On Topological Skyrme Model with Wess-Zumino Anomaly term and the Nucleus

Syed Afsar Abbas

Centre for Theoretical Physics
JMI University, 110025 - New Delhi, India
email: drafsarabbas@gmail.com

Frankfurt - September 2018

Electric Charge of the nucleus:

A well defined nuclear isospin group $SU(2)_T$ exists, as given a nucleus with isospin T there exist Isobaric Analog States in nuclei with $T_z = -T, -T+1, -T+2, ... + T$ values, where

$$T_z = \frac{(Z-N)}{2}; \quad \textit{in general} \quad T = |T_z|$$

The electric charge of the nucleus with mass number A is given as,

$$Q = T_z + \frac{A}{2} = T_z + \frac{Z+N}{2}$$

Taking isospin of individual nucleon as $t_z = \frac{+}{-}\frac{1}{2}$, the nuclear isospin could be anything within the range $\frac{Z-N}{2} \leq T \leq \frac{Z+N}{2}$. However not understood why isopsin T takes the lowest value.

• **All the models of the nucleus have FAILED to predict the above phenomenological nuclear charge expression.**

•• **Here we prove it within Skyrme-Wess-Zumino model.**

Electric charge in the Standard Model (SM)

- The **Gell-Mann-Nishijima relation** for "strong" group $SU(2) \otimes U(1)$ was suggested in 1953 as, $Q = T_3 + \frac{Y}{2}$ (hypercharge Y = B + S, is generator of the U(1) group).
- The three quarks as fundamental representation of SU(3) came **much later** to Gell-Mann (and independently to Zweig) in 1964. In SU(3) T_3 and Y are related as being generators of it. **Thus in QUARK MODEL electric charge is properly quantized**.
- Glashow in studying the weak interaction in 1961, incorporated electric charge in a larger electro-weak group $SU(2)_W \otimes U(1)_W$, where the subscript W refers a different groups defining the Weak and Electromagnetic interactions, in a partially unified manner.
- Glashow in 1961, **just copied** the Gell-Mann-Nishijima definition for his electro-weak (EW) group electric charge as

$$Q = {T_3}^W + \frac{Y_W}{2} \tag{1}$$

Here Y_W called weak-hypercharge and is put in by hand.
- **Hence electro-weak model electric charge NOT quantized.**

• **Glashow in 1961 had NO idea of Spontaneous Symmetry Breaking by the Englert-Brout-Higgs mechanism**, which came much later in 1964. In EW group in 1969 (Salam and Weinberg).
• Now the **Standard Model** $SU(3)_c \otimes SU(2)_L \otimes U(1)_{Y_W}$ is extension of the above electro-weak group $SU(2)_L \otimes U(1)_{Y_W}$. .
• Standard Model is the most successful model of particle physics.
• However the above definitions of the electric charge are **carried over in toto** to the Standard Model. Thus:
• **Electric charge NOT quantized and arbitrary in the SM**
This is consided a **major WEAKNESS of the Standard Model**

• IMPORTANT: this unquantized charge in the Standard Model:
•• Already exists prior to any Spontaneous Symmetry Beaking (SSB) through Englert-Brout-Higgs (EBH) field. (Only masses are generated by the SSB through EBH field in the SM).

•• It is immune or independent of the strong-colour group $SU(3)_c$.

•• Fixed and rigid values 2/3 and -1/3 (**no colour dependence)**

•• **Anomalies play no role other than being trivially satisfied by the above pre-fixed values of the hypercharge in the SM.**

Quantized Charge Standard Model (QCSM)

- We have to go beyond the above SM to get quantized charges.
- We take the same generation structure as that in the SM and the same Englert-Brout-Higgs (EBH) field as an $SU(2)_L$ group doublet,

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{2}$$

•• However, **major differences** with respect to the above SM are:

- We start with the complete group structure as $SU(N_c) \otimes SU(2)_L \otimes U(1)_{Y_W}$ *where* $N_c = 3$.
- We do not have pre-defined electric charge.
- We take the most general definition of the electric charge in terms of generators of the above group structure
- First we study the effects of SSB through the above EBH field,
- Next the role of anomaly cancellations.
- To ensure that all massless matter particles acquire mass through Yukawa couplings.
- Is all this rich enough to provide charge qunatization in this model structure? • Below we show that indeed it does!

The **first generation fermions** are assigned to the following representations for the group $SU(N_c) \otimes SU(2)_L \otimes U(1)_{Y_W}$.

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, (N_c, 2, Y_q); u_R, (N_c, 1, Y_u); d_R, (N_c, 1, Y_d)$$

$$l_R = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, (1, 2, Y_l); e_R, (1, 1, Y_e) \tag{3}$$

• Five unknown hypercharges above plus the unknown Y_ϕ (six). For $SU(2)_L \otimes U(1)_{Y_W}$ define the electric charge operator in the most general way in terms of the diagonal generators of the groups,

$$Q = T_3 + b\ Y \tag{4}$$

• In $SU(2)_L \otimes U(1)_{Y_W}$, symmetry has three massless generators W_1, W_2, W_3 of $SU(2)_L$ and $U(1)_Y$. SSB gave mass to $W^\pm$ and Z^0 gauge particles while ensuring zero mass for photons γ.
• Thus $U(1)_{em}$ is the exact consequent symmetry in the process $SU(2)_L \otimes U(1)_{Y_W} \to U(1)_{em}$.

• The EBH doublet field as above $\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

• Let the $T_3 = -\frac{1}{2}$ corresponding to the EBH field develop a nonzero vacuum expectation value $< \phi >_0$. As per the EBH mechanism for SSB, to ensure that one of the four generators $(W_1 W_2 W_3, X)$ is thereby left unbroken (meaning that what we ensure a massless photon as a generator of the $U(1)_{em}$ group), we demand:

$$Q < \phi >_0 = 0 \tag{5}$$

For the Q operator above,

$$T_3^{\phi} + b < \phi_0 > = 0 \tag{6}$$

• This fixes the unknown b and the **ELECTRIC CHARGE** is:

$$Q = T_3 + (\frac{1}{2Y_{\phi}})Y \tag{7}$$

•• **Note that Y_{ϕ} exists in the denominator of the expression for the electric charge.**

ANOMALIES: For theory to be renormalisable, we have to ensure that all the anomalies neutralise each other for all the particles known, For each generation cancellation of anomalies brings in the requirement for the satisfaction of the following three constraints:

$$(a)\, TrY[SU(N_C)]^2 = 0 \;\; \textit{which yields} \;\; 2Y_q = Y_u + Y_d \tag{8}$$

$$(b)\, TrY[SU(2)_L]^2 = 0 \;\; \textit{which gives} \;\; 2^2 Y_l + N_c[2^2 Y_q] = 0 \tag{9}$$

and thus

$$Y_q = -\frac{Y_l}{N_c} \tag{10}$$

$$(c) \qquad\qquad Tr[Y^3] = 0 \tag{11}$$

giving

$$2N_c Y_q^3 - N_c Y_u^3 - N_c Y_d^3 + 2Y_l^3 - Y_e^3 = 0 \tag{12}$$

We still need to have terms for Y_u, Y_d, Y_e, in addition to Y_q.

• Before SSB the matter particles are massless. We have to make them massive through this process of SBB by **Yukawa couplings**

$$\mathcal{L} = -\phi^\dagger \bar{q}_L u_R + \phi q_L \bar{d}_R + \phi e_L \bar{e}_R \tag{13}$$

• On demanding gauge invariance the above yields,

$$Y_u = Y_q + Y_\phi; \quad Y_d = Y_q - Y_\phi; \quad Y_e = Y_l - Y_\phi \tag{14}$$

• Now substituting Y_q and Y_u, Y_d, Y_e from above one obtains:

$$(Y_l + Y_\phi)^3 = 0 \tag{15}$$

The equation is reduced to two unknowns Y_l and Y_ϕ giving, $Y_l = -Y_\phi$. Putting this above gives, $Y_q = \frac{Y_\phi}{N_c}$. These yield,

$$Y_u = Y_\phi\left(\frac{1}{N_c} + 1\right) \tag{16}$$

And similarly for Y_d and Y_e. Finally, we get,

Quantized electric charges in the Quantized Charge Standard Model

$$Q(u) = \frac{1}{2}(1 + \frac{1}{N_c}); \quad Q(d) = \frac{1}{2}(-1 + \frac{1}{N_c})$$

$$Q(\nu_e) = 0; \quad Q(e) = -1 \tag{17}$$

•• **Note that though $U(1)_{em}$ does not know of colour, the electric charges are actually dependent upon colour itself**.

• The fact that the electric charge of the quark has colour dependence built into itself is a significant new result for the Quantized Charge Standard Model.

• However this is in direct conflict with the charges obtained in the Standard Model. These charges were always $Q(u) = \frac{2}{3}$ and $Q(d) = -\frac{1}{3}$ (i.e. indepndent of colour).

• In fact, below we show that **the colour dependent charges of the Quantized Charge Standard Model are the correct ones**, while the static charges $Q(u) = \frac{2}{3}$ and $Q(d) = -\frac{1}{3}$ of **the Standard Model are the wrong charges**.

The study of QCD in large N_c limit: While the number of quarks in $SU(N_c)$ scale as $\sim N_c$, the number of gluons scale as $\sim (N_c^2 - 1)$. So for large N_c, gluons will dominate over quarks. Also the field theory of $SU(N_c)$ for large N_c reduces to a theory of weekly interacting mesons. Thus this theory connects to the Skyrme model where baryons arise as topological structures in a Lagrangian composed of scalar mesons only.

• In this QCD, baryon has a finite size and has a mass going as:

$$M(baryon) \sim N_c \tag{18}$$

• Baryons are composed of N_c number of quarks. Composite baryons to be fermions N_c is an odd number like $N_c = 1, 3, 5...$

$$N_c = 2k + 1, \tag{19}$$

• Now assume that the proton is built up of $(k + 1)$ number of u-quarks and k number of d-quarks, and vice-versa for neutron.

• Now Witten et.al (Adkins, Nappi and Witten, Nucl. Phys. B228 (1983)228) took quark charges to be the same for any arbitrary N_c (i.e. independent of colour), $Q_u = 2/3$ *and* $Q_d = -1/3$

• Why is proton made up of 2 u-quarks and 1 d-quark for for $N_c = 3$? Because $N_c = 2k + 1$ and k=1 for 3-colours. So prorton is made up of (k+1) u-quark and k d-quark. **and k is colour dependent!**. So proton charge is colour dependent too.

• Thus in their model the proton and neutron charges are,

$$Q_p = (k+1)\frac{2}{3} + k\left(-\frac{1}{3}\right) = \left(\frac{k+2}{3}\right) = \frac{N_c + 3}{6}$$

$$Q_n = k\frac{2}{3} + (k+1)(-\frac{1}{3}) = \left(\frac{k-1}{3}\right) = \frac{N_c - 3}{6}$$

Now for $N_c = 3$ these gave apparently correct charges $Q_p = +1$ and $Q_n = 0$ - **but actually colour dependent**.

• Also for arbitrary N_c, these are not even integral. For $N_c = 5$, $Q_p = 4/3$, $Q_n = +1/3$ these charges actually blow up as $N_c \to \infty$.

• This is an **unsatisfactory behaviour of static charges of the SM** and in the work of Witten et. al. for QCD with any N_c (including 3), the colour dependence of proton charge is catastrophic.

• Now Witten et. al had unfortunately neglected the fundamental **Coulomb self-energy** term contribution to the baryon masses. But this should add as a QED contribution. And thus the QCD plus QED contributions to baryon mass are,

$$M(proton) \sim N_c + C\frac{\left(\frac{N_c+3}{6}\right)^2}{R} \tag{20}$$

where C is a constant and R is the finite size of proton. Now the baryon mass is blowing as ${N_c}^2$ due to the QED part.
• **This is messing up the whole analysis based on self-consistent QCD only - true for three-colours as well**. Thus here QCD plus QED tells us that there are no stable large number of colour baryons. This is thus what the conventional SM is telling us.

• **This is disastrous for the model of Witten et.al.**.

• **Thus the definition of electric charge in the Standrd Model is inconsistent with the structure of QCD**.

• Next our result of colour-dependent electric charges in Quantized Charge Standard Model, $Q_u = \frac{1}{2}\left(1 + \frac{1}{N_c}\right)$, $Q_d = \frac{1}{2}\left(-1 + \frac{1}{N_c}\right)$

• Now proton and neutron charges in the $SU(N_C)$ model are

$$Q_p = (k+1)\ \frac{1}{2}\left(1 + \frac{1}{N_c}\right) + k\frac{1}{2}\left(-1 + \frac{1}{N_c}\right) = 1 \qquad (21)$$

$$Q_n = k\ \ \frac{1}{2}\left(1 + \frac{1}{N_c}\right) + (k+1)\ \frac{1}{2}\left(-1 + \frac{1}{N_c}\right) = 0 \qquad (22)$$

• Thus $Q_p = 1$, $Q_n = 0$ for arbitrary N_c - it is independent of N_c.

• Hence the Coulomb self-energy term of the proton remains finite.

• **Thus the colour-dependent electric charge of the QCSM are the proper charges for quarks and proton**.

• **This is because in QCSM $M(proton) \sim N_c$. And this is the correct understanding of the relationship between the constituent quarks and the current quarks in QCD**.

• **Hence electric charges in the QCSM are consistent with QCD while those in the SM are NOT**.

Topological Skyrme model:

Let $U(x)$ is an element of the group $SU(2)_F$,

$$U(x)^{SU(2)} = exp((i\tau^a\phi^a/f_\pi), \quad (a = 1,2,3)$$

$$define \qquad L_\mu = U^\dagger \partial_\mu U \tag{23}$$

The the **Skyrme Lagrangian** is given as,

$$L_S = \frac{{f_\pi}^2}{4} Tr(L_\mu L^\mu) + \frac{1}{32e^2} Tr[L_\mu, L_\nu]^2$$

$$with\ topological\ current \quad W_\mu = \frac{1}{24\pi^2}\epsilon_{\mu\nu\alpha\beta} Tr[L_\nu L_\alpha L_\beta] \tag{24}$$

Independet of eqn of motion, this topological current is conserved, $\partial^\mu W_\mu = 0$; giving a conserved topological charge $q = \int W_0 d^3x$. The solitonic structure is obtained on making **Skyrme ansatz** as,

$$U_c(x)^{SU(2)} = exp((i/f_\pi \theta(r)\hat{r}^a\tau^a), \quad (a = 1,2,3) \tag{25}$$

This $U_c(x)$ is called the **Skyrmion**. But on quantization, the two flavour model Skyrmion has a well known boson-fermion ambiguity.

This is rectified by going to three flavours $SU(3)_F$ case,

$$U(x)^{SU(3)} = exp[\frac{i\lambda^a\phi^a(x)}{f_\pi}] \;\; (a = 1, 2..., 8) \qquad (26)$$

with ϕ^a the pseudoscalar octet of π, K and η mesons. But this has a spurious symmetry, not present in QCD. This is rectified by adding an anomaly term - the Wess-Zumino anomaly.

Skyrme-Wess-Zumino Model: In this model we supplement above Skyrme lagrangian with a Wess-Zumino effective action

$$\Gamma_{WZ} = \frac{-i}{240\pi^2}\int_\Sigma d^5x\epsilon^{\mu\nu\alpha\beta\gamma} Tr[L_\mu L_\nu L_\alpha L_\beta L_\gamma] \qquad (27)$$

on surface Σ. Thus with this anomaly term, the effective action is.

$$S_{eff} = \frac{f_\pi^2}{4}\int d^4x \; Tr \; [L_\mu L^\mu] + n \, \Gamma_{WZ} \qquad (28)$$

The winding number n is an integer $n \in Z$. The effective action is,

$$S_{eff} = \frac{f_\pi^2}{4}\int d^4x \; Tr \; [\partial_\mu U \partial^\mu U^\dagger] + n \, \Gamma_{WZ} \qquad (29)$$

Taking Q as charge operator, under a local electro-magnetic gauge transformation $h(x) = \exp(i\theta(x)Q)$ with small θ, one finds

$$\Gamma_{WZ} \to \Gamma_{WZ} - \int d^4x \partial_\mu x J^\mu(x) \tag{30}$$

whee J^μ is the Noether current arising from the WZ term. This coupling to the photon field is like,

$$J_\mu = \frac{1}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} Tr[Q(L_\nu L_\alpha L_\beta - R_\nu R_\alpha R_\beta)] \tag{31}$$

where $L_\mu = U^\dagger \partial_\mu U$, $R_\mu = U \partial_\mu U^\dagger$. With the electromagnetic field A_μ present, the gauge invariant form of effective action is,

$$\hat{S_{eff}} = \frac{f_\pi^2}{4} \int d^4x \; Tr\,[L_\mu L^\mu] + n\,\Gamma_{\hat{WZ}} \tag{32}$$

This means that when replacing the LHS by $\hat{\Gamma_{WZ}}$, then the RHS has two new terms involving $F_{\mu\nu}F^{\mu\nu}$. This allows us to interpret J_μ with the current carried by quarks. **With the charge operator Q, J_μ is found to be ISOSCALAR**. To obtain the baryon current, one replaces Q by $\frac{1}{N_c}$ (where N_c is the number of colours in $SU(N_c)$ - QCD for arbitrary number of colours), which is the baryon charge carried by each quark making up the baryon. For total antisymmetry, N_c number of quarks are needed to make up a baryon. Then $nJ_\mu \to J_\mu^B$ gives,

$$
\begin{aligned}
nJ_\mu^B(x) &= \frac{1}{48\pi^2}\left(\frac{n}{N_c}\right)\epsilon^{\mu\nu\alpha\beta}Tr[(L_\nu L_\alpha L_\beta - R_\nu R_\alpha R_\beta)] \\
&= \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}Tr[L_\nu L_\alpha L_\beta] \qquad (33)
\end{aligned}
$$

This is the same as the topological current of Skyrme. Thus the gauged WZ term gives rise to $J_\mu(x)$ which in turn gives the baryon charge. Thus though the WZ term Γ_{WZ} is zero for two-flavour case, but $J_\mu(x)$ still contibutes to the two-flavour case.

Next we embed the SU(2) Skyrme ansatz into $U(x)^{SU(3)}$ as follows for the SU(3) Skyrmion,

$$U_c(x)^{SU(2)} \;\rightarrow\; U_c(x)^{SU(3)} = \begin{pmatrix} U_c(x)^{SU(2)} & \\ & 1 \end{pmatrix} \qquad (34)$$

Next we insert the identity,

$$U(\vec{r},t)^{SU(3)} = A(t)U(\vec{r})_c^{SU(3)}A^{-1}(t) \qquad A \in SU(3)_F \qquad (35)$$

where A is the collective coordinate. Note that $U(\vec{r},t)$ is invariant under,

$$A \rightarrow Ae^{iY\alpha(t)} \qquad (36)$$

where

$$Y = \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \qquad (37)$$

The quantum dof manifest themselves in the WZ term as,

$$L_{WZ} = -\frac{1}{2}N_c B(U_c) tr(YA^{-1}A)$$

$$gauge - invariance \ \rightarrow L_{WZ} \rightarrow L_{WZ} + \frac{1}{3}N_c B(U_c)\dot{\alpha} \qquad (38)$$

In quantized theory A and Y are operators-from Noether's theorem

$$\hat{Y}\Psi = \frac{1}{3}N_c B\Psi$$

$$giving\ the\ right - hypercharge \ \ Y_R = \frac{1}{3}N_c B \qquad (39)$$

where B and N_c are integers. Note that this right-hypercharge was dictated by having defined SU(2) embedding in SU(3). With B = 1 and $N_c = 3$ one gets $Y_R = 1$. This identifies the nucleon hypercharge with the body-fixed hypercharge Y_R. Ultimately one obtains a tower of irreducible represenatations: (8,1/2), (10,3/2), 10,1/2), 27,3/2), ... of which **the lowest octet and decuptet are identified with the observed low energy baryons**. Hence we get all the low dimensional fermions as in the quark model.

Next study the significance of the fact, that **the Wess-Zumino term provides only isoscalar electric charge**. Hence **look at the structure of the electric charge in the $SU(2)_F$ SWZ model**. Define the electric charge operator in SU(2) as,

$$Q = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix} \tag{40}$$

It induces the following transformation,

$$U(x) \to e^{i\epsilon_0 \Lambda Q} U(x) e^{-i\epsilon_0 \Lambda Q} = e^{\frac{i\epsilon_0 \Lambda \tau_3 (q_1 - q_2)}{2}} U(x) e^{\frac{-i\epsilon_0 \Lambda \tau_3 (q_1 - q_2)}{2}} \tag{41}$$

ϵ_0 is the em coupling constant. The Noether current is,

$$\frac{J_\mu^{em}}{\epsilon_0} = \frac{iF_\pi^2}{8}\ Tr\ L_\mu (Q - U^\dagger Q U) - \frac{i}{8\epsilon_0^2}\ Tr\ [L_\nu, Q - U^\dagger Q U][L_\mu, L_\nu] \tag{42}$$

We obtain the gauge theory by replacing

$$\partial_\mu U \to D_\mu U = \partial_\mu U - i\epsilon_0 \Lambda_\mu [Q, U] \tag{43}$$

Next to get constraints on charges,

$$J_{\mu}^{em} = -i\epsilon_0(q_1 - q_2)(\pi_-\partial_\mu\pi_+ - \pi_+\partial_\mu\pi_-) + ..$$
$$\textit{from pion charges} \quad (q_1 - q_2) = 1 \tag{44}$$

Next the charges of baryons N *and* Δ with B=1 charge,

$$Q = \int d^4x \; J_0^{em}(\vec{x}, t) = \epsilon_0 L_\alpha \; Tr\tau_\alpha Q \tag{45}$$

$$Q = \epsilon_0(q_1 - q_2)L_3 \quad \textit{Giving} \quad Q = \epsilon_0 L_3 \tag{46}$$

L_3 is the third component of the isospiun operator, we get,

$$Q(proton) = +\frac{1}{2} \;\; and \;\; Q(neutron) = -\frac{1}{2} \tag{47}$$

• **This is in disagreement with experiment**. Thus the Skyrme Lagrangian fails to provide correct electric charges to proton and neutron. **As such this may be taken to mean that the Skyrme model is wrong**.

However one way to save the Skyrme model is to note that actually it is not providing charge of individual proton snd neutron but **is providing ISOVECTOR charge** $Q_p - Q_n = 1$ **of the nucleon**.

• **So what is quantized in the Skyrme model is not proton and neutron charges individually but the isovector charge of the whole nucleon** $N = \binom{p}{n}$.

This point of view will be supported by the fact that as we shall see below **the Wess-Zumino term in the complete Skyrme-Wess-Zumino model, shall give pure quantized isoscalar charge of the nucleon** $N = \binom{p}{n}$.

• This is completely different from what the Standard model and the Quark Model predict, which is independent electric charges of proton and neutron as per Gell-Mann-Nishijima expression: $Q = T_3 + \frac{Y}{2}$. In Skyrme-Wess-Zumino model it is quantized Isovector and isoscalar charges of the whole nucleon $N = \binom{p}{n}$. **Thus Skyrme-Wess-Zumino model representation is different from that of the Standard Model and Quark Model representation**.

• One should not be surprised by the basic difference of correct representations in the quark model and the Skyrme-Wess-Zumino model: the first one is based on the **Lie algebra** and the second one the **Lie group**.

• Thus the quark charges are determined by the diagonal genetrators of the Lie algebra; for example, Gell-Mann-Nishijima kind of charges.

• However quantum mechanically permitted states like $X = a|u> +b|d>$ are ruled out in Lie algebra by some putative superselection rule.

• **But these are permitted by the full Lie group**.

Thus the 2-flavour Skrmion-hedgehog (eqn. (25)) having $SU(2)_{isopspin} \otimes SU(2)_{spin}$ symmetry, develops a further K-spin invariance with K=I+J. Thus with I=J=1/2, in the special frame $K = I + J \rightarrow 0$, it is equivalent to the Skyrme-Wess-Zumino model in the large colour limit as,

$$\psi^{(0)} = \frac{1}{\sqrt{2}}(]u > \otimes| \uparrow> -|d > \otimes| \downarrow>) \tag{48}$$

This is constructed to be K-spin singlet: $K\psi^{(0)} = (I + J)\psi^{(0)} = 0$. We call this the "hedgehog quark" state. The symmetric N-fold tensor product

$$|0 >= \psi^{(0)} \otimes \psi^{(0)} \otimes \psi^{(0)} .... \psi^{(0)} \tag{49}$$

This is quark model analogue of the Skyrmion (but do not forget the difference - in the quark model this is a product of individual single quark states). For antisymmetry this symmetric state is multiplied by totally antisymmetrci state in N_C-colour space.

• **Thus Skyrme-Wess-Zumino model representation is EXPECTED to be different from that of the Standard Model and Quark Model representation**.

The equivalence between the Skyrme model and the hedgehog quark model is well established.

• Here we add a strong supporting evidence in favour of our **Quantized Charge Standard Model**.

In QCSM we had obtained quark charges as,
$Q(u) = \frac{1}{2}(1 + \frac{1}{N_c}); \quad Q(d) = \frac{1}{2}(-1 + \frac{1}{N_c})$

For $N_C \to \infty$ these reduce to $Q(u) = \frac{1}{2}, \;\; Q(d) = -\frac{1}{2}$, which are charges of proton and neutron respectively, in the Skyrme model (without the Wess-Zumino term).

• **Thus indeed the two models are fully equivalent to each other - but only in the QCSM**.

In our Skyrme-Wess-Zumino model we have the additional WZ term. With the WZ term, again let the field U be transformed by an electric charge operator Q as, $U(x) \to e^{i\Lambda\epsilon_0 Q}U(x)e^{-i\Lambda\epsilon_0 Q}$, Making $\Lambda = \Lambda(x)$ a local transformation the Noether current is

$$J_\mu{}^{em}(x) = j_\mu{}^{em}(x) + j_\mu{}^{WZ}(x) \tag{50}$$

where the first one is the standard Skyrme term and the second is the Wess-Zumino term

$$j_\mu{}^{WZ}(x) = \frac{\epsilon_0 N_c}{48\pi^2}\epsilon_{\mu\nu\lambda\sigma} TrV^\nu V^\lambda V^\sigma(Q + U^\dagger QU) \tag{51}$$

Remember that even though the WZ term vanishes for two flavours, its resulting contribution to electric charge does not. This term was of course missing in the original version of the Skyrme Lagrangian.
One finally obtains,

$$j_\mu{}^{WZ}(x) = \frac{\epsilon_0}{2}(q_1 + q_2)N_c J_\mu(x) \tag{52}$$

The WZ term correction to the electric charge is therefore,

$$\frac{\epsilon_0}{2}(q_1+q_2)N_c\int J_0(x)d^3x \;\; giving \quad \frac{\epsilon_0}{2}(q_1+q_2)N_c B(U_c) \tag{53}$$

Remember the right hypercharge $Y_R = 1$, and subsequently B=1 for $N_c = 3$. Note also the baryon in the Skyrme model with B=1 has three quarks. We thus obtain the charges of $N \;\; and \;\; \Delta$ if,

$$q_1 + q_2 = \frac{1}{3} \tag{54}$$

With earlier SKyrme term of $q_1 - q_2 = 1$ we obtain the charges as,

$$q_1 = \frac{2}{3}\,, \quad q_2 = -\frac{1}{3} \tag{55}$$

Amazing - fractional quark charges in $SU(2)_F$ itself. This is opposite to what happens in the $SU(3)_F$ quark model. There the smaller SU(2)-isospin group, one gets no fractional charges, and one has integral charges for nucleon N = $\binom{p}{n}$. Only in higher group $SU(3)_F$, one gets fractional charges for quarks. **This is a major diffrence between SWZ model and Quark Model.**

However most important to note that for the SU(2) case, the Skyrme Lagranguian (without the Wess-Zumino term) gave us pure isovector charges for proton and neutron. And next ,the Wess-Zumino term, is now giving us pure isoscalar charge. Then the correct quantized quark charges are obtained only after including both the original Skyrme lagrangian plus the Wess-Zumino anomaly term.

Rememeber above, the baryon number B was related to charge Q because as the electric charge in the Wess-Zumino term was pure isoscalar. This is what we have found here for two flavours,

•Next as $Q_p = q_1 + q_1 + q_2$; $Q_n = q_1 + q_2 + q_2$, hence necessarily, the main result for nucleon charge in Skyrme-Wess-Zumino model:

$$Q_p - Q_n = 1 \ \ (isovector); \quad Q_p + Q_n = 1 \ \ (isoscalar) \tag{56}$$

• **Again note that here no Gell-Mann-Nishijima expression of quark model for electric charge of proton and neutron, but quantized isovector and isoscalar charges of the nucleon.**

Using Z=1 for proton and N=1 for neutron the charges are

$$Q(p) = \left(\frac{Z=1}{2}\right)_{isovector} + \left(\frac{Z=1}{2}\right)_{isoscalar}$$
$$Q(n) = -\left(\frac{N=1}{2}\right)_{isovector} + \left(\frac{N=1}{2}\right)_{isoscalar} \quad (57)$$

Hence as per these skyrmions, this model gives right away the charge of a nucleus for arbitrary number of Z and N as,

$$Q = \frac{Z-N}{2} + \frac{Z+N}{2} \quad = \quad T_3 + \frac{A}{2} \quad (58)$$

This well known charge of the nucleus is obtained here, as nucleus is treated as made up of Z-protonic skyrmions and N-neutronic skyrmions. Note that we have obtained the fundamental nuclear charge equation directly in terms of the atomic mass number A, as a direct and basic result of the Skyrmion in the Skyrme-Wess-Zumino model. To belabour the point, this cannot be done for pure Skyrme model without the addition of the Wess-Zumino anomaly term. **This is the proper representation of the Nucleon in the nucleus as per the SWZ model.**

Next, in going to $SU(3)_F$, a the Noether current is,

$$J_\mu{}^{em}(x) = j_\mu{}^{em}(x) + j_\mu{}^{WZ}(x) \tag{59}$$

first one is the standard Skyrme term and second the WZ term

$$j_\mu{}^{WZ}(x) = \frac{N_c}{48\pi^2}\epsilon_{\mu\nu\lambda\sigma} TrL^\nu L^\lambda L^\sigma (Q + U^\dagger Q U) \tag{60}$$

As the charge operator can be simultaneously diagonalized along with the third component of isospin and hypercharge, we write it as, $Q = \begin{pmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{pmatrix}$. The electric charge of pseudoscalar octet mesons demand, $q_1 - q_2 = 1, \quad q_2 = q_3$. Hence one obtains

$$Q = (q_2 + \frac{1}{3})\mathbf{1}_{3x3} + \frac{1}{2}\lambda_3 + \frac{1}{2\sqrt{3}}\lambda_8 \tag{61}$$

Finally the total electric charge is,

$$Q = \frac{1}{2}L_3 + \frac{1}{2\sqrt{3}}L_8 + (q_2 + \frac{1}{3})N_c B(U_c) \tag{62}$$

The last term vanishes with down quark charge $q_2 = -\frac{1}{3}$, and one is left with the Gell-Mann-Nishijima expression of charge as

$$Q = t_3 + \frac{Y}{2} \tag{63}$$

This gives the electric charges of all the members of the baryon octet, conventionally supporting octet as lowest representation.
• But this is precisely what we do not want! As we saw above, the SU(2) nucleon charges are given in the Skyrme-Wess-Zumino model as isoscalar and isovector charges. Those are the properly quantized charges. Thus no Gell-Mann-Nishijima electric charges and $SU(3)_F$ should be consistent with separately quantized isovector and isoscalar charges. Out of all the members of the octet representation, the only one which has this property is Λ.
•Thus the SU(3) skyrmion is not octet or decuptet, or any member of the infinite ladder, but the spin half fermion $\mathcal{S} = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix}$
• **Thus the SWZ model is demanding revival of the long-ago discarded SAKATON of SU(3)**

• **So why is the well-accepted spin-half octet representation of the SWZ model WRONG and the CORRECT result forced upon us is that of a Sakaton?**

Answer: We saw above, that if **QED is IGNORED** (as done by Witten et. al) QCD in $N_C \to \infty$, *as for* $N_C = 3$, **gives inconstent result for the structure of baryons.**

•**Hence QCD is consistent only when QED is fully and properly incorporated with it**.

• **We note that the octet representation of the SWZ model, arises when QED was being ignored**. Later QED was incorporated, but incorrectly (charge of 2-flavour Skyrme model was interpreted wrongly). Only with proper and correct interpretation of the global nature of these groups that QED forced the proper interpretation of the representataion of the SWZ model.

Original Sakata Model: Sakata had extended the group $SU(2)_I$ to $SU(2)_I \times U(1)_Y$, and had taken Λ as a representation of the U(1) group. Thus it was natural to take $\mathcal{S} = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix}$ as the fundamental representation of a larger $SU(3)_F$ group. It is called **Sakaton** in anology with Nucleon of the isospin group.

•Note that the charges in Sakaton are all integral: 1,0,0 respectively.

•The Sakata Model predicted the mesons correctly as composites: $3 \times \overline{3} = 1 + 8$. However it failed to describe the baryons as $3 \times 3 \times \overline{3} = 3 + 3 + 6 + 15$.

•Also as both $n \; and \; \Lambda$ are neutral members of the fundamental triplet in Sakata model, they should have the same magnetic moment, $\mu_\Lambda = \mu_n$. This fails to match the expariment where, $\mu_\Lambda = -0.613 \; and \; \mu_n = -1.913$ in units of $\frac{e\hbar}{2m_p c}$, where mass is that of proton.

•Thus the fundamental triplet Sakaton was rejected.

However in our Skyrme-Wess-Zumino model with minimal symmetry breaking, the masses are:
$m_s = m_0$, *and* $(m_u = m_d) = m_o + aY$. With 'a' as negative in magnitute, $m_s > (m_u = m_d)$.

- **Hence magnetic moments of our skyrmionic Sakaton,** $\mathcal{S} = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix}$ **are succssfully obtained as,**

$$\begin{array}{ccc} Baryons & SWZ\ model & experiment \\ p & \frac{(4\mu_u - \mu_d)}{3} & 2.793 \\ n & \frac{(4\mu_d - \mu_u)}{3} & -1.913 \\ \Lambda & \mu_s & -0.614 \end{array} \tag{64}$$

- **Hence the SWZ model prediction of Sakaton as Skyrmion is good**

Hypernuclei

Physically as of now, one had assumed that hypernuclei reflect the presence of hyperons, arising in the spin $1/2$ octet, in the nucleus.

•However, this picture is unable to explain as to why the hypernuclei observed experimentally upto now, are predominantly made up of Λ's only - fortyone have a single Λ present, three have two-Λ and only one has a Σ meson?

•Our model here shows that actually the hypernuclei are a manifestation of the presence of Sakatons in a nucleus. Hence it predicts that strangeness in nuclei should arise from the Sakatons.

•**Thus the puzzling presence of only the Λ's in hypernuclei is actually a confirmation of our Sakaton model arising from the Skrme-Wess-Zumino model**.

A Very Heavy Scalar Meson

•Thus what we have shown is that Sakatons are physically as relevant as the quarks are in particle physics.

•But Sakatons are different from quarks in as much as they arise in the topological Skyrme model. Hence these are Skyrmions.

•Therefore the Skyrme model is not just one of the large number of phenomenological models of the hadrons, arising from some kind of an approximation of QCD.

• **Our analysis here has shown that the Sakatons are as basic and as fundamental as the fractionally charged quarks are to QCD**.

•Hence the prediction of a very heavy scalar meson which arises in the Topological Skyrme model, should be accorded a serious sonsideration as a genuine physical entity.