Fate of in-medium heavy quarks via a Lindblad equation

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Frankfurt, 25 September 2017

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Contents

DDB, JHEP08 (2017) 064, 1705.03567
Blaizot, DDB, Faccioli, Garberoglio, Nuc. Phys. A 946(2016) 49, 1503.03857

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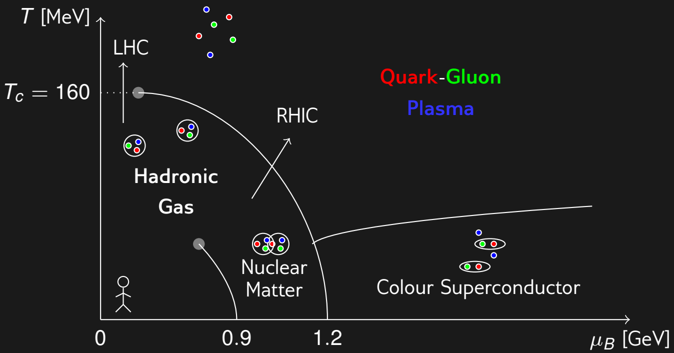
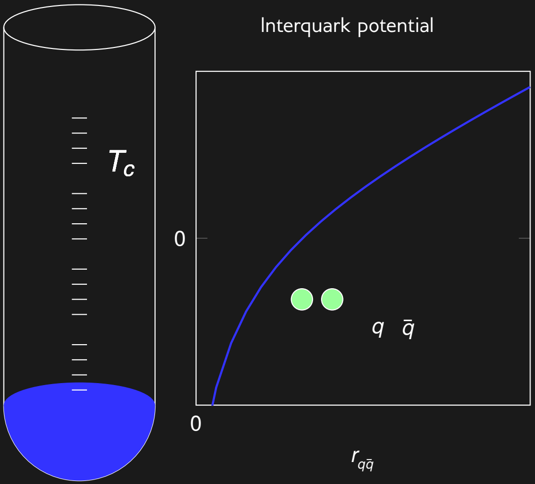
QCD phase diagram



Signatures of QGP: Heavy-quark bound states



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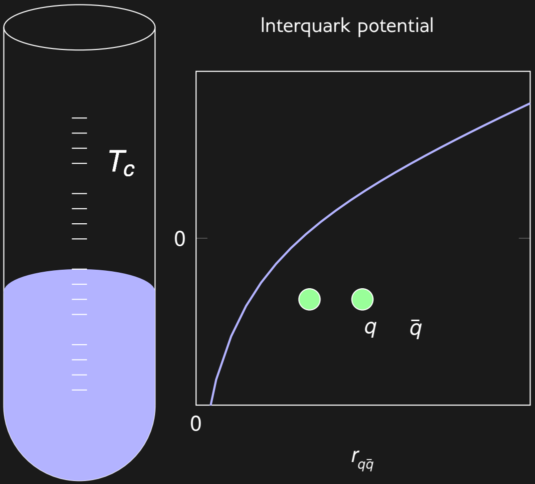

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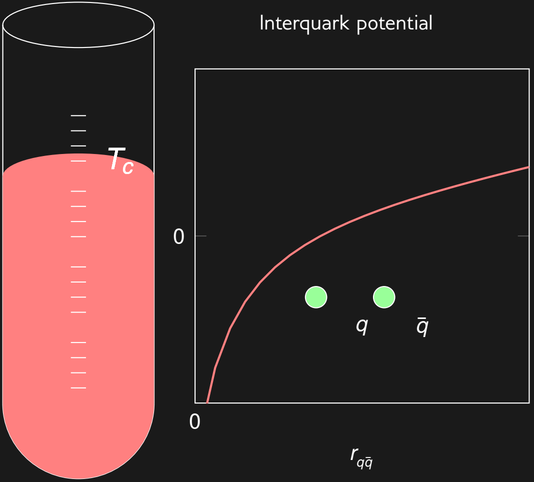

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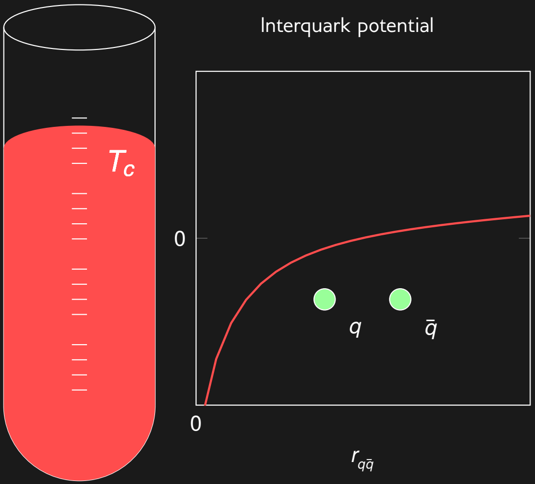

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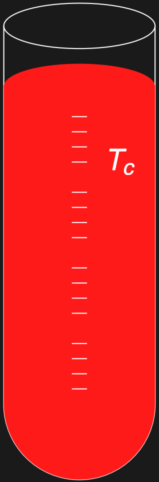

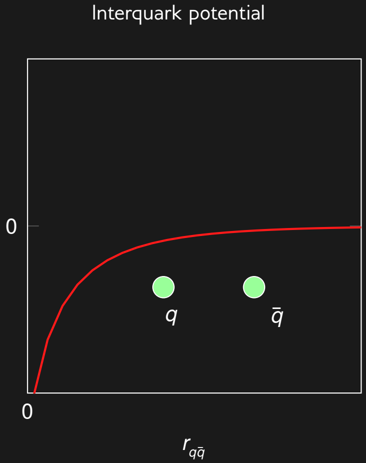

Interquark potential can be calculated using Lattice QCD, perturbative techniques, holographic approach

Interquark potential can be calculated using Lattice QCD, perturbative techniques, holographic approach

- Potential gets screened at high $T \Rightarrow$ Dissociation
  - studied via Schrödinger equation
- Potential develops an **imaginary part** $\Rightarrow$ Dissociation
  - This comes from Landau damping (here gluo-dissociation is not considered)
  - Schrödinger eqn not appropriate when deviation from unitary evolution is large

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Goal: To provide a unified framework for studying both processes:

- starting off from the underlying gauge theory
- controlling the real-time dynamics of the heavy quarks in the plasma

The model

We consider

- an Abelian plasma made of **relativistic light fermions** (quarks and antiquarks) in thermal equilibrium

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- Scales setting: (light quarks) $m \ll T \ll M$ (heavy quarks)

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- Light particles of the plasma are treated within *thermal field theory*

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Using the Coulomb gauge, the Hamiltonian of the system reads

$$\begin{aligned} H_{\mathrm{tot}} &= \frac{1}{2M}\sum_{i=1}^{N}\left(\mathbf{p}_i^2+\overline{\mathbf{p}}_i^2\right)+\int \mathrm{d}\mathbf{x}\,\overline{\psi}(\mathbf{x})\,\left(-\mathrm{i}\gamma^i\partial_i+m\right)\psi(\mathbf{x})+ \\ &+ \underbrace{\frac{1}{2}\iint \mathrm{d}\mathbf{x}\,\mathrm{d}\mathbf{y}\;j^0_{\mathrm{tot}}(\mathbf{x})\frac{1}{4\,\pi|\mathbf{x}-\mathbf{y}|}j^0_{\mathrm{tot}}(\mathbf{y})}_{\text{Coulomb interactions}} \end{aligned}$$

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$$j^0_{\mathrm{tot}}(\mathbf{x}) = \overbrace{g\sum_{i=1}^{N}\left(\delta(\mathbf{x}-\mathbf{q}_i)-\delta(\mathbf{x}-\bar{\mathbf{q}}_i)\right)}^{\text{heavy quarks+antiquarks}}+\overbrace{g\,\overline{\psi}(\mathbf{x})\,\gamma^0\psi(\mathbf{x})}^{\text{light particles}}$$

Open quantum systems

"Schrödinger" equation for closed quantum system (heavy particles + plasma)

$$\mathrm{i}\hbar \frac{d\rho_{\mathrm{tot}}}{dt}(t) = [H_{\mathrm{tot}}, \rho_{\mathrm{tot}}(t)] \qquad H_{\mathrm{tot}} = H \otimes \mathbb{I}_{\mathrm{env}} + \mathbb{I} \otimes H_{\mathrm{env}} + H_{\mathrm{int}}$$

- ρ_{tot} is the density operator of the total (closed) system
- $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$ for a pure state

Master equation for open quantum system (heavy particles)

$$\begin{aligned} \mathrm{i}\hbar \frac{d\rho}{dt}(t) &= \mathrm{Tr}_{\mathrm{env}} \left\{[H_{\mathrm{tot}}, \rho_{\mathrm{tot}}(t)]\right\} \\ &= [H, \rho(t)] + \mathrm{Tr}_{\mathrm{env}} \left\{[\mathbb{I} \otimes H_{\mathrm{env}} + H_{\mathrm{int}}, \rho_{\mathrm{tot}}(t)]\right\} \\ &\equiv [H, \rho(t)] + \mathrm{i}\, \mathcal{D}\rho(t) \end{aligned}$$

$$\rho \equiv \mathrm{Tr}_{\mathrm{env}}\rho_{\mathrm{tot}}$$

Lindblad equation

Most general master equation in the Markovian limit (negligible memory effects)

$$\dot{\rho} = -\frac{\mathrm{i}}{\hbar}[H,\rho] + \frac{1}{2\hbar}\sum_{\mu}\Big([L_\mu\rho, L_\mu^\dagger] + [L_\mu, \rho L_\mu^\dagger]\Big)$$

L_μ, $L_\mu^\dagger$ are the Lindblad operators

Path integral from Trotter decomposition

$$\langle q|\hat{\rho}(t+\Delta t)|q'\rangle = \langle q|\hat{\rho}(t)|q'\rangle - \frac{\mathrm{i}}{\hbar}\langle q|[\hat{H},\hat{\rho}(t)]|q'\rangle + \text{Lindbladian terms}$$

$$\Rightarrow \rho(t,q,q') = \int \mathrm{d}q_0 \int \mathrm{d}q_0' \, P(q,q',t|q_0,q_0',t_0)\,\rho(t_0,q_0,q_0')$$

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Path integral

$$P(q, q', t | q_0, q_0', t_0) = \int_{(q_0,t_0)}^{(q,t)} \mathcal{D}q\mathcal{D}p \int_{(q_0',t_0)}^{(q',t)} \mathcal{D}q'\mathcal{D}p' \, \exp\left[\frac{\mathrm{i}}{\hbar} S[q, p; q', p']\right]$$

$$S[q, p; q', p'] = \int_{t_0}^{t} \mathrm{d}\tau \left[\dot{q}p - H_{\text{eff}}(q, p) - \dot{q}'p' + H_{\text{eff}}^*(q', p') \right.$$
$$\left. -\mathrm{i} \sum_{\mu} L_\mu(q, p) L_\mu^*(q', p') \right]$$

$H_{\text{eff}} = H - \frac{\mathrm{i}}{2} \sum_\mu L_\mu^\dagger L_\mu$

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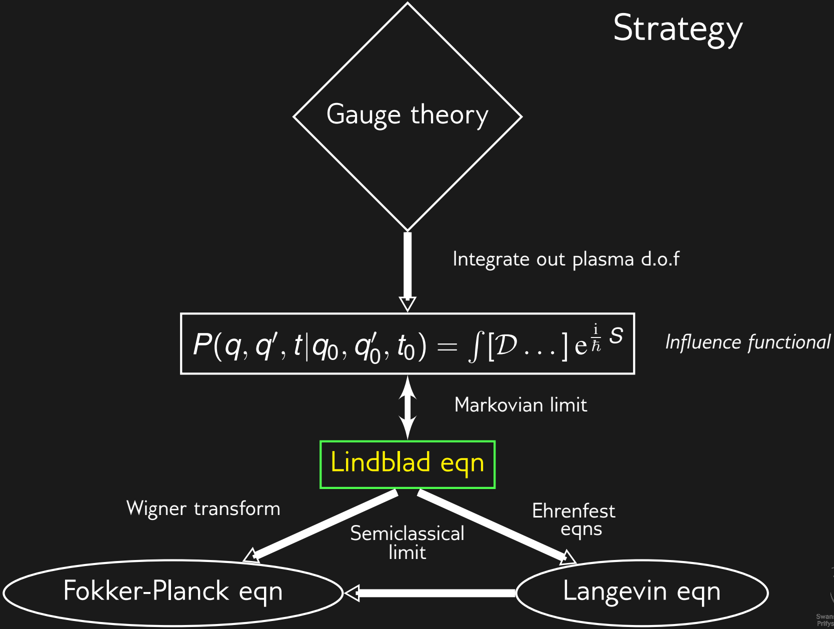
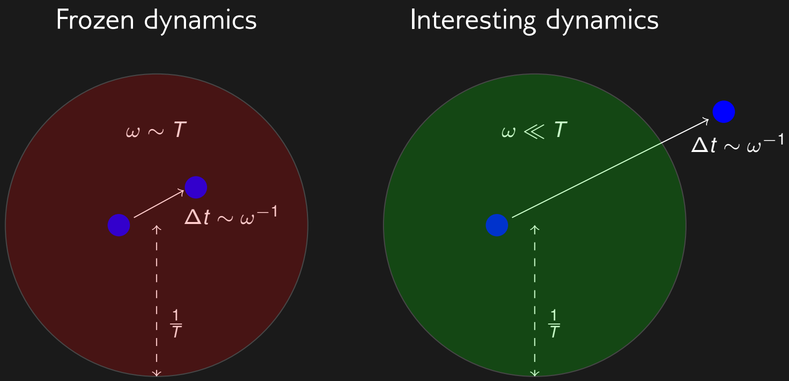
Strategy


Gauge theory
Integrate out plasma d.o.f
$P(q, q', t|q_0, q'_0, t_0) = \int [\mathcal{D} \ldots] \, e^{\frac{i}{\hbar} S}$
Influence functional
Markovian limit
Lindblad eqn
Wigner transform
Semiclassical limit
Ehrenfest eqns
Fokker-Planck eqn
Langevin eqn

Markovian approximation:

Low frequency expansion $(\omega/T \ll 1)$

$\tau_{\rm env} \ll \tau_{\rm sys}$



Low frequency expansion of the gluon (photon) correlation function

$$\Delta(\omega) \approx \Delta(\omega = 0) + \omega\, \Delta'(\omega = 0)$$

$$\Delta(t_x - t_y) \approx \delta(t_x - t_y)\Delta(\omega = 0) + \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t_x}\delta(t_x - t_y)\Delta'(\omega = 0)$$

Other approximations:

- Perturbative expansion up to order g^2
- Hard Thermal Loop calculation of the photon self-energy

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The real (V) and imaginary (W) parts of the potential come from this correlation functions at $\omega = 0$

$$V(\mathbf{r}) = -\Delta^{\mathrm{R}}(\omega = 0, \mathbf{r}) \qquad W(\mathbf{r}) = -\Delta^{<}(\omega = 0, \mathbf{r})$$

Lindblad eqn for a single heavy quark

$$\mathbf{r} = \frac{1}{2}(\mathbf{q} + \mathbf{q}'), \qquad \mathbf{y} = \mathbf{q} - \mathbf{q}'$$

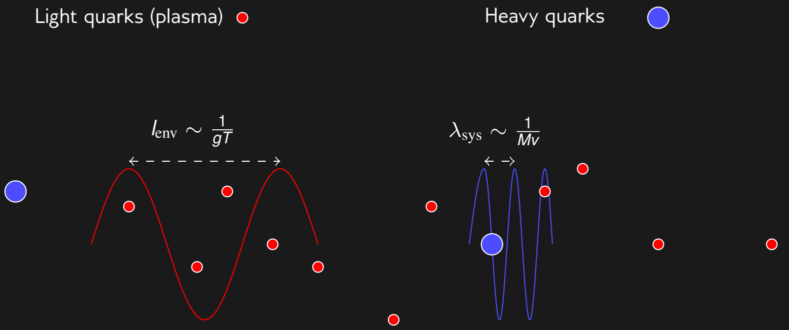
- $W(\mathbf{r})$ is the imaginary part of the interquark potential
- The Lindblad matrix elements depend only on W

$$\frac{\partial \rho(t, \mathbf{r}, \mathbf{y})}{\partial t} = \Bigg(\overbrace{\frac{\mathrm{i}\hbar}{M} \frac{\partial}{\partial \mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{y}} - \frac{\mathrm{i}}{\hbar}\left(V_{\mathrm{ext}}(\mathbf{r} + \mathbf{y}/2) - V_{\mathrm{ext}}(\mathbf{r} - \mathbf{y}/2)\right)}^{\text{Schrödinger}}$$

$$\underbrace{-\frac{g^2}{\hbar}(W(\mathbf{y}) - W(0))}_{\text{diffusion, decoherence}} \underbrace{- \frac{g^2 \hbar}{2MT} \frac{\partial W(\mathbf{y})}{\partial \mathbf{y}} \cdot \frac{\partial}{\partial \mathbf{y}}}_{\text{dissipation}} \Bigg) \rho(t, \mathbf{r}, \mathbf{y})$$

Resolution of the system by the plasma

The (irreversible) interaction between the system and the plasma is determined by $W(\mathbf{r})$, which has a characteristic correlation length $l_{\text{env}} \sim \frac{1}{gT}$ (screening from Landau damping)



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- $l_{\mathrm{env}} \lesssim \lambda_{\mathrm{sys}} \Rightarrow$ Medium as a bound-state sieve
- $l_{\mathrm{env}} \ll \lambda_{\mathrm{sys}} \Rightarrow$ Medium heavily perturbs the system
  All bound states decay at the same rate

Fokker-Planck and Langevin equations

Wigner function

$$\rho(t,\mathbf{r},\mathbf{p}) = \int \mathrm{d}\mathbf{y}\, \rho(t,\mathbf{r},\mathbf{y}) \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\mathbf{p}\cdot\mathbf{y}}$$

Fokker-Planck equation for one heavy quark (semiclassical limit):

$$\left[\partial_t + \frac{\mathbf{p}}{M}\cdot\partial_{\mathbf{r}} - \partial_{\mathbf{r}} V_{\mathrm{ext}}(\mathbf{r})\cdot\partial_{\mathbf{p}}\right]\rho(t,\mathbf{r},\mathbf{p}) = \gamma\left[MT\,\nabla^2_{\mathbf{p}} + \partial_{\mathbf{p}}\cdot\mathbf{p}\right]\,\rho(t,\mathbf{r},\mathbf{p})$$

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Corresponding Langevin equation for one heavy quark:

$$M\ddot{\mathbf{r}} + M\gamma\dot{\mathbf{r}} + \nabla_{\mathbf{r}} V_{\mathrm{ext}}(\mathbf{r}) = \boldsymbol{\eta}(\mathbf{r},t) \qquad \gamma \sim W''(\mathbf{r}=0)$$

(γ is space-depent in the many-quark case)
Noise vector corresponds to a stochastic force

$$\langle\boldsymbol{\eta}(\mathbf{r},t)\rangle = 0\,, \qquad \langle\eta_i(\mathbf{r},t)\eta_j(\mathbf{r},t')\rangle = 2M\gamma T\delta_{ij}\delta(t-t')$$

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Langevin dynamics

- High temperature $\leftrightarrow$ melted $q\bar{q}$ pairs
- Low temperature $\leftrightarrow$ $q\bar{q}$ pairs strongly bound
- Medium temperature $\leftrightarrow$ $q\bar{q}$ pairs faintly bound

Langevin dynamics

Pros

- Cheap simulations of many $q\bar{q}$ pairs
- Easy to study pair dissociation and formation

Cons

- Initial conditions are *classical*, hence the bound states are not quantum
- Dynamics is correct only in the semiclassical limit

Lindblad eqn for a $q\bar{q}$ pair (CoM frame)

$$\mathbf{r} = \frac{1}{2}(\mathbf{q} + \mathbf{q}'), \qquad \mathbf{y} = \frac{1}{2}(\mathbf{q} - \mathbf{q}')$$

$$\frac{\partial \rho(t, \mathbf{r}, \mathbf{y})}{\partial t} = \Bigg(\overbrace{\frac{\mathrm{i}\hbar}{M}\frac{\partial}{\partial \mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{y}} - \frac{\mathrm{i}}{\hbar}(V(\mathbf{r} + \mathbf{y}/2) - V(\mathbf{r} - \mathbf{y}/2))}^{\text{Schrödinger}}$$

$$\underbrace{-\frac{g^2}{\hbar}(2W(\mathbf{y}) - 2W(\mathbf{r}) + W(\mathbf{r} + \mathbf{y}) + W(\mathbf{r} - \mathbf{y}) - 2W(0))}_{\text{diffusion, decoherence}}$$

$$-\underbrace{\frac{g^2\hbar}{2MT}\left(\frac{\partial W(\mathbf{y})}{\partial \mathbf{y}} \cdot \frac{\partial}{\partial \mathbf{y}} - \frac{\partial W(\mathbf{r})}{\partial \mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{r}} - \frac{\partial^2 W(\mathbf{r})}{\partial \mathbf{r}^2}\right)}_{\text{dissipation}} \Bigg) \rho(t, \mathbf{r}, \mathbf{y})$$

Quantities of interest

- Probability of having the state $|\psi\rangle$ at time t

$$P(\psi, t|\psi_0, t_0) = \int \mathrm{d}q \int \mathrm{d}q' \psi(q')\psi^*(q)\rho(t, q, q')$$

- Linear entropy (proxy of thermal entropy $S = -\mathrm{Tr}\,[\rho \ln \rho]$)

$$S_L = \mathrm{Tr}\rho - \mathrm{Tr}\rho^2 = 1 - \mathrm{Tr}\rho^2$$

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Pure states $\rho = \rho^2 \Rightarrow S_L = 0$

Non-pure states $\rho \neq \rho^2 \Rightarrow 0 < S_L \leq 1$

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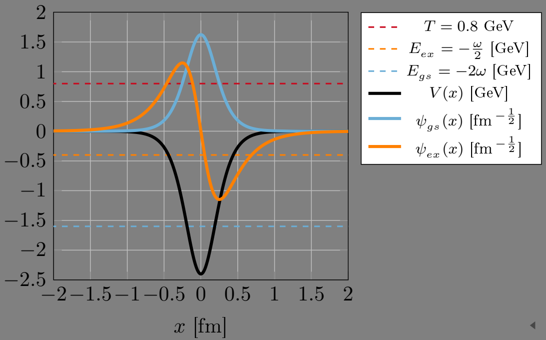
Numerical results for a $q\bar{q}$ pair in 1D

Pöschl-Teller potential:

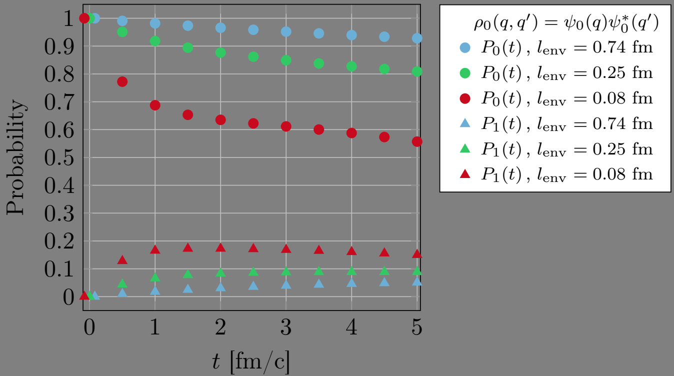
$$V(x) = -\frac{\omega}{2} j(j+1)\mathrm{sech}^2\left[\sqrt{\frac{M\omega}{2\hbar^2}}x\right] \qquad j = 2 \text{ (bound states)}$$

$$W(x) = -\frac{T}{2}\exp\left[-\frac{1}{2}\left(\frac{x}{l_{\mathrm{env}}}\right)^2\right] \qquad l_{\mathrm{env}} \sim \frac{1}{gT}$$



Starting off with the ground state

$\lambda_{\rm sys} = 0.16$ fm



Ground state melts with $P = 1 - P_0 - P_1 \sim 10\%$ after $\Delta t = 5$ fm/c when $l_{\rm env} = 0.25$ fm

Starting off with the ground state

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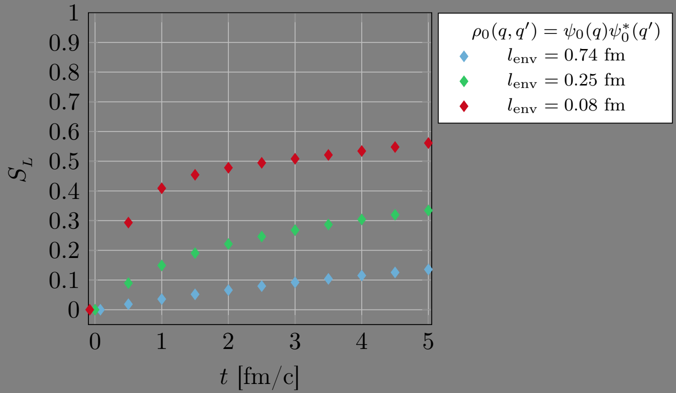
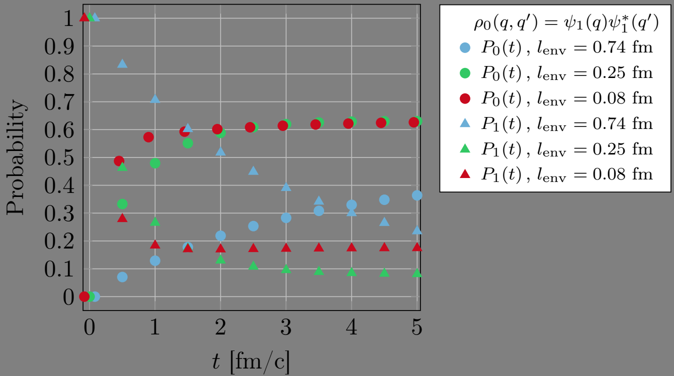


The smaller l_{env} , the more rapidly the linear entropy initially increases

Starting off with the excited state

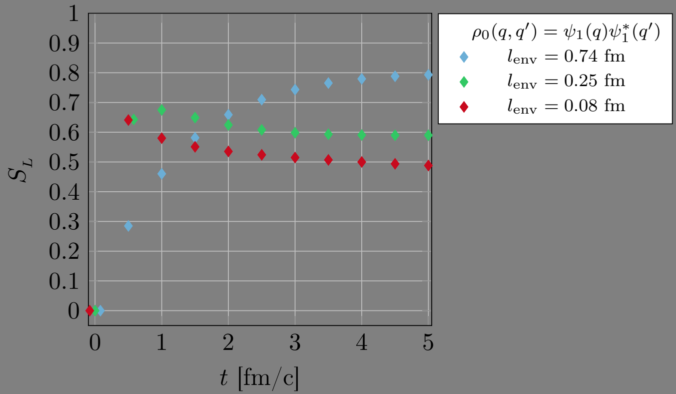
$\lambda_{\text{sys}} = 0.38$ fm



Excited state melts with $P = 1 - P_0 - P_1 \sim 30\%$ after $\Delta t = 5$ fm/c when $l_{\text{env}} = 0.25$ fm

Starting off with the excited state

$\lambda_{\text{sys}} = 0.38\,\text{fm}$



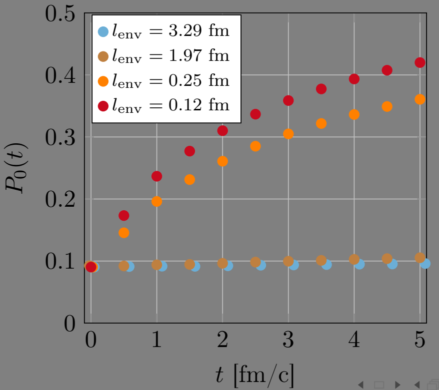
Notice that the linear entropy does not increase monotonically like the thermal entropy

Starting off with a thermal scattering state

$$\psi_{\text{scatt}}(x) \sim e^{-\frac{1}{2}\left(\frac{x}{\delta}\right)^2 + \frac{i}{\hbar} x p} \qquad \delta = \sqrt{2\langle \hat{x}^2 \rangle} = \sqrt{2}\,\lambda_{\text{sys}}$$

$$\lambda_{\text{sys}} = \frac{h}{p_{\text{th}}} = \frac{\sqrt{2}h}{\sqrt{mT}} = 1.77\,\text{fm} \qquad m = 1.2\,\text{GeV}$$

Recombination effects!



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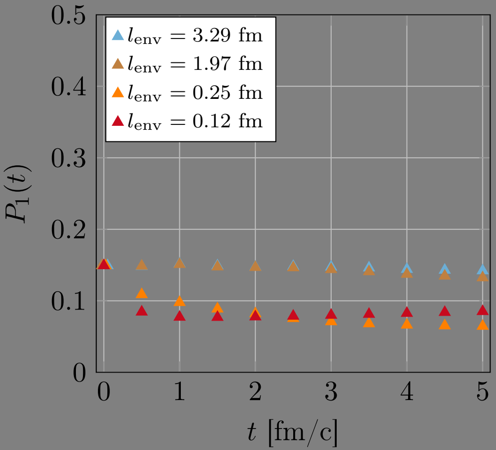

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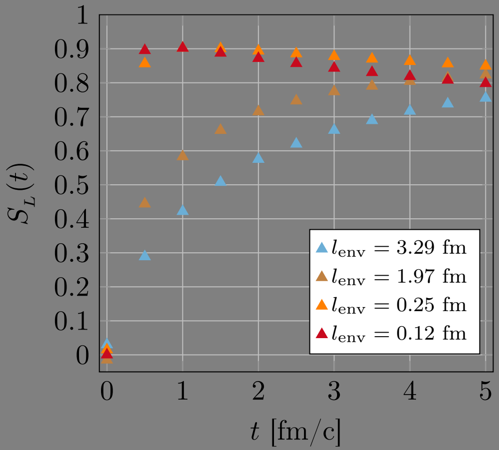


Time evolution of the density matrix

- $l_{\text{env}} \sim \lambda_{\text{sys}}$

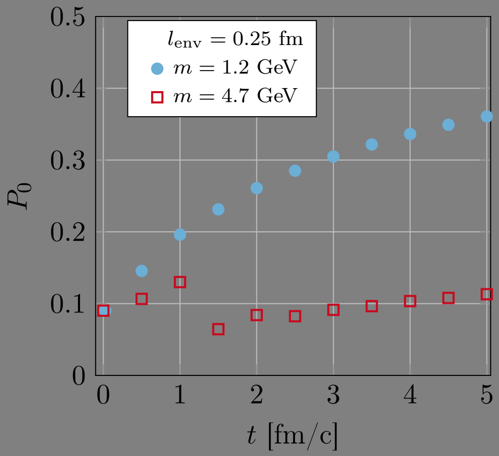
- $l_{\text{env}} < \lambda_{\text{sys}}$

- $l_{\text{env}} \ll \lambda_{\text{sys}}$

Mass dependence of recombination

Experimental fact: Υ ($b\bar{b}$) recombination $\ll$ J/Ψ ($c\bar{c}$) recombination

$$\lambda = \frac{h}{p_{\text{th}}} = \frac{\sqrt{2}h}{\sqrt{mT}} \qquad \lambda_b = 0.89 \text{ fm} \qquad \lambda_c = 1.77 \text{ fm}$$



Conclusions and outlook

- A Lindblad equation for a heavy $q\bar{q}$ pair can be derived from the gauge theory
- This equation allows us to study dissociation, recombination and quantum decoherence of bound states
- Solution goes beyond Langevin/Fokker-Planck dynamics

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Next steps:

- Solve the Lindblad equation in 3D (and for more than 2 particles)
- Is it possible to implement the initial quantum conditions (maybe by weighting somehow the classical paths) in a Langevin equation ?
- Derive the Lindblad equation in QCD

\huge{Thank you}

\end{document}

Backup material

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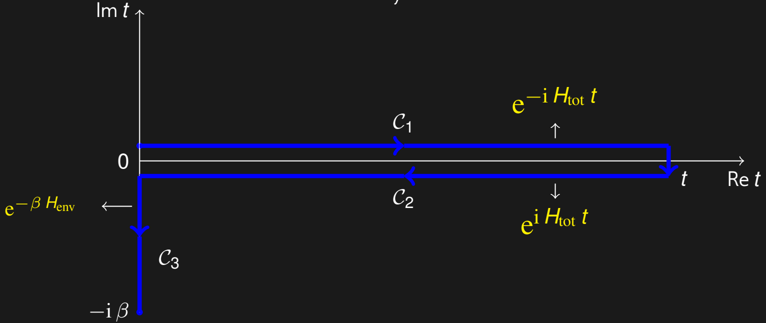
Keldysh formalism

$$\rho_{\rm tot}(t) = e^{-i H_{\rm tot} t}\, \rho_{\rm tot}(0)\, e^{i H_{\rm tot} t}\,; \qquad \rho_{\rm tot}(0) = |\mathbf{Q}_0\rangle\langle \mathbf{Q}_0| \otimes \frac{e^{-\beta H_{\rm env}}}{Z_{\rm env}}$$

Keldysh contour



Keldysh formalism

- Trotter decomposition brings to

$$P(\mathbf{Q}_f, t|\mathbf{Q}_i, 0) = \int_{\mathbf{Q}_i}^{\mathbf{Q}_f} \mathcal{D}\mathbf{Q} \int \mathcal{D}\left(\overline{\psi}\psi\right) \mathrm{e}^{\mathrm{i}\, S[\mathbf{Q},\psi,\overline{\psi}]}$$

The action of the system is defined on the Keldysh contour

$$S[\mathbf{Q}, \psi, \overline{\psi}] = \int_{\mathcal{C}} \mathrm{d}t \left[\frac{M}{2} \sum_{j=1}^{N} \left(\dot{q}_j^2 + \dot{\bar{q}}_j^2 \right) + \int \mathrm{d}\mathbf{x}\, \overline{\psi}(\mathbf{x}, t)(\, \mathrm{i}\gamma^\mu \partial_\mu - m\,)\psi(\mathbf{x}, t) + \right.$$

$$\left. -\ \frac{1}{2} \iint \mathrm{d}\mathbf{x}\ \mathrm{d}\mathbf{y}\, j^0_{\mathrm{tot}}(\mathbf{x}, t) \frac{1}{4\,\pi|\mathbf{x} - \mathbf{y}|} j^0_{\mathrm{tot}}(\mathbf{y}, t) \right]$$

Eliminating plasma degrees of freedom

1^{st} step

Hubbard-Stratonovich transformation
$\longrightarrow$ Introducing a Coulomb field A_0

$$\exp\left[-\frac{\mathrm{i}}{2}\, j^0_{\mathrm{tot}} \cdot K \cdot j^0_{\mathrm{tot}}\right] = \mathcal{N} \int DA_0 \, \exp\left[\frac{\mathrm{i}}{2}\, A_0 \cdot K^{-1} \cdot A_0 - \mathrm{i}\, A_0 \cdot j^0_{\mathrm{tot}}\right]$$

where

$$K(\mathbf{x}-\mathbf{y}) = \frac{1}{4\,\pi|\mathbf{x}-\mathbf{y}|}\;; \qquad K^{-1}(\mathbf{x}-\mathbf{y}) = -\delta(\mathbf{x}-\mathbf{y})\,\nabla^2_{\mathbf{y}}$$

2^{nd} step

To eliminate the field of the light quarks by performing the Gaussian integral below

$$\int \mathcal{D}\left(\overline{\psi}\psi\right)\ \exp\left[\mathrm{i}\int_{\mathcal{C}} \mathrm{d}^4x\ \overline{\psi}(x)\left(\mathrm{i}\gamma^\mu\partial_\mu - m - g\,\gamma^0 A_0(x)\right)\psi(x)\right] =$$

$$= \det(\mathrm{i}\gamma^\mu\partial_\mu - m - g\,\gamma^0 A_0) =$$

$$= \exp\left[\mathrm{Tr}\ \ln\left[\mathrm{i}\gamma^\mu\partial_\mu - m - g\,\gamma^0 A_0\right]\right]$$

3^{rd} step (first approximation)

Expand

$$\det\left[\mathrm{i}\gamma^\mu\partial_\mu - m - g\,\gamma^0 A_0\right]$$

to second order in g ($g \ll 1$) and perform the Gaussian integral over A_0

$$\int \mathcal{D}A_0 \ \exp\left[-\frac{1}{2}A_0\cdot\Delta_c^{-1}\cdot A_0 - \mathrm{i}\,A_0\cdot j^0\right] = \exp\left[\frac{1}{2}j^0\cdot\Delta_c\cdot j^0\right]$$

where

$$-\Delta_c^{-1}(x-y) \;=\; \delta_c(t_x^{\mathcal{C}} - t_y^{\mathcal{C}})\,K^{-1}(\mathbf{x}-\mathbf{y}) + \Pi_{00}^{\mathcal{C}}(x-y)$$

$$\Delta_c(x-y) \;=\; \mathrm{i}\langle\, T_c\,[A_0(x)A_0(y)\,]\rangle$$

We obtain the *Feynman-Vernon influence functional*

$$P(\mathbf{Q}_f, t|\mathbf{Q}_i, 0) = \int_{\mathbf{Q}_i}^{\mathbf{Q}_f} \mathcal{D}\mathbf{Q} \exp\left[\mathrm{i}\left(\Phi[\mathbf{Q}] + \frac{M}{2}\int_{\mathcal{C}_1\cup\mathcal{C}_2} \mathrm{d}t \sum_{j=1}^{N}\left(\dot{\mathbf{q}}_j^2 + \dot{\bar{\mathbf{q}}}_j^2\right)\right)\right]$$

$$\begin{aligned}\Phi[\mathbf{Q}] &= \frac{1}{2}\iint_{\mathcal{C}} \mathrm{d}^4x\, \mathrm{d}^4y\, j_0(x)\Delta_{\mathcal{C}}(x-y)j_0(y)\\ &= \frac{1}{2}\int_0^t \mathrm{d}t_x\, \mathrm{d}t_y \int \mathrm{d}\mathbf{x}\, \mathrm{d}\mathbf{y}\, (-1)^{\mathrm{a}+\mathrm{b}} j_{\mathrm{a}}^0(t_x, \mathbf{x})\Delta_{\mathrm{ab}}(t_x - t_y, \mathbf{x}-\mathbf{y})j_{\mathrm{b}}^0(t_y, \mathbf{y})\end{aligned}$$

with

$$j_{\mathrm{a}}^0(t_x, \mathbf{x}) = g\sum_{i=1}^{N}\left(\delta(\mathbf{x}-\mathbf{q}_{\mathrm{i,a}}) - \delta(\mathbf{x}-\bar{\mathbf{q}}_{\mathrm{i,a}})\right)$$

Second approximation: low frequency expansion of the gluon correlation function

$$\Delta(\omega) \approx \Delta(\omega = 0) + \omega\,\Delta'(\omega = 0)$$

$$\Delta(t_x - t_y) \approx \delta(t_x - t_y)\Delta(\omega = 0) + \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t_x}\delta(t_x - t_y)\Delta'(\omega = 0)$$

Performing this expansion up to o(ω) in
$\Phi[\mathbf{Q}] = \Phi_{QQ}[\mathbf{Q}] + \Phi_{\bar{Q}\bar{Q}}[\mathbf{Q}] + \Phi_{Q\bar{Q}}[\mathbf{Q}]$
and introducing the two real quantities

$$V(\mathbf{r}) \equiv -\Delta^{\mathrm{R}}(\omega = 0, \mathbf{r})\,; \qquad W(\mathbf{r}) = -\Delta^{<}(\omega = 0, \mathbf{r})$$

where $\Delta^R = \Delta_{11} - \mathrm{i}\,\Delta^<$ and $\Delta^< = -\mathrm{i}\,\Delta_{12}$, we get

$$\begin{aligned}
\Phi_{QQ}[\mathbf{Q}] \;\; = \;\; & \frac{g^2}{2} \sum_{i,j=1}^{N} \int_{t_i}^{t_f} \mathrm{d}t \Big[V(\mathbf{q}_{j,2} - \mathbf{q}_{i,2}) - V(\mathbf{q}_{j,1} - \mathbf{q}_{i,1}) \\
& -\mathrm{i} W(\mathbf{q}_{j,2} - \mathbf{q}_{i,2}) - \mathrm{i} W(\mathbf{q}_{j,1} - \mathbf{q}_{i,1}) + 2\,\mathrm{i}\, W(\mathbf{q}_{j,1} - \mathbf{q}_{i,2}) \\
& + \frac{\beta}{2}(\dot{\mathbf{q}}_{i,2} + \dot{\mathbf{q}}_{j,1}) \cdot \frac{\partial}{\partial \mathbf{q}_{i,2}} W(\mathbf{q}_{j,1} - \mathbf{q}_{i,2}) \Big]
\end{aligned}$$

and similarly for $\Phi_{\bar{Q}\bar{Q}}[\mathbf{Q}]$ and $\Phi_{Q\bar{Q}}[\mathbf{Q}]$

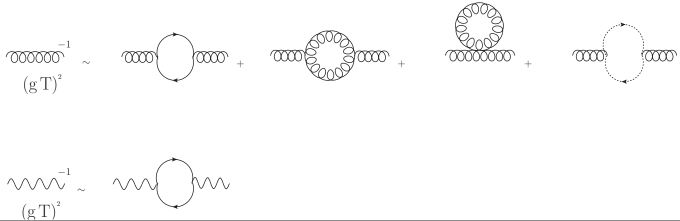
$\rightarrow$ the conditional probability depends only on V and W

3^{rd} approximation: Hard thermal loop

Slow heavy particles exchange **soft** gluons, which have momentum $|\mathbf{k}| \lesssim gT$ and frequency $\omega \lesssim gT$
(consistency with low frequency approximation and with the expansion of the fermionic determinant up to $o(g^2)$*)*

Inverse propagators of soft particles are of the **same order** of 1-loop self energies

$$(g\,T)^2 \sim \Delta^{-1}_{\mu\nu}(k \sim g\,T) \sim \Pi_{\mu\nu}(k \sim g\,T)$$



Gluon self-energy contains the information about collision effects and the screening of the interactions in the plasma

Similarity between HTL 1-loop **gluon** and **photon** self-energy

$$\Pi^{ab}_{\mu\nu}(\omega,\mathbf{q}) = \frac{g^2 T^2}{3} \overbrace{\left(N_c + \frac{N_f}{2}\right)}^{=4} \delta^{ab} \int \frac{\mathrm{d}\Omega}{4\pi} \left(\frac{\mathrm{i}\,\omega\,\widehat{K}_\mu \widehat{K}_\nu}{\mathrm{i}\,\omega + \mathbf{q}\cdot\hat{\mathbf{k}}} + \delta_{\mu 4}\delta_{\nu 4} \right)$$

$$\Pi_{\mu\nu}(\omega,\mathbf{q}) = \frac{e^2 T^2}{3} \int \frac{\mathrm{d}\Omega}{4\pi} \left(\frac{\mathrm{i}\,\omega\,\widehat{K}_\mu \widehat{K}_\nu}{\mathrm{i}\,\omega + \mathbf{q}\cdot\hat{\mathbf{k}}} + \delta_{\mu 4}\delta_{\nu 4} \right) \; ; \quad \widehat{K} \equiv (\hat{\mathbf{k}}, -\mathrm{i})$$

We work with a QED plasma, eventually going to QCD by changing the constant in front of the integral

Using the HTL approximation we obtain

$$V(r) = \int \frac{\mathrm{d}\mathbf{k}}{(2\,\pi)^3} \frac{\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}}{\mathbf{k}^2 + m_D^2} = \frac{e^{-m_D r}}{4\pi\, r}$$

$$W(r) = -\int^{\Lambda} \frac{\mathrm{d}\mathbf{k}}{(2\,\pi)^3} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} \frac{\pi\, T\, m_D^2}{|\mathbf{k}|(\mathbf{k}^2 + m_D^2)^2}$$

where $\Pi_{00}^{\mathrm{HTL}}(0, \mathbf{k}) = m_{\mathrm{D}}^2 = \frac{4}{3} g^2 T^2$

- $V(r)$ is the screening potential between the heavy quarks
- $W(r)$ originates from the collisions between the light fermions of the plasma and the heavy particles

Indeed $-\frac{g^2 T}{2} W(r = 0)$ is the rate of collisions between a heavy quark and the particles of the plasma