SU(3) & SU(4) extended linear sigma model

ABDEL NASSER TEWFIK

Egyptian Center for Theoretical Physics (ECTP), MTI, Cairo-Egypt

World Laboratory for Cosmology And Particle Physics (WLCAPP), Egypt

Goethe Universität Frankfurt,

Institut für Theoretische Physik

30th September 2016

Contents

Partly published in

1. 1608.01034 [hep-ph], Int.J.Adv.Res.Phys.Sci. 3 (2016) 4-14
2. 1606.09153 [hep-ph], Indian J. Phys. (2016)
3. 1604.08174 [hep-lat], European Phys. J. A (2016)
4. 1604.00043 [hep-ph], Adv.High Energy Phys. 2016 (2016) 1381479
5. 1510.02673 [nucl-th], J.Phys.Conf.Ser. 668 (2016) no.1, 012082
6. 1509.07114 [hep-ph], Adv.High Energy Phys. 2015 (2015) 563428
7. 1501.01124 [hep-ph], Phys.Rev. C91 (2015) 015206
8. 1412.2395 [hep-ph], Phys.Rev. C91 (2015) 015204
9. 1411.1871 [hep-ph], J.Phys. G42 (2015) 015004
10. 1406.7488 [hep-ph], Phys.Rev. C90 (2014) 015204
11. 1405.0577 [hep-ph], Phys.Rev. C89 (2014) 055210

Abdel Magied Diab

Sigma Model

was introduced by Gell-Mann and Levy in 1960
Long before the invention of QCD

Il Nuovo Cimento 16, 705 (1960)

The name σ-model comes from a field corresponding to the spinless meson scalar *σ* introduced earlier by Schwinger.

It describes a physical system with the Lagrangian

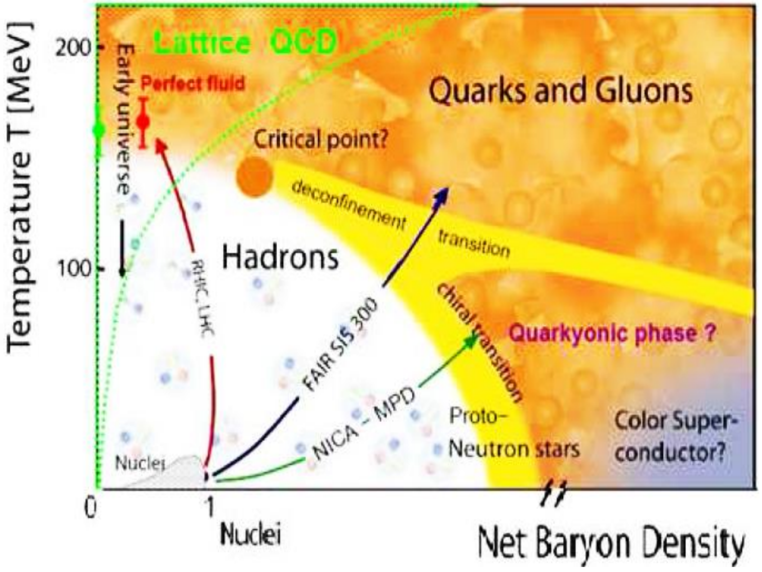
$$\mathcal{L}(\phi_1, \phi_2, \ldots, \phi_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij}\, \mathrm{d}\phi_i \wedge *\mathrm{d}\phi_j$$ Wedge Product

fields φ_i represent map from base manifold space-time (worldsheet) to target (Riemannian) manifold of the scalars linked together by internal symmetries,

scalars gij determine linear and non-linear properties.

Why Sigma Model?

- LSM is one of lattice QCD alternatives.
- It doesn't need supercomputers.
- One can use PC and easily computational techniques.
- Various symmetry-breaking scenarios can be investigated in a more easy way, for instance, various properties of strongly interacting matter can be studied
    - QCD Equation of State
    - Chiral phase structure of masses
    - QGP transport properties
    - ….etc.



LSM: Chiral symmetry breaking

Vector field under unitary transformation $\vec{\Phi} \Longrightarrow e^{-i\ \theta^a\ T^a_{ij}}\ \vec{\Phi}$

θ^a is rotational angle T^a_{ij} being a matrix generating transformation.

Vector transformation

$$\Psi \Longrightarrow e^{-i\ \frac{\bar{\tau}}{2}\vec{\theta}}\ \Psi \ \approx \ (1 - i\frac{\bar{\tau}}{2}\vec{\theta})\ \Psi$$

$$\bar{\Psi} \Longrightarrow e^{+i\ \frac{\bar{\tau}}{2}\vec{\theta}}\ \bar{\Psi} \ \approx \ (1 + i\frac{\bar{\tau}}{2}\vec{\theta})\ \bar{\Psi}.$$

τ Pauli matrices

Axial-vector transformation

$$\Psi \Longrightarrow e^{-i\gamma_5\ \frac{\bar{\tau}}{2}\vec{\theta}}\ \Psi \ \approx \ (1 - i\gamma_5\frac{\bar{\tau}}{2}\vec{\theta})\ \Psi$$

$$\bar{\Psi} \Longrightarrow e^{-i\ \gamma_5\frac{\bar{\tau}}{2}\vec{\theta}}\ \bar{\Psi} \ \approx \ (1 - i\gamma_5\frac{\bar{\tau}}{2}\vec{\theta})\ \bar{\Psi}.$$

γ Gell-Mann matrices

LSM: Chiral symmetry breaking

For a massless fermion: $\mathcal{L}_D = \bar{\psi}\,(i\gamma_\mu \partial^\mu)\,\psi.$

Both vector and axialvector transformations are invariant:

$$\bar{\psi}(i\gamma_\mu \partial^\mu)\psi \Longrightarrow \bar{\psi}(i\gamma_\mu \partial^\mu)\psi$$

For a massive fermion: $\mathcal{L}_D = \bar{\psi}(i\gamma_\mu \partial^\mu - m^2)\psi$

Vector transformation is invariant, WHILE axialvector NOT

$$m\,\bar{\psi}\,\psi \Longrightarrow e^{-i\,\gamma^5 \frac{\vec{\tau}}{2}\vec{\theta}}\, m\,\bar{\psi}\,\psi \;\approx\; (1 - i\gamma^5 \frac{\vec{\tau}}{2}\vec{\theta})\, m\,\bar{\psi}\,\psi$$

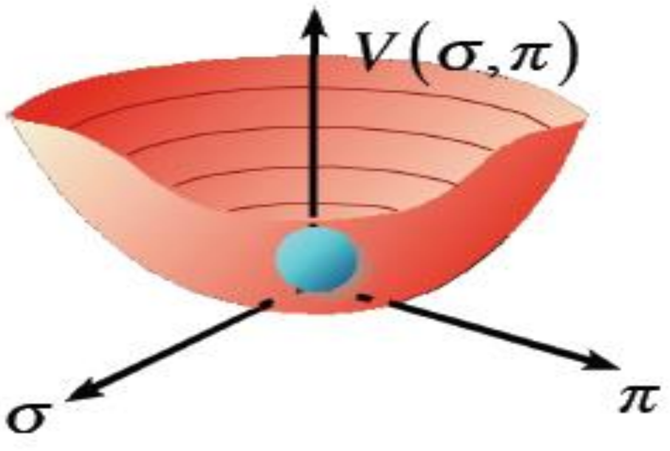
$$= m\,\bar{\psi}\,\psi - 2im\,\bar{\theta}(\bar{\psi}\gamma_5 \frac{\tau}{2}\psi)$$

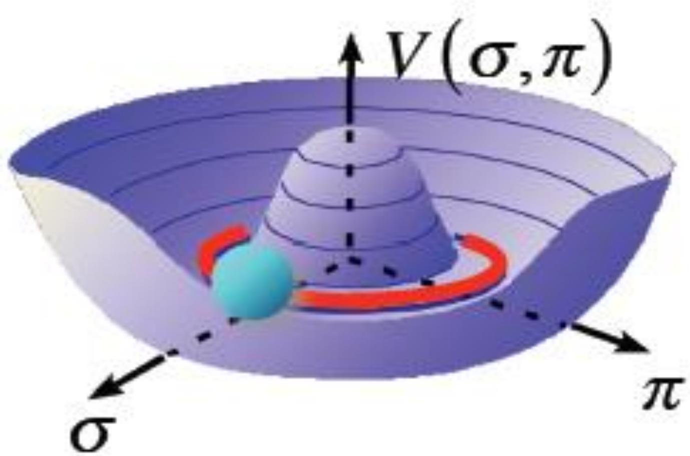
LSM: chiral symmetry

Lagrangian of massless fermions is invariant under chiral transformation BUT massive ones cause spontaneous symmetry breaking

Minimum energy configuration is given as shown in the potential energy

Minimum energy (density) is given by an any point on the circle (1,1)





Ground state is right in the middle (0,0) and the potential plus ground state are still invariant under rotations

Ground state is away from center. The point at center is local maximum of potential and thus unstable

WLCAPP World Laboratory Cosmology And Particle Physics

SU(2) LSM: Lagrangian

EGYPTIAN CENTER FOR THEORETICAL PHYSICS
ECTP
CENTER OF EXCELLENCE

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial_\mu\psi + \frac{1}{2}(\partial_\mu\pi\partial^\mu\pi + \partial_\mu\sigma\partial^\mu\sigma)$$

K. E. of nucleons

K. E. of mesons

$$- g_\pi \left[(i\bar{\psi}\gamma_5\bar{\tau})\bar{\pi} + (i\bar{\psi}\psi)\sigma\right]$$

nucleons-mesons interactions

$$- \frac{\lambda}{4}\left((\bar{\pi}^2 + \sigma^2) - f_\pi^2\right)$$

Interaction term

Pion-Nucleon Potential

Int.J.Mod.Phys. E6 (1997) 203

U(Nf) LSM: Lagrangian

For any Nf flavors, chiral Lagrangian of U(Nf)r × U(Nf)ℓ linear-sigma model

$$\mathcal{L}_{chiral} = \mathcal{L}_q + \mathcal{L}_m$$

Fermion

$$\mathcal{L}_q = \sum_f \overline{q}_f \left[i\gamma^\mu D_\mu - g T_a (\sigma_a + i\gamma_5 \pi_a) \right] q,$$

Meson

$$\mathcal{L}_m = \mathrm{Tr}(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2 \Phi^\dagger \Phi) - \lambda_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \mathrm{Tr}(\Phi^\dagger \Phi)^2 + c[\mathrm{Det}(\Phi) + \mathrm{Det}(\Phi^\dagger)] + \mathrm{Tr}[H(\Phi + \Phi^\dagger)],$$

where Φ is 3×3 matrix includes the nonet meson states as

$$\Phi = \sum_{a=0}^{N_f^2-1} T_a (\sigma_a - i\pi_a).$$

Phys. Rev. D 62, 085008 (2000).

Physical scalar and pseudoscalar meson fields

By using Pauli and Gell-Mann Matrices, we find

For $N_f = 2$,

$$T_a \sigma_a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\sigma_0 + \frac{1}{\sqrt{2}} a_0^0 & a_0^+ \\ a_0^- & \frac{1}{\sqrt{2}}\sigma_0 - \frac{1}{\sqrt{2}} a_0^0 \end{pmatrix},$$

$$T_a \pi_a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi_0 + \frac{1}{\sqrt{2}} \pi^0 & \pi^+ \\ \pi^- & \frac{1}{\sqrt{2}}\pi_0 - \frac{1}{\sqrt{2}} \pi^0 \end{pmatrix}.$$

Physical scalar and pseudoscalar meson fields

For $N_f = 3$,

$$T_a\sigma_a = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{6}}\sigma_8 + \frac{1}{\sqrt{3}}\sigma_0 & a_0^- & \kappa^- \\ a_0^+ & -\frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{6}}\sigma_8 + \frac{1}{\sqrt{3}}\sigma_0 & \bar{\kappa}^0 \\ \kappa^+ & \kappa^0 & -\frac{2}{\sqrt{3}}\sigma_8 + \frac{1}{\sqrt{3}}\sigma_0 \end{pmatrix}$$

$$T_a\pi_a = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\pi_8 + \frac{1}{\sqrt{3}}\pi_0 & \pi^- & K^- \\ \pi^+ & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\pi_8 + \frac{1}{\sqrt{3}}\pi_0 & \bar{K}^0 \\ K^+ & K^0 & -\frac{2}{\sqrt{3}}\pi_8 + \frac{1}{\sqrt{3}}\pi_0 \end{pmatrix}$$

For $N_f = 4$,

$$T_a \sigma_a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{a0}{\sqrt{2}} + \frac{\sigma 0}{2} + \frac{\sigma 15}{2\sqrt{3}} + \frac{\sigma 8}{\sqrt{6}} & a_0^+ & k^+ & D_0^{*0} \\ a_0^- & -\frac{a0}{\sqrt{2}} + \frac{\sigma 0}{2} + \frac{\sigma 15}{2\sqrt{3}} + \frac{\sigma 8}{\sqrt{6}} & k^0 & D_0^{*-} \\ k^- & k00 & \frac{\sigma 0}{2} + \frac{\sigma 15}{2\sqrt{3}} - \frac{2\,\sigma 8}{\sqrt{6}} & D_{S0}^{*-} \\ \bar{D}_0^{*0} & D_0^{*+} & D_{S0}^{*+} & \frac{\sigma 0}{2} - \frac{3\,\sigma 15}{2\sqrt{3}} \end{pmatrix}$$

$$T_a \pi_a : \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\pi 00}{\sqrt{2}} + \frac{\pi 0}{\sqrt{3}} + \frac{\pi 15}{2\sqrt{3}} + \frac{\pi 8}{\sqrt{6}} & \pi^+ & K^+ & D^0 \\ \pi^- & -\frac{\pi 00}{\sqrt{2}} + \frac{\pi 0}{\sqrt{3}} + \frac{\pi 15}{2\sqrt{3}} + \frac{\pi 8}{\sqrt{6}} & K^0 & D^- \\ K^- & \bar{K}^0 & \frac{\pi 0}{\sqrt{3}} + \frac{\pi 15}{2\sqrt{3}} - \frac{2\,\pi 8}{\sqrt{6}} & D_S^- \\ \bar{D}^0 & D^+ & D_S^+ & \frac{\pi 0}{\sqrt{3}} - \frac{3\,\pi 15}{2\sqrt{3}} \end{pmatrix}$$

Thermodynamic potential

In nf=2 LSM, the transformations

$$q \to (1 + i\,\theta)q \;\Rightarrow\; \delta q = iq$$

$$\bar{q} \to (1 - i\theta)\bar{q} \;\Rightarrow\; \delta\bar{q} = -i\bar{q}$$

leads to Noether's current

$$j^{\mu} = \frac{\partial\, L}{\partial_{\mu}\,\psi}\delta\psi + \frac{\partial\,\mathcal{L}}{\partial_{\mu}\bar{\psi}}\delta\bar{\psi} = \bar{\psi}\gamma^{\mu}\psi$$

Its 0-th component is conserved charge density ≡ quark number density, μ is inserted

$$\mathcal{L} \to \mathcal{L} + \mu\,\bar{q}\,\gamma^{0} q$$

Mean Field Approximation:

All fields are treated as constants in space and imaginary time τ.

Average fields are defined by

$$\bar{\phi} = \frac{T}{V}\int_{0}^{\beta} d\,\tau \int d^{3}\,\phi(\nabla x)$$

LSM: Free Energy

σ and π replaced by time-space independence averaged values, thus

$$\mathcal{Z} = e^{-\beta V U(\langle\sigma\rangle,\langle\vec{\pi}\rangle)} \int \mathcal{D}\bar{q}\mathcal{D}q \exp\left\{ - \int_x \bar{q}\left[\gamma^0\partial_\tau - \vec{\gamma}\cdot\nabla + g(\langle\sigma\rangle + i\gamma^5\vec{\tau}\cdot\langle\vec{\pi}\rangle) - \mu\gamma^0\right]q \right\}$$

Fourier transform fields ψ's and then calculation space-time integral, Helmholtz' free energy reads

$$\mathcal{F} = -T\sum \text{tr} \log\left[\beta\left(i\gamma^0(\omega_n + i\mu) + \vec{\gamma}\cdot\mathrm{p} + g\sigma + ig\gamma^5\vec{\tau}\cdot\vec{\pi}\right)\right]$$

When expanding log around $g\sigma$ where all γ matrices are traceless

$$\mathcal{F} = -2TN_f\sum \log\left[\beta^2\left((\omega_n + i\mu)^2 + \mathrm{p}^2 + g^2(\sigma^2 + \vec{\pi}^2)\right)\right]$$

With dispersion relation $\omega^2 = \mathrm{p}^2 + m^2$ and $m^2 = g^2(\sigma^2 + \vec{\pi}^2)$

$$\mathcal{F} = -TN_f\sum \log\left[\beta^2\left(\omega_n^2 + (\omega + \mu)^2\right)\right] + \log\left[\beta^2\left(\omega_n^2 + (\omega - \mu)^2\right)\right]$$

$$\mathcal{F} = -2N_f V \int \frac{\mathrm{dp}}{(2\pi)^3}\left\{\omega + T\log\left[1 + e^{-(\omega+\mu)\beta}\right] + T\log\left[1 + e^{-(\omega-\mu)\beta}\right]\right\}$$

Grand-canonical partition function

In thermal equilibrium, the grand-canonical partition function can be defined by path integral over quark, antiquark and meson field, where chemical potential is included, we well

$$\mathcal{Z} = \mathrm{Tr}\exp[-(\hat{\mathcal{H}} - \sum_f \mu_f \hat{\mathcal{N}}_f)/T]$$

$$= \int \prod_a \mathcal{D}\sigma_a \mathcal{D}\pi_a \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left[\int_x (\mathcal{L} + \sum_f \mu_f \bar{\psi}_f \gamma^0 \psi_f)\right],$$

where $\int_x \equiv i \int_0^{1/T} dt \int_V d^3x$ and μ_f is the chemical potential

Mean field approximation

The thermodynamic potential reads

$N_f = 2$, $$\Omega = \mathcal{U}(\phi,\phi^*) + U(\sigma_l) + \Omega_{\bar{q}q}(\phi,\phi^*,\sigma_l),$$

$N_f = 3$, $$\Omega = \mathcal{U}(\phi,\phi^*) + U(\sigma_l,\sigma_s) + \Omega_{\bar{q}q}(\phi,\phi^*,\sigma_l,\sigma_s).$$

$N_f = 4$, $$\Omega = \mathcal{U}(\phi,\phi^*) + U(\sigma_l,\sigma_s,\sigma_c) + \Omega_{\bar{q}q}(\phi,\phi^*,\sigma_l,\sigma_s,\sigma_c).$$

Polyakov-loop Potential

Pure mesonic potential

Quarks-antiquarks potential

PLS: pure mesonic potential

$$U(\sigma) = \frac{\lambda}{4}(\sigma^2 - v^2)^2 - c\sigma \ .$$

For $N_f = 2$,

where g, λ, v, and c, are model parameters

Phys.Rev.D76:074023,2007

For $N_f = 3$,

$$U(\sigma_l, \sigma_s) = -h_l\sigma_l - h_s\sigma_s + \frac{m^2(\sigma_l^2 + \sigma_s^2)}{2} - \frac{c\,\sigma_l^2\sigma_s}{2\sqrt{2}} + \frac{\lambda_1\,\sigma_l^2\sigma_s^2}{2} + \frac{(2\lambda_1 + \lambda_2)\sigma_l^4}{8} + \frac{(\lambda_1 + \lambda_2)\sigma_s^4}{4}$$

where m_2, h_l, h_s, λ_1, λ_2, c, and g are model parameters

Phys.Rev. D81 (2010) 074013

SU(4) pure mesonic potential

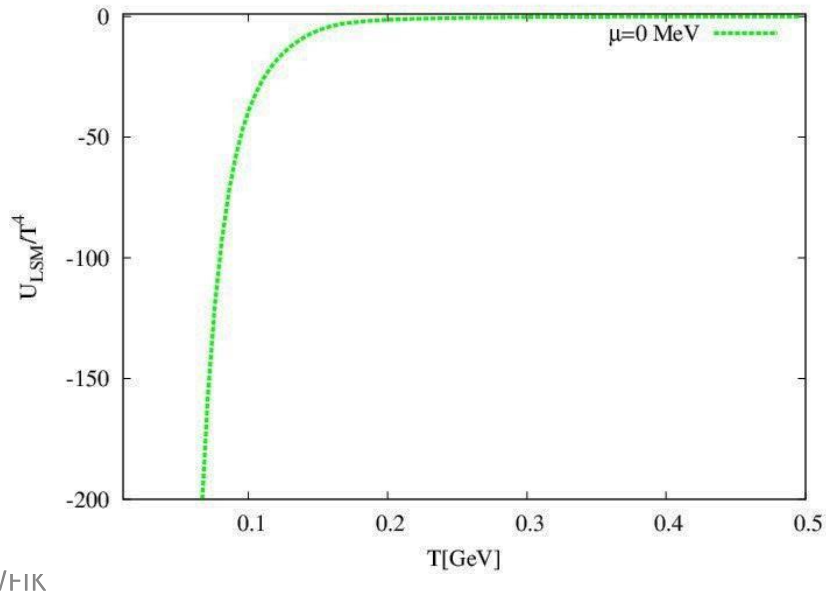
For $N_f = 4$,

$$U(\sigma_l, \sigma_s, \sigma_c) = \frac{1}{8}\Big(-8 h_c \sigma_c + 2\lambda_1 \sigma_c^4 + 2\lambda_2 \sigma_c^4 - 8 h_l \sigma_l + 4\lambda_1 \sigma c^2 \sigma l^2 + 2\lambda_1 \sigma l^4 + \lambda_2 \sigma_l^4 -$$

$$2\left(4 h_s + c\,\sigma_c \sigma_l^2\right) \sigma s + 4\lambda_1 \left(\sigma_c^2 + \sigma_l^2\right) \sigma_s^2 + 2(\lambda_1 + \lambda_2)\sigma_s^4 + 4 m^2 \left(\sigma_c^2 + \sigma_l^2 + \sigma_s^2\right)\Big)$$

A few remarks are now in order:

- This potential is sensitive to T, μ, and B through σ_f
- At μ =0, T-dependence ➔
- It counts for valence quark contributions



qq thermodynamic potential at B=0

WLCAPP World Laboratory Cosmology And Particle Physics

EGYPTIAN CENTER FOR THEORETICAL PHYSICS ECTP CENTER OF EXCELLENCE

SU(3) quarks and antiquarks contributions

At B≠0

Fukushima, PLB591,277(2004)

$$\Omega_{\bar{q}q}(T,\mu_f) = -2T\sum_{f=l,s}\int_0^\infty \frac{d^3\vec{P}}{(2\pi)^3}\left\{\ln\left[1+3\left(\phi^*+\phi\, e^{-\frac{E_f-\mu_f}{T}}\right)e^{-\frac{E_f-\mu_f}{T}}+e^{-3\frac{E_f-\mu_f}{T}}\right]\right.$$
$$\left.+\ln\left[1+3\left(\phi+\phi^*\, e^{-\frac{E_f+\mu_f}{T}}\right)e^{-\frac{E_f+\mu_f}{T}}+e^{-3\frac{E_f+\mu_f}{T}}\right]\right\},$$

where the dispersion relation $E_f = \sqrt{\vec{P}^2+m_f^2}$

Phys. Rev. D 77, 114028 (2008)

Quark masses are coupled to the sigma field via Yukawa coupling g

N_f = 2, $m_l = g\sigma_l$

N_f = 3, $m_l = g\sigma_l/2$ and $m_s = g\sigma_s/\sqrt{2}$

N_f = 4 $m_l = g\sigma_l/2,\ m_s = g\sigma_s/\sqrt{2}$ and $m_c = g\sigma_c/\sqrt{2}$

SU(3) qq thermodynamic potential at B≠0

At B≠0

$$\Omega_{\bar{q}q}(T,\mu_f,B) = -2\sum_{f=l,s}\frac{|q_f|BT}{(2\pi)^2}\int_0^\infty dP_z\left\{\ln\left[1+3\left(\phi+\phi^* e^{-\frac{E_f-\mu_f}{T}}\right)e^{-\frac{E_f-\mu_f}{T}}+e^{-3\frac{E_f-\mu_f}{T}}\right]\right.$$
$$\left.+\ln\left[1+3\left(\phi^*+\phi e^{-\frac{E_f+\mu_f}{T}}\right)e^{-\frac{E_f+\mu_f}{T}}+e^{-3\frac{E_f+\mu_f}{T}}\right]\right\}$$
$$-\;4\sum_{f=l,s}\frac{|q_f|BT}{(2\pi)^2}\sum_{\nu=1}^{(\nu_{max})_f}\int_0^\infty dP_z\left\{\ln\left[1+3\left(\phi+\phi^* e^{-\frac{E_{B,f}-\mu_f}{T}}\right)e^{-\frac{E_{B,f}-\mu_f}{T}}+e^{-3\frac{E_{B,f}-\mu_f}{T}}\right]\right.$$
$$\left.+\ln\left[1+3\left(\phi^*+\phi e^{-\frac{E_{B,f}+\mu_f}{T}}\right)e^{-\frac{E_{B,f}+\mu_f}{T}}+e^{-3\frac{E_{B,f}+\mu_f}{T}}\right]\right\}$$

where the dispersion relation $E_{B,f} = \left[P_z^2+m_f^2+|q_f|(2n+1-\sigma)B\right]^{1/2}$

Landau quantization and why 3 is added?

WLCAPP World Laboratory Cosmology And Particle Physics

Polyakov-loop Potential

EGYPTIAN CENTER FOR THEORETICAL PHYSICS
CENTER OF EXCELLENCE

Polyakov-loop potential introduces the gluons degrees-of-freedom and the dynamics of the quark-gluon interactions to the QCD matter.

Polynomial-logarithmic parameterization of the Polyakov-loop potential, they even included higher-order terms,

$$\frac{\mathcal{U}_{\text{PolyLog}}(\phi,\phi^*,T)}{T^4} = \frac{-a(T)}{2}\,\phi^*\phi + b(T)\,\ln\left[1-6\,\phi^*\phi+4\,(\phi^{*3}+\phi^3)-3\,(\phi^*\phi)^2\right] + \frac{c(T)}{2}\,(\phi^{*3}+\phi^3)+d(T)\,(\phi^*\phi)^2.$$

$$x(T) = \frac{x_0 + x_1\,(T0/T) + x_2\,(T0/T)^2}{1+x_3\,(T0/T)+x_4\,(T0/T)^2}, \qquad b(T) = b_0\,(T0/T)^{b_1}\left(1-e^{b_2 (T0/T)^{b_3}}\right),$$

where the coefficients x=a, c and d

Phys. Rev. D 88, 074502 (2013).

WLCAPP World Laboratory Cosmology And Particle Physics

Landau quantization

ECTP Egyptian Center for Theoretical Physics — Center of Excellence

$$E_{B,f} = \left[P_z^2 + m_f^2 + |q_f|(2n+1-\sigma)B\right]^{1/2}$$

where P_z is z-component of P, q_f is electric charge of f quark, n is quantization number and σ is related to spin number S, $\sigma=\pm S/2$.

2n+1-σ can be replaced by sum over Landau levels,

$$\upsilon_{max,f} = \left\lfloor \frac{\mu_f^2 - \Lambda_{QCD}^2}{2\,|q_f|\,\boldsymbol{B}} \right\rfloor$$

How Landau levels are occupied?

At $eB=10m_\pi^2$

At μ=0 and T=100MeV, u- (62), d- and s-quarks (124 levels) each
At μ=200MeV, u- (59), d- and s-quarks (118 levels) each

At μ=0 and T=100MeV, MLL for u- (3), d- and s-quarks (6 levels) each
At μ=200MeV, MLL for u- (2), d- and s-quarks (4 levels) each

Increasing eB fills up LLL first and #levels decreases.
Increasing eB allows LLL to accommodate more quarks

Landau quantization (cont.) and factor 3

Population of MLL depends on T, q_f and eB.
Thus MLL occupations of u- and d-quarks depend on their q_f, greatly.

At T=50MeV and eB=1m_π^2, u- (31), d- and s-quarks (62 levels) each
At μ=100MeV and eB=15m_π^2, u- (2), d- and s-quarks (4 levels) each

Population of Landau levels is most sensitive to eB and q_f

$$\Omega_{\bar{q}q}(T,\mu_f) = -2T\sum_{f=l,s}\int_0^\infty \frac{d^3\vec{P}}{(2\pi)^3}\left\{\ln\left[1+3\left(\phi^*+\phi\, e^{-\frac{E_f-\mu_f}{T}}\right)e^{-\frac{E_f-\mu_f}{T}}+e^{-3\frac{E_f-\mu_f}{T}}\right] + \ln\left[1+3\left(\phi+\phi^*\, e^{-\frac{E_f+\mu_f}{T}}\right)e^{-\frac{E_f+\mu_f}{T}}+e^{-3\frac{E_f+\mu_f}{T}}\right]\right\},$$

Emergence of 3 in Boltzmann exponent at zero Polyakov loops is an emergence of a statistical confinement that only 3-quark states and not 1- or 2-quark states are allowed in the statistical sum for the partition function

Modifications due to magnetic field

Landau quantization

$$E_{B,f} = \left[P_z^2 + m_f^2 + |q_f|(2n+1-\sigma)B\right]^{1/2}$$

$$\int \frac{d^3p}{(2\pi)^3} \longrightarrow \frac{|q_f|B}{2\pi} \sum_{\nu=0}^{(\nu_{max})_f} \int \frac{dP_z}{2\pi}(2-\delta_{0\nu})$$

$$\begin{aligned}\Omega_{\bar{q}q}(T,\mu_f,B) \;=\; & -2\sum_{f=l,s}\frac{|q_f|BT}{(2\pi)^2}\int_0^\infty dP_z\left\{\ln\left[1+3\left(\phi+\phi^* e^{-\frac{E_f-\mu_f}{T}}\right)\, e^{-\frac{E_f-\mu_f}{T}} + e^{-3\frac{E_f-\mu_f}{T}}\right]\right.\\ & \left. + \ln\left[1+3\left(\phi^*+\phi e^{-\frac{E_f+\mu_f}{T}}\right)\, e^{-\frac{E_f+\mu_f}{T}} + e^{-3\frac{E_f+\mu_f}{T}}\right]\right\}\\ & -\; 4\sum_{f=l,s}\frac{|q_f|BT}{(2\pi)^2}\sum_{\nu=1}^{(\nu_{max})_f}\int_0^\infty dP_z\left\{\ln\left[1+3\left(\phi+\phi^* e^{-\frac{E_{B,f}-\mu_f}{T}}\right)\, e^{-\frac{E_{B,f}-\mu_f}{T}} + e^{-3\frac{E_{B,f}-\mu_f}{T}}\right]\right.\\ & \left. + \ln\left[1+3\left(\phi^*+\phi e^{-\frac{E_{B,f}+\mu_f}{T}}\right)\, e^{-\frac{E_{B,f}+\mu_f}{T}} + e^{-3\frac{E_{B,f}+\mu_f}{T}}\right]\right\}\end{aligned}$$

Thermodynamic potential

SU(4) thermodynamic potential

$$\Omega = \mathcal{U}(\phi, \phi^*) + U(\sigma_l, \sigma_s, \sigma_c) + \Omega_{\bar{q}q}(\phi, \phi^*, \sigma_l, \sigma_s, \sigma_c).$$

has the parameters

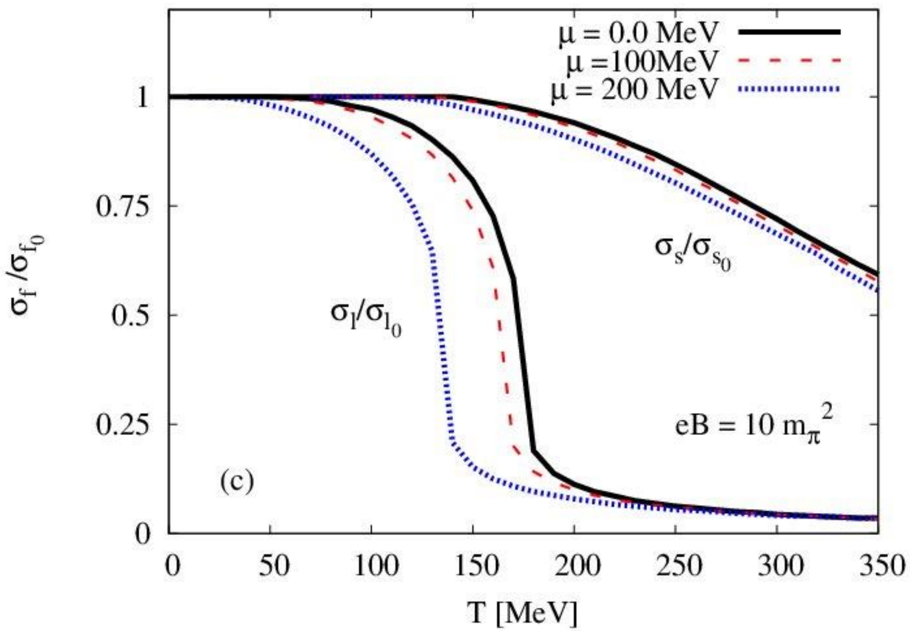
$$\sigma_l, \sigma_s, \sigma_c \qquad \phi \text{ and } \phi^*$$

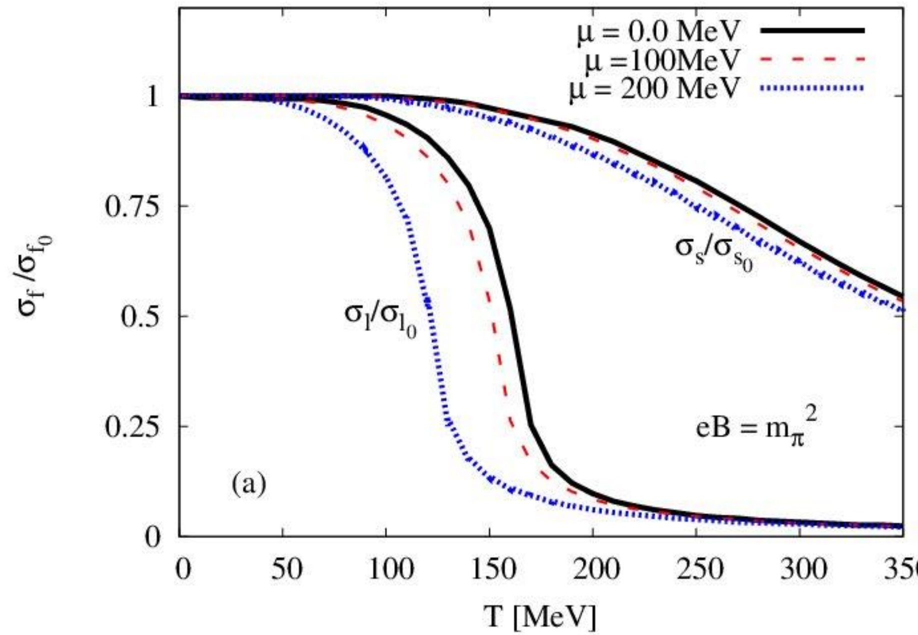
The analysis of the order parameters is given by minimizing the real part of thermodynamic potential $\mathrm{Re}\,\Omega$

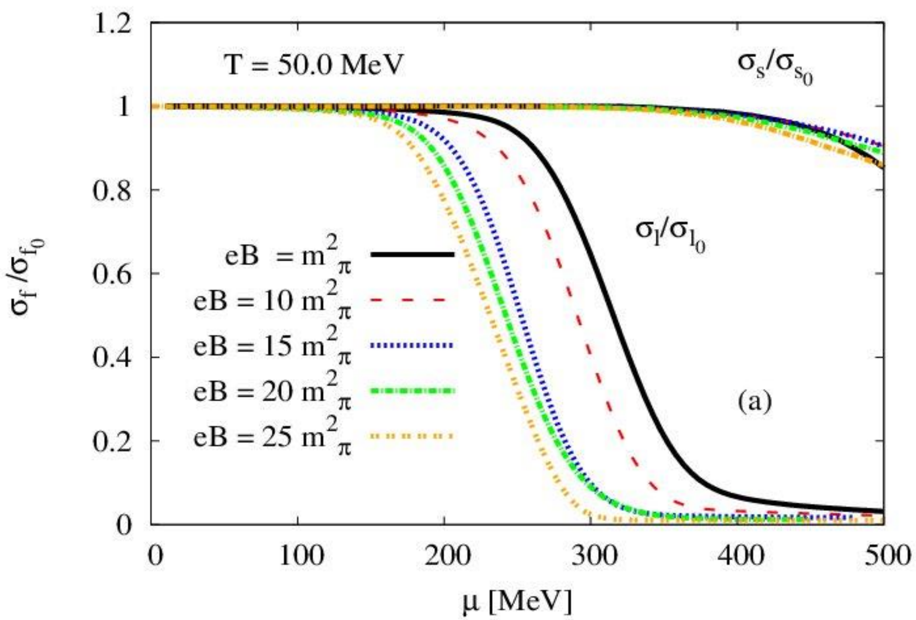
The solutions of these equations can be determined by minimizing the real potential at a saddle point,

For SU(2) $$\frac{\partial \Omega}{\partial \sigma_x} = \frac{\partial \Omega}{\partial \sigma_y} = \frac{\partial \Omega}{\partial \phi} = \frac{\partial \Omega}{\partial \phi^*} \Big|_{\sigma_x=\bar{\sigma}_x, \sigma_y=\bar{\sigma}_y, \phi=\bar{\phi}, \phi^*=\bar{\phi}^*} = 0$$

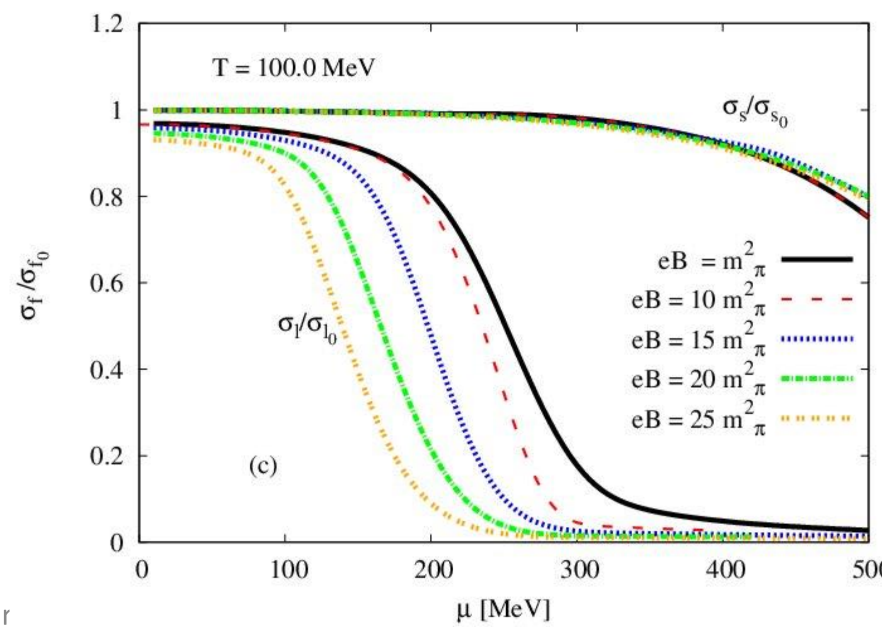
The behavior of the chiral condensate $\sigma_l, \sigma_s, \sigma_c$ and the Polyakov-loop expectation values ϕ and ϕ^* as functions of T and μ shall be presented

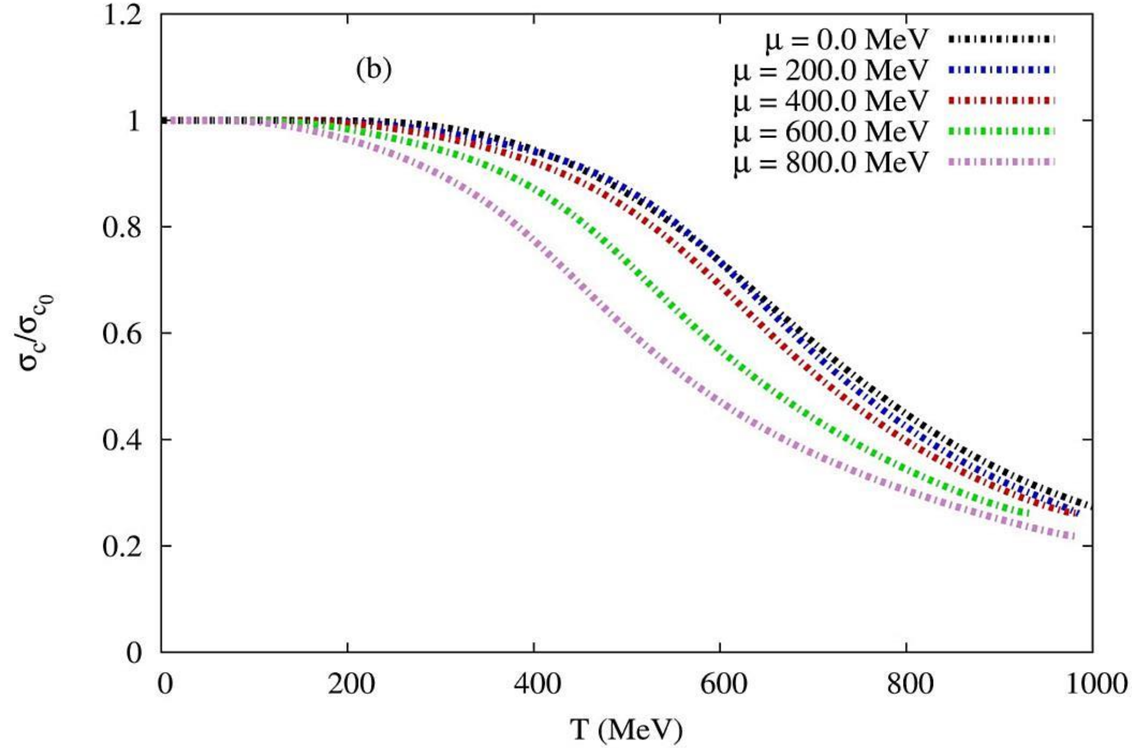

μ = 0.0 MeV
μ =100MeV
μ = 200 MeV
σ_s/σ_s0
σ_l/σ_l0
eB = 10 m_π^2
(c)
σ_f /σ_f0
T [MeV]

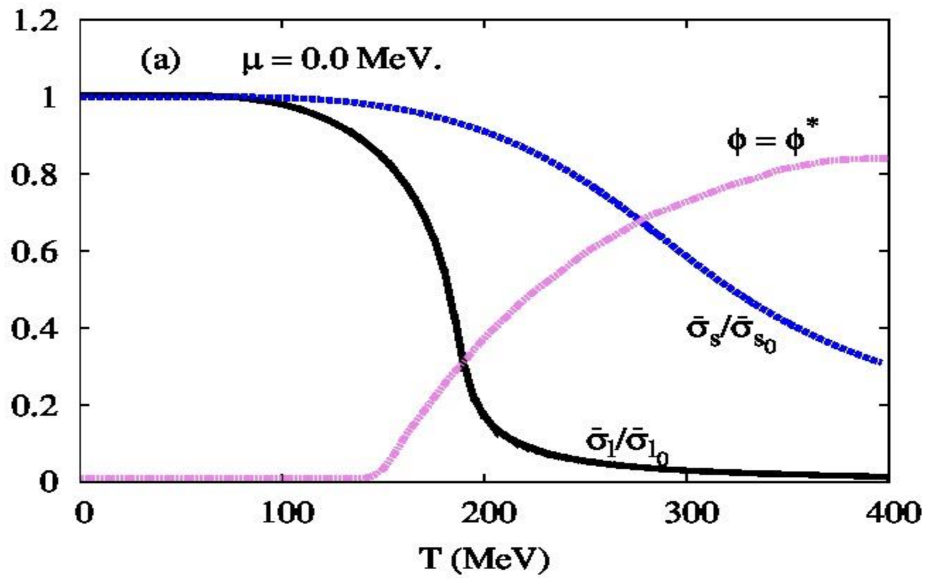

μ = 0.0 MeV
μ =100MeV
μ = 200 MeV
σ_s/σ_s0
σ_l/σ_l0
eB = m_π^2
(a)
σ_f /σ_f0
T [MeV]

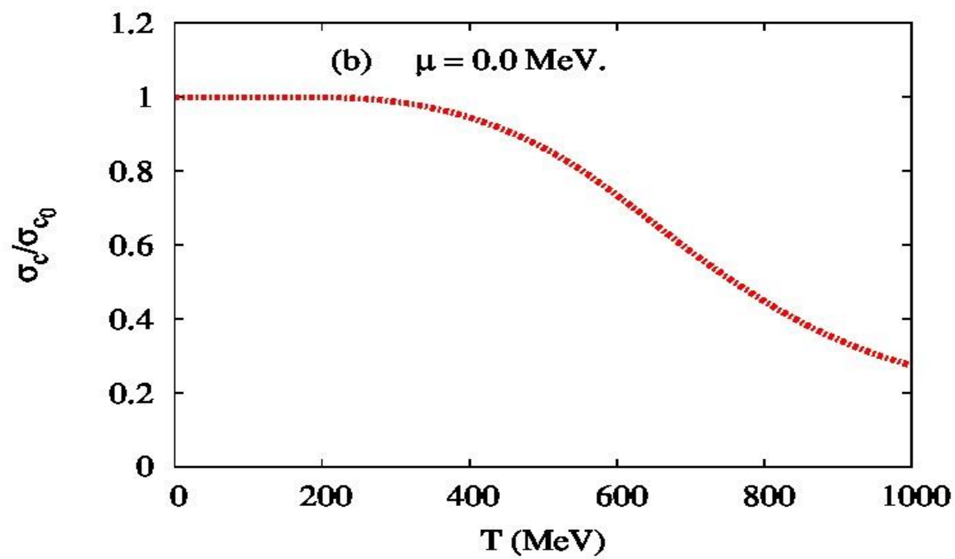

T = 50.0 MeV
σ_s/σ_s0
σ_l/σ_l0
eB = m^2_π
eB = 10 m^2_π
eB = 15 m^2_π
eB = 20 m^2_π
eB = 25 m^2_π
(a)
σ_f /σ_f0
μ [MeV]

er

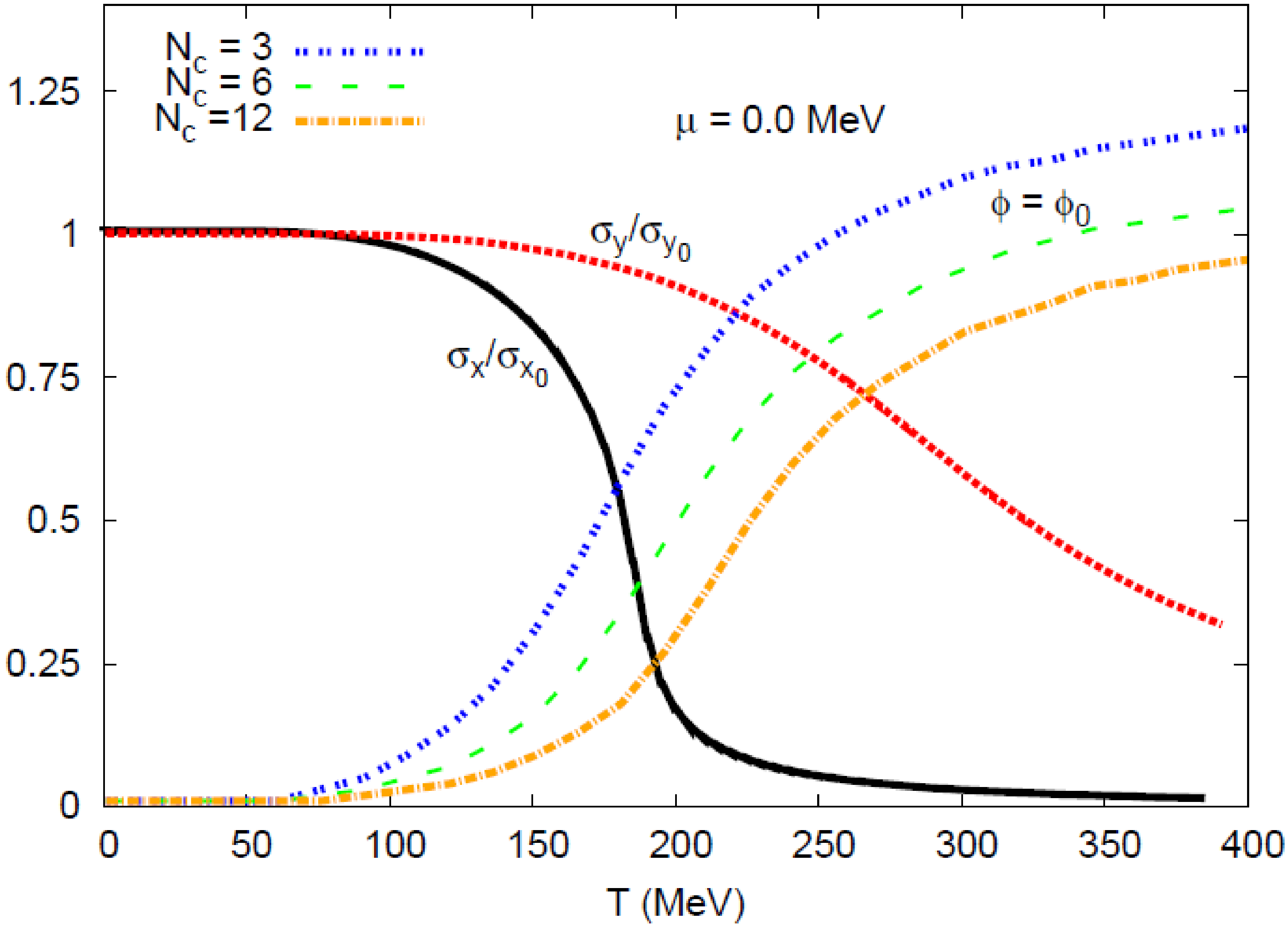

T = 100.0 MeV
σ_s/σ_s0
σ_l/σ_l0
eB = m^2_π
eB = 10 m^2_π
eB = 15 m^2_π
eB = 20 m^2_π
eB = 25 m^2_π
(c)
σ_f /σ_f0
μ [MeV]

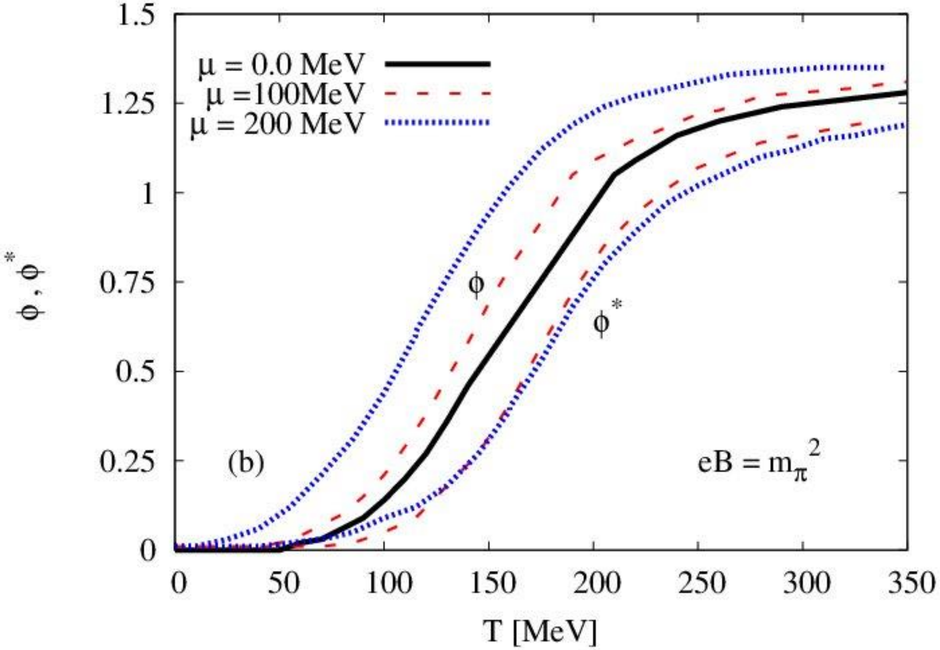

(b)
μ = 0.0 MeV
μ = 200.0 MeV
μ = 400.0 MeV
μ = 600.0 MeV
μ = 800.0 MeV
σ_c/σ_{c_0}
T (MeV)
0
0.2
0.4
0.6
0.8
1
1.2
0
200
400
600
800
1000

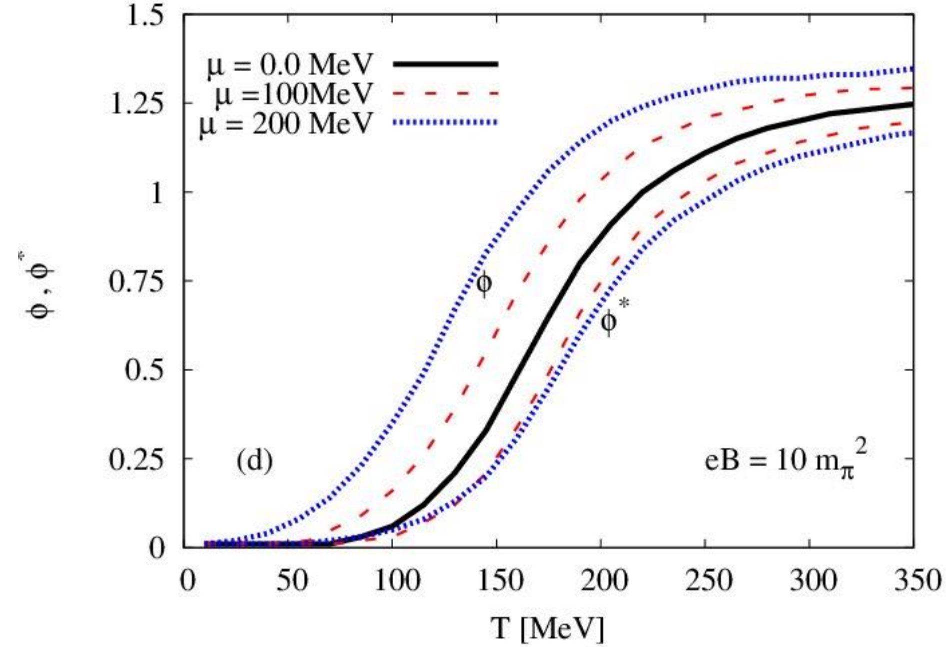

(a) μ = 0.0 MeV.
$\phi = \phi^*$
$\bar{\sigma}_s/\bar{\sigma}_{s_0}$
$\bar{\sigma}_l/\bar{\sigma}_{l_0}$
T (MeV)
0
0.2
0.4
0.6
0.8
1
1.2
0
100
200
300
400

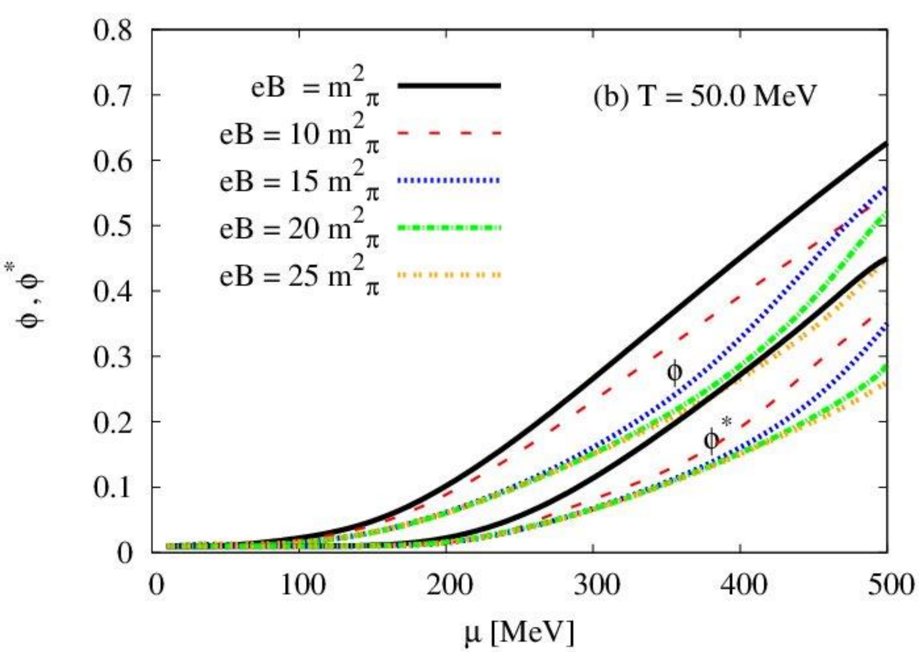

(b) μ = 0.0 MeV.
σ_c/σ_{c_0}
T (MeV)
0
0.2
0.4
0.6
0.8
1
1.2
0
200
400
600
800
1000

SU(3) PLSM: Chiral condensate at large Nc

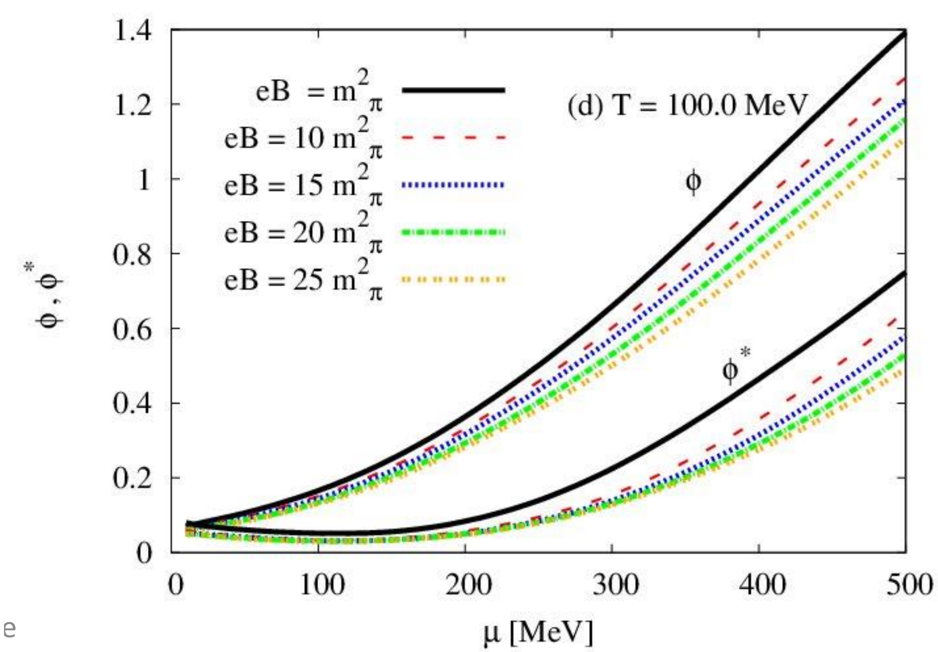



μ = 0.0 MeV
μ =100MeV
μ = 200 MeV
φ
φ*
(b)
eB = mπ2
φ , φ*
T [MeV]

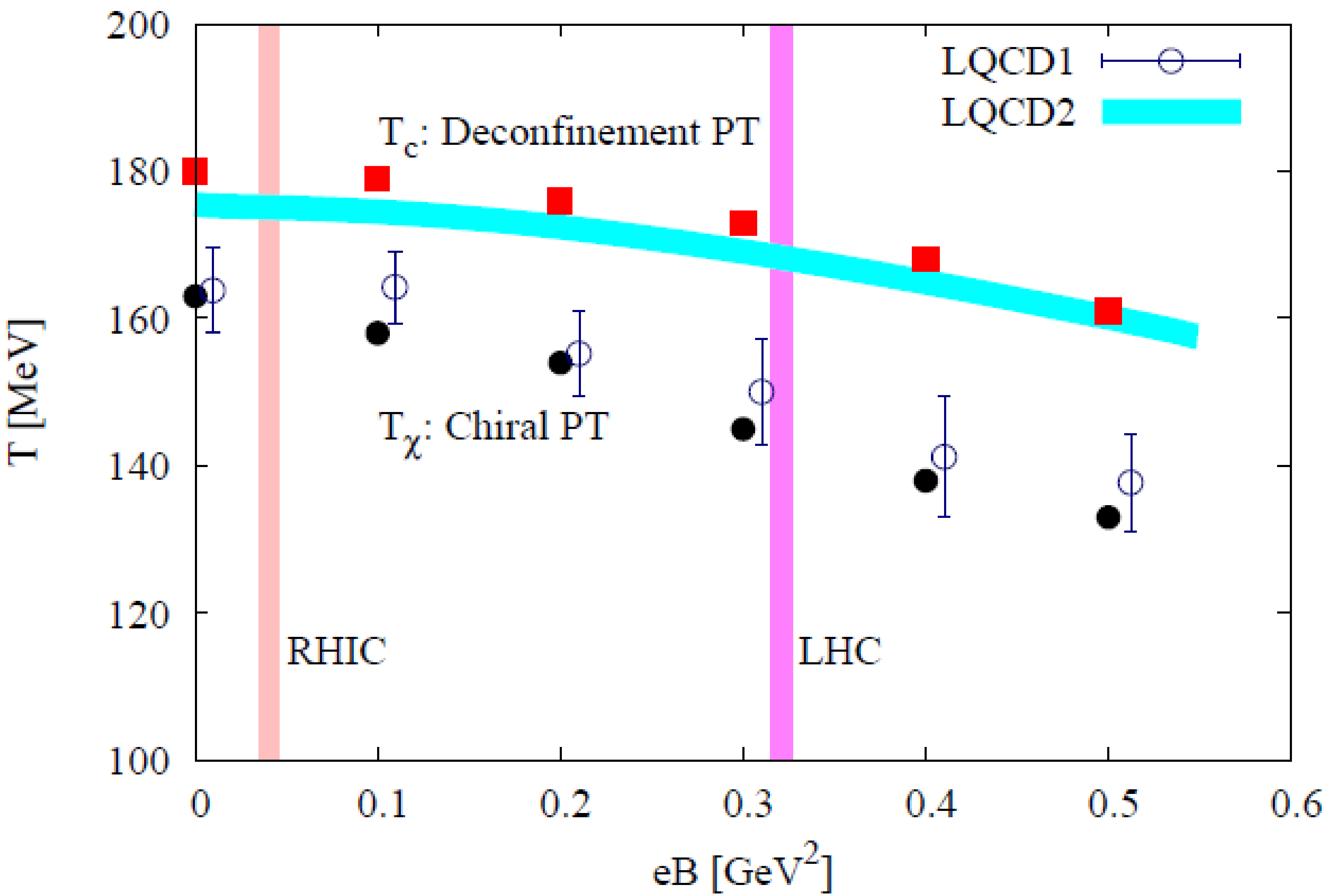

μ = 0.0 MeV
μ =100MeV
μ = 200 MeV
φ
φ*
(d)
eB = 10 mπ2
φ , φ*
T [MeV]


eB = m2π
eB = 10 m2π
eB = 15 m2π
eB = 20 m2π
eB = 25 m2π
(b) T = 50.0 MeV
φ
φ*
φ , φ*
μ [MeV]

e

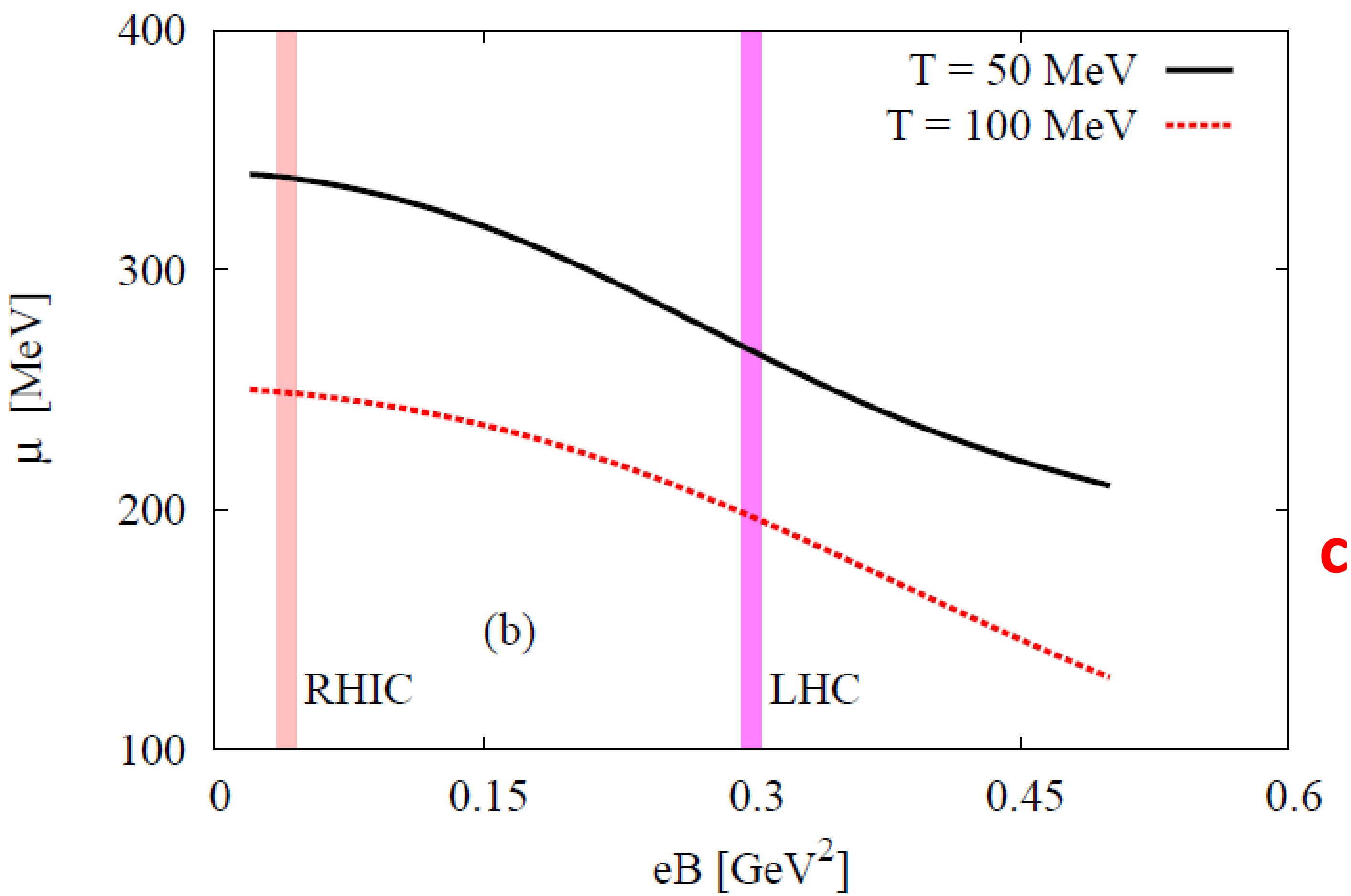

eB = m2π
eB = 10 m2π
eB = 15 m2π
eB = 20 m2π
eB = 25 m2π
(d) T = 100.0 MeV
φ
φ*
φ , φ*
μ [MeV]

SU(3) PLSM: T_c vs. eB (mag. Catalysis)



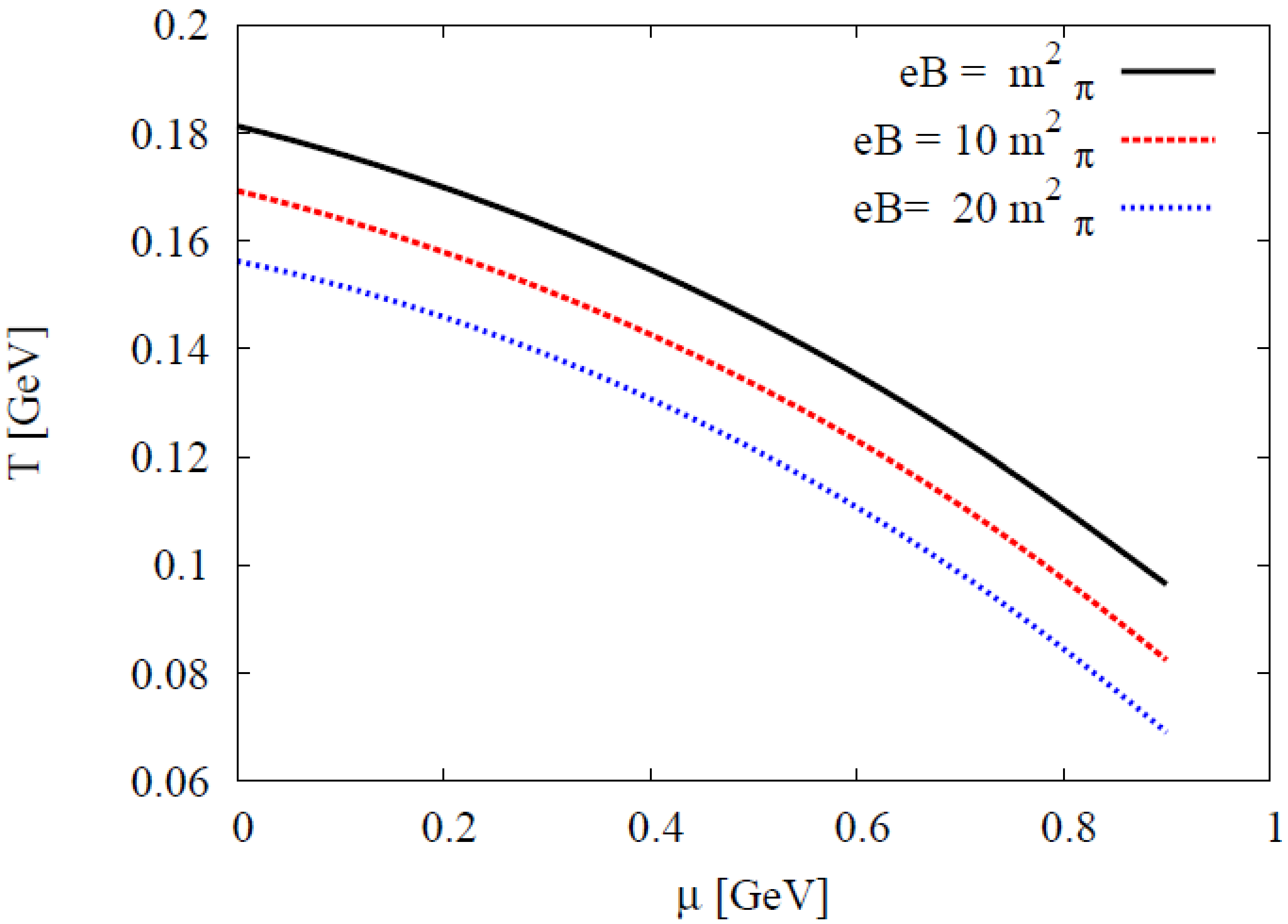
To appear in PRC

SU(3) PLSM: μ_c vs. eB

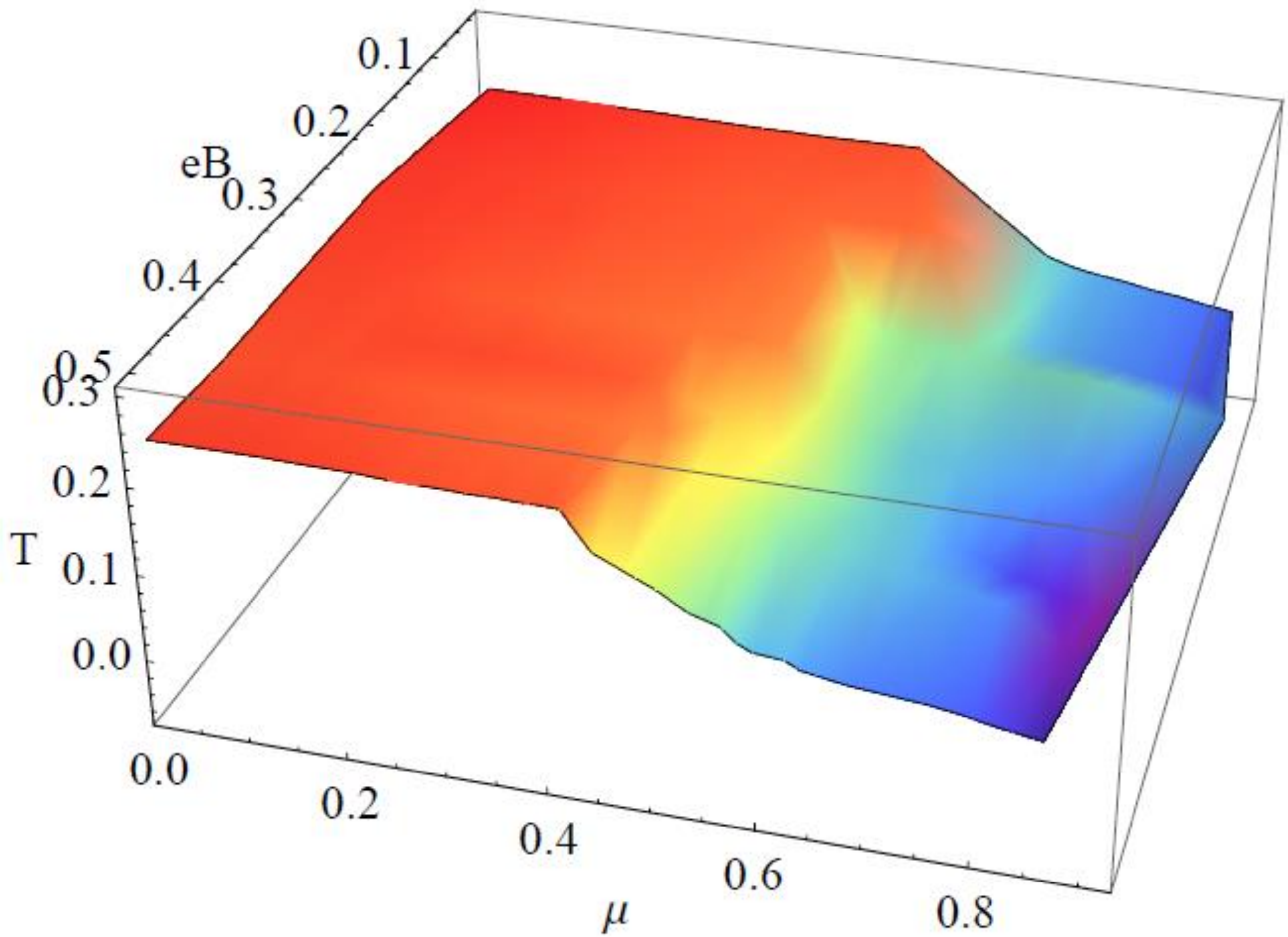


chiral

SU(3) PLSM: T_c vs. μ



SU(3) PLSM: T_c vs. eB vs. μ



SU(3) PLSM: Magnetization

In finite magnetic field, the free energy is modified,

$$\mathcal{M} = \frac{\partial \mathcal{F}}{\partial\,(e\,B)}$$

Positive $\mathcal{M}$: para-magnetic,

- most color charges align towards the direction of eB

negative $\mathcal{M}$: dia-magnetic

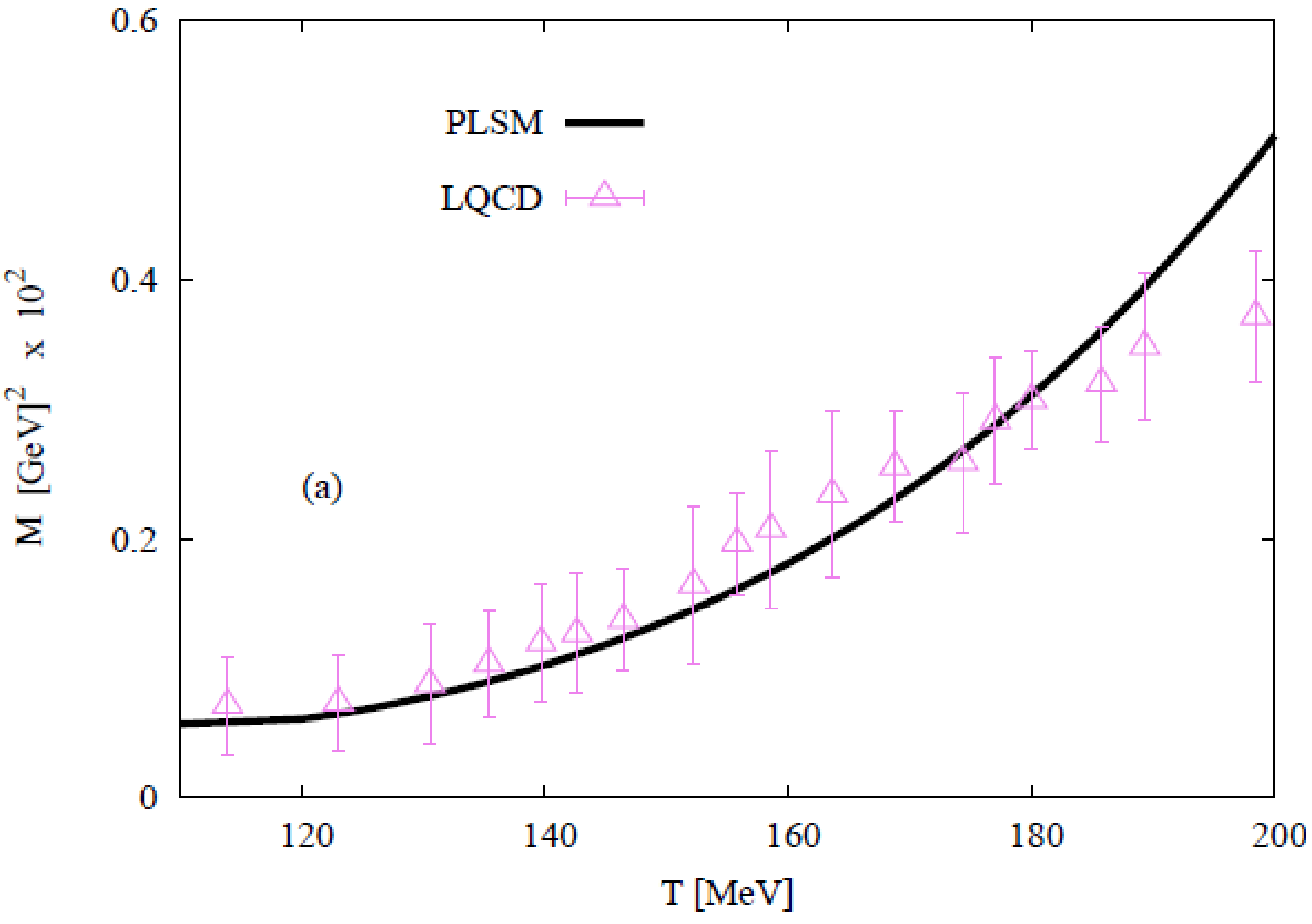
- Color charges align oppositely to the direction of eB,
- produce an induced current spreading as small loops attempting to cancel out the effects of the applied eB,

Magnetic susceptibility: χ_B is the derivative of $\mathcal{M}$ wrt $e\,B$

Magnetic permeability: $\mu_r = 1 + \chi_B$

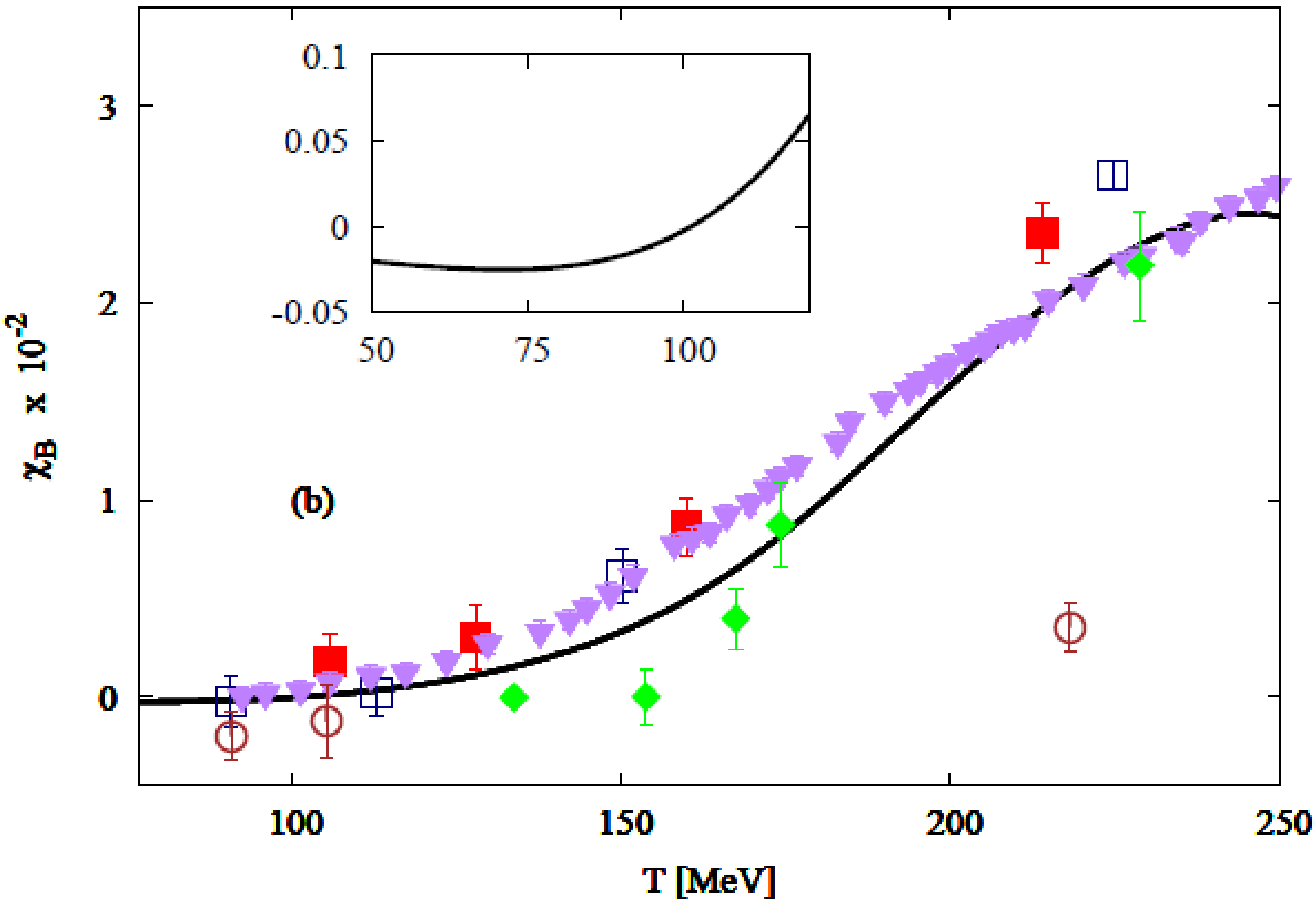
1606.09153 [hep-ph]

SU(3) PLSM: Magnetization



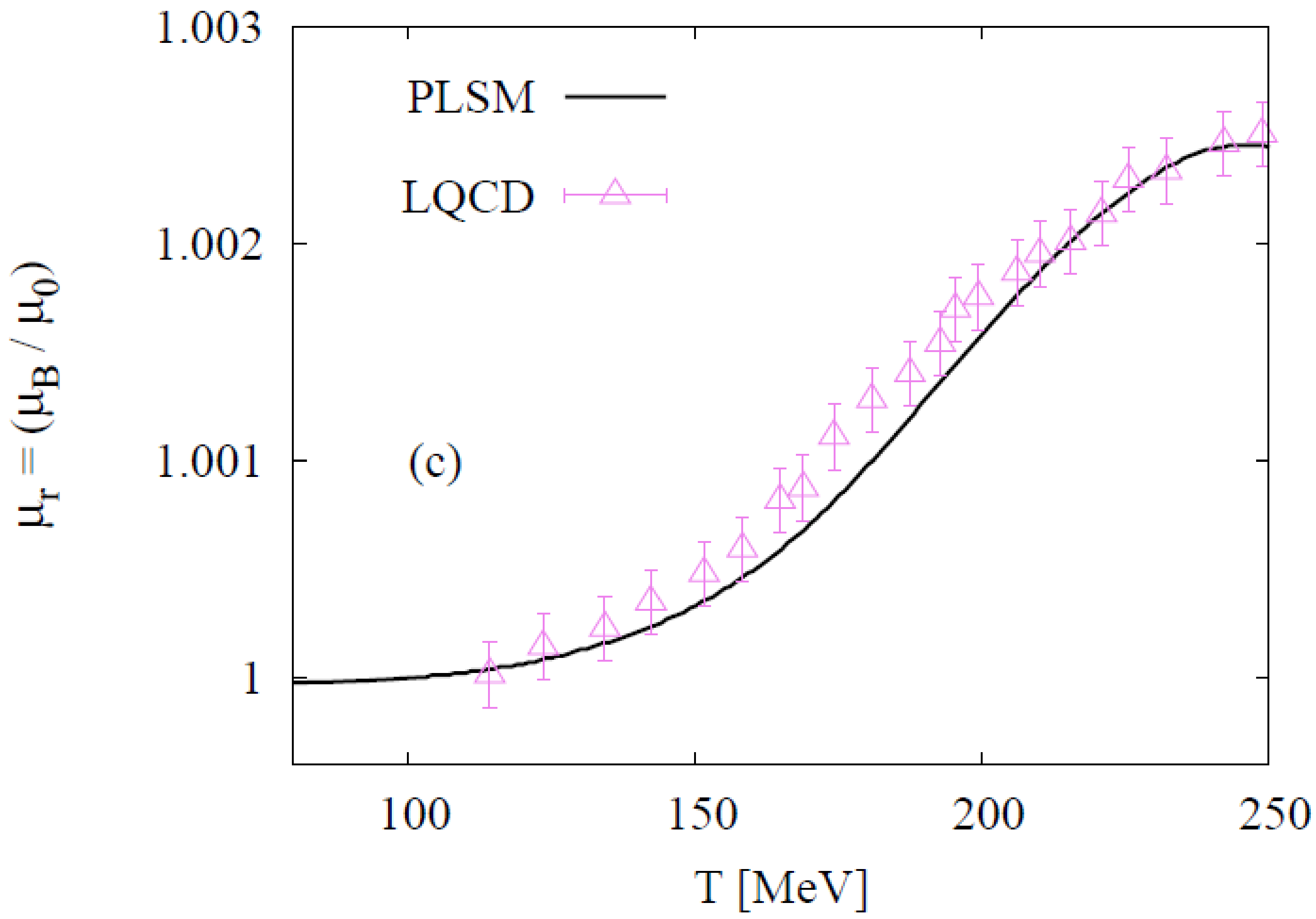
1606.09153 [hep-ph]

SU(3) PLSM: Susceptibility



1606.09153 [hep-ph]

SU(3) PLSM: Permeability



Abdel Nasser TAWFIK

1606.09153 [hep-ph]

SU(3) PLSM: Fluctuations & Correlations

First we reproduce available lattice data → fixing PLSM parameters
Then, confront PLSM calculations with STAR & ALICE measurements

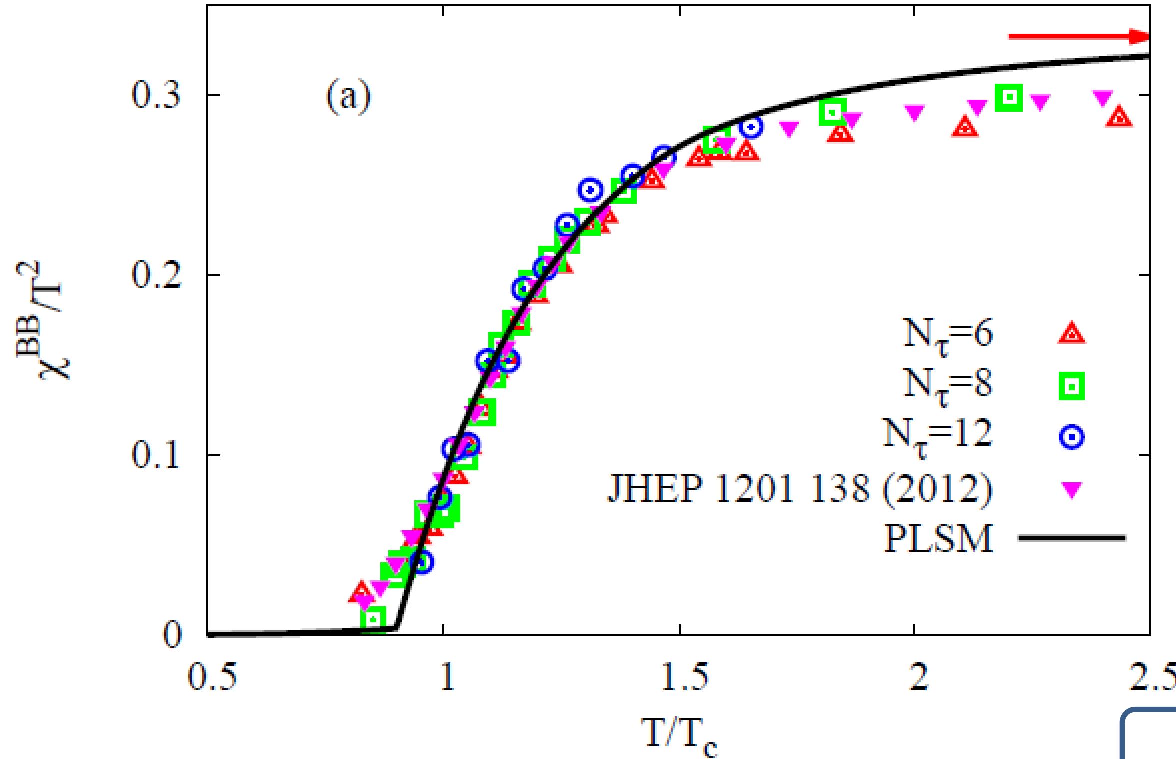
$$\chi_{ijk}^{BQS} = \frac{\partial^{i+j+k}(P/T^4)}{\partial(\mu_B/T)^i \partial(\mu_Q/T)^j \partial(\mu_S/T)^k}$$

$$\sigma^2 = \langle(\delta N)^2\rangle = V T^3 \chi_2$$

$$S = \frac{\langle(\delta N)^3\rangle}{\sigma^3} = \frac{V T^3 \chi_3}{(V T^3 \chi_2)^{3/2}},$$

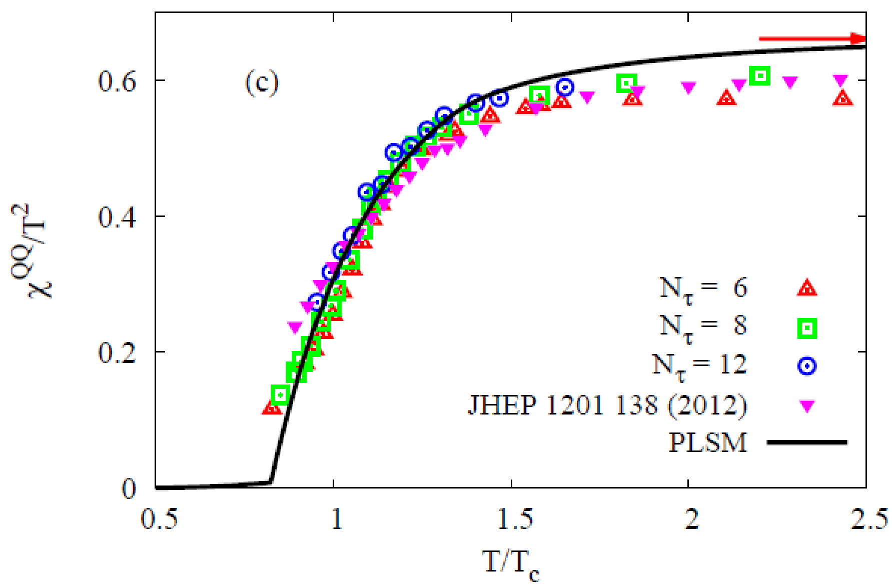
$$\kappa = \frac{\langle(\delta N)^4\rangle}{\sigma^4} - 3 = \frac{V T^3 \chi_4}{(V T^3 \chi_2)^2}.$$

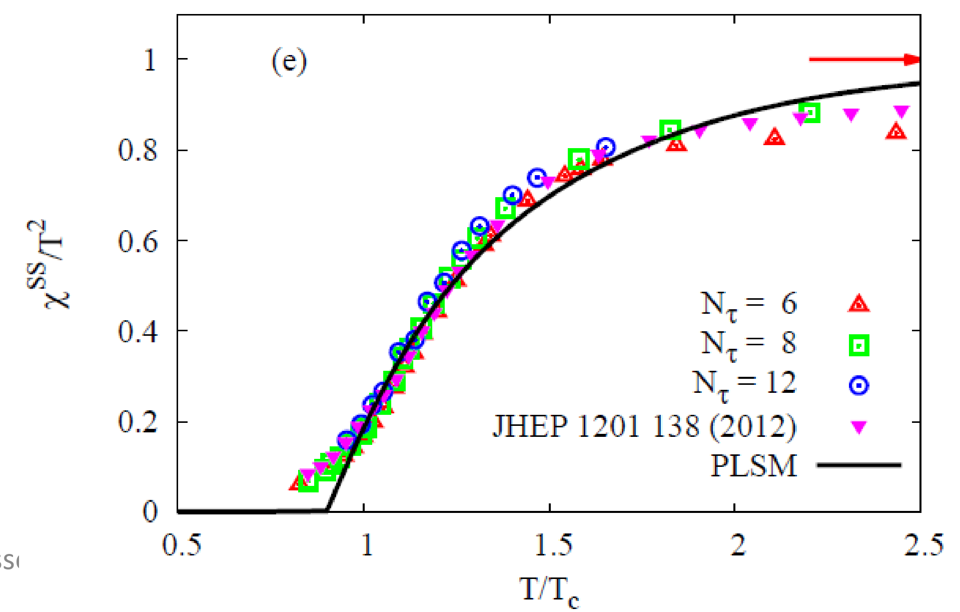
Not yet published



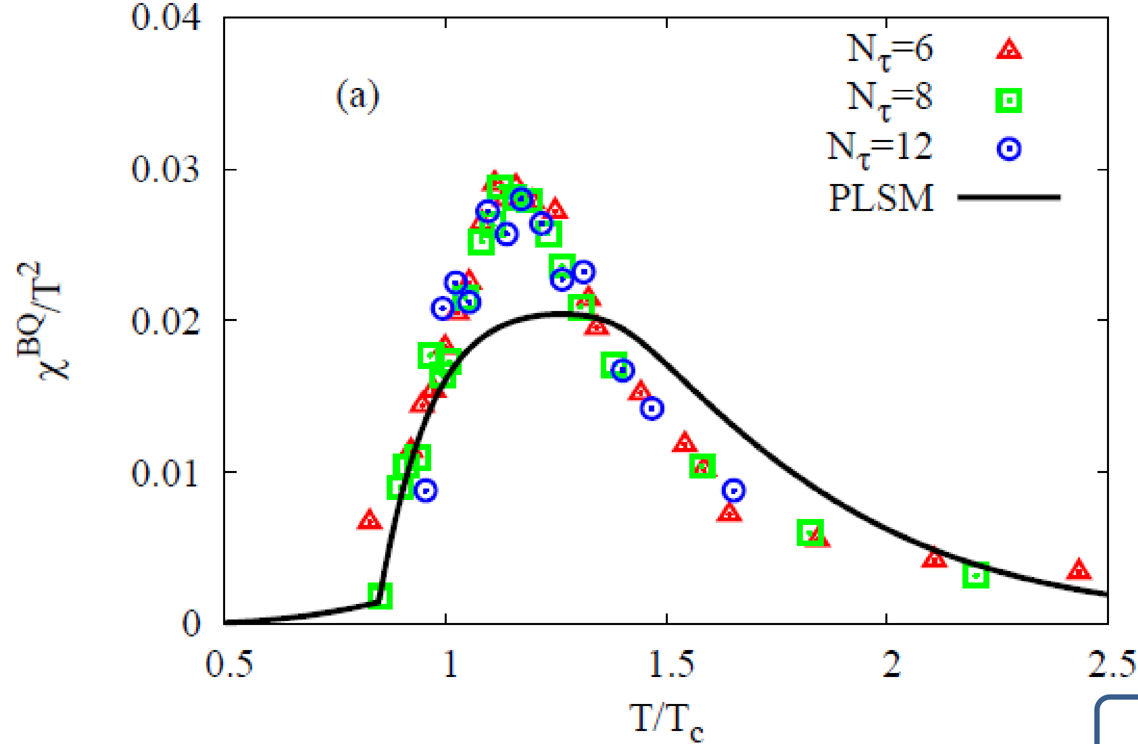
Fluctuations

Not yet published

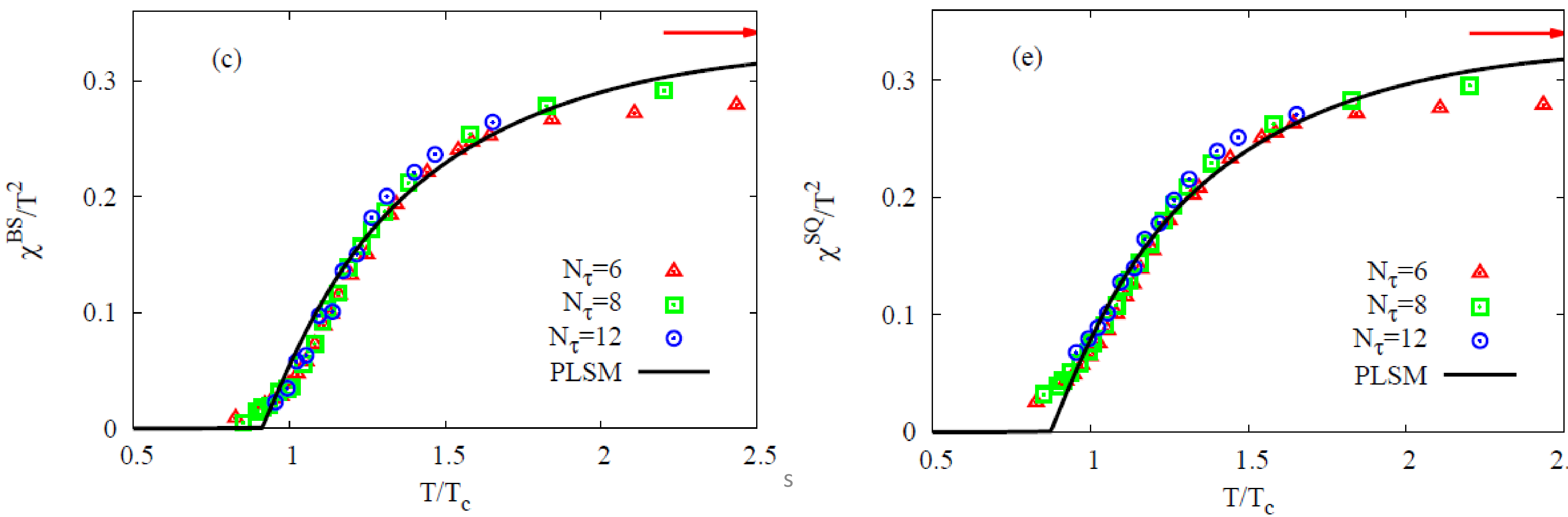




Correlations

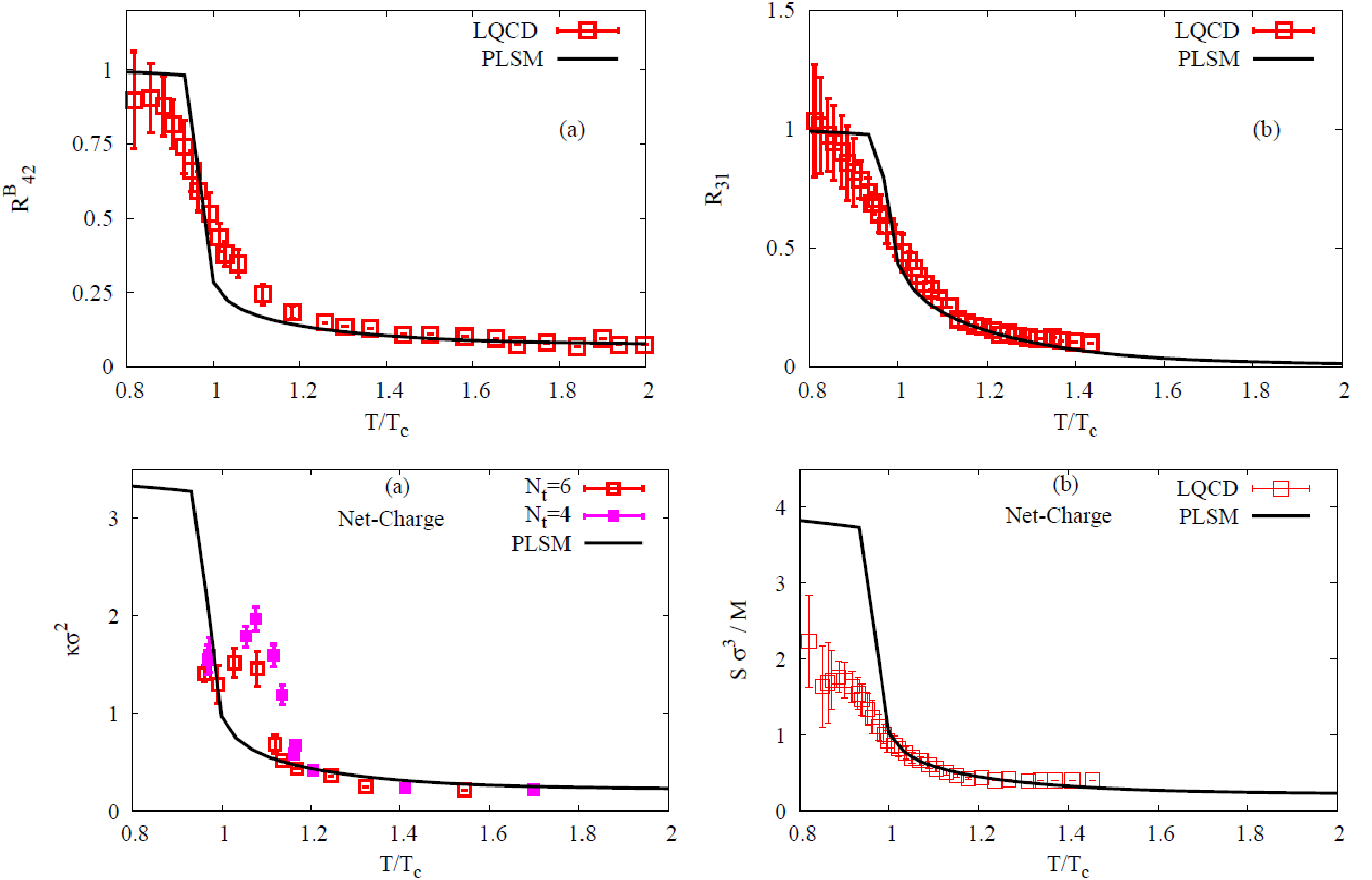


Not yet published



Not yet published

Higher-Order Moments



SU(3) PLSM: Freezeout Temperature

From the phenomenological relation $\mu_B = \frac{a}{1 + b\sqrt{s_{NN}}}$

$$a = 1.308 \pm 0.028 \text{ GeV and } b = 0.273 \pm 0.008 \text{ GeV}^{-1}$$

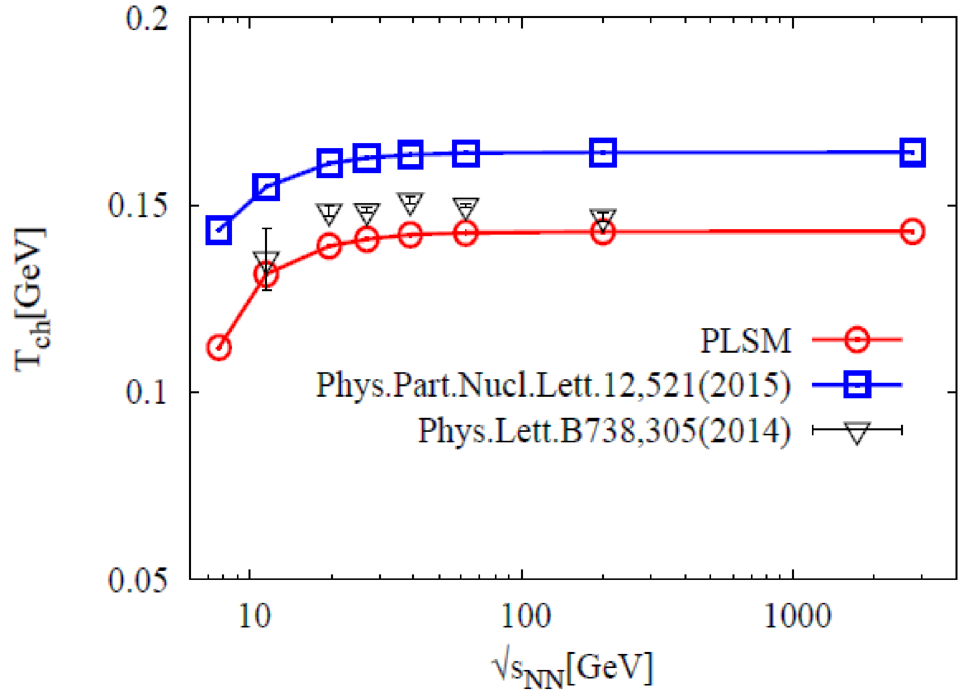
corresponding freezeout temperature can be estimated from PLSM from best reproduction of $S\sigma$ measured by STAR and ALICE

$$T_{ch} = c - d\,\mu_B^2 - e\,\mu_B^4$$

$$c = 0.143 \text{ GeV}$$

$$d = 0.051 \text{ GeV}^{-1}$$

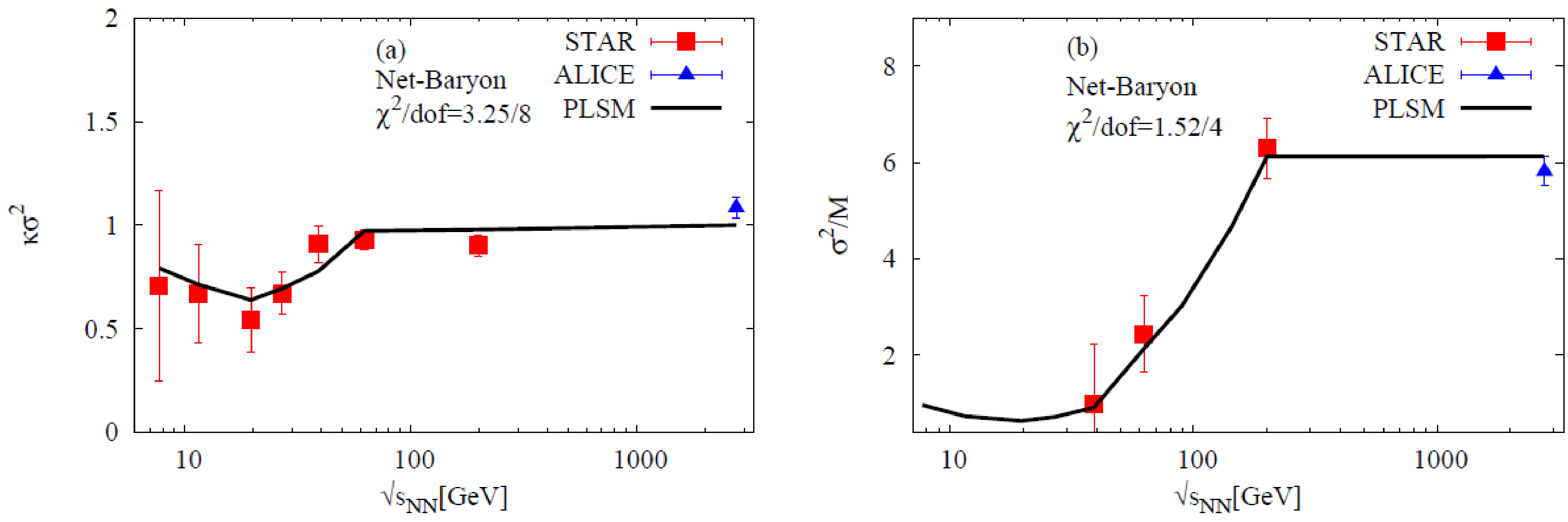
$$e = 0.686 \text{ GeV}^{-3}$$



Not yet published

SU(3) PLSM: RHIC/LHC higher-order moments

At fixed μ_B and resulted T_{ch}, both $\kappa\,\sigma^2$ and σ^2/M shall be calculated. No further fitting has been done.



Abdel Nasser TAWFIK

Not yet published

WLCAPP World Laboratory Cosmology And Particle Physics

SU(3) PLSM: Phase Structure

EGYPTIAN CENTER FOR THEORETICAL PHYSICS — E C T P — CENTER OF EXCELLENCE

$$m^2_{i,ab} = \frac{\partial^2 \Omega(T,\mu_f)}{\partial \zeta_{i,a} \partial \zeta_{i,b}}\Big|_{\min} = \nu_c \sum_{f=l,s} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{q,f}} \Big[(n_{q,f} + n_{\bar{q},f}) \left(m^2_{f,ab} - \frac{m^2_{f,a} m^2_{f,b}}{2E^2_{q,f}} \right) - (b_{q,f} + b_{\bar{q},f}) \left(\frac{m^2_{f,a} m^2_{f,b}}{2E_{q,f} T} \right) \Big].$$

where i stands for scalar, pseudoscalar, vector and axial-vector mesons and a and b range from $0, \ldots, 8$. In vacuum, the mesonic sectors are formulated in the non–strange and strange the meson fields $\zeta_{i,a}$, $m^2_{f,a} \equiv \partial m^2_f / \partial \zeta_{i,a}$ $\zeta_{i,a} \partial \zeta_{i,b}$, $m^2_{f,ab} \equiv \partial m^2_f / \partial \zeta_{i,a} \partial \zeta_{i,b}$

$$b_{q,f}(T,\mu_f) = n_{q,f}(T,\mu_f)(1 - n_{q,f}(T,\mu_f))$$

$$N_{q,f} = \frac{\Phi e^{-E_{q,f}/T} + 2\Phi^* e^{-2E_{q,f}/T} + e^{-3E_{q,f}/T}}{1 + 3(\phi + \phi^* e^{-E_{q,f}/T}) e^{-E_{q,f}/T} + e^{-3E_{\bar{q},f}/T}},$$

$$N_{\bar{q},f} = \frac{\Phi^* e^{-E_{\bar{q},f}/T} + 2\Phi e^{-2E_{\bar{q},f}/T} + e^{-3E_{\bar{q},f}/T}}{1 + 3(\phi^* + \phi e^{-E_{\bar{q},f}/T}) e^{-E_{\bar{q},f}/T} + e^{-3E_{\bar{q},f}/T}},$$

$$m^2_{i,ab} = \frac{\partial^2 \Omega(T,\mu_f)}{\partial \zeta_{i,a} \partial \zeta_{i,b}}\Big|_{\min} = \nu_c \sum_{f=l,s} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{q,f}} \Big[(N_{q,f} + N_{\bar{q},f}) \left(m^2_{f,ab} - \frac{m^2_{f,a} m^2_{f,b}}{2E^2_{q,f}} \right) + (B_{q,f} + B_{\bar{q},f}) \left(\frac{m^2_{f,a} m^2_{f,b}}{2E_{q,f} T} \right) \Big].$$

Phys.Rev. C91 (2015)

SU(3) PLSM: Phase Structure

For example, scalar mesons

$$m_{a_0}^2 = m^2 + \lambda_1 \left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) + \frac{3\lambda_2}{2}\bar{\sigma}_x^2 + \frac{\sqrt{2}c}{2}\bar{\sigma}_y,$$

$$m_{\kappa}^2 = m^2 + \lambda_1 \left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) + \frac{\lambda_2}{2}\left(\bar{\sigma}_x^2 + \sqrt{2}\bar{\sigma}_x\bar{\sigma}_y + 2\bar{\sigma}_y^2\right) + \frac{c}{2}\bar{\sigma}_x,$$

$$m_{\sigma}^2 = m_{s,00}^2 \cos^2\theta_s + m_{s,88}^2 \sin^2\theta_s + 2m_{s,08}^2 \sin\theta_s \cos\theta_s,$$

$$m_{f_0}^2 = m_{s,00}^2 \sin^2\theta_s + m_{s,88}^2 \cos^2\theta_s - 2m_{s,08}^2 \sin\theta_s \cos\theta_s,$$

where

$$m_{s,00}^2 = m^2 + \frac{\lambda_1}{3}\left(7\bar{\sigma}_x^2 + 4\sqrt{2}\bar{\sigma}_x\bar{\sigma}_y + 5\bar{\sigma}_y^2\right) + \lambda_2\left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) - \frac{\sqrt{2}c}{3}\left(\sqrt{2}\bar{\sigma}_x + \bar{\sigma}_y\right),$$

$$m_{s,88}^2 = m^2 + \frac{\lambda_1}{3}\left(5\bar{\sigma}_x^2 - 4\sqrt{2}\bar{\sigma}_x\bar{\sigma}_y + 7\bar{\sigma}_y^2\right) + \lambda_2\left(\frac{\bar{\sigma}_x^2}{2} + 2\bar{\sigma}_y^2\right) + \frac{\sqrt{2}c}{3}\left(\sqrt{2}\bar{\sigma}_x - \frac{\bar{\sigma}_y}{2}\right),$$

$$m_{s,08}^2 = \frac{2\lambda_1}{3}\left(\sqrt{2}\bar{\sigma}_x^2 - \bar{\sigma}_x\bar{\sigma}_y - \sqrt{2}\bar{\sigma}_y^2\right) + \sqrt{2}\lambda_2\left(\frac{\bar{\sigma}_x^2}{2} - \bar{\sigma}_y^2\right) + \frac{c}{3\sqrt{2}}\left(\bar{\sigma}_x - \sqrt{2}\bar{\sigma}_y\right).$$

Phys.Rev. C91 (2015)

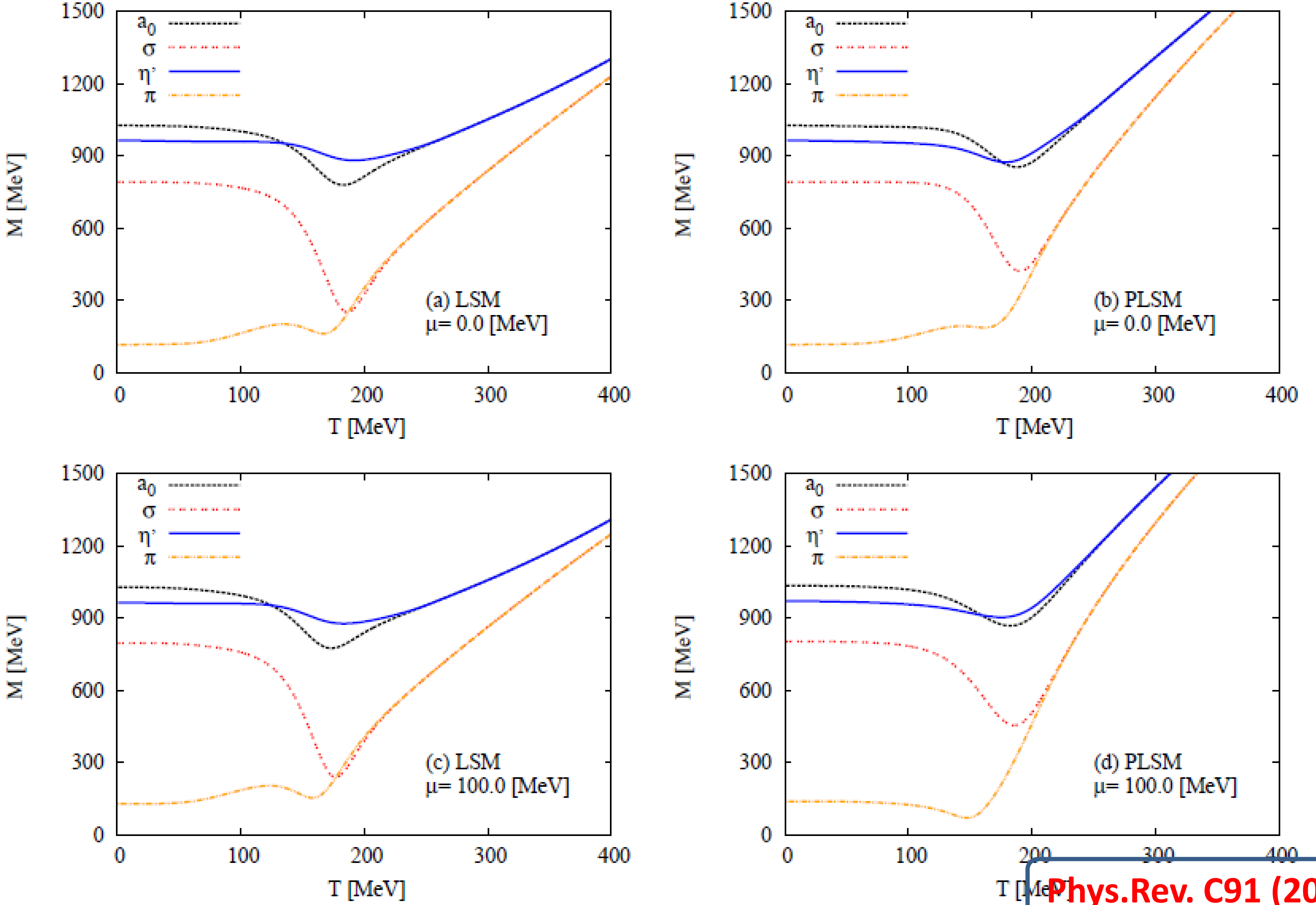
Sector	Symbol	PDG [33]	PLSM	PNJL [26, 27]	Lattice QCD	
					Hot QCD[30]	PACS-CS [31]
Scalar $J^{PC}=0^{++}$	a_0	$a_0(980^{\pm 20})$	1026	837		
	κ	$K_0^*(1425^{\pm 50})$	1115	1013		
	σ	$\sigma(400-1200)$	800	700		
	f_0	$f_0(1200-1500)$	1284	1169		
Pseudoscalar $J^{PC}=0^{-+}$	π	$\pi^0(134.97^{\pm 6.9})$	120	126	$134^{\pm 6}$	$135.4^{\pm 6.2}$
	K	$K^0\ (497.614^{\pm 24.8})$	509	490	$422.6^{\pm 11.3}$	$498^{\pm 22}$
	η	$\eta(547.853^{\pm 27.4})$	553	505	$579^{\pm 7.3}$	$688^{\pm 32}$
	η'	$\eta'(957.78^{\pm 60})$	965	949	–	–
Vector $J^{PC}=1^{-}$	ρ	$\rho(775.49^{\pm 38.8})$	745	–	$756.2^{\pm 36}$	$597^{\pm 86}$
	ω_X	$\omega(782.65^{\pm 44.7})$	745	–	$884^{\pm 18}$	$861^{\pm 23}$
	K^*	$K^*(891.66^{\pm 26})$	894	–	$1005^{\pm 93}$	$1010.2^{\pm 77}$
	ω_y	$\phi(1019.455^{\pm 51})$	1005	–	–	–
Axial-Vector $J^{PC}=1^{++}$	a_1	$a_1(1030-1260)$	980	–		
	f_{1x}	$f_1(1281^{\pm 60})$	980	–		
	K_1^*	$K_1^*(1270^{\pm 7})$	1135	–		
	f_{1y}	$f_1(1420^{\pm 71.3})$	1315	–		

Phys.Rev. C91 (2015)

WLCAPP World Laboratory Cosmology And Particle Physics

SU(3) (P)LSM: Phase Structure

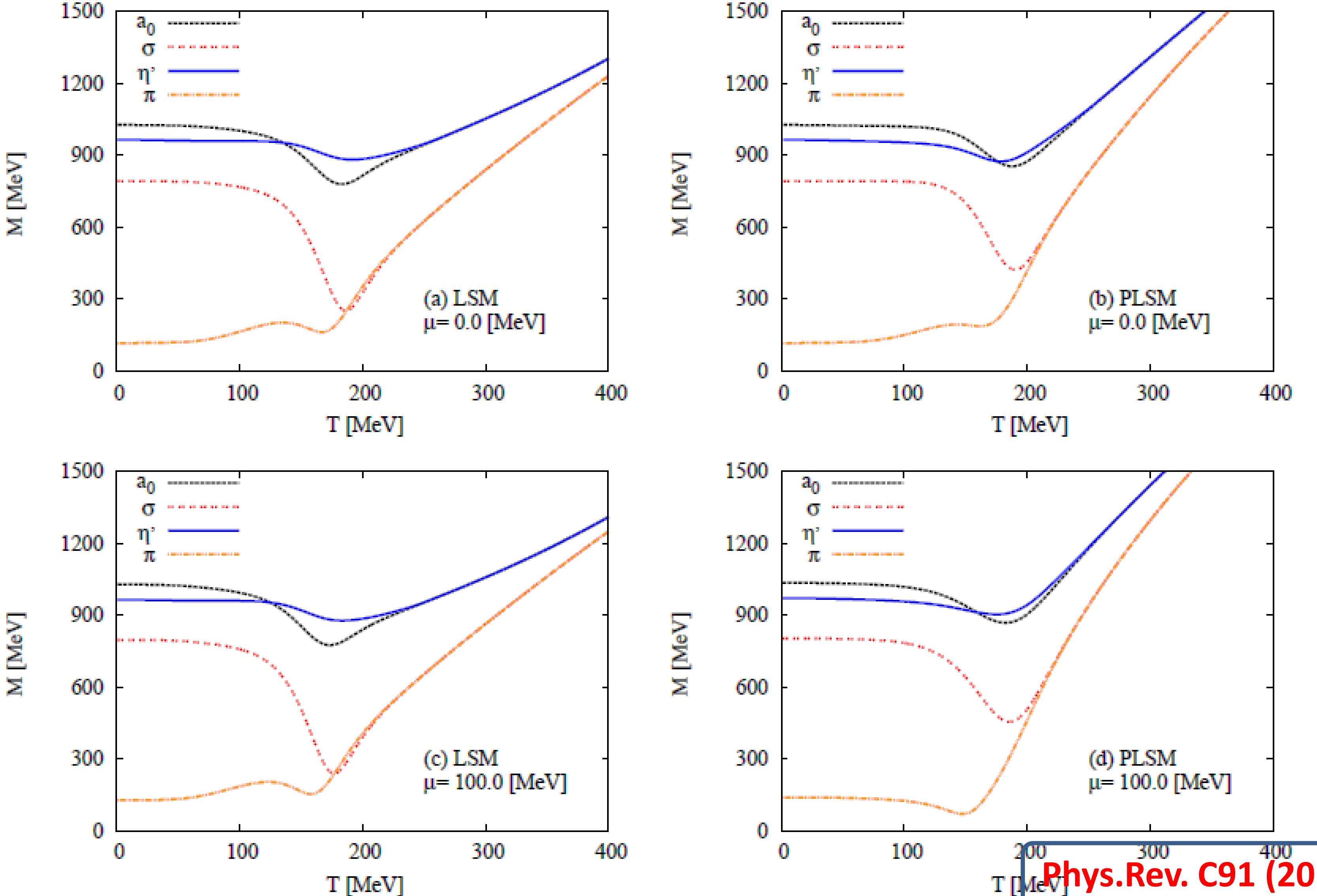
EGYPTIAN CENTER FOR THEORETICAL PHYSICS
ECTP
CENTER OF EXCELLENCE



Phys.Rev. C91 (2015)

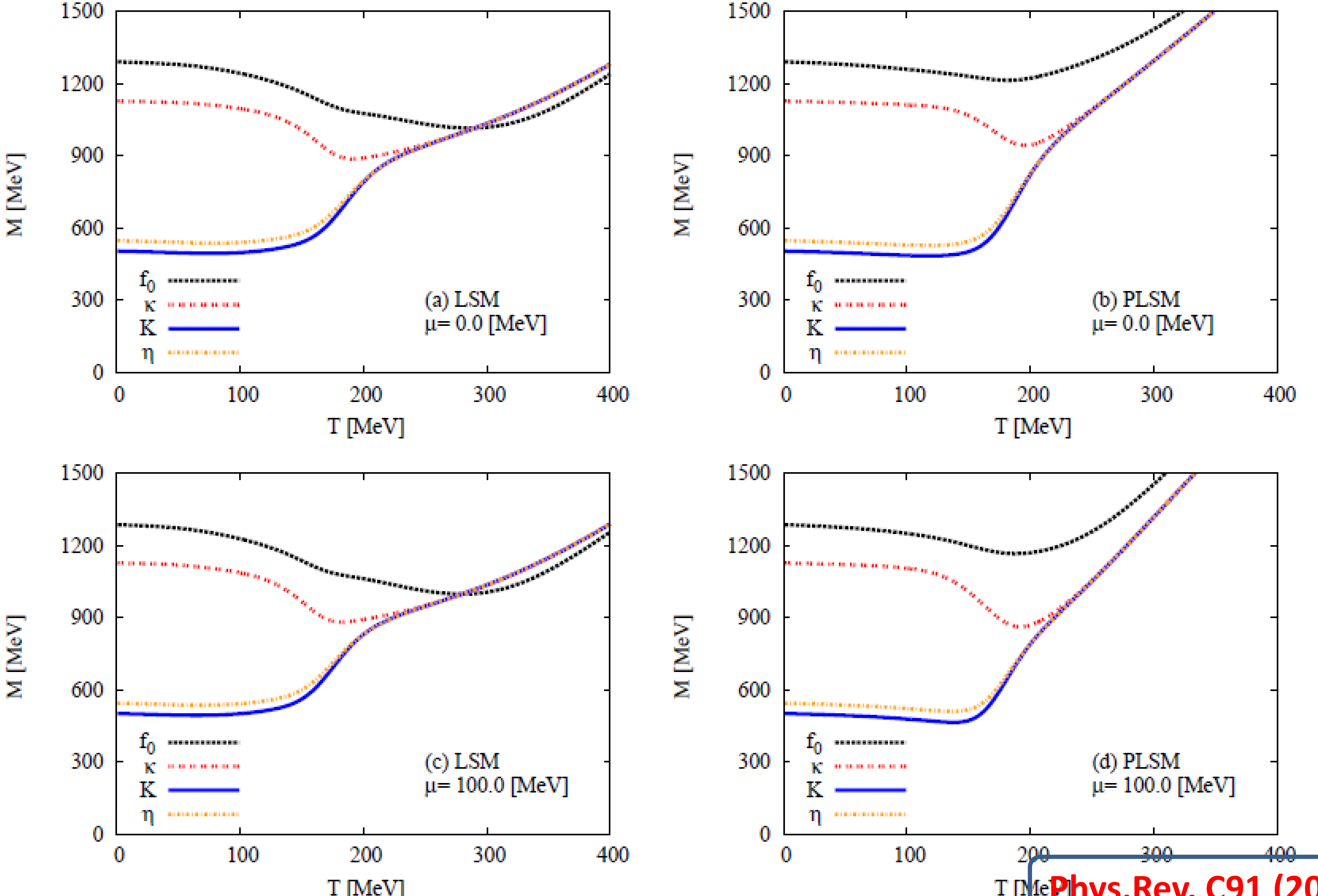
WLCAPP World Laboratory Cosmology And Particle Physics

SU(3) (P)LSM: Phase Structure



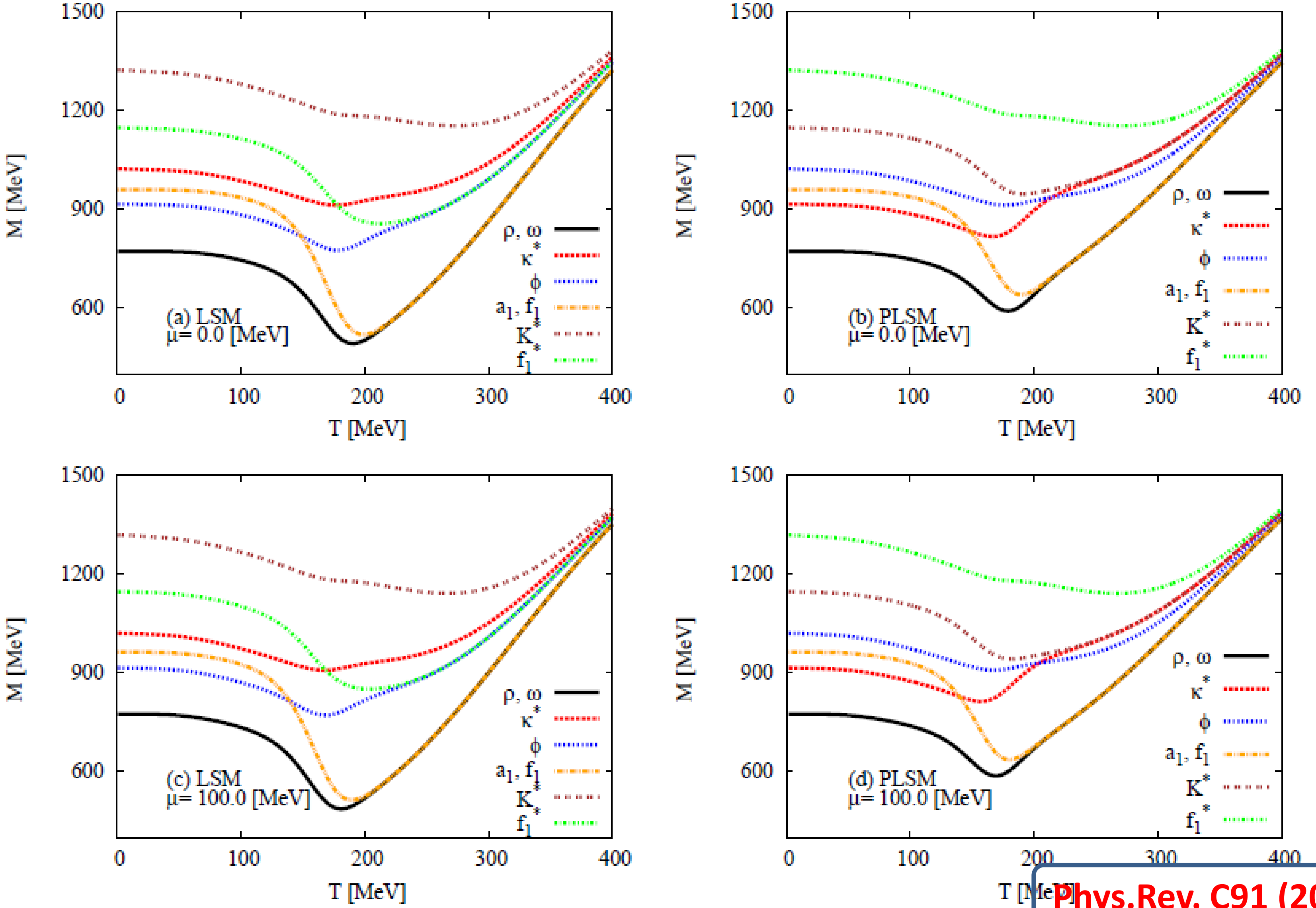
Phys.Rev. C91 (2015)

SU(3) (P)LSM: Phase Structure



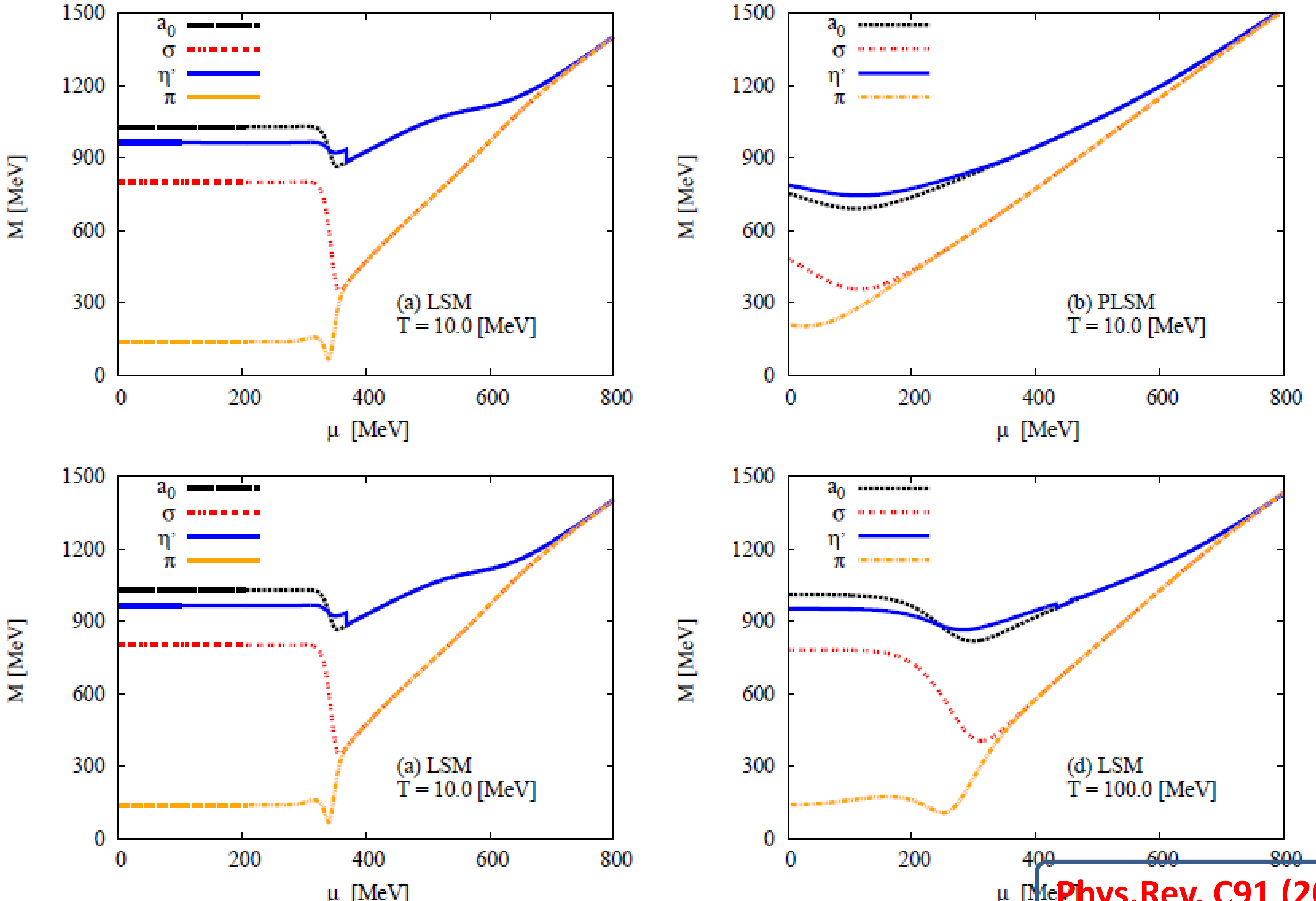
Phys.Rev. C91 (2015)

SU(3) (P)LSM: Phase Structure



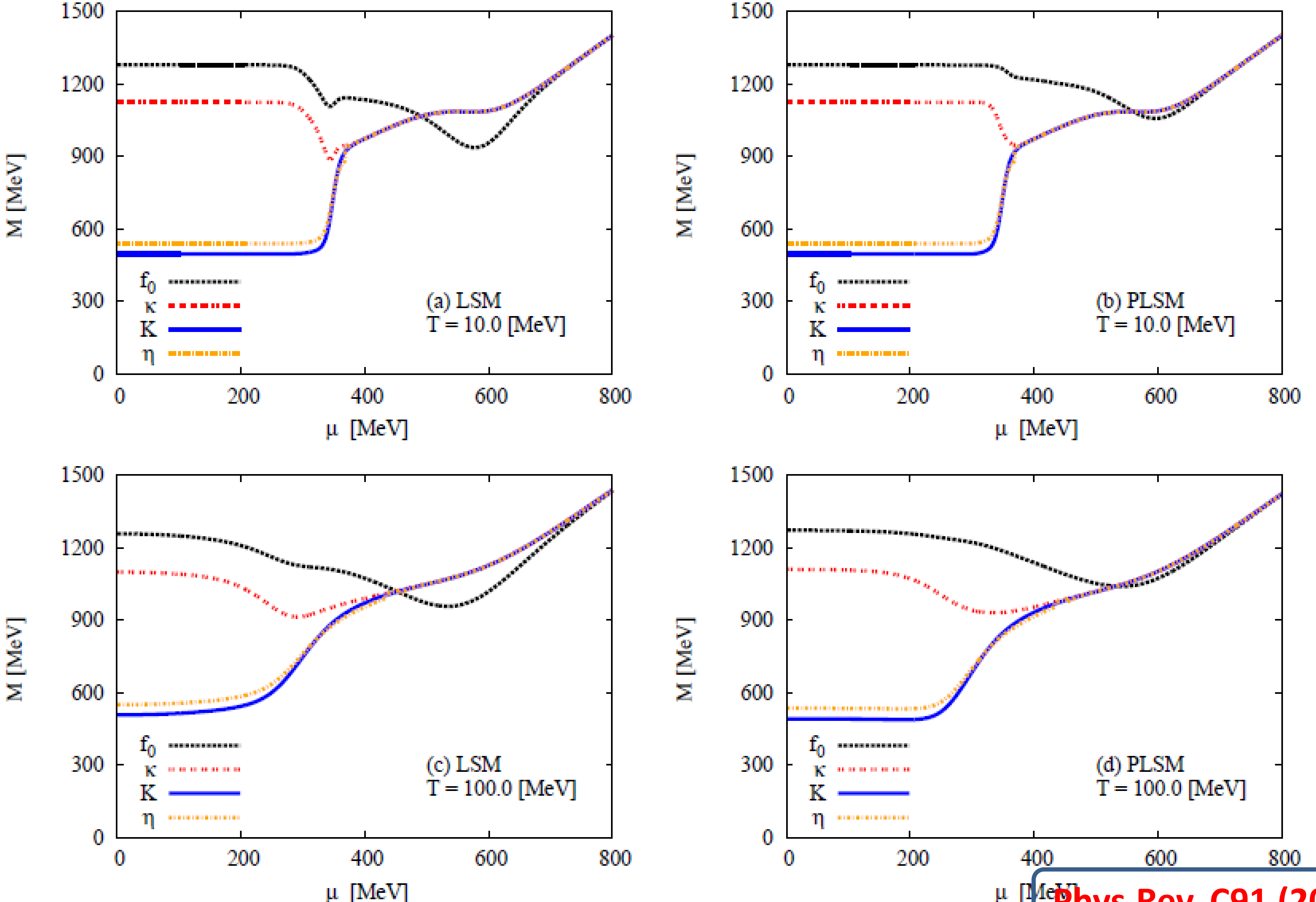
Phys.Rev. C91 (2015)

SU(3) (P)LSM: Phase Structure



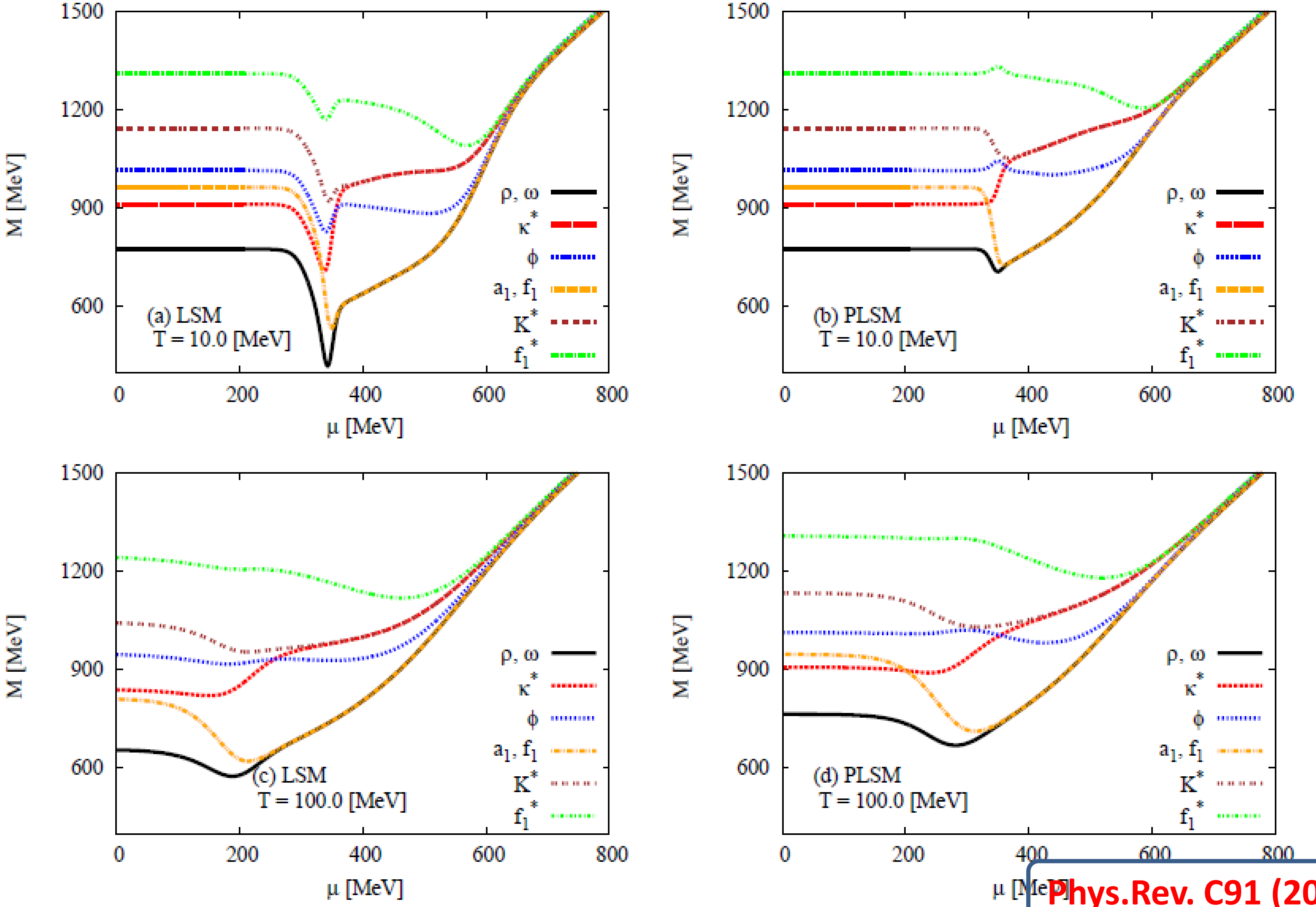
Phys.Rev. C91 (2015)

SU(3) (P)LSM: Phase Structure



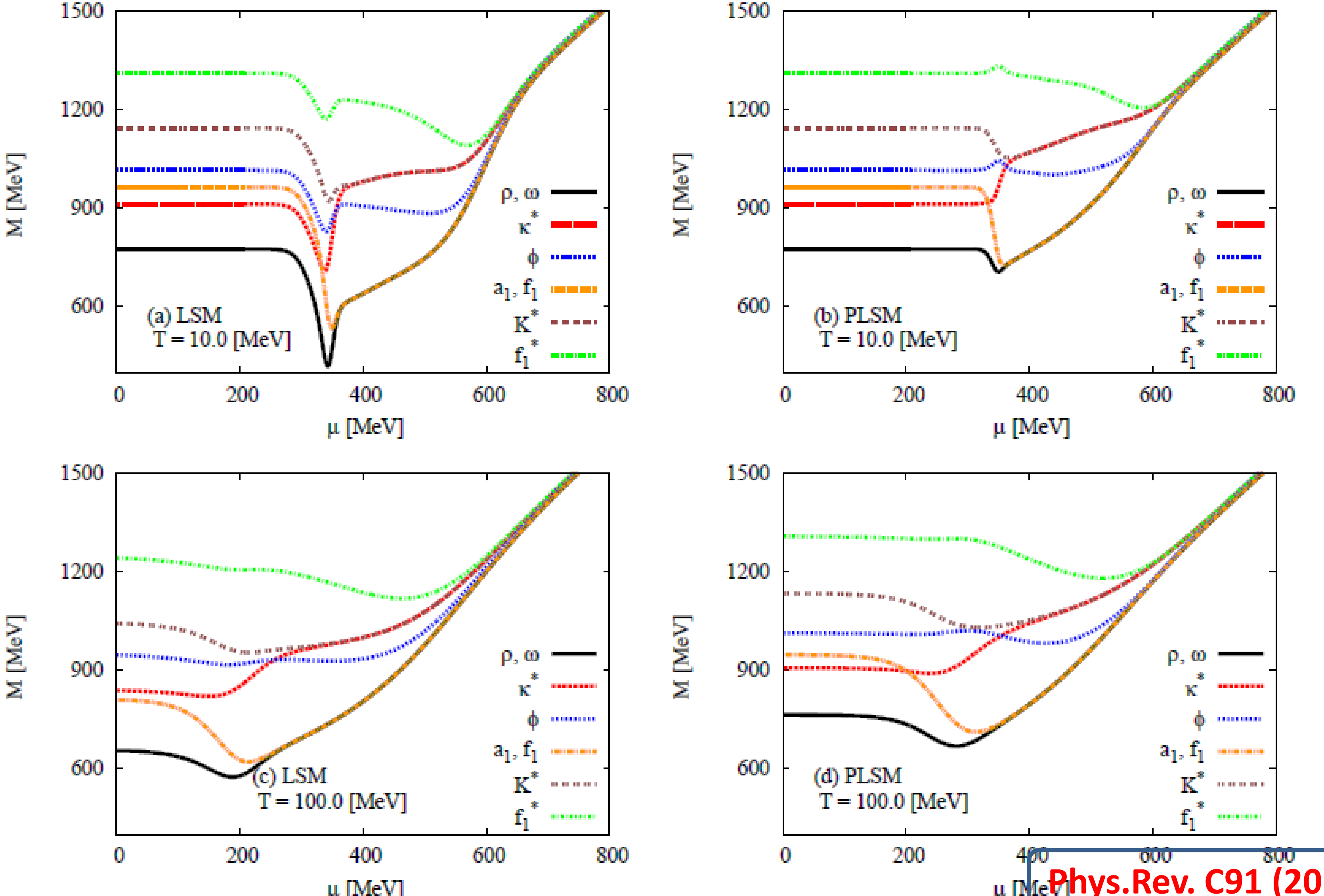
Phys.Rev. C91 (2015)

SU(3) (P)LSM: Phase Structure



Phys.Rev. C91 (2015)

SU(3) (P)LSM: Phase Structure

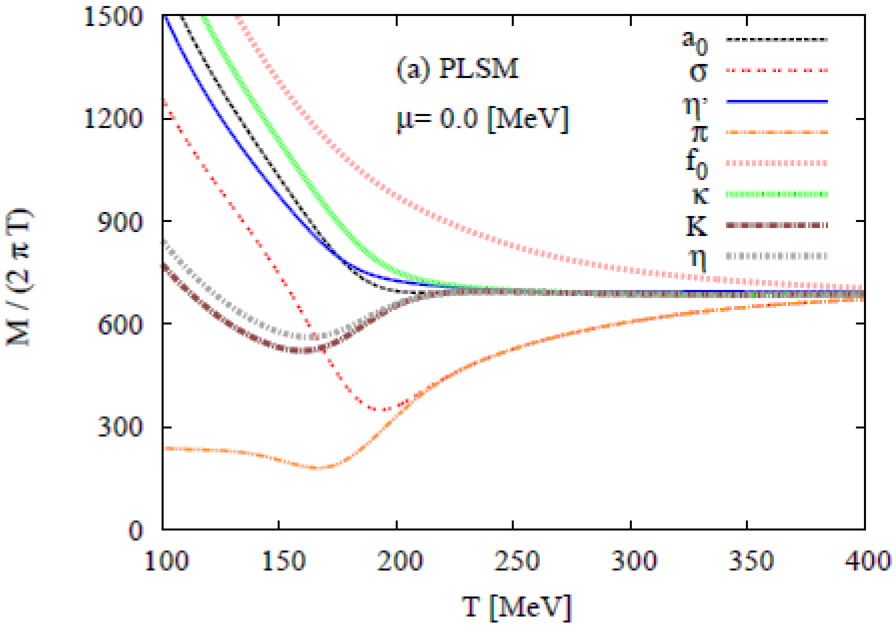


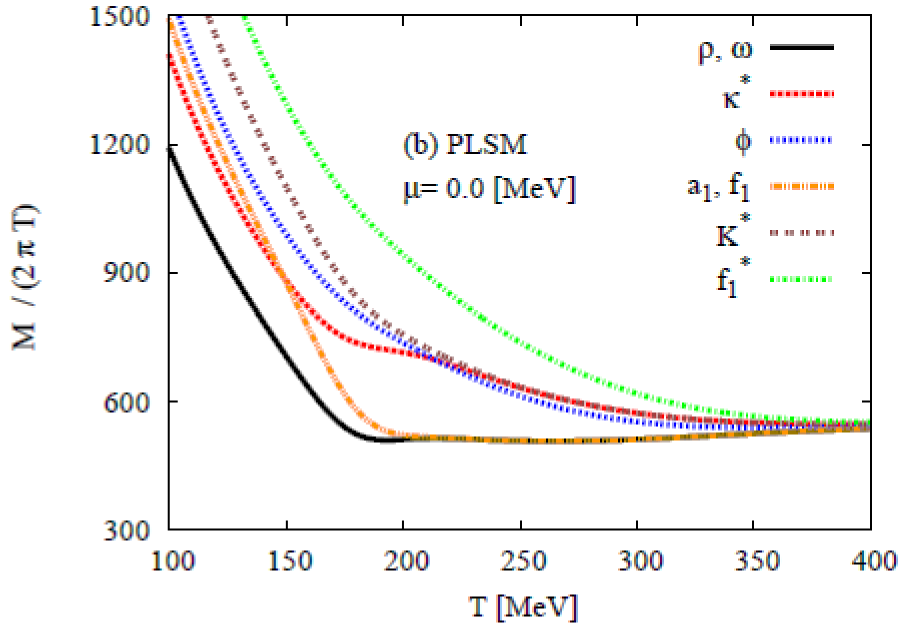
Phys.Rev. C91 (2015)

SU(3) PLSM: Phase Structure

	Scalar mesons				Pseudoscalar mesons				Vector mesons				Axial-vector mesons			
meson	a_0	κ	σ	f_0	π	K	η	η'	ρ	K_0^*	ω	ϕ	a_1	K_1	f_1	f_1^*
$T_{Dissolving}^{Meson}$ [MeV]	200	250	320	320	320	230	235	300	195	300	195	300	205	250	205	350

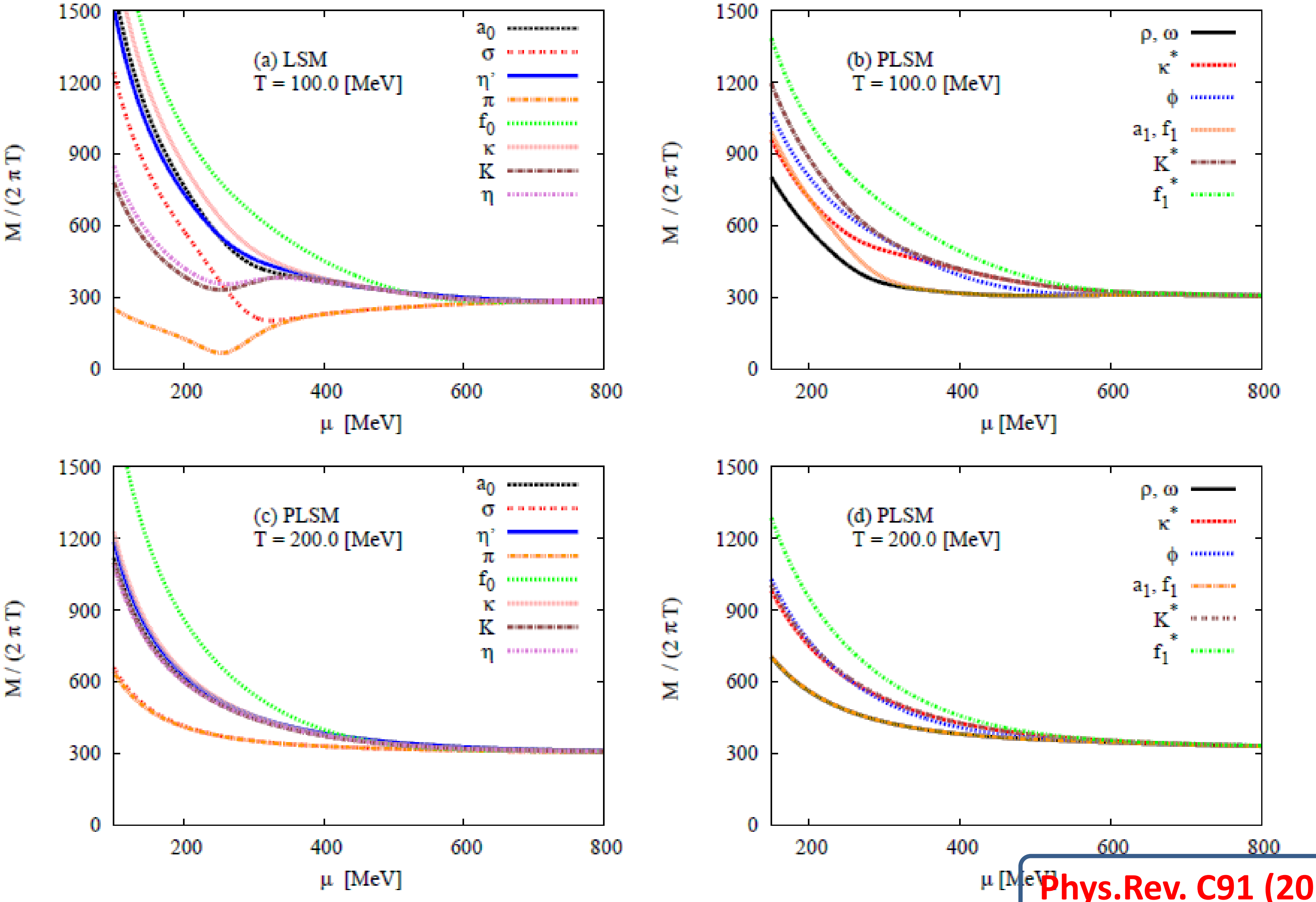
Tab. III: The approximative dissolving temperature corresponding to the different meson states.





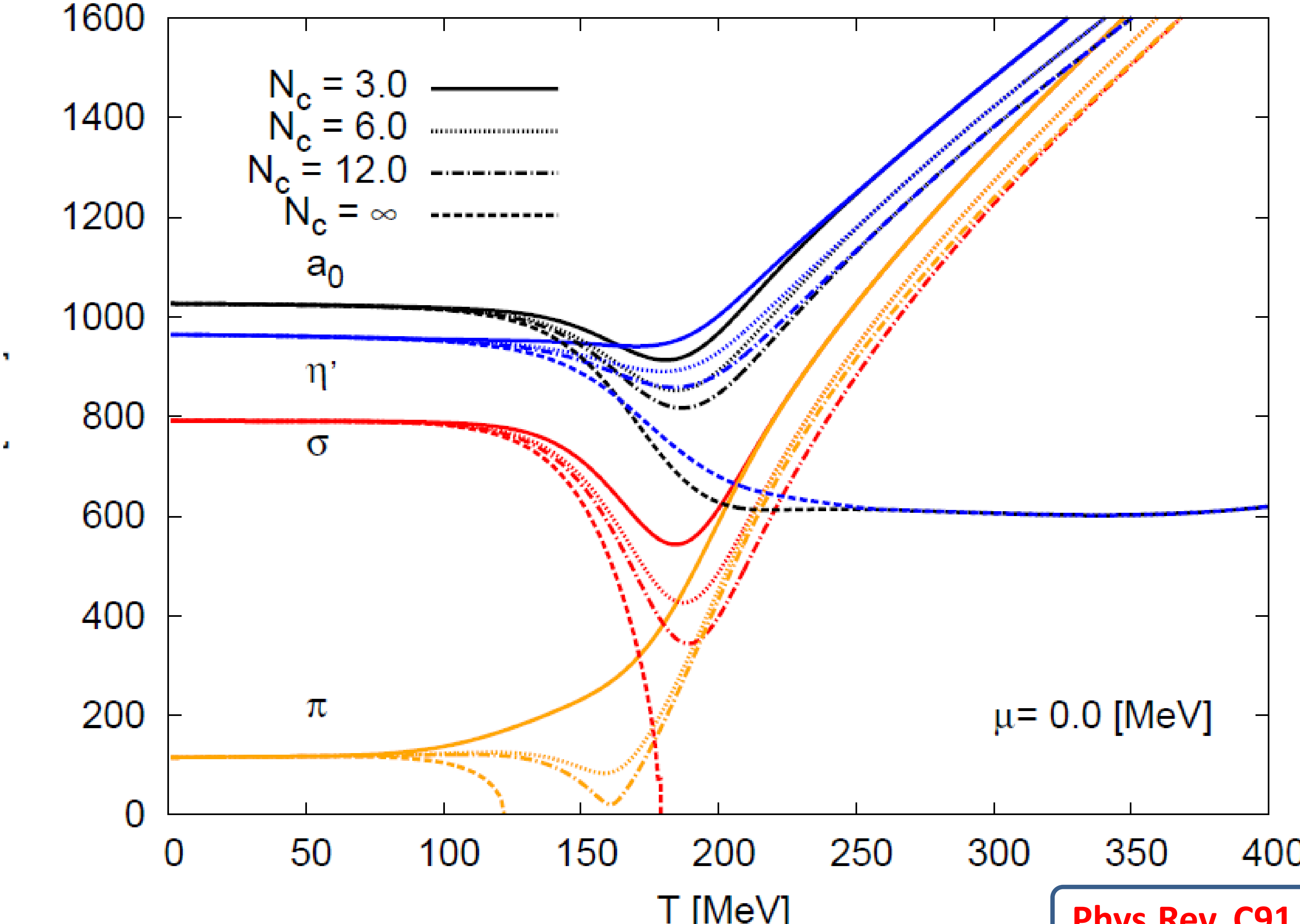
Phys.Rev. C91 (2015)

SU(3) PLSM: (P)hase Structure



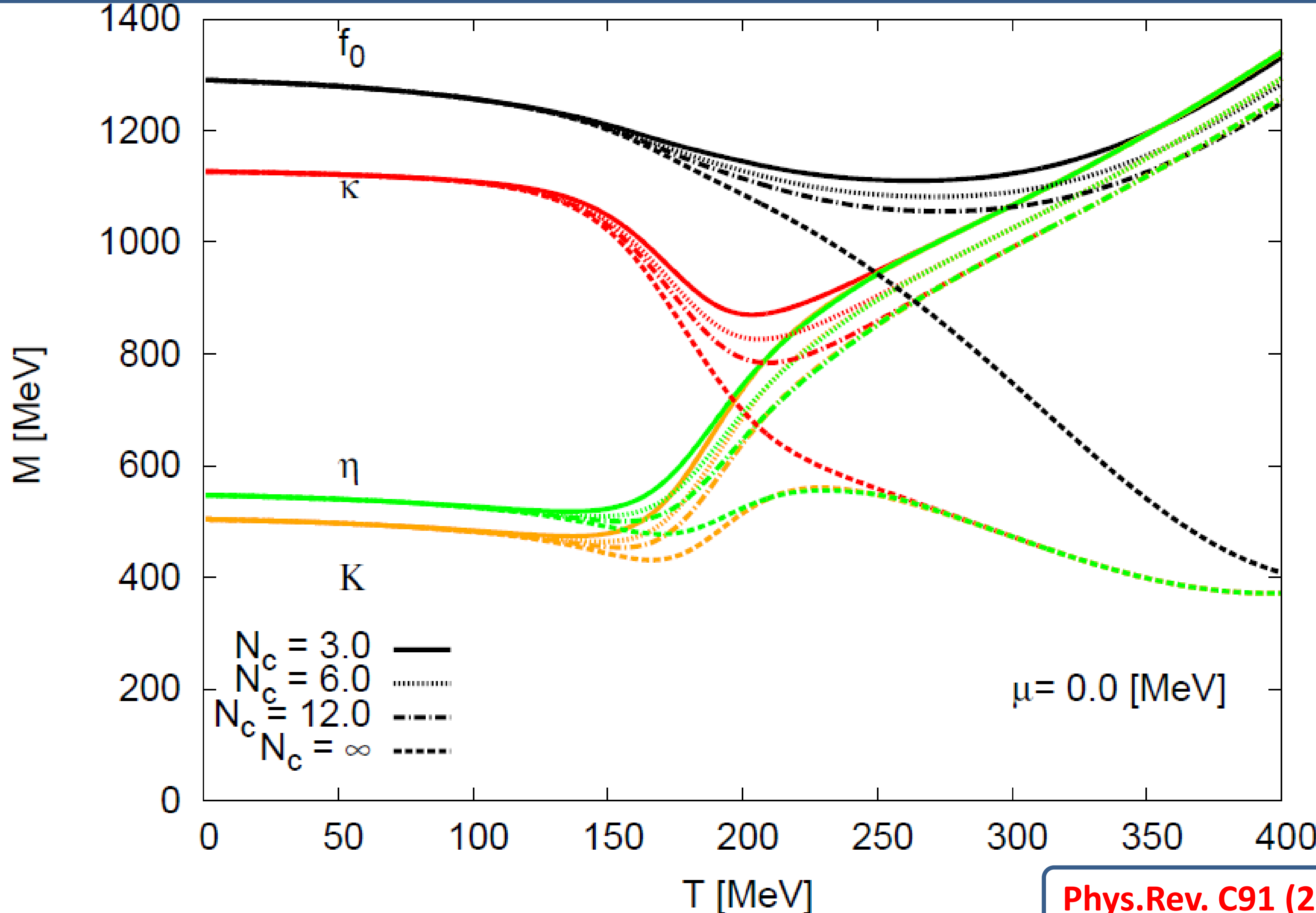
Phys.Rev. C91 (2015)

SU(3) PLSM: Phase Structure



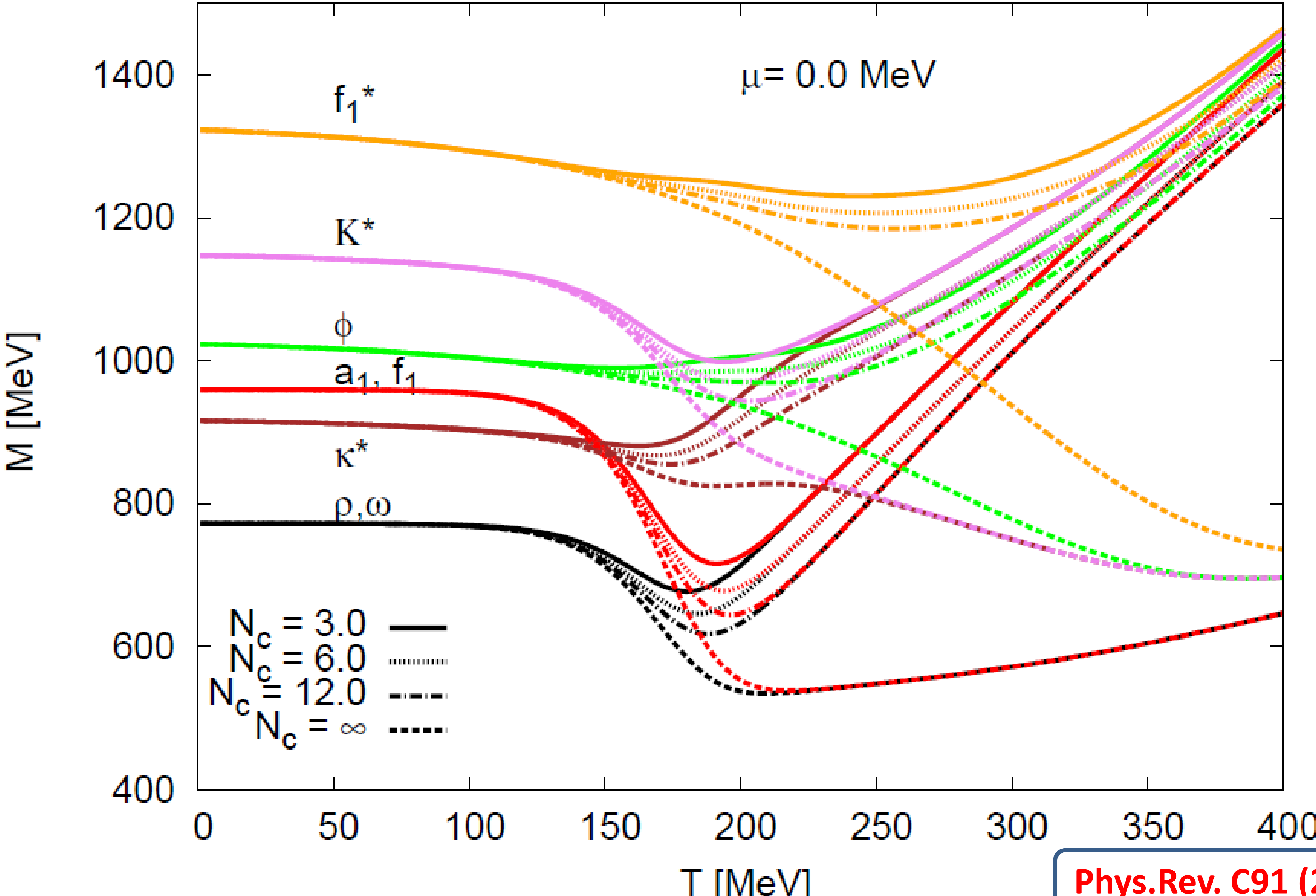
Phys.Rev. C91 (2015)

SU(3) PLSM: Phase Structure



Phys.Rev. C91 (2015)

SU(3) PLSM: Phase Structure



Phys.Rev. C91 (2015)

WLCAPP World Laboratory Cosmology And Particle Physics

SU(3) PLSM: Phase Structure at eB≠0

EGYPTIAN CENTER FOR THEORETICAL PHYSICS
ECTP
CENTER OF EXCELLENCE

$$\mathcal{L}_{chiral} = \mathcal{L}_f + \mathcal{L}_m \qquad \mathcal{L}_f = \bar{q}\left[i\partial\!\!\!/ - g\, T_a\, \left(\sigma_a + i\,\gamma_5\,\pi_a + \gamma_\zeta V_a^\zeta + \gamma_\zeta\gamma_5 A_a^\zeta\right)\right] q,$$

scalars ($J^{pc} = 0^{++}$) and pseudoscalars ($J^{pc} = 0^{-+}$)

vectors ($J^{pc} = 1^{--}$) and axial-vectors ($J^{pc} = 1^{++}$)

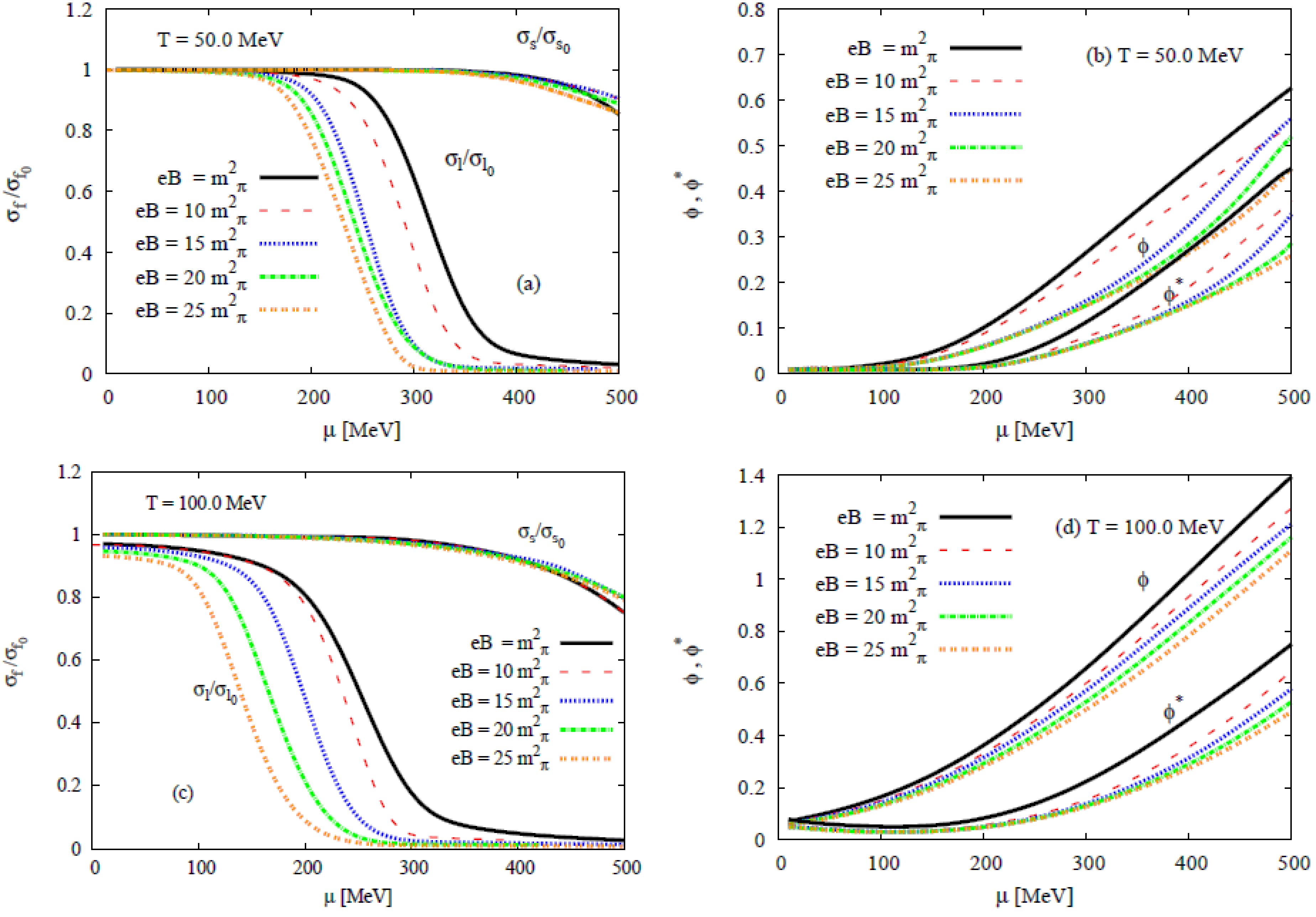
$$\begin{aligned}
\mathcal{L}_{SP} &= \mathrm{Tr}(\partial_\mu\Phi^\dagger\partial^\mu\Phi - m^2\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 + \mathrm{Tr}[H(\Phi+\Phi^\dagger)],\\
\mathcal{L}_{AV} &= -\frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right]\\
&+ i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\})\\
&+ g_3[\mathrm{Tr}(L_\mu L_\nu L^\mu L^\nu) + \mathrm{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4[\mathrm{Tr}\,(L_\mu L^\mu L_\nu L^\nu) + \mathrm{Tr}\,(R_\mu R^\mu R_\nu R^\nu)]\\
&+ g_5\,\mathrm{Tr}\,(L_\mu L^\mu)\;\mathrm{Tr}\,(R_\nu R^\nu) + g_6[\mathrm{Tr}(L_\mu L^\mu)\;\mathrm{Tr}(L_\nu L^\nu) + \mathrm{Tr}(R_\mu R^\mu)\;\mathrm{Tr}(R_\nu R^\nu)],\\
\mathcal{L}_{Int} &= \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\,\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\,\mathrm{Tr}[|L_\mu\Phi|^2 + |\Phi R_\mu|^2] + 2h_3\,\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger),\\
\mathcal{L}_{U(1)_A} &= c[\mathrm{Det}(\Phi) + \mathrm{Det}(\Phi^\dagger)] + c_0[\mathrm{Det}(\Phi) - \mathrm{Det}(\Phi^\dagger)]^2 + c_1[\mathrm{Det}(\Phi) + \mathrm{Det}(\Phi^\dagger)]\,\mathrm{Tr}[\Phi\Phi^\dagger].
\end{aligned}$$

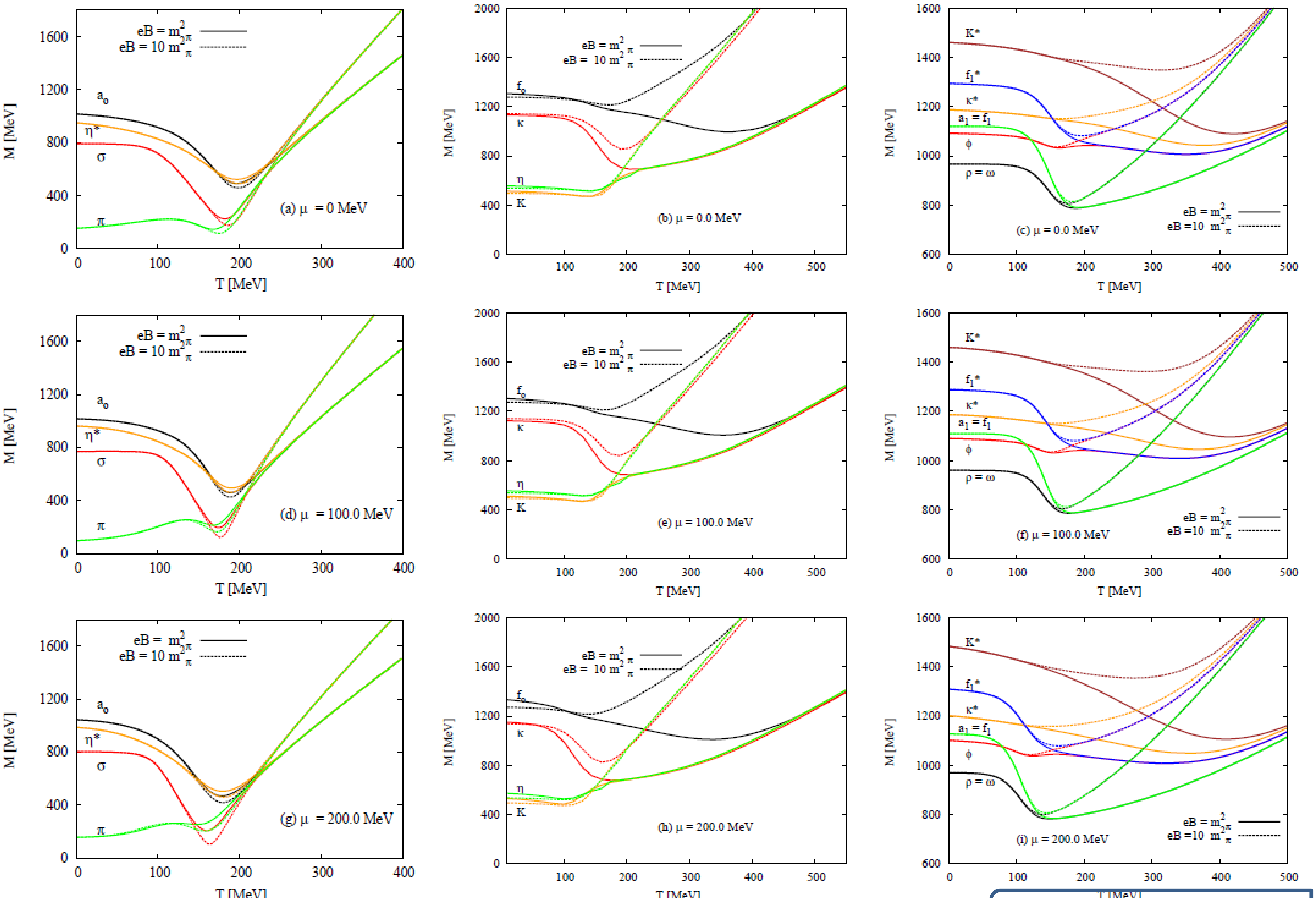
$$\Phi = \sum_{a=0}^{N_f^2-1} T_a(\sigma_a + i\pi_a), \qquad L^\mu = \sum_{a=0}^{N_f^2-1} T_a\,(V_a^\mu + A_a^\mu), \qquad R^\mu = \sum_{a=0}^{N_f^2-1} T_a\,(V_a^\mu - A_a^\mu). (6)$$

Not yet Published

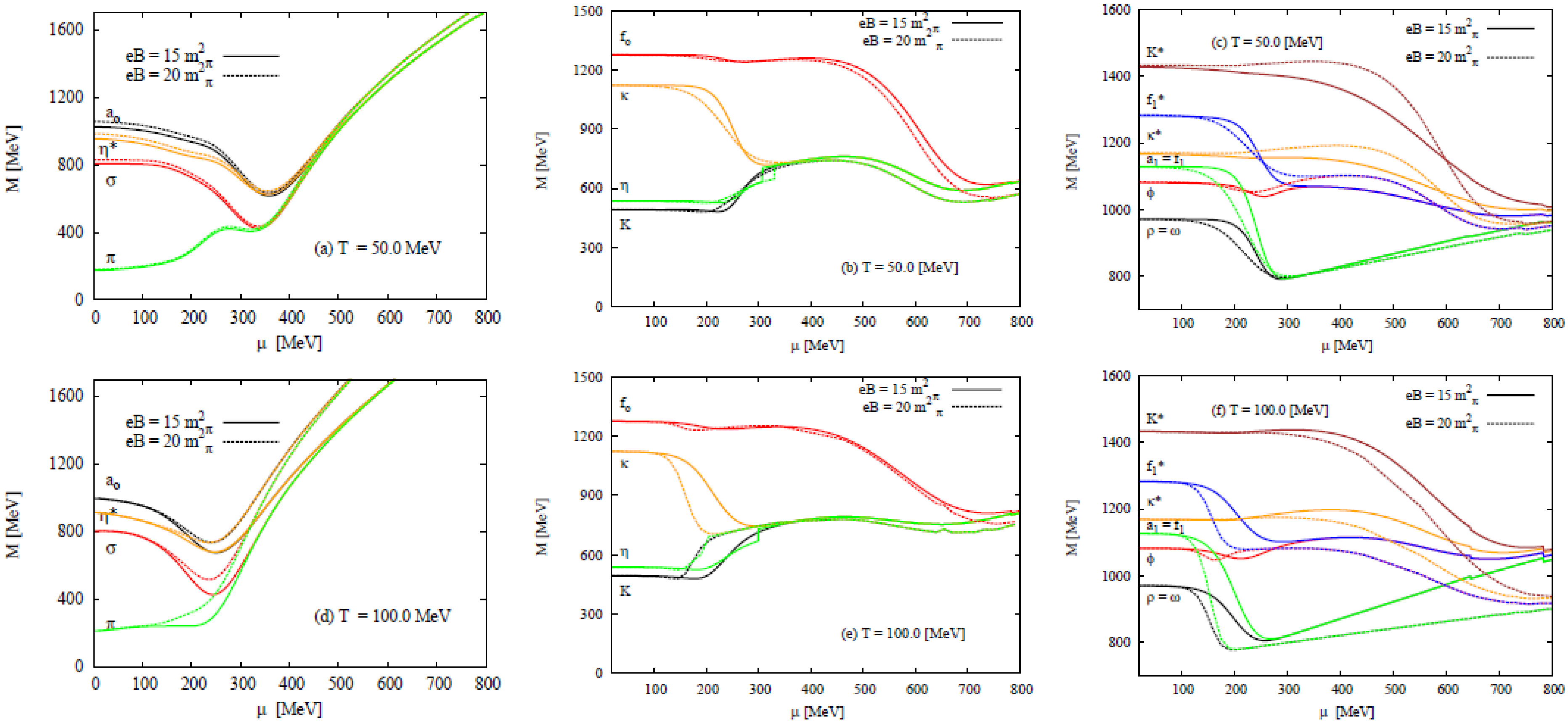
$$
\begin{aligned}
\Omega_{\bar{q}q}(T,\mu_f,B) \;=\; & -2\sum_{f=l,s}\frac{|q_f|B\,T}{(2\pi)^2}\int_0^\infty dP_z\left\{\ln\left[1+3\left(\phi+\phi^* e^{-\frac{E_f-\mu_f}{T}}\right)\, e^{-\frac{E_f-\mu_f}{T}}+e^{-3\frac{E_f-\mu_f}{T}}\right]\right.\\
& \left.+\ln\left[1+3\left(\phi^*+\phi e^{-\frac{E_f+\mu_f}{T}}\right)\, e^{-\frac{E_f+\mu_f}{T}}+e^{-3\frac{E_f+\mu_f}{T}}\right]\right\}\\
& -\;4\sum_{f=l,s}\frac{|q_f|B\,T}{(2\pi)^2}\sum_{\nu=1}^{(\nu_{max})_f}\int_0^\infty dP_z\left\{\ln\left[1+3\left(\phi+\phi^* e^{-\frac{E_{B,f}-\mu_f}{T}}\right)\, e^{-\frac{E_{B,f}-\mu_f}{T}}+e^{-3\frac{E_{B,f}-\mu_f}{T}}\right]\right.\\
& \left.+\ln\left[1+3\left(\phi^*+\phi e^{-\frac{E_{B,f}+\mu_f}{T}}\right)\, e^{-\frac{E_{B,f}+\mu_f}{T}}+e^{-3\frac{E_{B,f}+\mu_f}{T}}\right]\right\}
\end{aligned}
$$

Reminder to Tch



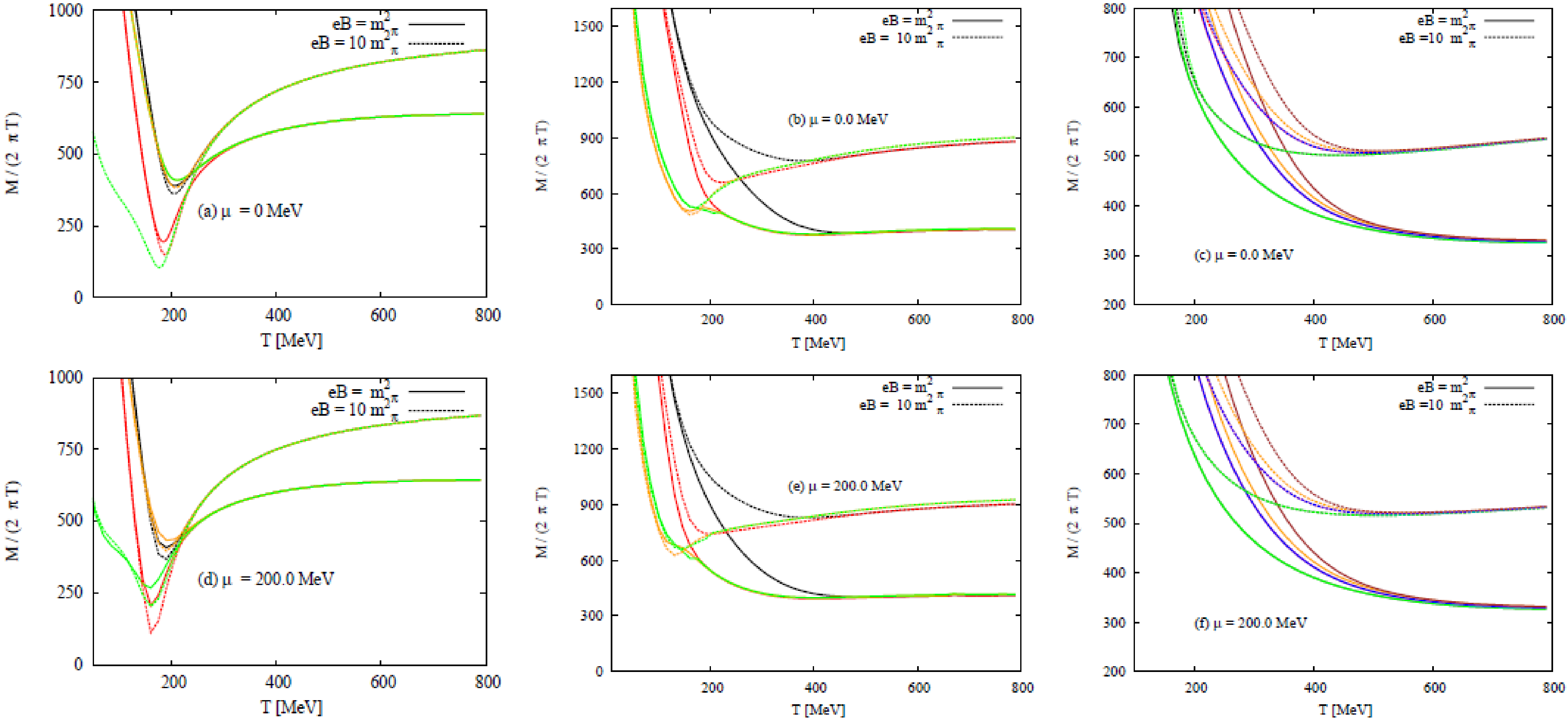


Not yet Published



Abdel Nasser TAWFIK

Not yet Published



Comparison	Scalar Mesons	Pseudoscalar Mesons	Vector Mesons	Axialvector Mesons
Meson States	a_0 κ σ f_0	π K η η'	ρ K_0^* ω ϕ	a_1 K_1 f_1 f_1^*
T_c^d in MeV	430 450 470 450	320 230 335 240	495 495 495 495	495 495 495 495

SU(3) dissolving T < SU(4) dissolving T

Not yet Published

SU(3) PLSM: Viscosity at eB(=)≠0

Boltzmann-Uehling-Uhlenbeck (BUU) Equation

eB=0

$$\zeta(T,\mu) = \frac{1}{9T}\sum_f \int \frac{d^3p}{(2\pi)^3}\,\frac{\tau_f}{E_f^2}\left[\frac{|\vec{p}|^2}{3} - c_s^2 E_f^2\right]^2 f_f(T,\mu),$$

$$\eta(T,\mu) = \frac{1}{15T}\sum_f \int \frac{d^3p}{(2\pi)^3}\,\frac{p^4}{E_f^2}\,\tau_f f_f(T,\mu).$$

eB≠0

$$\zeta(T,\mu,eB) = \frac{1}{9\,T}\sum_f \frac{|q_f|B}{2\pi}\sum_\nu \int \frac{dp}{2\pi}\,(2-\delta_{0\nu})\,\frac{\tau_f}{E_{B,f}^2}\left[\frac{|\vec{p}|^2}{3} - c_s^2 E_{B,f}^2\right]^2 f_f(T,\mu),$$

$$\eta(T,\mu,eB) = \frac{1}{15\,T}\sum_f \frac{|q_f|B}{2\pi}\sum_\nu \int \frac{dp}{2\pi}\,(2-\delta_{0\nu})\,\frac{p^4}{E_{B,f}^2}\,\tau_f f_f(T,\mu).$$

1608.01034 [hep-ph]

WLCAPP World Laboratory Cosmology And Particle Physics

SU(3) PLSM: Viscosity at eB≠0: BUU

EGYPTIAN CENTER FOR THEORETICAL PHYSICS — CENTER OF EXCELLENCE

symmetric energy-momentum tensor $T^{\mu\nu} = -p\, g^{\mu\nu} + \mathcal{H}\, u^{\mu}\, u^{\nu} + \Delta T^{\mu\nu}$

$u^{\nu|\mu}$ being four velocity, p is the pressure, and $\mathcal{H} = p + \epsilon$ is the enthalpy density

$\epsilon = -p + T s + \epsilon^{\text{field}}$ $\qquad \epsilon^{\text{field}} = eB \cdot \mathcal{M}$

When adding a dissipative part $\Delta T^{\mu\nu}$ to the energy-momentum tensor

$$\Delta T^{\mu\nu} = \eta\left(D^{\mu} u^{\nu} + D^{\nu} u^{\mu} + \frac{2}{3}\Delta^{\mu\nu}\partial_{\sigma} u^{\sigma}\right) - \zeta \Delta^{\mu\nu}\partial_{\sigma} u^{\sigma},$$

and the Landau-Lifshitz condition, $u_{\mu}\, \Delta T^{\mu\nu} = 0$

the hydrodynamic expansion reads

$$\delta T^{ij} = \sum_f \int d\Gamma^* \frac{p^i\, p^j}{E_f}\left[- \mathcal{A}_f\, \partial_{\sigma} u^{\sigma} - \mathcal{B}_f\, p_f^{\nu} D_{\nu}\left(\frac{\mu}{T}\right) + \mathcal{C}_f\, p_f^{\mu} p_f^{\nu}\left(D^{\mu} u^{\nu} + D^{\nu} u^{\mu} + \frac{2}{3}\Delta^{\mu\nu}\partial_{\sigma} u^{\sigma}\right)\right] f_f^{eq}$$

Because of eB≠0, the phase space distributions become modified

$$\int d\Gamma^* \equiv \int \frac{d^3 k}{(2\pi)^3} \longrightarrow \frac{|q_f| B}{2\pi} \sum_{\nu} \int \frac{dk_z}{2\pi}(2 - \delta_{0\nu})$$

$$p_f^i p_f^j p_f^{\sigma} p_f^{\rho} = |p_f|^4 (\delta_{ij}\delta_{\sigma\rho} + \delta_{i\sigma}\delta_{j\rho} + \delta_{i\rho}\delta_{j\sigma})$$

1608.01034 [hep-ph]

SU(3) PLSM: Viscosity: BUU

Bulk and shear viscosity read

$$\zeta = \frac{1}{3}\sum_f \frac{|q_f|B}{2\pi}\sum_\nu \int \frac{dk_z}{2\pi}(2-\delta_{0\nu})\frac{|p|^2}{E_f} f_f \mathcal{A}_f$$

$$\eta = \frac{2}{15}\sum_f \frac{|q_f|B}{2\pi}\sum_\nu \int \frac{dk_z}{2\pi}(2-\delta_{0\nu})\frac{|p|^4}{E_f} f_f \mathcal{C}_f$$

For an out-of-equilibrium state, the four velocity gets modified and

$$f_f(x,p) = f^{eq}\left(u_i p^i/T\right)\left[1+\phi_f(x,p)\right]$$

$$\phi_f = \left[-\mathcal{A}_f\,\partial_\sigma u^\sigma - \mathcal{B}_f\, p_f^\nu D_\nu\left(\frac{\mu}{T}\right) + \mathcal{C}_f\, p_f^\mu p_f^\nu\left(D^\mu u^\nu + D^\nu u^\mu + \frac{2}{3}\Delta^{\mu\nu}\partial_\sigma u^\sigma\right)\right]$$

Boltzmann master equations is needed to determine A_f anf C_f

$$\frac{\partial f_f(x,t,p)}{\partial t} = \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x^i}\frac{\partial x^i}{\partial t} + \frac{\partial}{\partial p^i}\frac{\partial p^i}{\partial t}\right) f_f(x,t,p) \equiv \mathbf{C}\left[f_f\right]$$

$$\mathbf{C} = \sum_{\{i\}\{j\};f}\sum_\nu \frac{|q_f|B}{2\pi}(2-\delta_{0\nu})\frac{1}{S}\int\left(\frac{dk_z}{2\pi}\right)_{\{i\}}\left(\frac{dk_z}{2\pi}\right)_{\{j\}} W(\{i\}|\{j\})F[f_f]$$

Statistical factor → S

BE or FD statistics → $F[f_f]$

Abdel Nasser TAWFIK

1608.01034 [hep-ph]

SU(3) PLSM: Viscosity at eB≠0: BUU

$\mathcal{A}_f = \mathcal{A}_f^{\text{par}} - bE_{b,f}$ is a particular solution conserving Landau-Lifshitz condition

$$\zeta = \frac{1}{3}\sum_f \frac{|q_f|B}{2\pi}\sum_\nu \int \frac{dk_z}{2\pi}(2-\delta_{0\nu})\left[\frac{|\vec{p}|^2}{3} - c_s^2 E_{B,f}^2\right] f_f \mathcal{A}_f^{\text{par}},$$

$$\eta = \frac{2}{15}\sum_f \frac{|q_f|B}{2\pi}\sum_\nu \int \frac{dk_z}{2\pi}(2-\delta_{0\nu})\frac{|p|^4}{E_{B,f}} f_f \mathcal{C}_f^{\text{par}}.$$

In relaxation time approximation, quark distributions can be expressed in their equilibrium one and an arbitrary infinitesimal $f = f^{eq} + \delta f$

$$\mathcal{A}_f^{\text{par}} = \frac{\tau_f}{3T}\left[\frac{|\vec{p}|^2}{3} - c_s^2 E_{B,f}^2\right],$$

$$\mathcal{C}_f^{\text{par}} = \frac{\tau_f}{2TE_f}$$

In local rest-frame of the fluid, bulk and shear viscosity at eB≠0 read

$$\zeta(T,\mu,eB) = \frac{1}{9T}\sum_f \frac{|q_f|B}{2\pi}\sum_\nu \int \frac{dk_z}{2\pi}(2-\delta_{0\nu})\frac{\tau_f}{E_{B,f}^2}\left[\frac{|\vec{p}|^2}{3} - c_s^2 E_{B,f}^2\right]^2 f_f(T,\mu,eB),$$

$$\eta(T,\mu,eB) = \frac{1}{15T}\sum_f \frac{|q_f|B}{2\pi}\sum_\nu \int \frac{dk_z}{2\pi}(2-\delta_{0\nu})\frac{p^4}{E_{B,f}^2}\tau_f f_f(T,\mu,eB).$$

1608.01034 [hep-ph]

SU(3) PLSM: Viscosity at eB(=)≠0

Green-Kubo (GK) Correlation

eB=0

$$\zeta(T,\mu) = \frac{3}{2T}\sum_f \int \frac{d^3p}{(2\pi)^3}\frac{\tau_f}{E_f^2}\left[\frac{|\vec{p}|^2}{3} - c_s^2\, E_f^2\right]^2 f_f(T,\mu)\Big[1 - f_f(T,\mu)\Big],$$

$$\eta(T,\mu) = \frac{2}{15T}\sum_f \int \frac{d^3p}{(2\pi)^3}\frac{|\vec{p}|^4\tau_f}{E_f^2}\, f_f(T,\mu)\Big[1 - f_f(T,\mu)\Big],$$

eB≠0

$$\zeta(T,\mu,eB) = \frac{3}{2\,T}\sum_f \frac{|q_f|B}{2\pi}\sum_\nu \int \frac{dp}{2\pi}\,(2-\delta_{0\nu})\,\frac{\tau_f}{E_{B,f}^2} \left[\frac{|\vec{p}|^2}{3} - c_s^2 E_{B,f}^2\right]^2 f_f(T,\mu,eB)\Big[1 - f_f(T,\mu,eB)\Big],$$

$$\eta(T,\mu,eB) = \frac{2}{15\,T}\sum_f \frac{|q_f|B}{2\pi}\sum_\nu \int \frac{dp}{2\pi}\,(2-\delta_{0\nu})\,\frac{|\vec{p}|^4\tau_f}{E_{B,f}^2} f_f(T,\mu,eB)\Big[1 - f_f(T,\mu,eB)\Big].$$

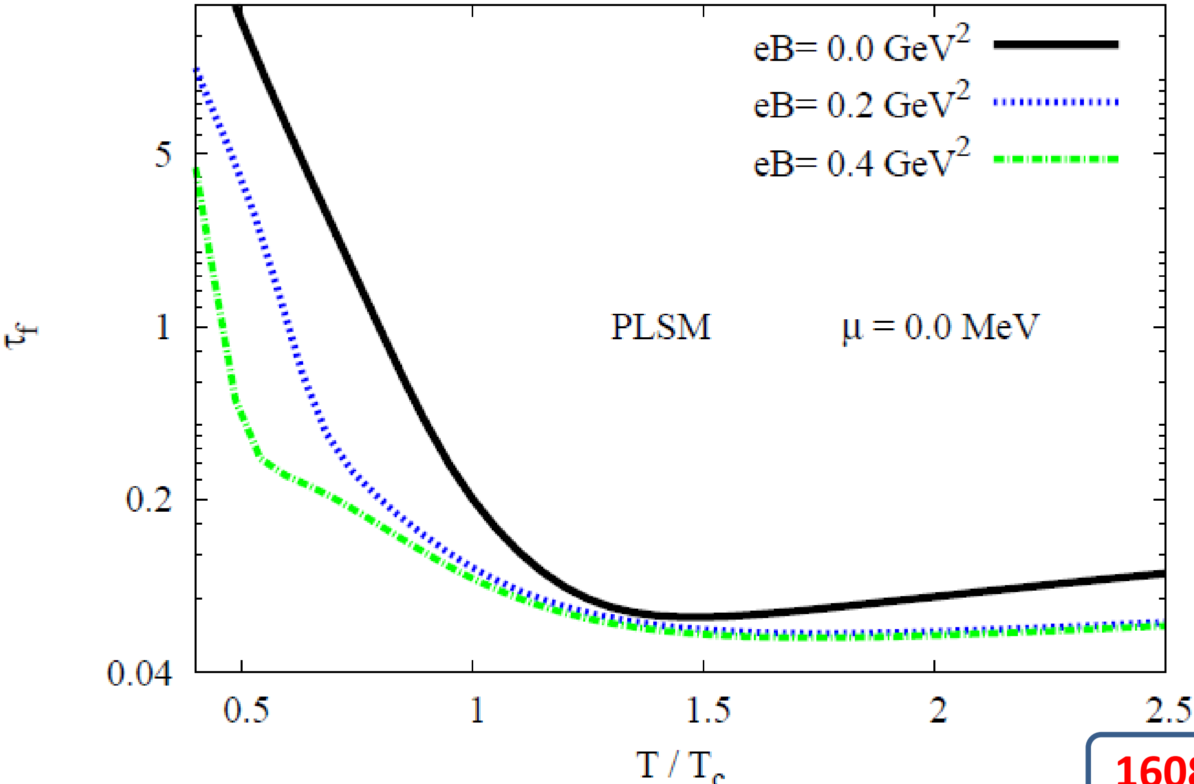
1608.01034 [hep-ph]

Relaxation Time

It can be determined from the thermal average of total elastic scattering and relative cross section

$$\tau = \left[n_f \left\langle v_{rel}(T)\, \sigma_{tr}(T)\right\rangle\right]^{-1}$$

The cross section is given as $\sigma_{tr,i}(T) \;=\; \frac{4}{15}\frac{\langle p\rangle_i}{\rho_i(4-\mu_i/T)}\,\frac{1}{\eta/s}$



1608.01034 [hep-ph]

Electrical/Thermal Relaxation Time

Relaxation rates for **electrical** conduction and viscous properties have a universal scale

$$\tau \simeq \frac{m_q^{2/3}}{(\alpha_s T)^{5/3}},$$

Relaxation rates for **thermal** conductivity scales as

$$\tau \simeq \frac{1}{\alpha_s T}$$

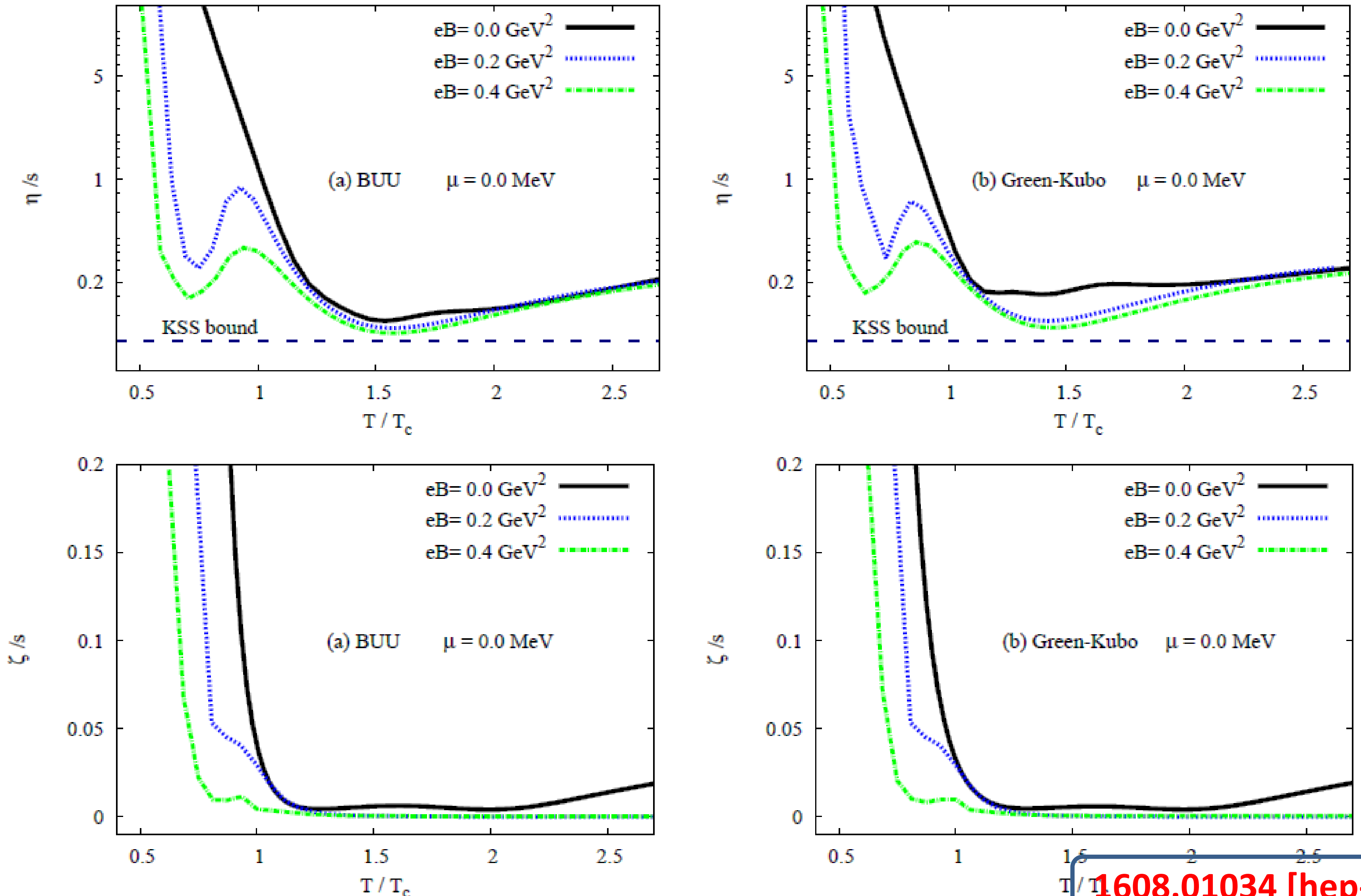
Relaxation time in both partonic and hadronic phases scales with T

$$T < T_c : \begin{cases} n \propto e^{-m/T} \\ \sigma \simeq \text{const.} \end{cases} \Rightarrow \tau \propto e^{m/T},$$

$$T > T_c : \begin{cases} n \propto T^3 \\ \sigma \propto T^{-2} \end{cases} \Rightarrow \tau \propto T^{-1}.$$

1608.01034 [hep-ph]

SU(3) PLSM Viscosity: BUU & GK at eB(=)≠0

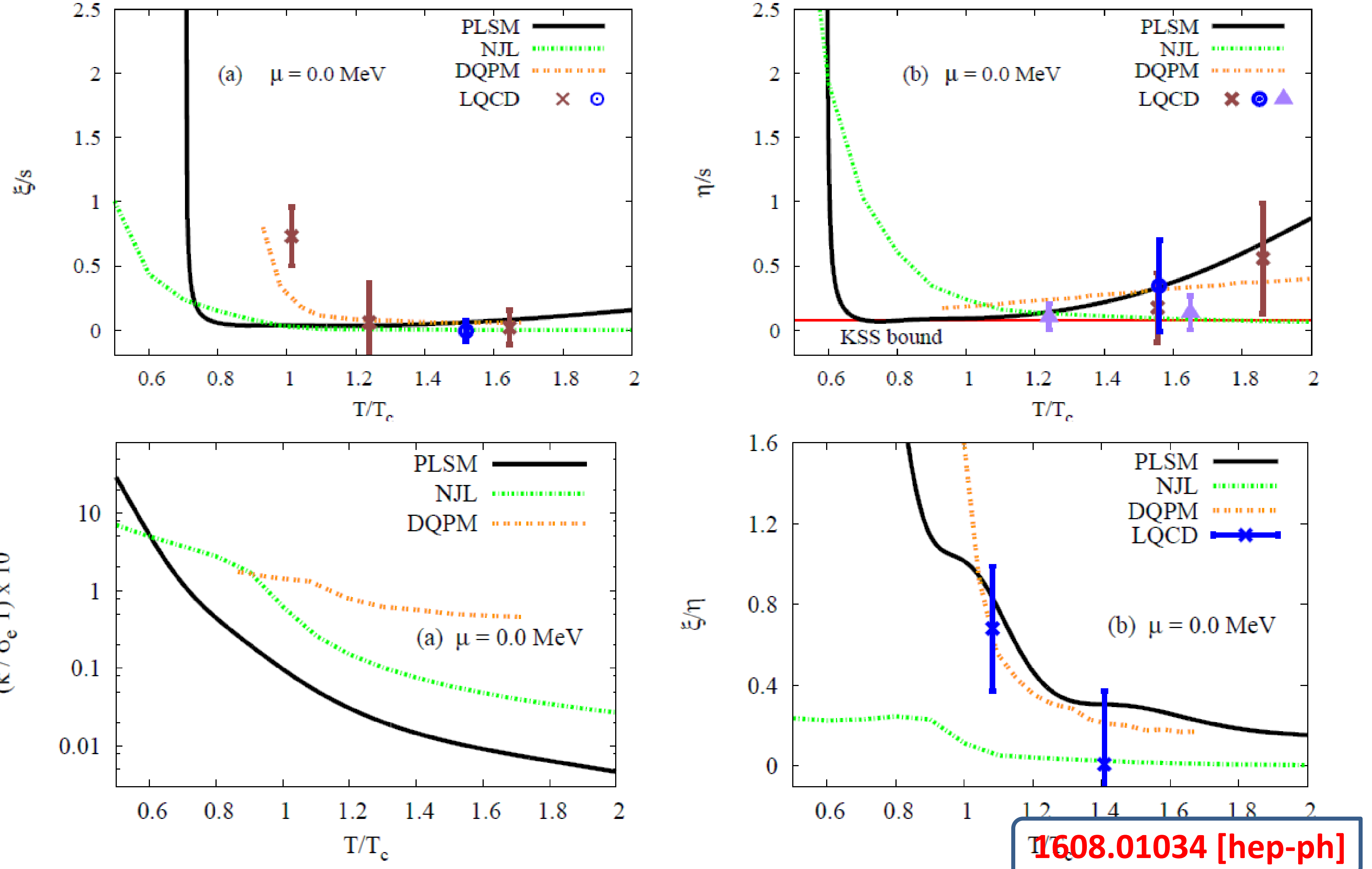


1608.01034 [hep-ph]

WLCAPP World Laboratory Cosmology And Particle Physics

SU(3) PLSM Viscosity: BUU & GK at eB=0

EGYPTIAN CENTER FOR THEORETICAL PHYSICS
CENTER OF EXCELLENCE



1608.01034 [hep-ph]

SU(3) PLSM: Conductivity

Electric current density $j_z = n\, e\, \bar{v}_z \quad \bar{v}_z = e\mathcal{E}\tau/m$

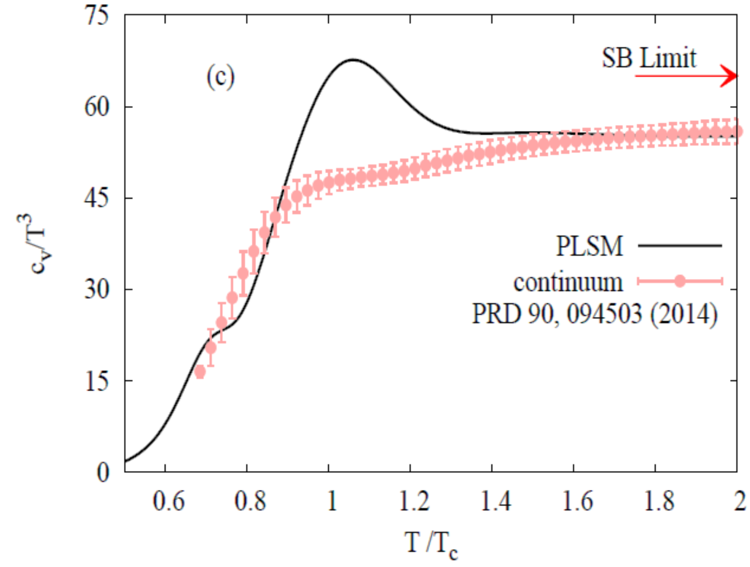
Drude-Lorentz Conductivity
$$\sigma_{el} = \sum_f e_f^2 \frac{n_f(T,\mu)\,\tau_f(T,\mu)}{m_f(T,\mu)}$$
$$e_f^2 = \frac{4\pi}{137} q^2$$

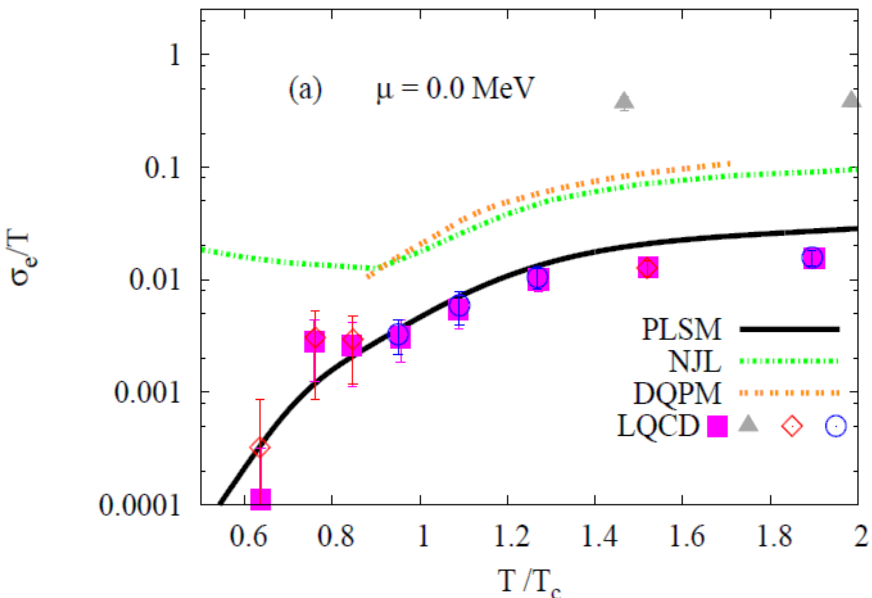
Heat conductivity ∝ to heat flow and indicates the rate of energy change taking place in relativistic fluid

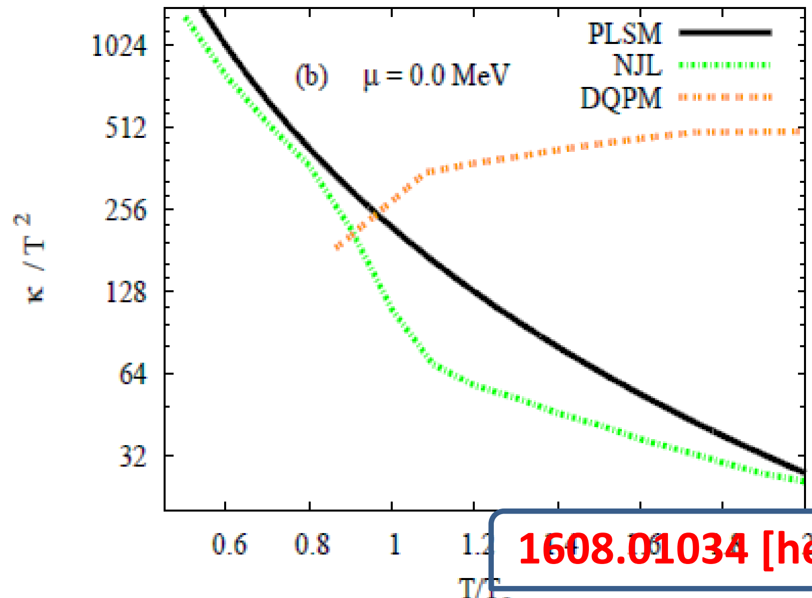
$$\kappa(T,\mu) = \frac{1}{3}\nu_{rel}\, c_v(T,\mu) \sum_f \tau_f(T,\mu)$$
$$\nu_{rel} = \sqrt{(p_1 p_2)^2 - (m_1 m_2)^2}/E_1 E_2$$ **Relative velocity**

1608.01034 [hep-ph]

SU(3) PLSM: Conductivity

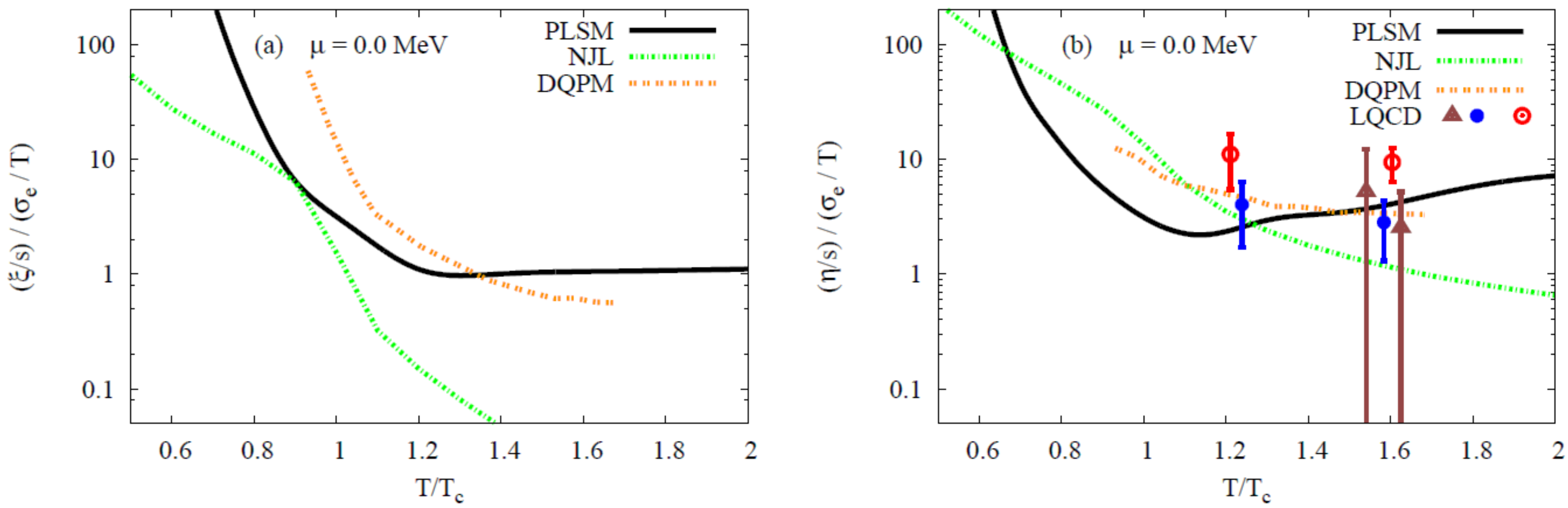






1608.01034 [hep-ph]

SU(3) PLSM: Conductivity



1608.01034 [hep-ph]

Summary and Outlook

- Good agreement with available lattice QCD
- Inverse Magnetic Catalysis
- Detailed study for the phase structure of the model
- Reasonable precision for various meson spectra
- First confrontation to experimental results
- Predictions for viscosity and conductivity properties
- Extending SU(4) at finite T, mu and eB
- Comparing with lattice QCD and experiments
- Electromagnetic effects at finite T and mu

Thank you!