Event-by-event pre-equilibrium dynamics in high-energy heavy-ion collisions

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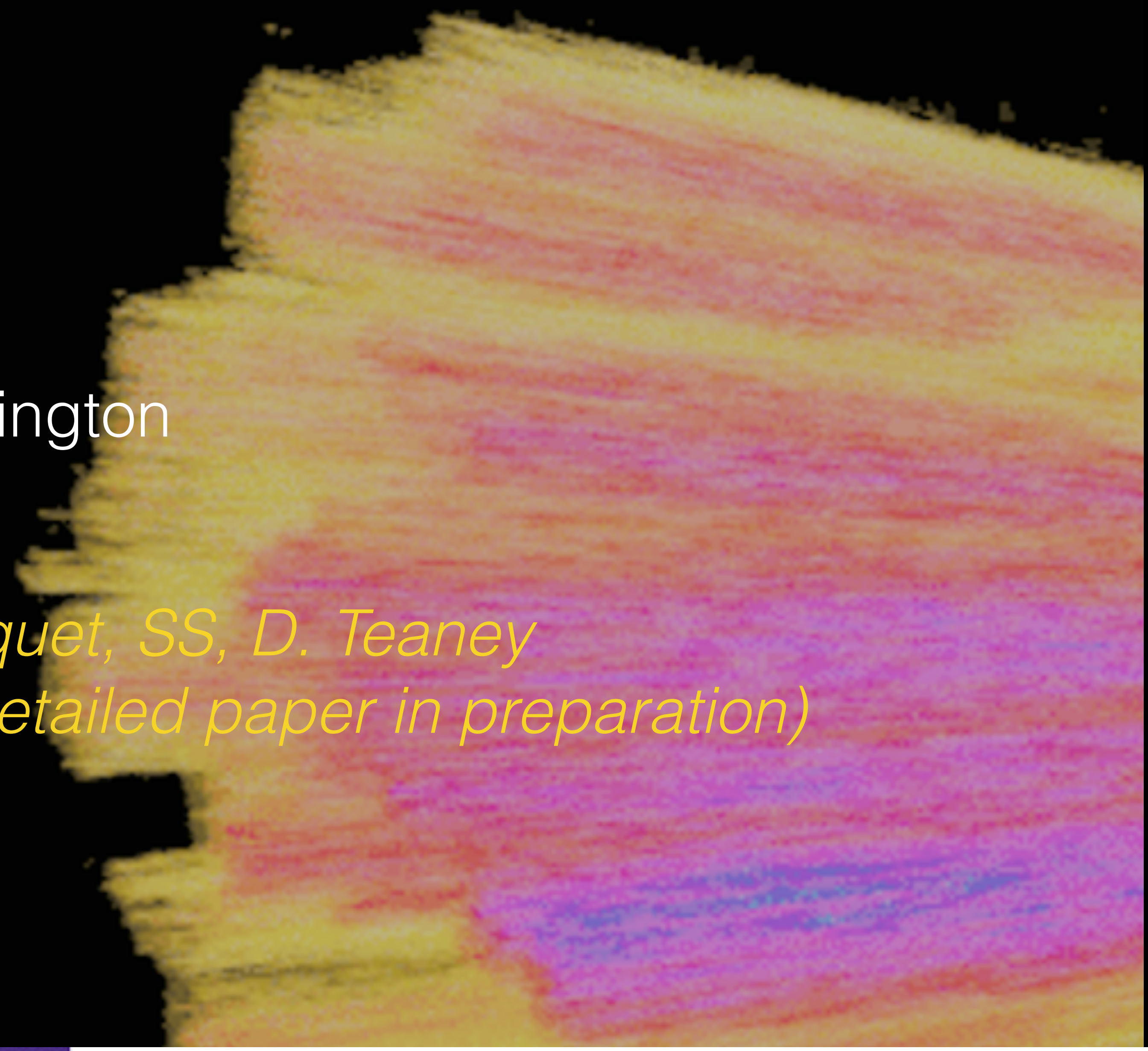
Based on
A. Kurkela, A. Mazeliauskas, J.-F. Paquet, SS, D. Teaney
(QM proceeding arXiv:1704.05242; detailed paper in preparation)



Outline

Early time dynamics & equilibration process

— microscopic dynamics & “bottom-up” thermalization

Description of early-time dynamics by macroscopic d.o.f.

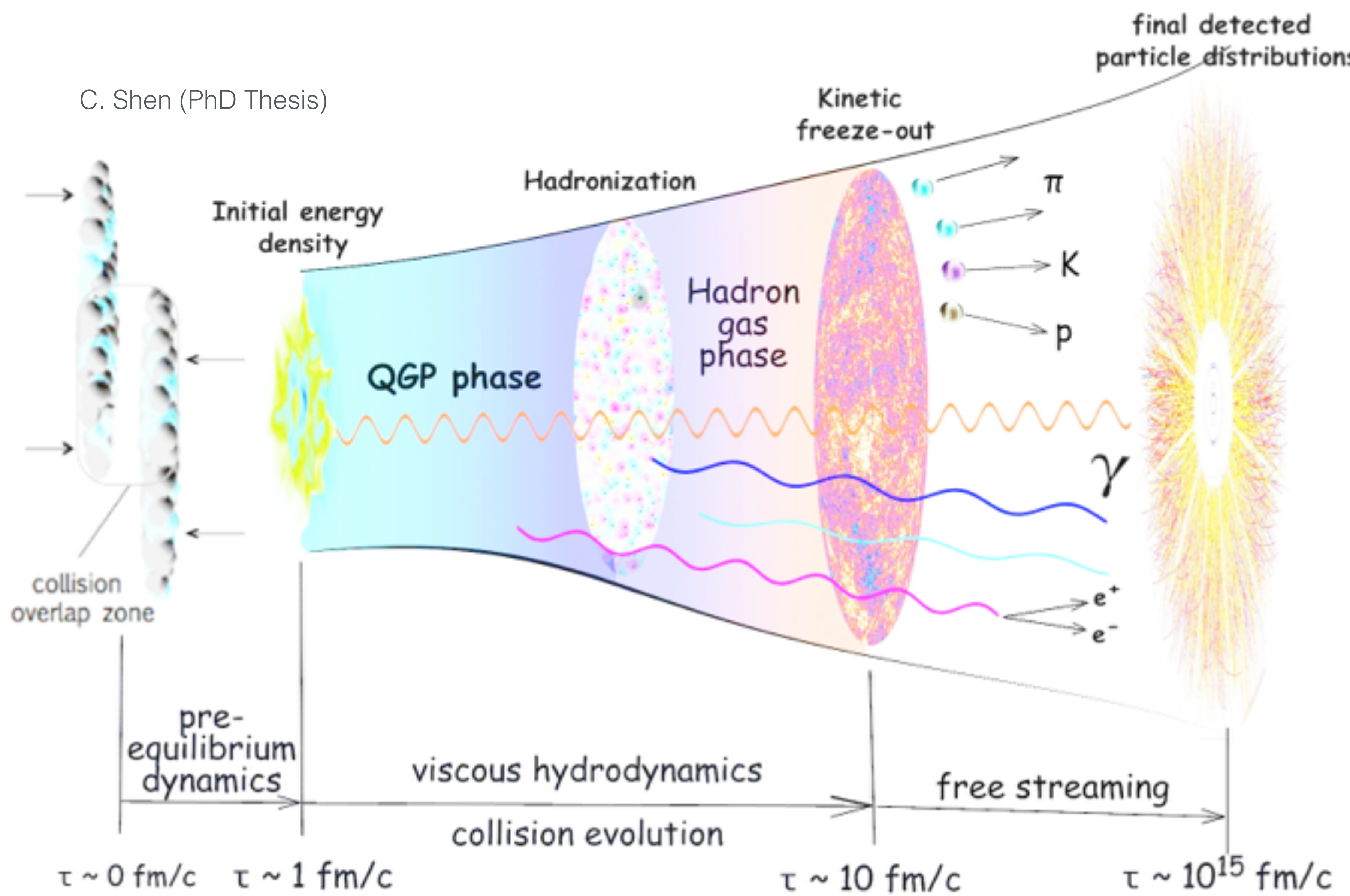
— energy momentum tensor & non-eq. response function

Event-by-event simulation of pre-equilibrium dynamics

— consistent matching to rel. visc. hydrodynamics

Conclusions & Outlook

Space-time picture of HIC

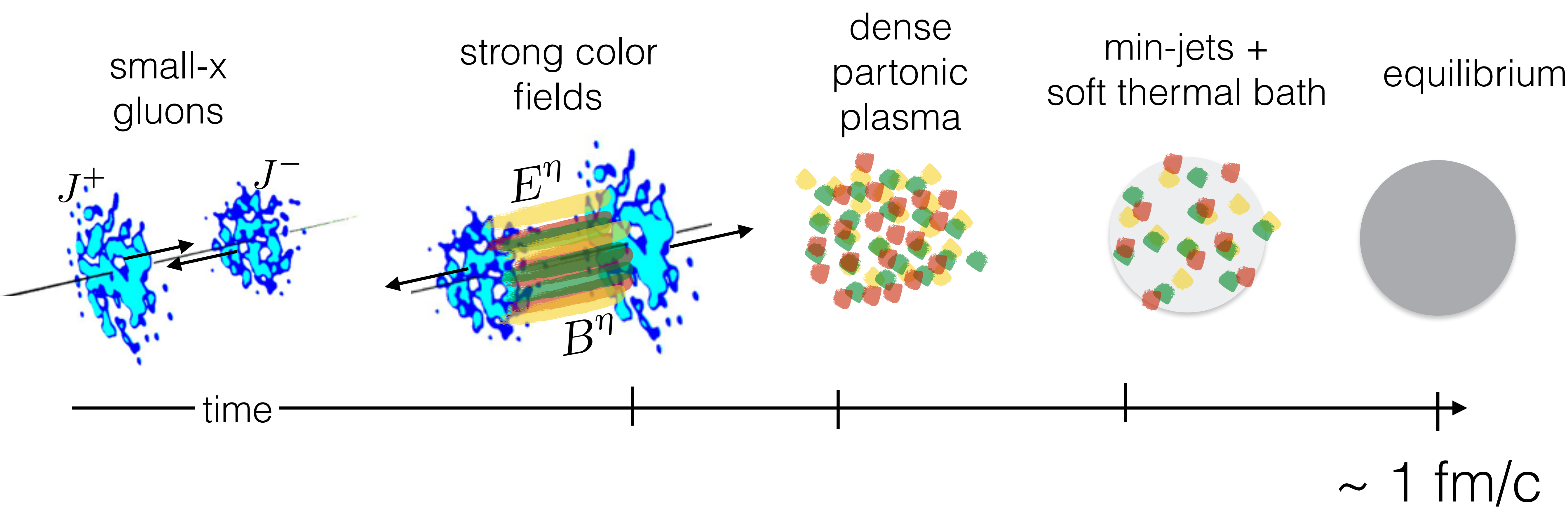


Extremely successful phenomenology based on hydrodynamic models of space-time evolution starting from $\tau \sim 1$fm/c

Challenge: Include theor. description of pre-equilibrium stage for complete description of space-time dynamics

Early time dynamics & equilibration process

Canonical picture at weak coupling:



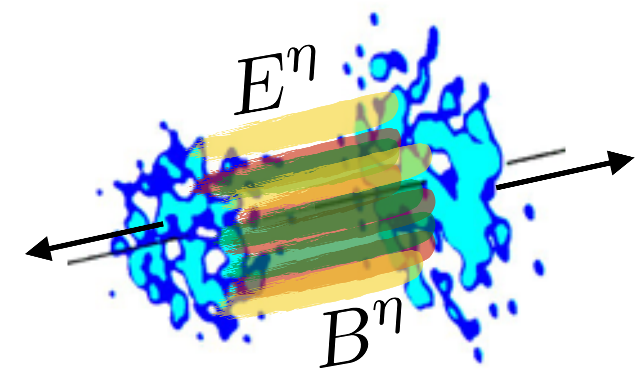
Starting from before the collisions sequence of processes eventually leads to the formation of an equilibrated QGP

Early time dynamics ($0<\tau<1/Q_s$)

Because of high phase-space density of small-x gluons particle initial particle production and early time dynamics described in terms of classical field theory to leading order

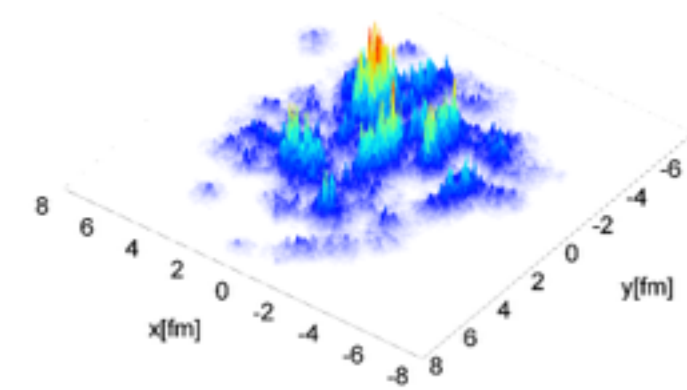
$$D_\mu F^{\mu\nu} = J^\nu$$

Strong boost invariant classical fields E^η, B^η created immediately after the collision



Decoherence of classical fields occurs on a time scale $\tau \sim 1/Q_s$ where quasi-particle description starts to become applicable

-> Basis for microscopic initial state calculations e.g. IP-Glasma



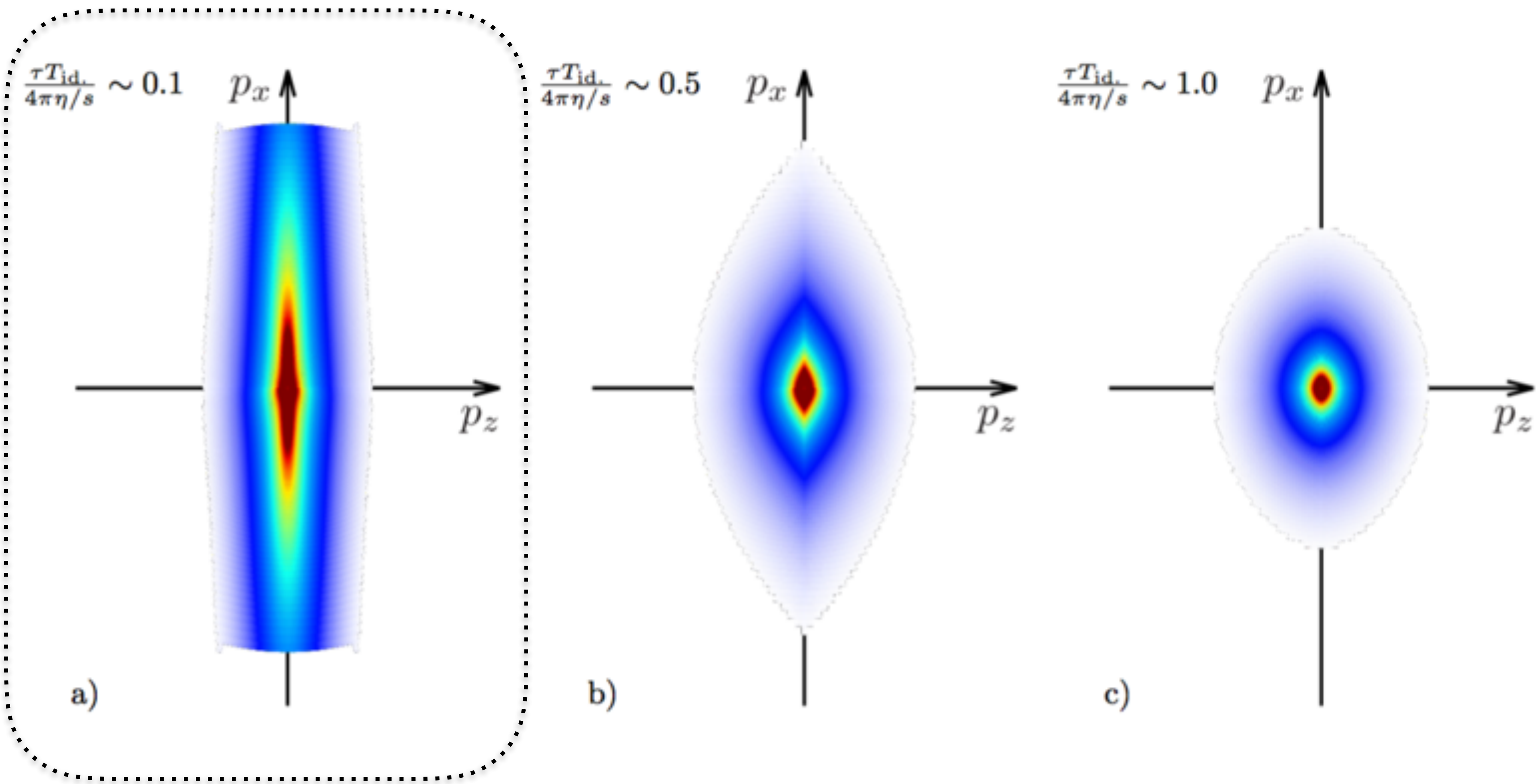
IP-Glasma

Challenge to understand subsequent equilibration process

Equilibration process at weak coupling

Equilibration proceeds as three step process described by "bottom-up" scenario

Baier, Mueller, Schiff, Son PLB502 (2001) 51-58



Phase I: Quasi-particle description becomes applicable.
Elastics scattering dominant but insufficient to isotropize system

c.f. Berges,Boguslavski,SS, Venugopalan, PRD 89 (2014) no.7, 074011

Equilibration process at weak coupling

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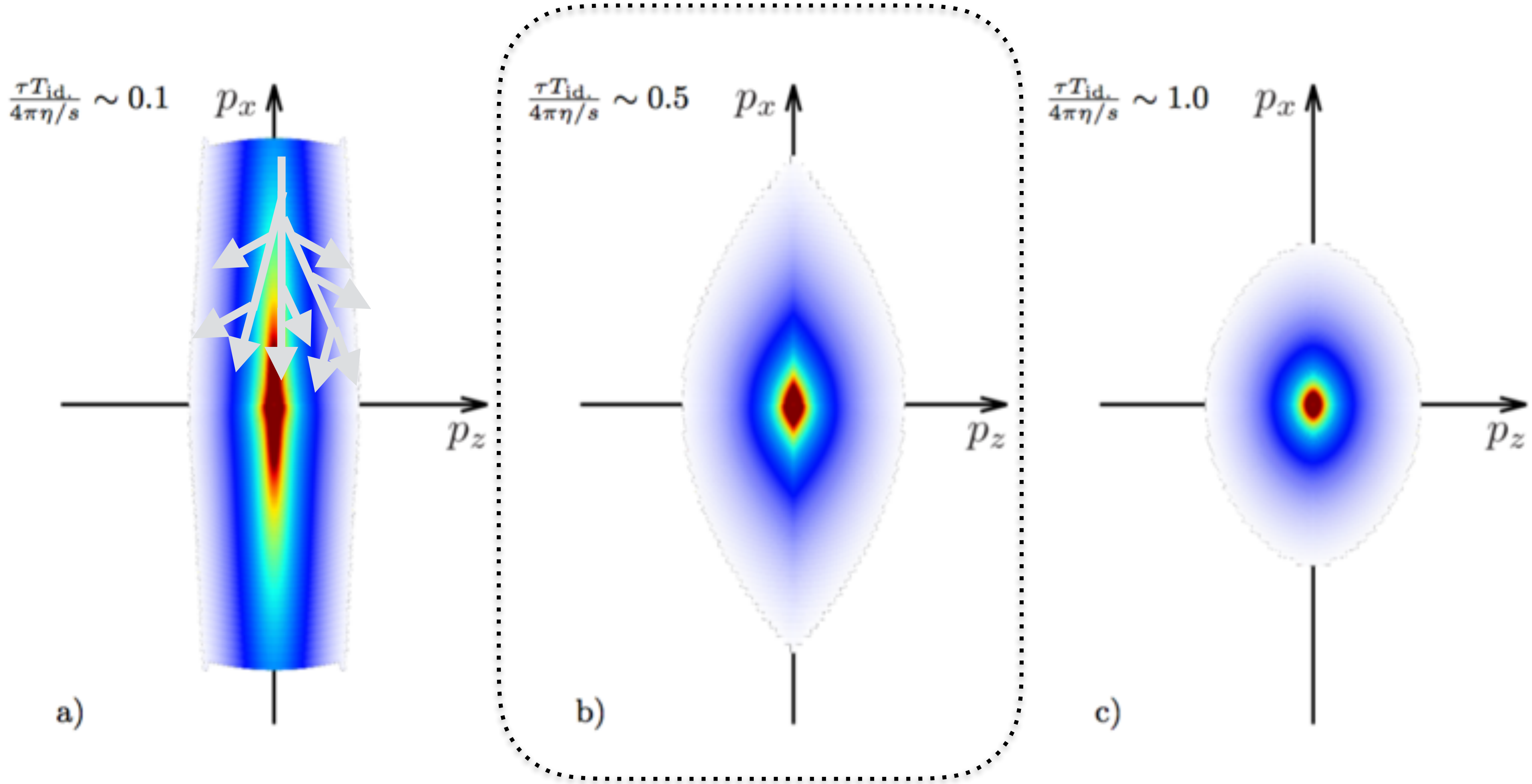


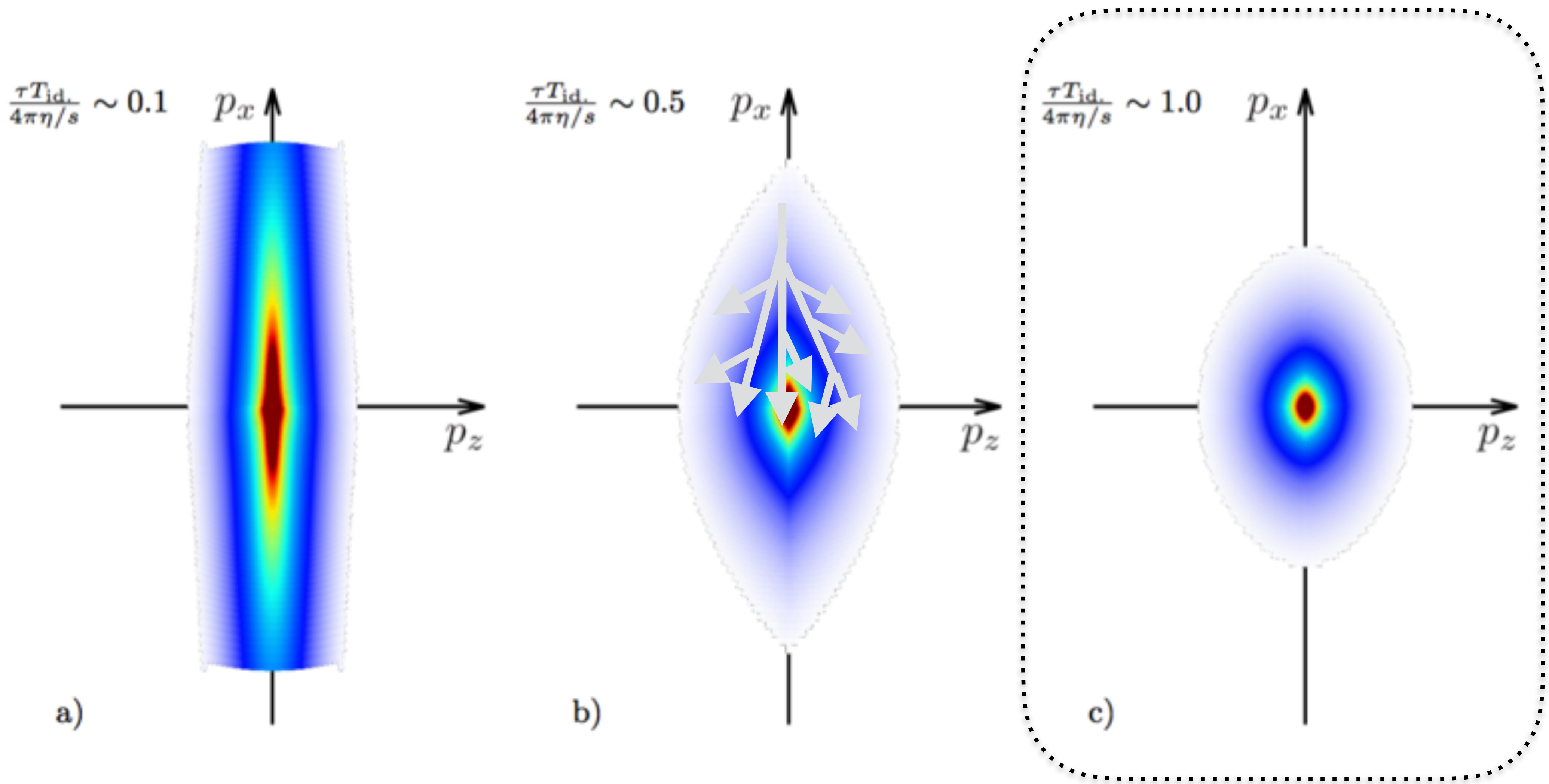
Phase II: Mini-jets undergo a radiative break-up cascade eventually leading to formation of soft thermal bath

c.f. Kurkela, Zhu PRL 115 (2015) 182301

Equilibration process at weak coupling

Equilibration proceeds as three step process described by "bottom-up" scenario

Baier, Mueller, Schiff, Son PLB502 (2001) 51-58



Phase III: Quenching of mini-jets in soft thermal bath transfers energy to soft sector leading to isotropization of plasma

c.f. Kurkela, Zhu PRL 115 (2015) 182301

Equilibration process at weak coupling

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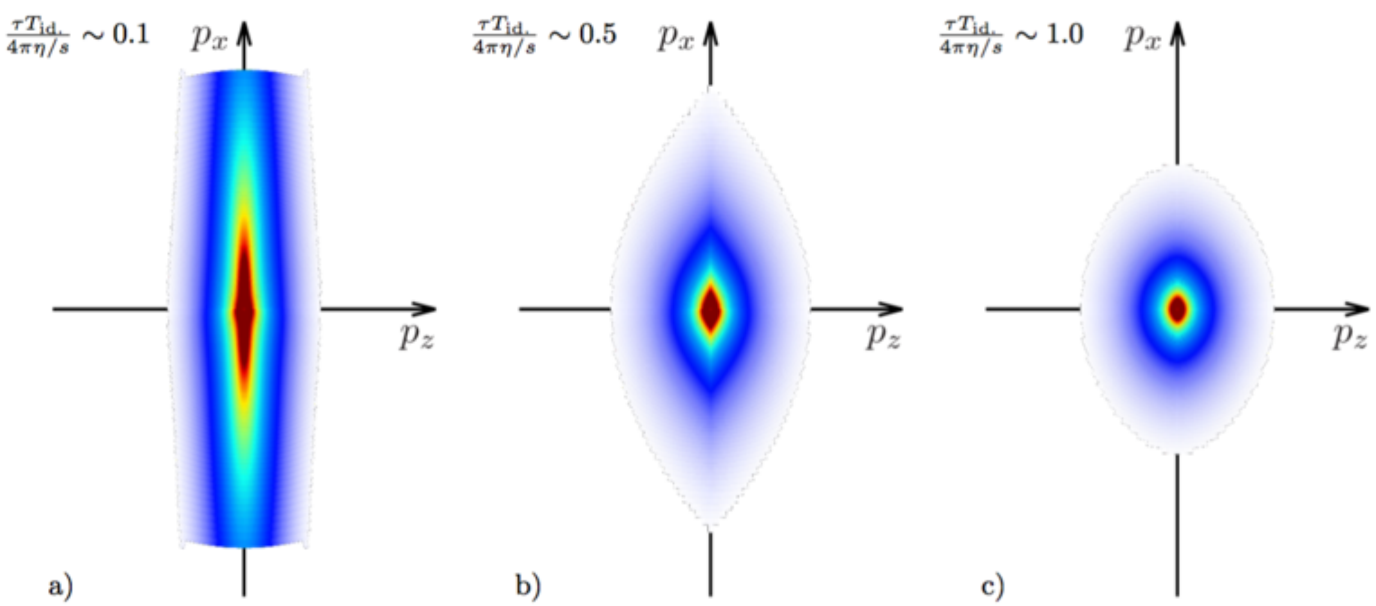
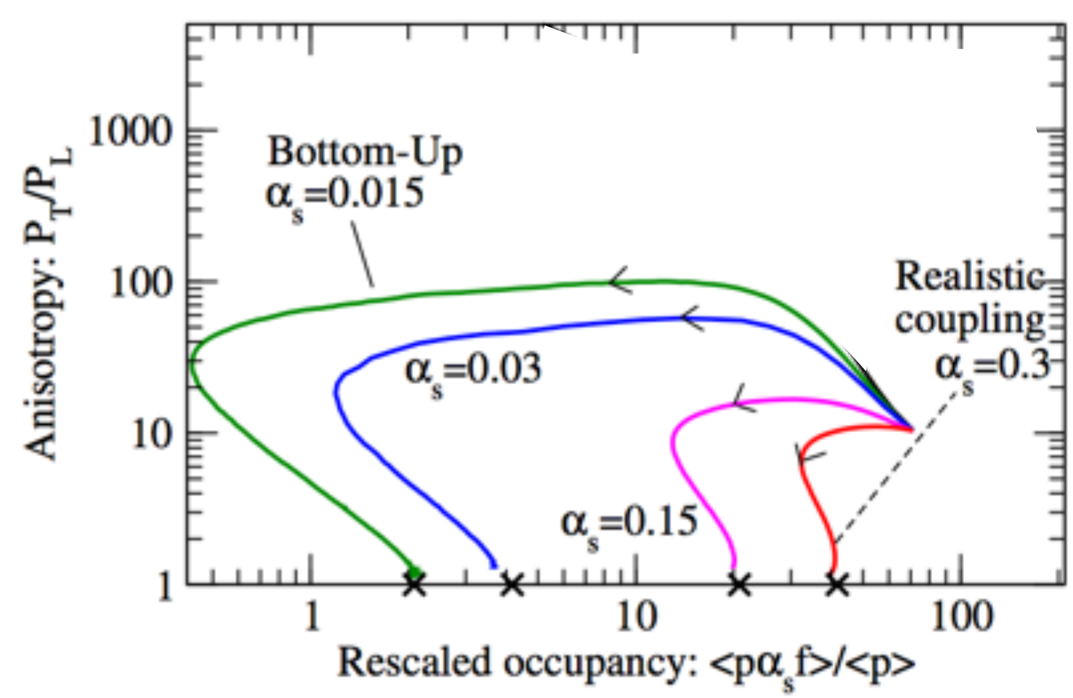




Beyond very early times equilibration process similar to jet-energy loss

Equilibration time determined by the time-scale for a mini-jet (p~Q_s) to loose all its energy

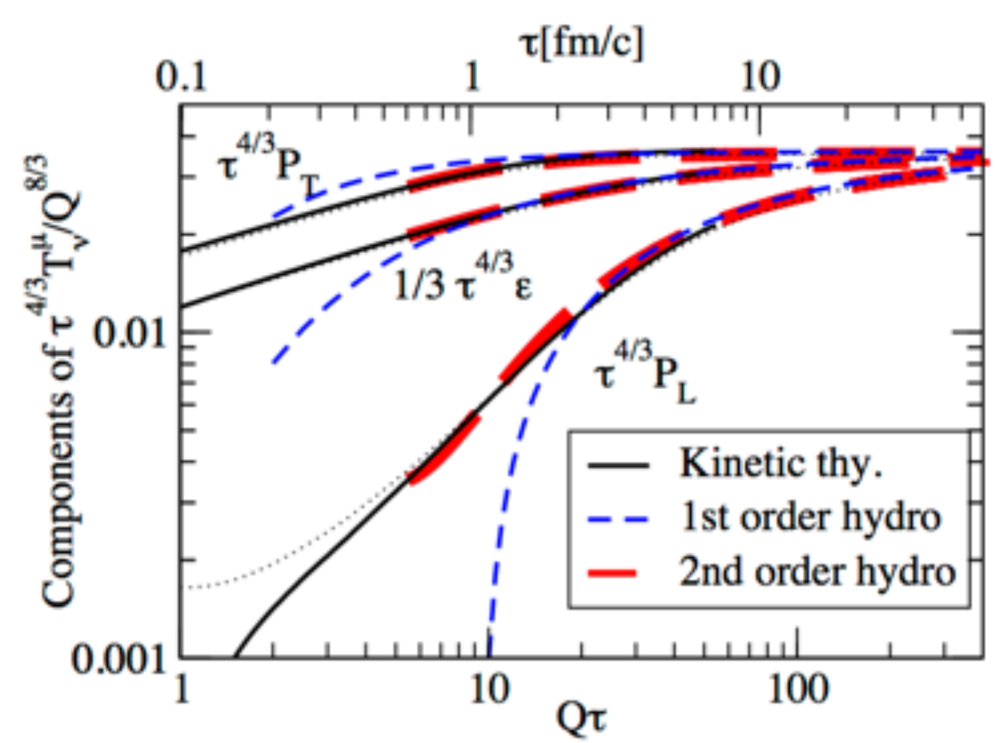
see also Xu, Greiner, Phys.Rev. C71 (2005) 064901

Hydrodynamic behavior

Extrapolations from weak-coupling limit to realistic values of α_s (~0.3) at RHIC & LHC energies yield results consistent with phenomenological estimates

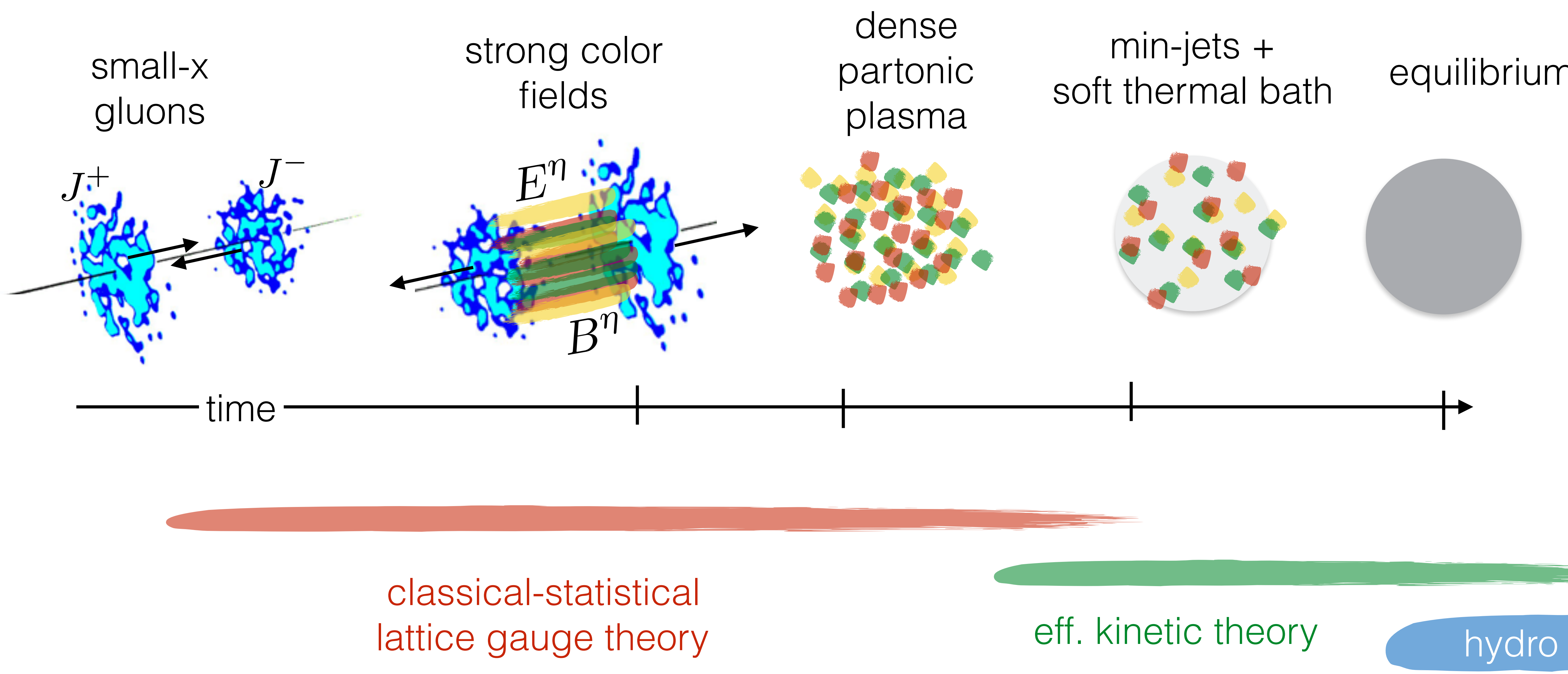
Viscous hydrodynamics applicable on time scales ~1 fm/c

Similar to strong coupling picture viscous hydrodynamics becomes applicable when pressure anisotropies are still O(1)



Kurkela, Zhu PRL 115 (2015) 182301

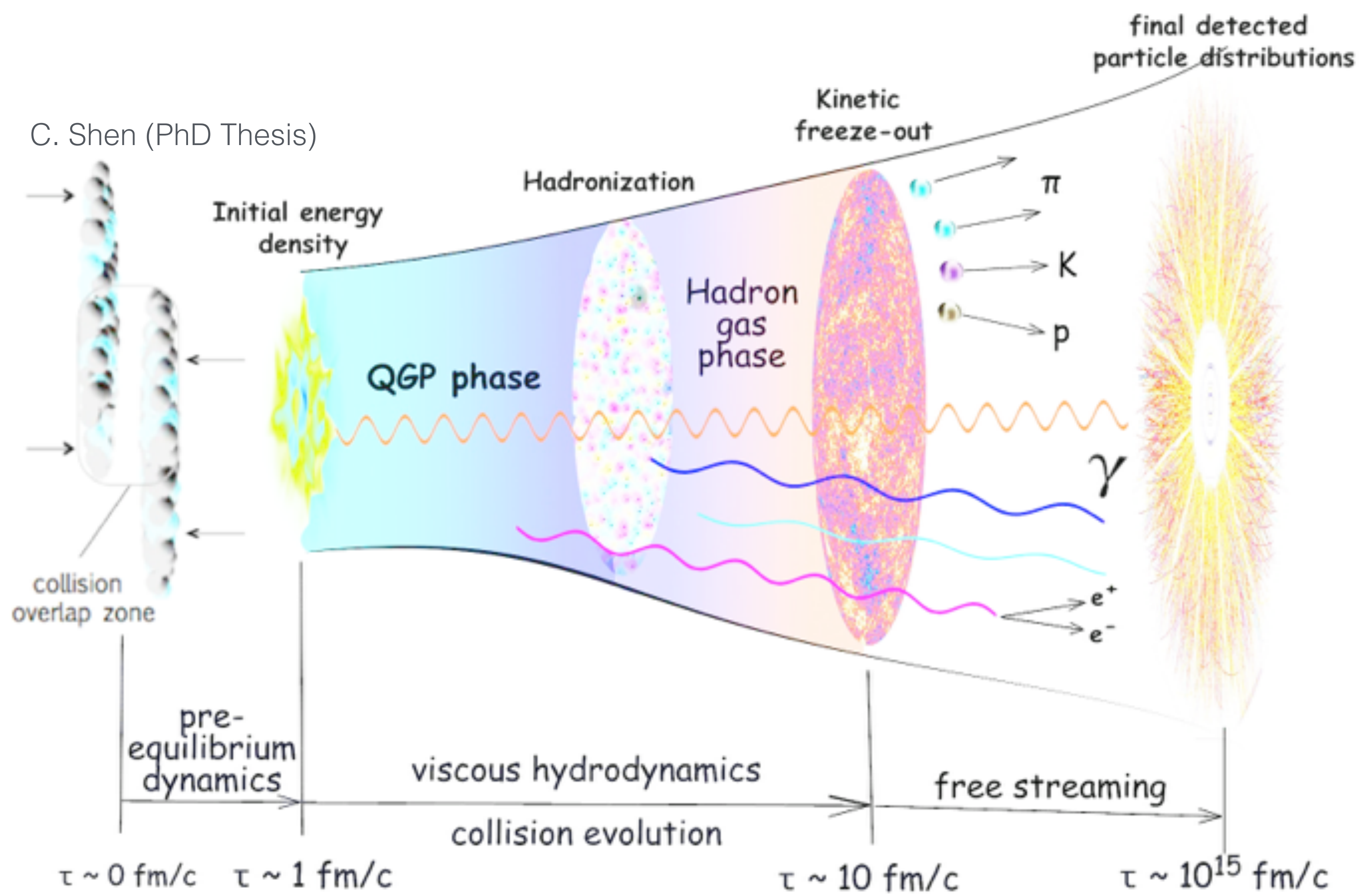
Early time dynamics & equilibration process



By combination of weak-coupling methods a complete description of early-time dynamics can be achieved

Event-by-event pre-equilibrium dynamics

<u>Goal:</u> Event-by-event initial conditions for hydro evolution from weakly coupled pre-equilibrium evolution

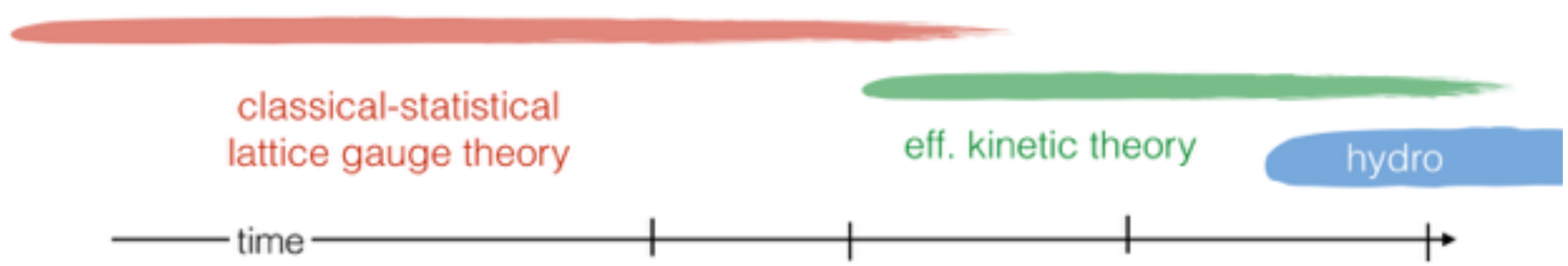


-> Eliminate uncertainties in extraction of QGP transport properties due to artificial time scale τ_{Hydro} when hydro simulation starts

Event-by-event pre-equilibrium dynamics

Challenge: Different degrees of freedom relevant at different times

classical fields, quasi-particles, energy-momentum tensor



Brute force calculation extremely challenging (but possible e.g. with BAMPS)

Greif, Greiner, Schenke, SS, Xu, Phys.Rev. D96 (2017) no.9, 091504

Ultimately we are only interested in calculation of energy-momentum tensor

Exploit memory loss to use macroscopic degrees of freedom
for description of pre-equilibrium dynamics

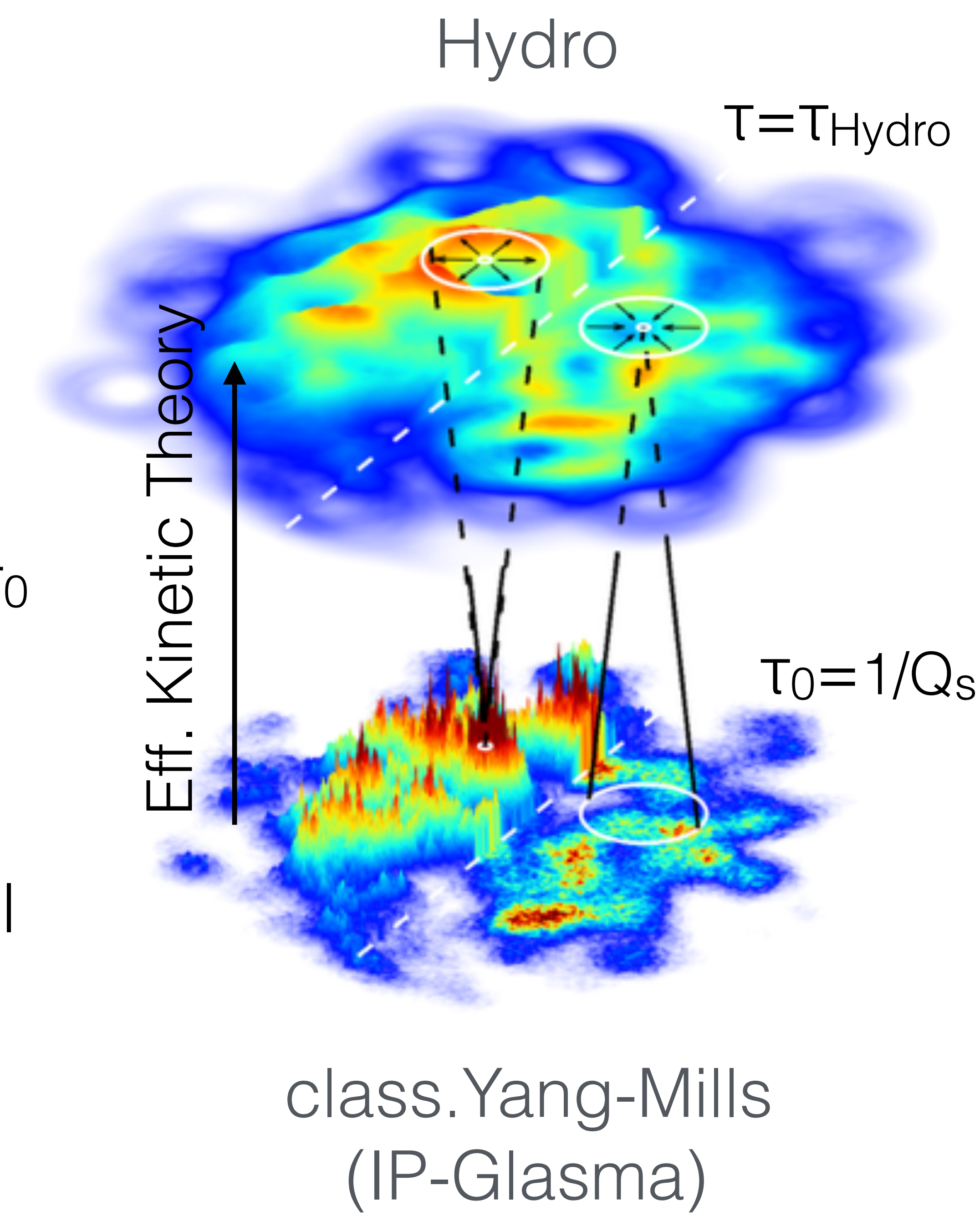
Macroscopic pre-equilibrium evolution

Extract energy-momentum tensor $T^{\mu\nu}(x)$ from initial state model (e.g. IP-Glasma)

Evolve $T^{\mu\nu}$ from initial time $\tau_0 \sim 1/Q_s$ to hydro initialization time τ_{Hydro} using eff. kinetic theory description

Causality restricts contributions to $T^{\mu\nu}(x)$ to be localized from causal disc $|x-x_0| < \tau_{Hydro}-\tau_0$ useful to decompose into a local average $T^{\mu\nu}_{BG}(x)$ and fluctuations $\delta T^{\mu\nu}(x)$

Since in practice size of causal disc is small $\tau_{Hydro}-\tau_0 << R_A$ fluctuations $\delta T^{\mu\nu}(x)$ around local average $T^{\mu\nu}_{BG}(x)$ are small and can be treated in a linearized fashion



Keegan,Kurkela, Mazeliauskas, Teaney JHEP 1608 (2016) 171
Kurkela, Mazeliauskas, Paquet, SS, Teaney (in preparation)

Macroscopic pre-equilibrium evolution

Effective kinetic description needs phase-space distribution $f(\tau, p, x)$

Memory loss: Details of initial phase-space distribution become irrelevant as system approaches local equilibrium

Can describe evolution of $T^{\mu\nu}$ in kinetic theory in terms of a representative phase-space distribution

$$f(\tau, p, x) = f_{BG}(Q_s(x)\tau, p/Q_s(x)) + \delta f(\tau, p, x)$$

where f_{BG} characterizes typical momentum space distribution, and δf can be chosen to represent local fluctuations of initial energy momentum tensor, e.g. energy density $\delta T^{\tau\tau}$ and momentum flow $\delta T^{\tau i}$

Energy perturbations:

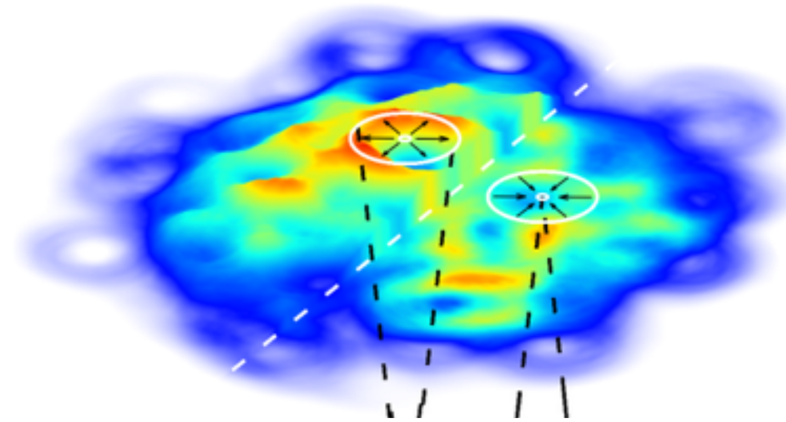
$$\delta f_s(\tau_0, p, x) \propto \frac{\delta T^{\tau\tau}(x)}{T^{\tau\tau}_{BG}(x)} \times \frac{\partial}{\partial Q_s(x)} f_{BG}\Big(\tau_0, p/Q_s(x)\Big)$$

local amplitude

representative form of phase-space distribution

Macroscopic pre-equilibrium evolution

Energy-momentum tensor on the hydro surface can be reconstructed directly from initial conditions according to

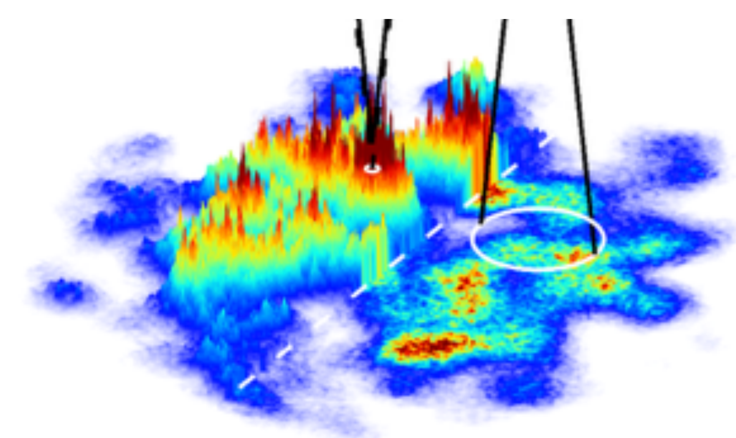


$$T^{\mu\nu}(\tau,x) = T^{\mu\nu}_{BG}\Big(Q_s(x)\tau\Big) + \int_{Disc} G^{\mu\nu}_{\alpha\beta}\Big(\tau,\tau_0,x,x_0,Q_s(x)\Big)\ \delta T^{\alpha\beta}(\tau_0,x_0)$$

non-equilibrium evolution of (local) average background

non-equilibrium Greens function of energy-momentum tensor

Effective kinetic theory simulations only need to be performed once to compute background evolution and Greens functions



Scaling variables

Background evolution and Greens functions still depend on variety of variables e.g. $Q_s(x)$ (local energy scale), α_s, (coupling constant) …

-> Identify appropriate scaling variables to reduce complexity

Since ultimately evolution will match onto visc. hydrodynamics, check wether hydrodynamics admits scaling solution

1st order hydro: $$T^{\tau\tau}(\tau) = T^{\tau\tau}_{Ideal}(\tau)\Big(1 - \frac{8}{3}\frac{\eta/s}{T_{eff}\tau} + ...\Big)$$

where $T^{\tau\tau}_{Ideal}(\tau)$ is the Bjorken energy density and $T_{eff} = \tau^{-1/3} \lim_{\tau\to\infty} T(\tau)\tau^{1/3}$

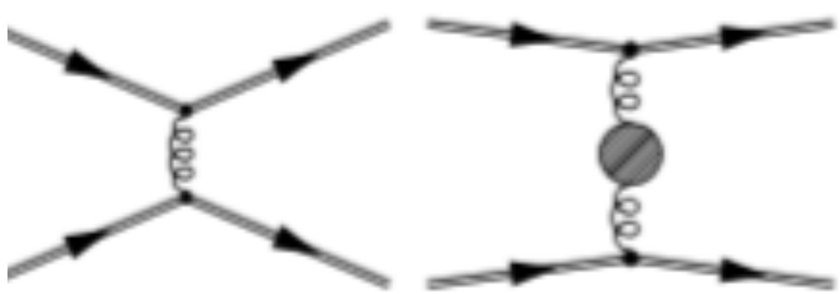
Natural candidate for scaling variable is $x_s = T_{eff}\tau/(\eta/s)$

(age of system / equilibrium relaxation rate)

Background — Kinetic theory simulation

Numerical simulation of background evolution in effective kinetic theory of Arnold, Moore, Yaffe

$$\left(\partial_\tau - \frac{p_z}{\tau}\right) f(\tau, |\mathbf{p}_\perp|, p_z) = \mathcal{C}[f] = \mathcal{C}_{2\leftrightarrow 2}[f] + \mathcal{C}_{1\leftrightarrow 2}[f]$$



elast. 2<->2 scattering
screened by Debye mass

collinear 1<->2 Bremsstrahlung
incl. LPM efffect
via eff. vertex re-summation

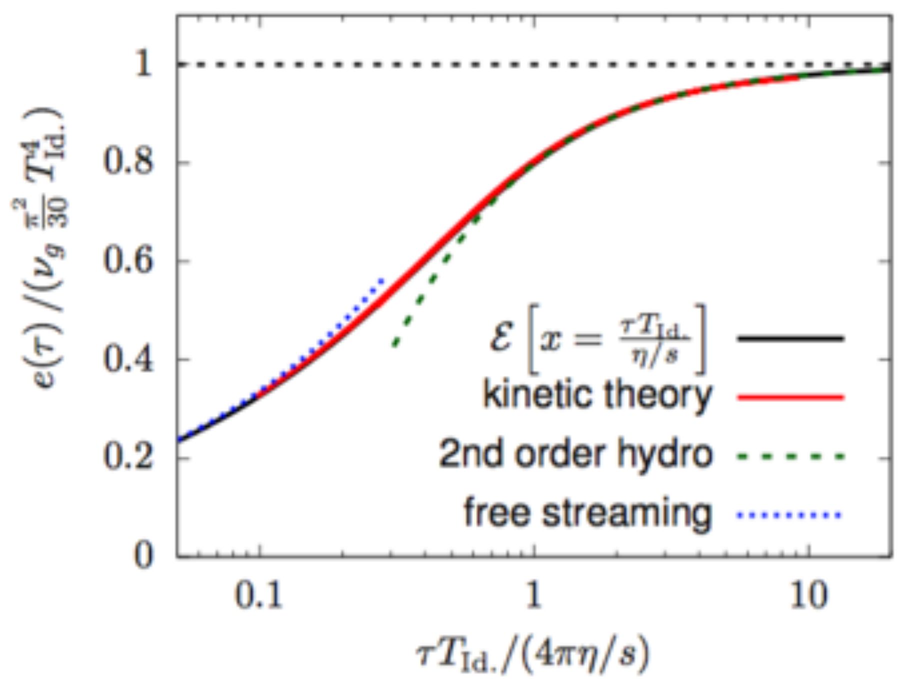
Solved directly as integro-differential equation for a discrete set of momenta

see Keegan,Kurkela,Mazeliauskas,Teaney JHEP 1608 (2016) 171 for details

Background — Scaling & Equilibration time

Non-equilibrium evolution of background
$T^{\mu\nu}$ is a unique function of x_s

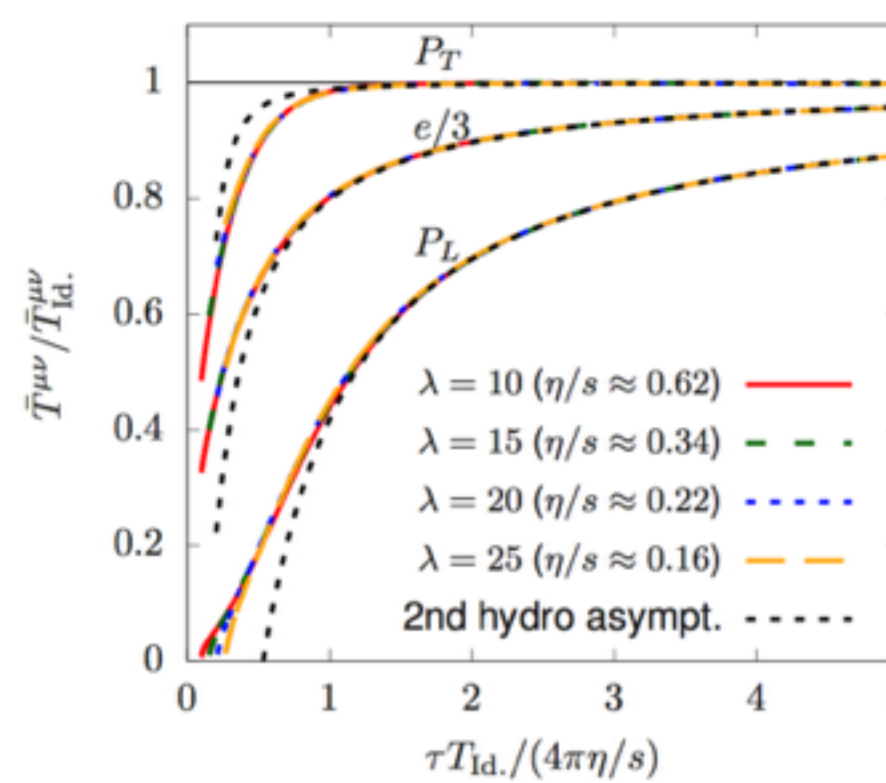
Scaling property extends beyond hydrodynamic regime in the relevant range of (large) couplings



Estimate of minimal time scale
for applicability of visc. hydrodynamics

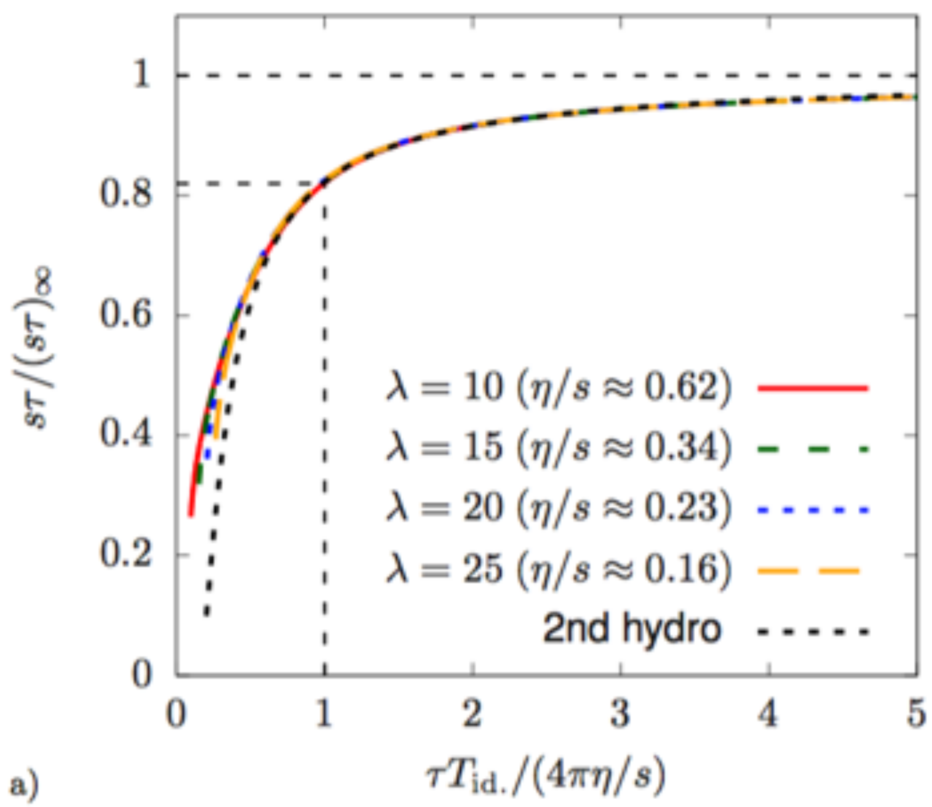
$$\tau_{\text{hydro}} \approx 0.85\,\text{fm}\left(\frac{4\pi(\eta/s)}{2}\right)^{\frac{3}{2}}\left(\frac{1.6\,\text{GeV}}{\langle\tau e^{3/4}\rangle}\right)^{1/2}$$

e.g. T_{Initial} ~0.75 GeV, η/s ~2/4π, τ_{Hydro}~0.85 fm/c



Kurkela, Zhu PRL 115 (2015) 182301
Kurkela, Mazeliauskas, Paquet, SS, Teaney (in preparation)

Background — Entropy prod. & hydro bounds



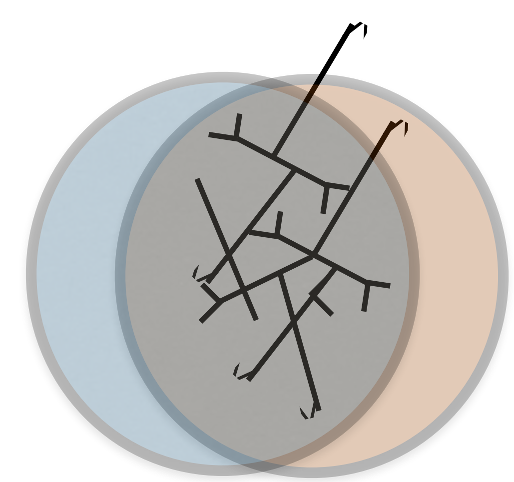
Significant amount of entropy production (x3) during the pre-equilibrium phase

Entropy <-> Multiplicity:
Important to relate properties of microscopic initial state (e.g. Q_s) to experimental data

Naive criterion for formation of hydrodynamic QGP:
equilibration time (τ_{Eq}) << system size (R)

$$\tau_{Eq} \simeq (4\pi\eta/s)^{3/2} \left(\frac{\frac{4}{3}\frac{\pi^2}{30}\nu_{eff}}{\langle s\tau\rangle_\infty} \right)^{1/2} \qquad \langle s\tau\rangle_\infty \simeq \frac{S}{N}\frac{1}{\pi R^2}\frac{dN}{dy}$$

$$\frac{\tau_{eq}}{R} \simeq \left(\frac{4\pi\eta/s}{2}\right)^{3/2} \left(\frac{dN/dy}{25}\right)^{-1/2} \left(\frac{S/N}{7}\right)^{-1/2} \left(\frac{\nu_{eff}}{16}\right)^{1/2}$$



Greens functions

Greens functions describe evolution of energy/momentum perturbations on top of a (locally) homogenous boost-invariant background

-> Description of perturbations in Fourier space

Decomposition in a complete basis of tensors leaves a total of 10 independent functions, e.g. for energy perturbations

energy response

$$\tilde{G}^{\tau\tau}_{\tau\tau}(\tau,\tau_0,\mathbf{k}) = \tilde{G}^{s}_{s}(\tau,\tau_0,|\mathbf{k}|)\ ,$$

momentum response

$$\tilde{G}^{\tau i}_{\tau\tau}(\tau,\tau_0,\mathbf{k}) = \frac{\mathbf{k}^i}{|\mathbf{k}|}\tilde{G}^{v}_{s}(\tau,\tau_0,|\mathbf{k}|)\ ,$$

shear stress response

$$\tilde{G}^{ij}_{\tau\tau}(\tau,\tau_0,\mathbf{k}) = \tilde{G}^{t,\delta}_{s}(\tau,\tau_0,|\mathbf{k}|)\ \delta^{ij} + \tilde{G}^{t,k}_{s}(\tau,\tau_0,|\mathbf{k}|)\ \frac{\mathbf{k}^i\mathbf{k}^j}{|\mathbf{k}|^2}\ ;$$

Numerically computed in eff. kinetic theory by solving linearized Boltzmann equation on top of non-equilibrium background

$$\left(\partial_\tau + \frac{i\mathbf{p}_\perp\mathbf{k}_\perp}{p} - \frac{p_z}{\tau}\right)\ \widetilde{\delta f}(\tau,|\mathbf{p}_\perp|,p_z;\ \mathbf{k}_\perp) = \delta\mathcal{C}[f,\widetilde{\delta f}]$$

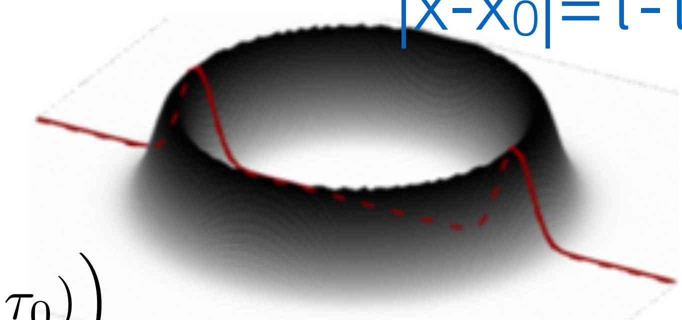
and computing appropriate moments of δf

Greens functions

Free-streaming:

coordinate space

Energy-momentum perturbations propagate as a concentric wave traveling at the speed of light



energy/momentum response:

$$G_s^{s/v}(\tau,\tau_0,\mathbf{x}-\mathbf{x}_0) = \frac{1}{2\pi(\tau-\tau_0)}\delta\Big(|\mathbf{x}-\mathbf{x}_0| - (\tau-\tau_0)\Big)$$

Hydrodynamic response functions in the limit of large times $x_s >> 1$ and small wave-number $k\,(\tau-\tau_0) << x_s^{1/2}$

(c.f. Vredevoogd,Pratt PRC79 (2009) 044915, Keegan,Kurkela, Mazeliauskas, Teaney JHEP 1608 (2016) 171)

energy response: $$\tilde{G}_s^s(\tau,\tau_0,k) = \tilde{G}_s^s(\tau,\tau_0,k=0)\left(1-\frac{1}{2}k^2(\tau-\tau_0)^2\,\tilde{s}_s^{(2)}+...\right),$$

momentum response: $$\tilde{G}_s^v(\tau,\tau_0,k) = \tilde{G}_s^s(\tau,\tau_0,k=0)\left(\quad k(\tau-\tau_0)\,\tilde{s}_v^{(1)}+...\right),$$

shear response: determined by hydrodynamic constitutive relations

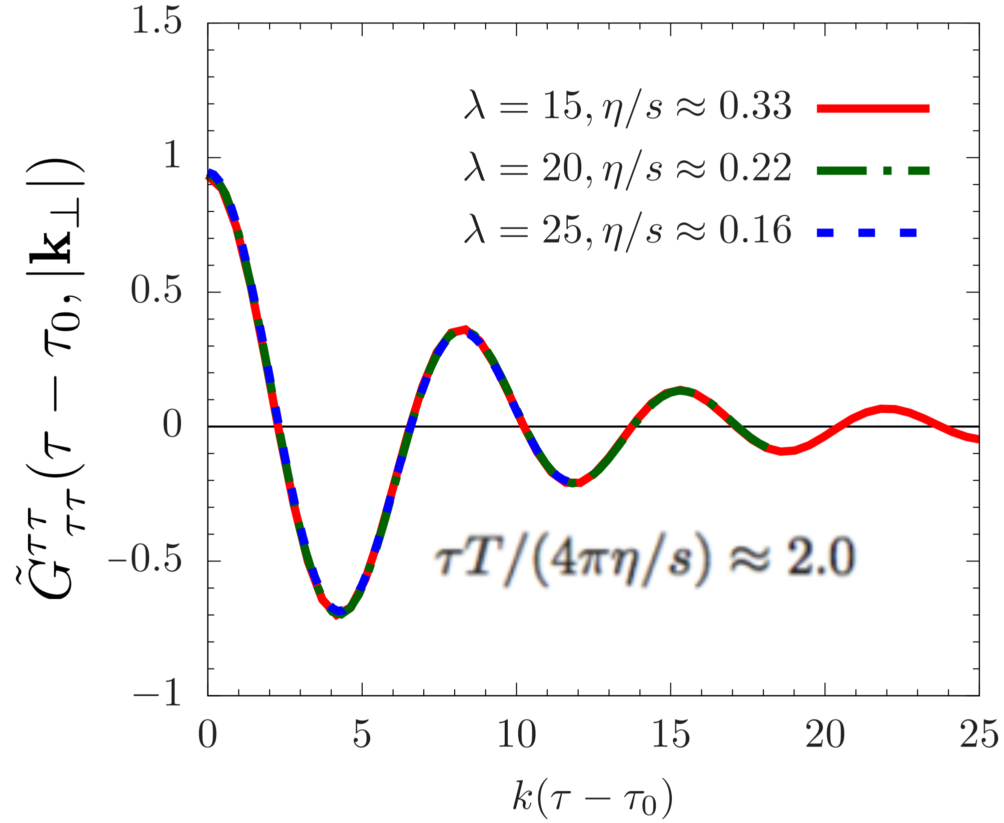
$$\tilde{G}_s^s(\tau,\tau_0,k=0) = \left(\frac{T^{\tau\tau}(\tau_0)}{T^{\tau\tau}(\tau)}\right)\left(\frac{3T^{\tau\tau}(\tau)-T^{\eta}_{\eta}(\tau)}{3T^{\tau\tau}(\tau_0)-T^{\eta}_{\eta}(\tau_0)}\right) \qquad \tilde{s}_s^{(2)}=\frac{1}{2}, \quad \tilde{s}_v^{(1)}=\frac{1}{2}+\frac{1}{2}\frac{\eta/s}{\tau T_{\rm id.}},$$

background evolution

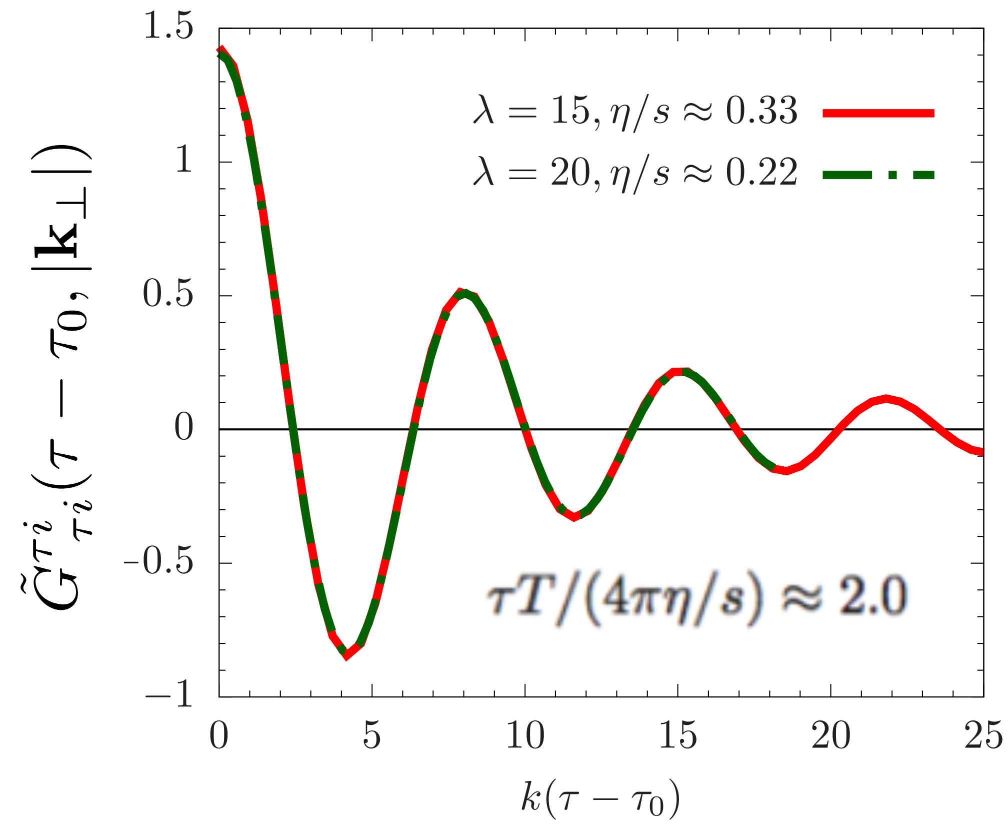
"long wave-length constants"

Greens functions — Scaling variables

Energy response
to energy perturbation



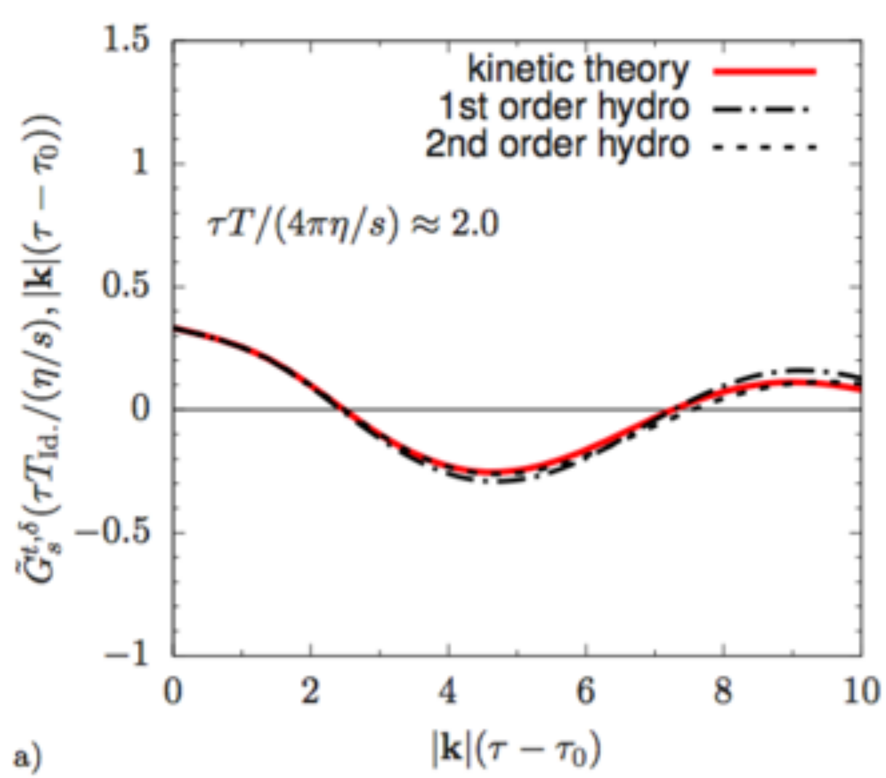
Momentum response
to momentum perturbations



Non-equilibrium Greens functions show universal scaling in $x_s = T_{eff}\tau/(\eta/s)$ and $k(\tau - \tau_0)$ beyond hydro limit

Greens functions — Scaling variables

Shear stress response to energy perturbation



Non-equilibrium Greens functions show universal scaling in $x_s = T_{eff}\tau/(\eta/s)$ and $k(\tau-\tau_0)$ beyond hydro limit

Satisfy hydrodynamic constitutive relations for sufficiently large times $x_s >> 1$ and long wave-length $k(\tau-\tau_0) << x_s^{1/2}$

KoMPoST

Scaling properties ensure that pre-equilibrium evolution of energy momentum tensor can be expressed in terms of

Background: $T^{\mu\nu}_{BG}(x_s)$ Greens-functions: $G^{\mu\nu}_{\alpha\beta}\left(x_s, \frac{x-x_0}{\tau-\tau_0}\right)$

computed once and for all in numerical kinetic theory simulation

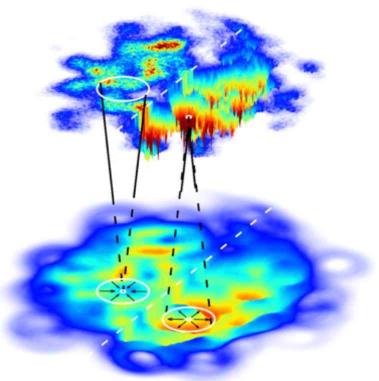
Dependence of coupling constant **α_s** has been re-expressed in terms of physical parameter **η/s**, can now perform event-by-event simulations for variety of macroscopic physical parameters

General framework for event-by-event pre-equilibrium dynamics (KoMPoST):

Input: Out-of-equilibrium energy-momentum tensor; η/s

non-equilibrium evolution in linear response formalism

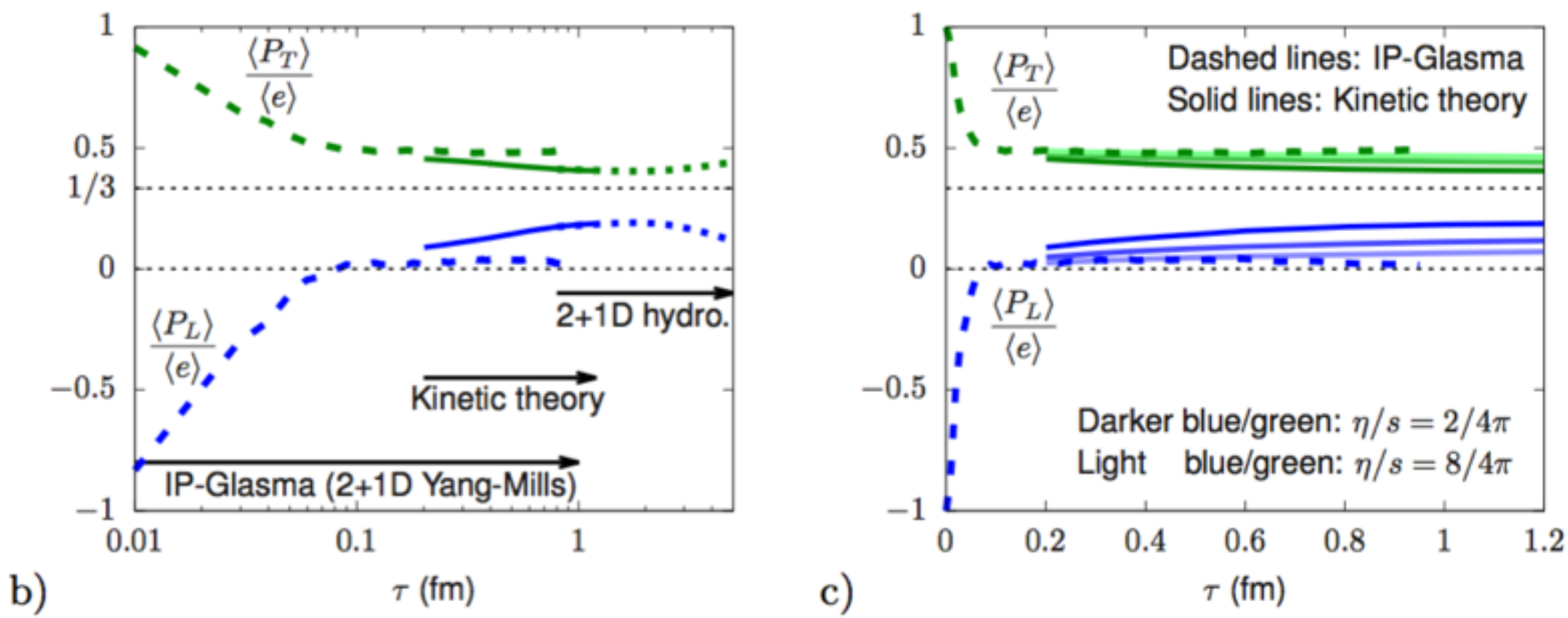
Output: Energy-momentum tensor at τ_{Hydro} when visc. hydro becomes applicable



Event-by-event pre-equilibrium evolution

1) Evolve class. Yang-Mills fields to early time τ_0 = 0.2 fm/c (IP-Glasma)
2) Macroscopic pre-equilibrium evolution to hydro initialization time τ_{Hydro}
3) Hydrodynamic evolution from τ_{Hydro} ($\eta/s = 2/(4\pi)$ | conformal EoS)

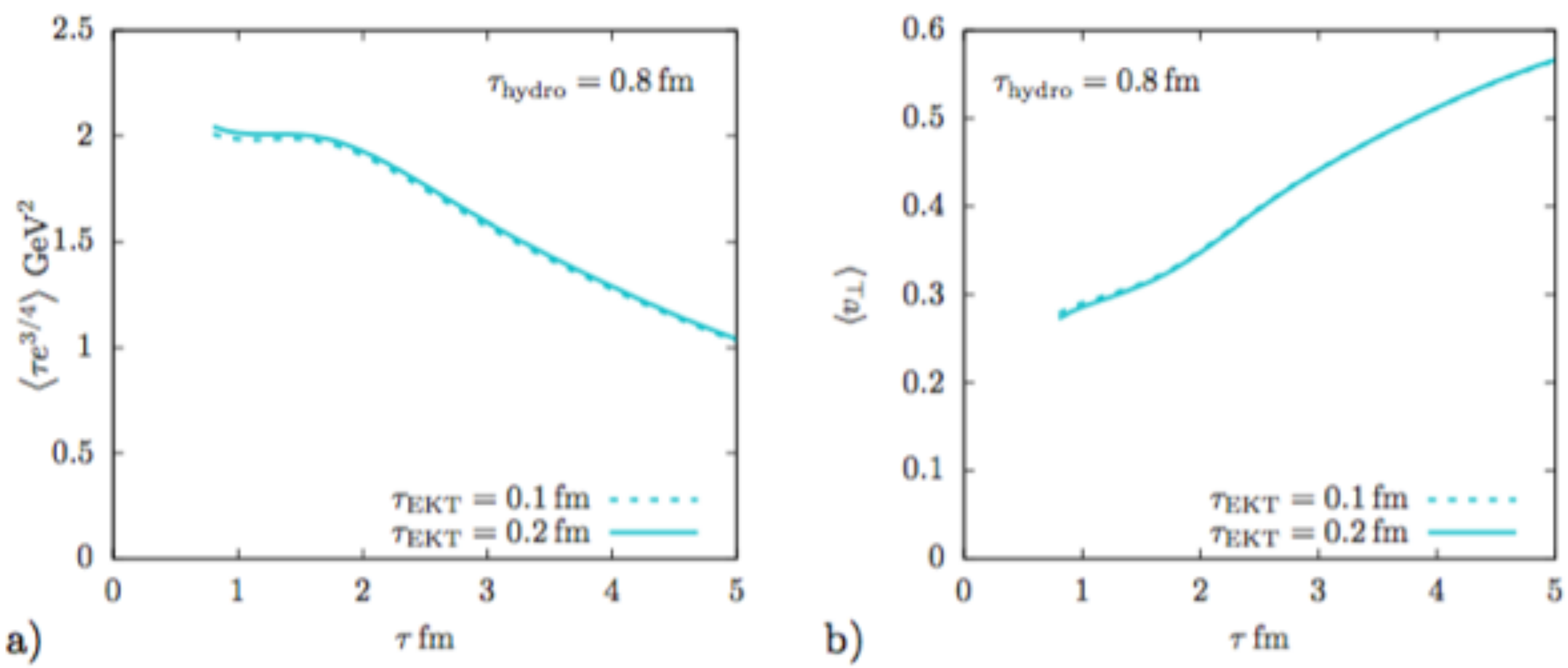
Energy/pressure evolution in central Pb+Pb collision



Based on combination of weak-coupling methods can consistently describe early-time dynamics until onset of hydro

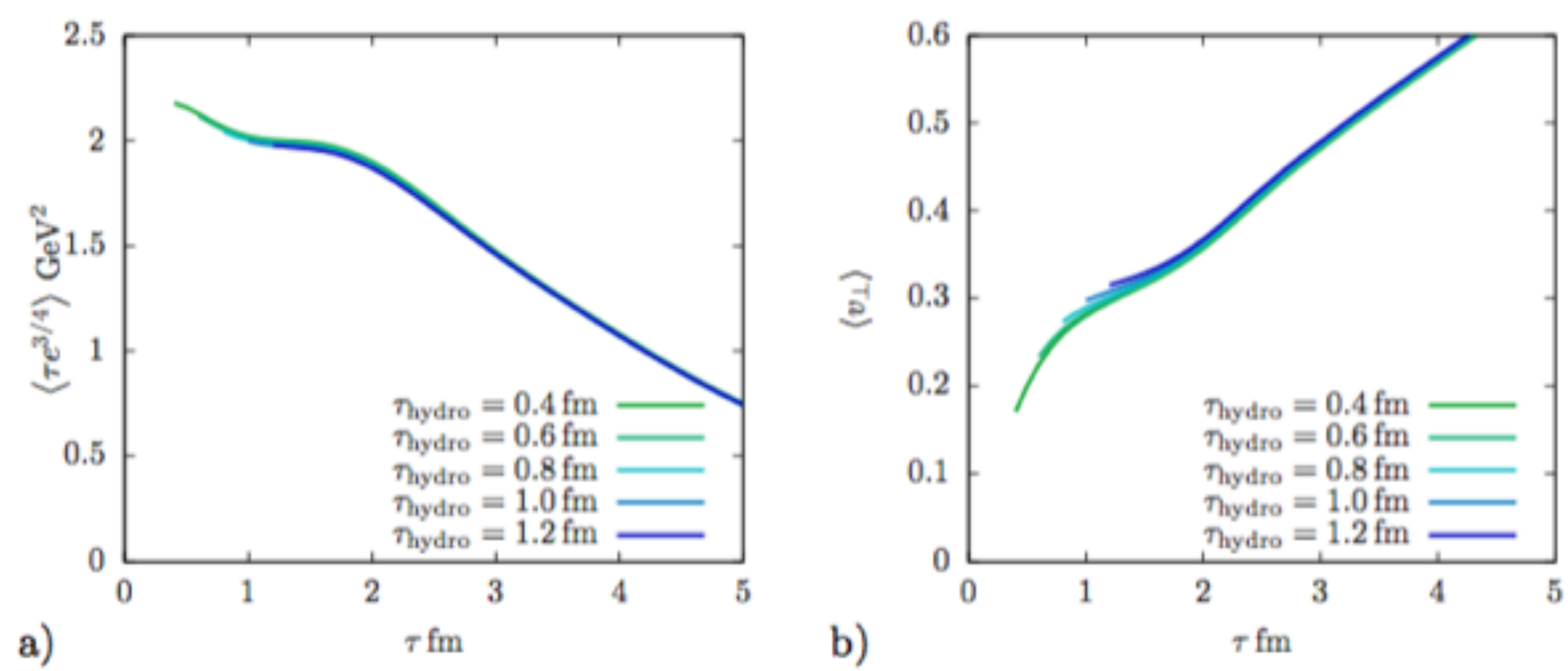
Event-by-event pre-equilibrium evolution

Energy density & radial flow in central Pb+Pb collision



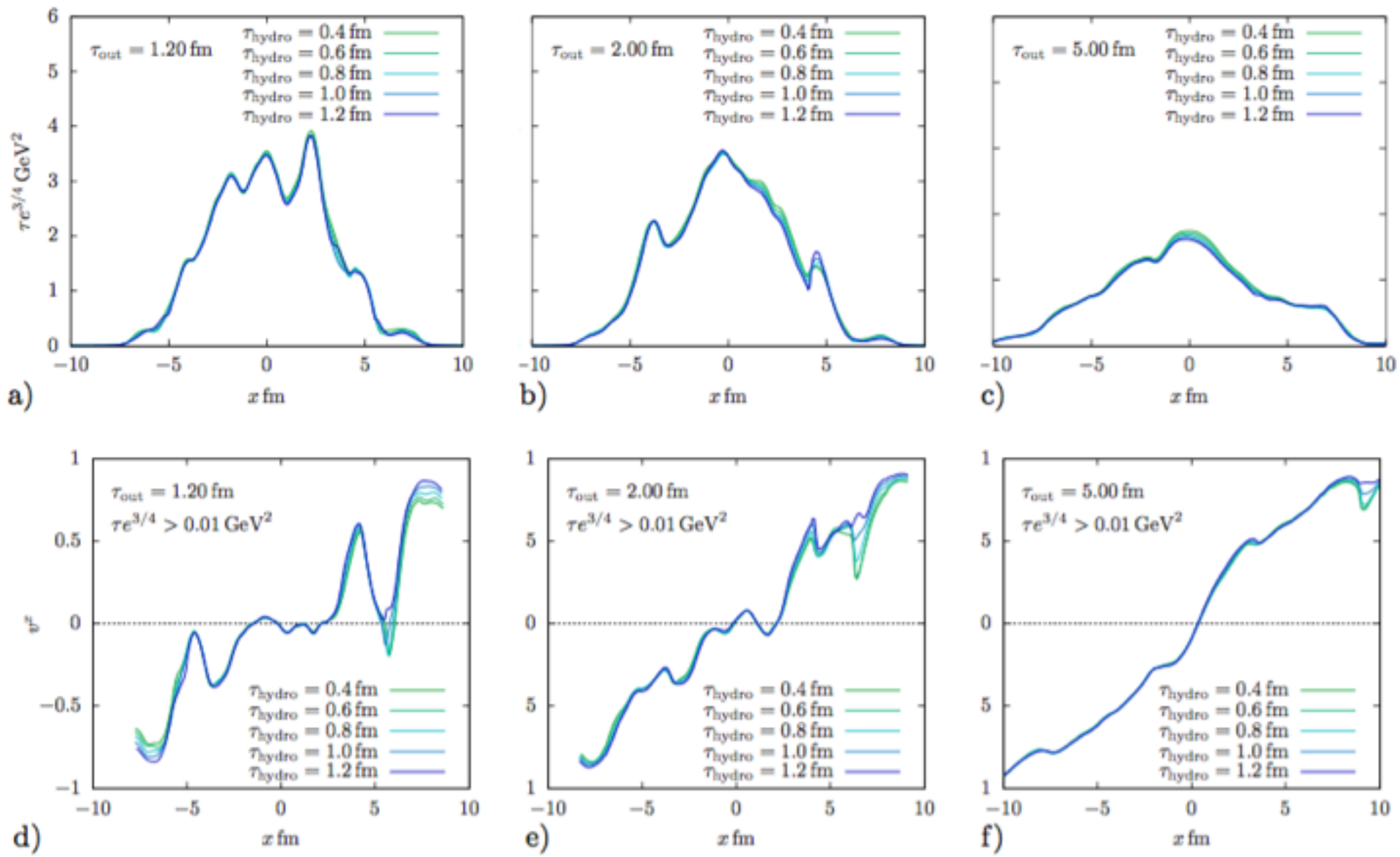
Overlap in the range of validity ensures smooth transition from CYM to EKT to Hydro

No sensitivity to switching times τ_{EKT} , τ_{Hydro} in sensible range



Event-by-event pre-equilibrium evolution

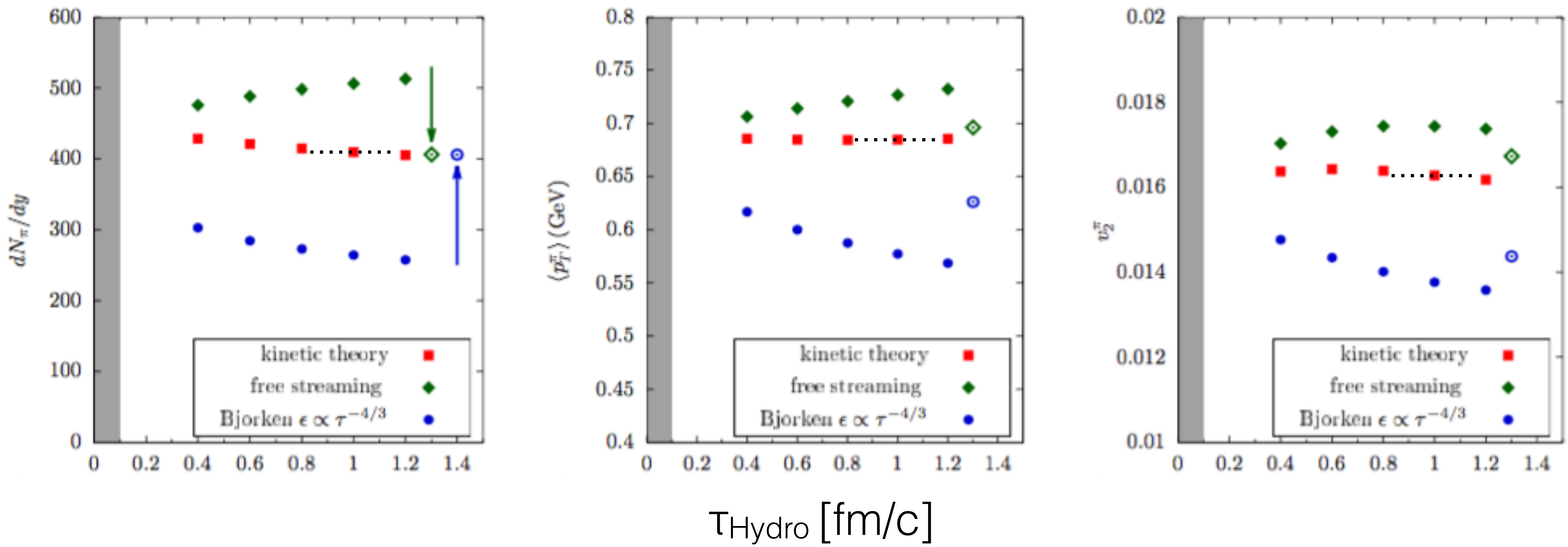
Energy density profile in Pb+Pb collision



Even with QCD EoS sensitivity to switching time τ_{Hydro} from pre-equilibrium to hydro is negligible

Event-by-event pre-equilibrium evolution

Hadronic observables in single (MC-Glauber) Pb+Pb event:



Very little to no sensitivity to switching time τ_{Hydro} from pre-equilibrium to hydro for dN/dy, <p_T>, <v_2>, …

Conclusions & Outlook

Significant progress in understanding early time dynamics of heavy-ion collisions from weak-coupling perspective

Development of macroscopic description of pre-equilibrium dynamics which enables event-by-event description of heavy-ion collisions from beginning to end

Description in macroscopic framework is completely general and can be used beyond weak coupling limit

-> Direct comparisons with other theoretical approaches (e.g. strong coupling limit) possible

Several interesting directions beyond bulk phenomenology

Quarks: chemical equilibration, electro-magnetic probes, anomalous transport

Non-eq: small systems, unified description of soft & hard physics