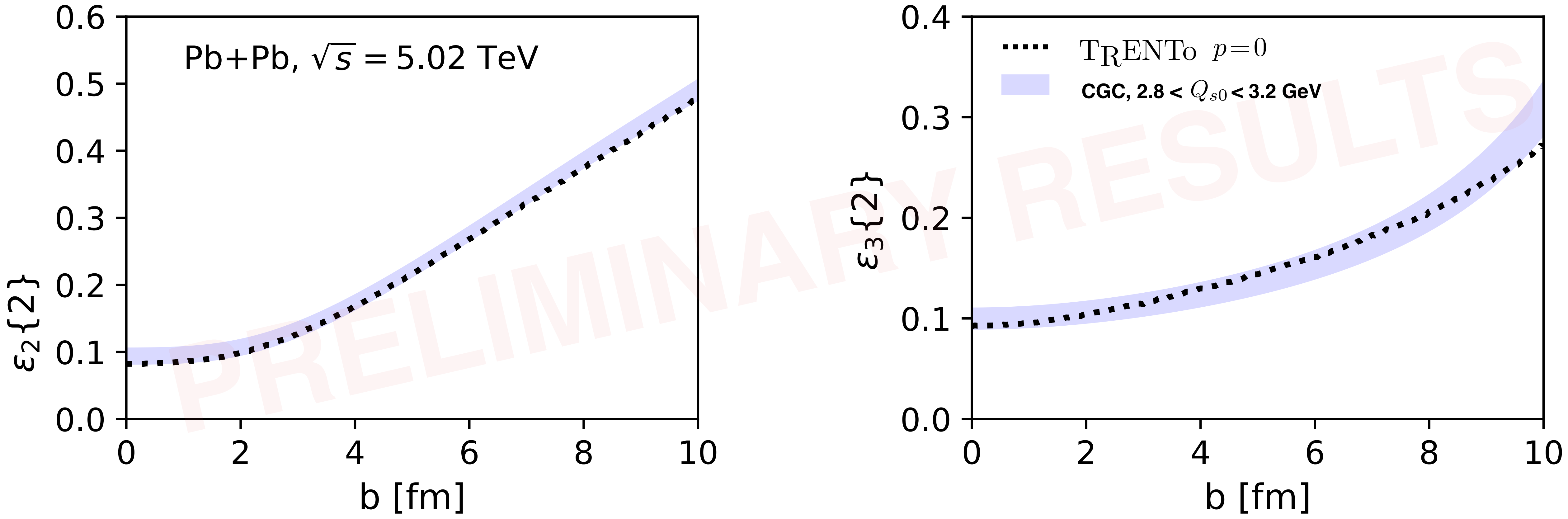
An application: eccentricity fluctuations

- In the picture proposed by Blaizot et al. (*[Blaizot, Broniowski & Ollitrault '14]*), ε_n fluctuations can be characterized in terms of **n-point correlators** of the energy density distribution by assuming that, for a given impact parameter we have:

$$\epsilon(x_\perp) = \langle \epsilon(x_\perp) \rangle + \delta\epsilon(x_\perp) \quad \text{with} \quad \langle \epsilon(x_\perp) \rangle \gg \delta\epsilon(x_\perp)$$

- To leading order in $\delta\epsilon(x_\perp)$, we have the following expression for the **mean squared eccentricities**:

$$\langle \Delta\varepsilon_n^2 \rangle = \frac{\int_{z_1 z_2} z_1^n \bar{z}_2^n \mathrm{Cov}[\epsilon(z_1, z_2)]}{\left(\int_z |z|^n \langle \epsilon(z) \rangle \right)^2}$$



Conclusions

Conclusions

We have performed an exact analytical calculation of the covariance of the energy momentum tensor of the **Glasma** at $\tau = 0^+$, in the framework of the **Color Glass Condensate**.

- We find remarkably **long-range correlations** in comparison to naive expectations and previous calculations (such as the one performed in the Glasma Graph approximation).
- The modifications introduced in the MV model will prove useful in subsequent phenomenological applications of our results.

This work presents a wide variety of applications and potential **follow-up projects:**

- Computation of **time evolution** of our result towards thermalization time $\tau \sim 1/Q_s$, where it can serve as input for **hydro QGP simulations**.

$$T^{\mu\nu} = T_0^{\mu\nu} + T_1^{\mu\nu}\tau + T_2^{\mu\nu}\tau^2 + \dots$$

- Analytical calculation of **eccentricity fluctuations** (directly related to experimentally measured anisotropic flow coefficients).
- Computation of **dilute-dense limit**, appropriate for p-A processes.