Thermalization and hydrodynamization in weakly coupled heavy-ion collisions

Aleksi Kurkela

AK, Zhu PRL 115 (2015) 18, 182301
AK, Lu PRL 113 (2014) 18, 182301
AK, Moore JHEP 1111 (2011) 120
AK, Moore JHEP 1112 (2011) 044

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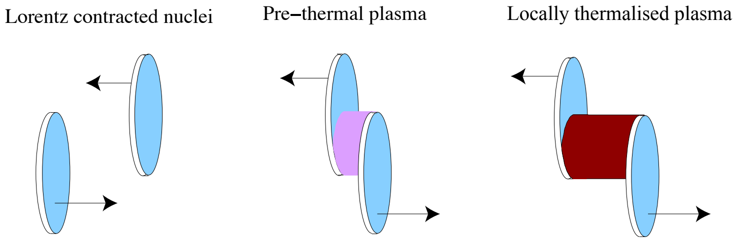
Frankfurt, Nov 2015

What: Pre-equilibrium dynamics in HIC
How: Weak coupling. classical Yang-Mills and kinetic thy.
New: Smooth and automatic approach to hydrodynamics

Where are we at?



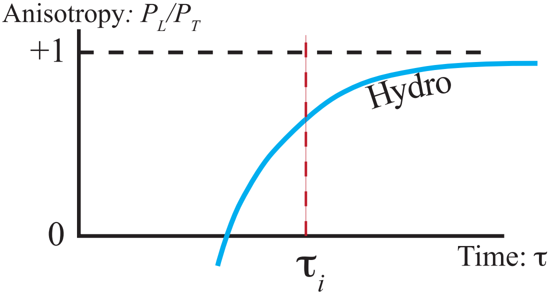
- Soft physics of HIC described by relativistic hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0$$

- Gradient expansion around local thermal equilibrium

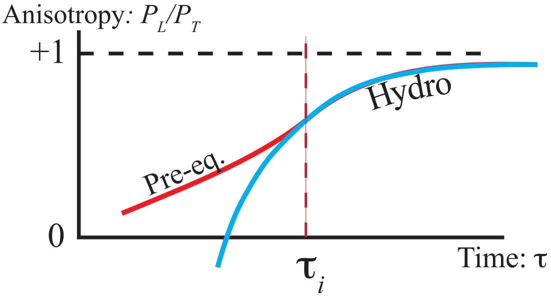
$$T^{\mu\nu} = T^{\mu\nu}_{\text{eq.}} - \eta 2 \nabla^{<\mu} u^{\nu>} + \ldots$$

Where are we at?



- Strong anisotropy $P_L/P_T \ll 1$, sign of large corrections
- At early times *pre-equilibrium* evolution
- Hydro simulations start at *intialization time* τ_i

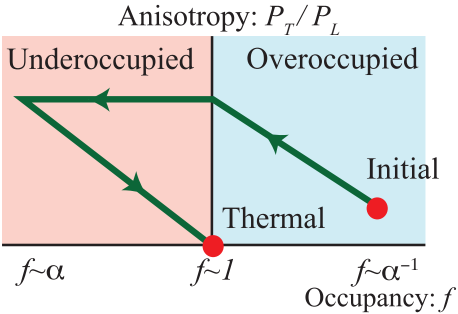
Where are we at?



- If prethermal evolution converges smoothly to hydro, independence of unphysical τ_i
- Explicit example: Strong coupling $\mathcal{N} = 4$ SYM
  Chesler, Yaffe PRL 106 (2011) 021601; van der Schee et al. PRL 111 (2013) 22, 222302, arXiv:1507.08195

This has proven to be challenging in QCD, even at weak coupling

Bottom-up thermalization at weak coupling



- Color Glass Condensate: Initial condition overoccupied

McLerran, Venugopalan PRD49 (1994) 2233-2241 , PRD49 (1994) 3352-3355 ; Gelis et. al Int.J.Mod.Phys. E16 (2007) 2595-2637 , Ann.Rev.Nucl.Part.Sci. 60 (2010) 463-489

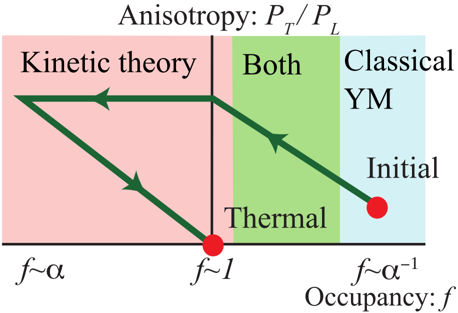
$$f(Q_s) \sim 1/\alpha_s, \qquad Q_s \sim 2\text{GeV}$$

- Expansion makes system underoccupied before thermalizing

Baier et al Phys.Lett. B502 (2001) 51-58; AK, Moore JHEP 1111 (2011) 120

$$f(Q_s) \ll 1$$

Bottom-up thermalization at weak coupling

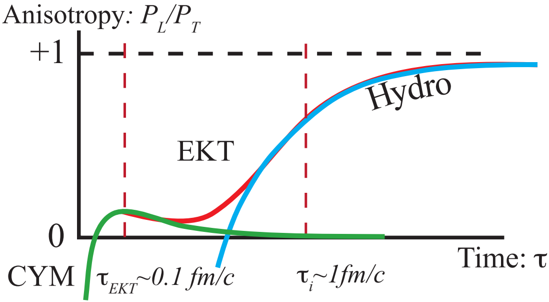


- Degrees of freedom:
  - $f \gg 1$: Classical Yang-Mills theory (CYM)
  - $f \ll 1/\alpha_s$: (Semi-)classical particles, Eff. Kinetic Theory (EKT)

- Transmutation of fields to particles: Field-particle duality
  Son, Mueller PLB582 (2004) 279-287; Jeon PRC72 (2005) 014907; Mathieu et al EPJ. C74 (2014) 2873 ; AK, Moore PRD89 (2014) 7, 074036

$$1 \ll f \ll 1/\alpha_s$$

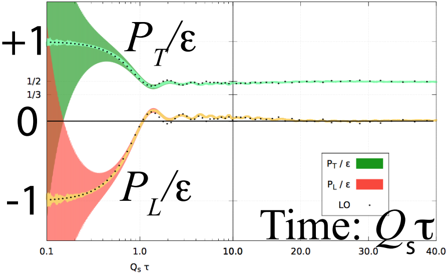
Strategy at weak coupling



Strategy: Switch from CYM to EKT at τ_{EKT}, $1 \ll f \ll 1/\alpha_s$

From EKT to hydro at τ_i, $P_L/P_T \sim 1$

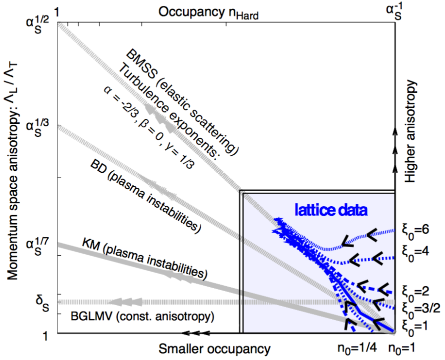
Early times $0 < Q_s\tau \lesssim 1$: classical evolution

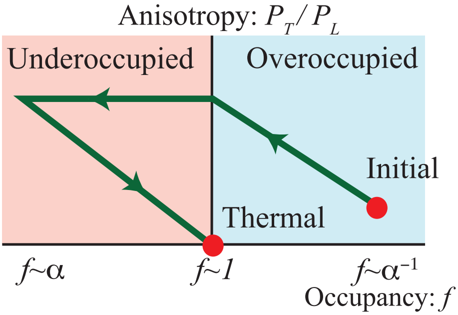


Epelbaum & Gelis, PRL. 111 (2013) 23230

- Melting of the coherent boost invariant CGC fields
  - The initial condition from CGC: MV-model, JIMWLK
  - After $\tau \sim 1/Q_s$, fields decohere, $P_L > 0$

Later times $Q_s\tau > 1$: classical evolution





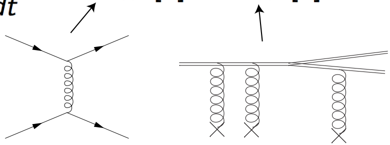
Berges et al. Phys.Rev. D89 (2014) 7, 074011

- Numerical demonstration of classical/overoccupied part of the diagram
- Classical theory never thermalises or isotropizes
- Before $f \sim 1$, must switch to kinetic theory

Effective kinetic theory of Arnold, Moore, Yaffe

JHEP 0301 (2003) 030

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$

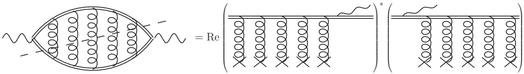


- Based on quasiparticle form of the spectral function:

$$p^2 \gg m_D^2 \equiv 2N_c g^2 \int_{\mathbf{p}} f(p)/p$$

- Soft and collinear divergences lead to nontrivial matrix elements

soft: screening, Hard-loop; collinear: LPM, ladder resum

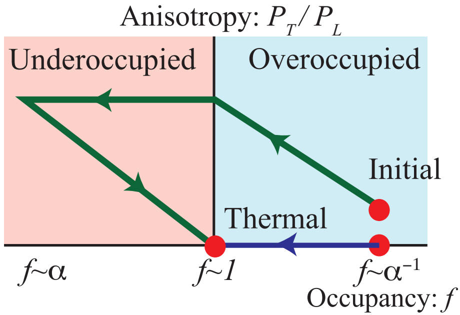


- No free parameters; LO accurate in the $\alpha_s \to 0$, $\alpha_s f \to 0$ limit.

- Used for transport coefficients in QCD, jet energy loss

Arnold et al. JHEP 0305 (2003) 051; Moore, York PRD79 (2009) 054011; Ghiglieri, Teaney 1502.03730; AK, Wiedemann PLB740 (2015) 172-178; Iancu, Wu 1506.07871

Outline

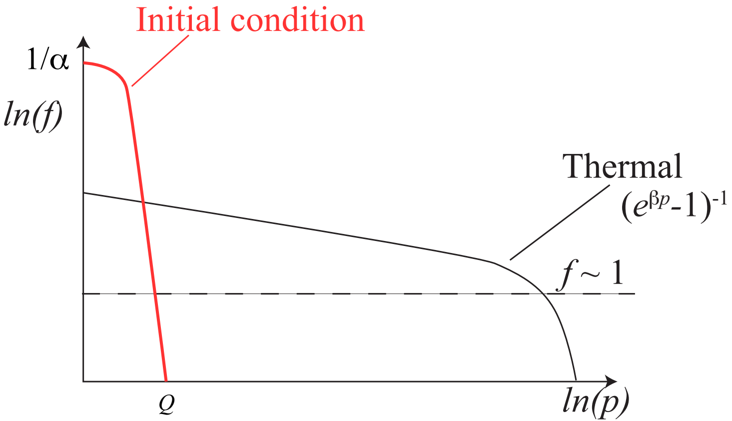


- Isotropic overoccupied: Transmutation of d.o.f's
- Isotropic underoccupied: Radiative break-up
- Application to HIC: effect of longitudinal expansion

Overoccupied cascade

AK, Moore JHEP 1112 (2011) 044

What happens if you have too many soft gluons, $f \sim 1/\alpha_s$.
No longitudinal expansion.



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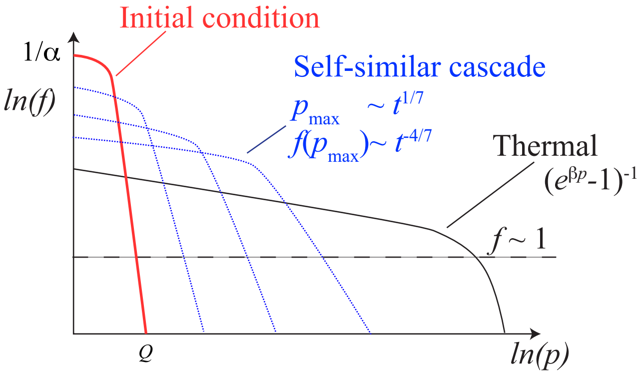
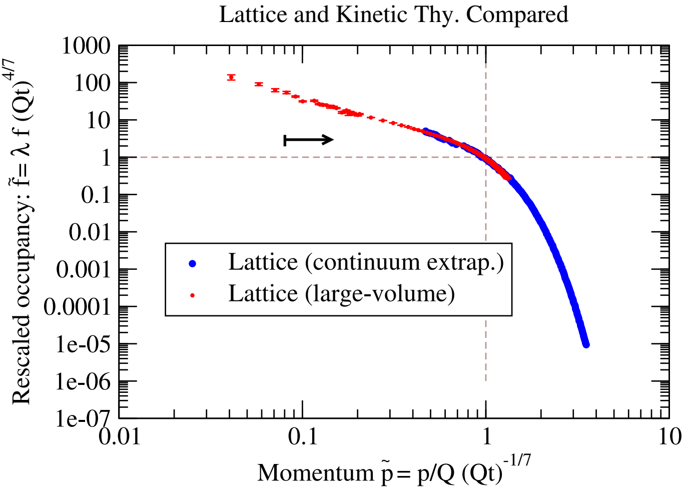


$$\tau_{\text{init}} \sim [\sigma n(1+f)]^{-1} \sim \left(\frac{Q}{T}\right)^7 \frac{1}{\alpha_s^2 T} \ll \frac{1}{\alpha_s^2 T} \sim \tau_{\text{them.}}$$

Overoccupied cascade

AK, Lu, Moore, PRD89 (2014) 7, 074036



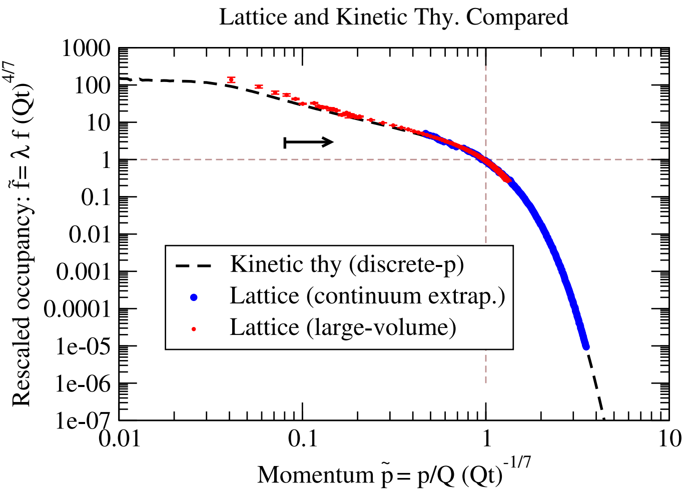
Form of cascade from classical lattice simulation,

$$1 \ll f \lesssim 1/\alpha_s$$

Large-volume: (Qa)=0.2, (QL)=51.2, Cont. extr.: down to (Qa)=0.1, (QL)=25.6, Qt=2000, $\tilde{m} = 0.08$

Overoccupied cascade

AK, Lu, Moore, PRD89 (2014) 7, 074036

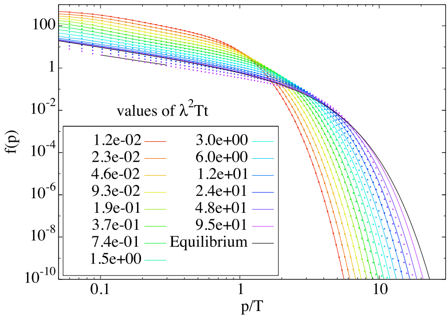


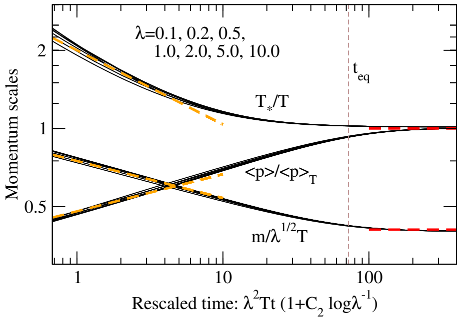
Same system, very different degrees of freedom

$$1 \lesssim f \ll 1/\alpha_s$$

Numerical demonstration of field-particle duality

Ending of the overoccupied cascade AK, Lu PRL 113 (2014) 18, 182301





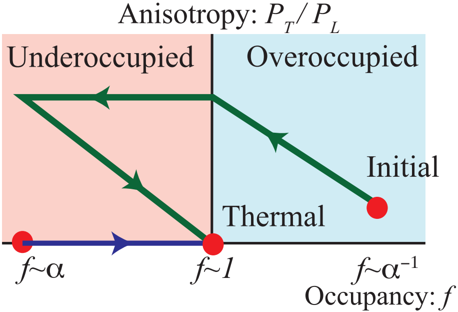
Thermal equilibrium reached once $f \sim 1, p \sim T$ (or $t \sim \frac{1}{\alpha_s^2 T}$).

Therm. time through the approach of $\langle p \rangle - \langle p \rangle_T \sim \exp(-t/t_{\rm eq})$

$$t_{\rm eq} \approx \frac{72.}{1 + 0.12 \log \lambda^{-1}} \frac{1}{\lambda^2 T}$$

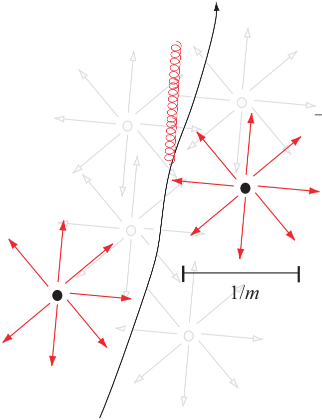
$\lambda = 4\pi N_c \alpha_s$

Outline

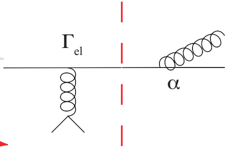


- Isotropic overoccupied: Transmutation of d.o.f's reheating?
- Isotropic underoccupied: Radiative break-up inflaton decay?
- Application to HIC: effect of longitudinal expansion

Underoccupied cascade: Formation of thermal bath



- Soft modes quick to emit



$$\Gamma_{\rm el} \sim \alpha_s^2 \frac{n}{m_D^2} \sim \alpha_s^2 \frac{\int_{\mathbf{p}} f}{\alpha_s \int_{\mathbf{p}} f/p}$$

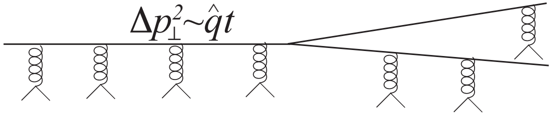
$$n_{\rm soft} \sim \alpha_s \, \Gamma_{\rm el} \, t$$

- Low-p: easy to thermalize
- Can dominate the dynamics

  scattering, screening, ...

$\Rightarrow$ Few energetic "jets" propagating in thermal bath

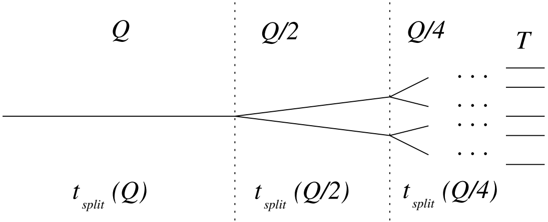
Underoccupied cascade: Radiational breakup



- In vacuum: on-shell splitting kin. disallowed
- In medium:
  - frequent soft scatterings with medium, mom. diffusion: $\Delta p^2 \sim \hat{q}t$
  - Scatterings lead to virtuality: $P^2 \sim \hat{q}t$
  - Now offshell particle may split collinearly: $t_f \sim Q/P^2 \sim \sqrt{Q/\hat{q}}$
  - Splitting time (per particle) $t_{\text{split}}(Q) \sim \frac{1}{\alpha_s} t_f \sim \frac{1}{\alpha_s}\sqrt{\frac{Q}{\hat{q}}}$

QED: Landau, Pomeranchuk, Migdal 1953.
QCD: Baier Dokshitzer Mueller Peigne Schiff hep-ph/9607355

Underoccupied cascade: Radiational breakup



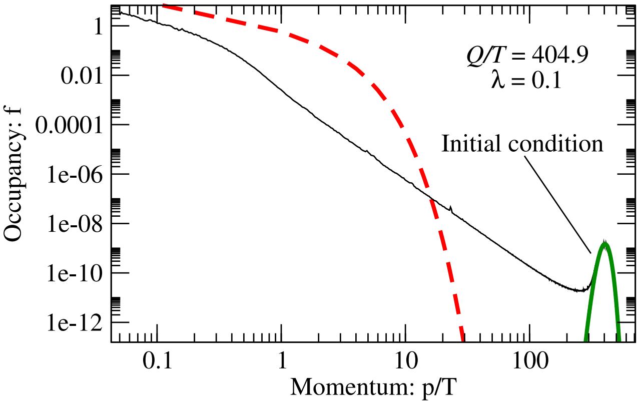
- Successive splittings happen in faster times scales:

$$t_{\text{quench}}(Q) \sim t_{\text{split}}(Q) + t_{\text{split}}(Q/2) + t_{\text{split}}(Q/4) + \ldots \sim t_{\text{split}}(Q)$$

- Once the parton has had time to split it cascades its energy to IR. T increases.

Radiational breakup

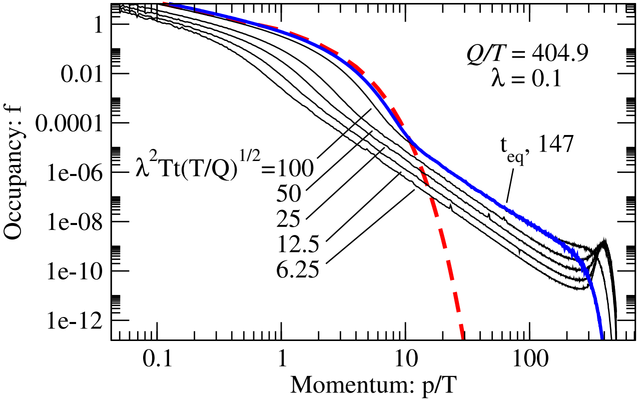
AK, Lu, PRL 113 (2014) 18, 182301



- Start with an underoccupied initial condition $p \sim Q$
- after a very short time, an IR bath is created (1$\leftrightarrow$ 2–processes)

Radiational breakup

AK, Lu, PRL 113 (2014) 18, 182301



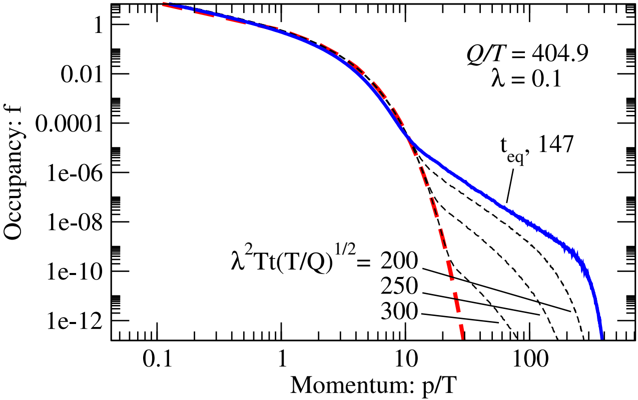
- More energy flows to the IR, temperature increases, "Bottom-up"
- When "bottom" reaches final T, "up" is quenched

AK, Moore JHEP 1112 (2011) 044

$$t_{\rm eq} \sim (Q/T)^{1/2} \frac{1}{\lambda^2 T}$$

Radiational breakup

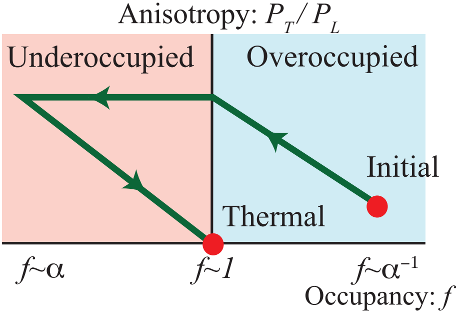
AK, Lu, PRL 113 (2014) 18, 182301



- Hardest scales reach equilibrium last.

Close resemblance to Blaizot, Iancu, Mehtar-tani for jets PRL 111 (2013) 052001

Outline



- Isotropic overoccupied: Transmutation of d.o.f's
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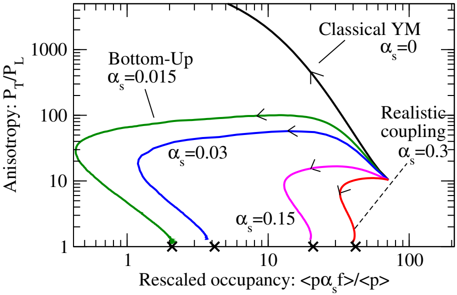
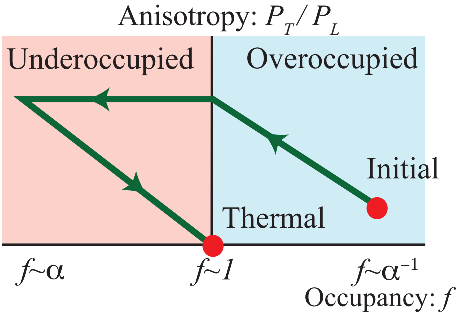
Route to equilibrium in EKT

AK, Zhu, PRL 115 (2015) 18, 182301

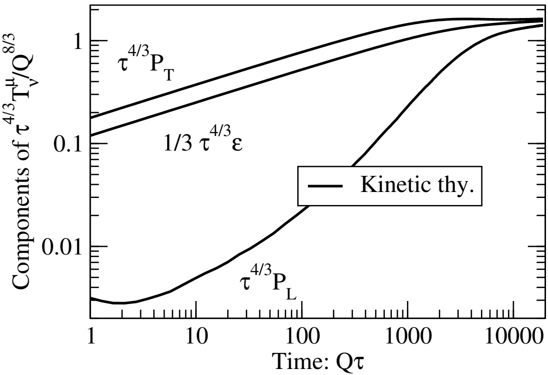




- Initial condition ($f \sim 1/\alpha_s$) from classical field theory calculation
  Lappi PLB703 (2011) 325-330
- In the classical limit ($\alpha_s \to 0, \alpha_s f$ fixed), no thermalization
- At small values of couplings, clear Bottom-Up behaviour
- Features become less defined as α_s grows

Smooth approach to hydrodynamics AK, Zhu, PRL 115 (2015) 18, 182301

$$\alpha_s = 0.03$$



- Kinetic theory converges to hydro smoothly and automatically

Smooth approach to hydrodynamics AK, Zhu, PRL 115 (2015) 18, 182301

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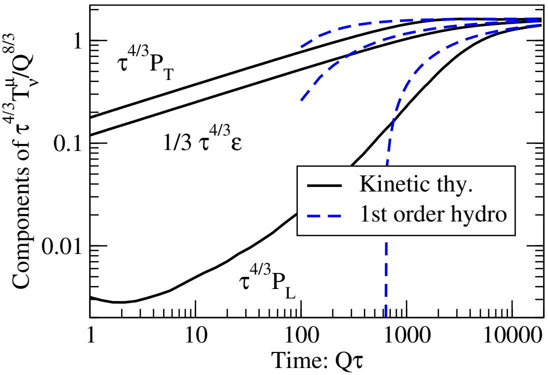


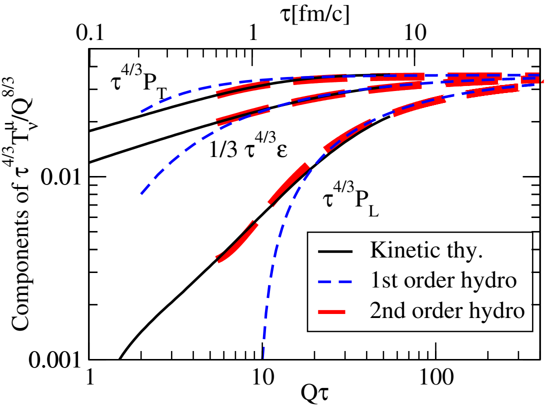
- Kinetic theory converges to hydro smoothly and automatically
- Approach to hydro fixed by perturbative η/s

Arnold et al. JHEP 0305 (2003) 051

$$\partial_\tau \epsilon = -\frac{4}{3}\frac{\epsilon}{\tau} + \frac{4\eta}{3\tau^2}, \qquad P_L = \frac{\epsilon}{3} - \frac{4\eta}{3\tau}$$

Smooth approach to hydrodynamics AK, Zhu, PRL 115 (2015) 18, 182301

$$\alpha_s = 0.3$$



- For realistic couplings, hydrodynamics reached around $\lesssim 1\text{fm/c}$.
- Hydro seems to give a good description even when $P_L/P_T \sim 1/5$

Caveats

- Fermions — Underway
- Transverse dynamics, preflow — Underway
- Plasma instabilities, anisotropic screening
  AK, Moore, JHEP 1112 (2011) 044 , JHEP 1111 (2011) 120
  - Numerically small effect? — Berges et al. Phys.Rev. D89 (2014) 7, 074011
- Improved initial CYM simulations for initial condition of EKT

and

- Potentially large NLO corrections
  Caron-Huot, Moore PRL 100 (2008) 052301, Ghiglieri et al. JHEP 1305 (2013) 010, JHEP 1412 (2014) 029, 1502.03730, 1509.07773
  - But $T(\tau_i) \sim 3T_c$ in perturbative region

Qualitative $\Rightarrow$ Quantitative

Where are we going?

- Combination of classical Yang-Mills simulations and effective kinetic theory allows to follow the time evolution from highly occupied initial condition to thermal equilibrium.

- Weak coupling thermalization extrapolated to realistic couplings shows agreement with hydro around

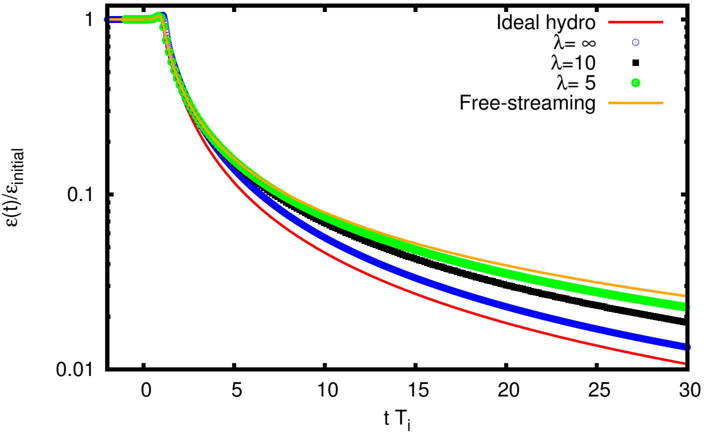
$$\tau_{\rm i} \sim 1 {\rm fm/c}$$

- Unified description of soft and hard physics: hydro, jets, etc.

Weakly or strongly coupled thermalization?

Apples to apples comparison of weak and strong coupling

Energy density

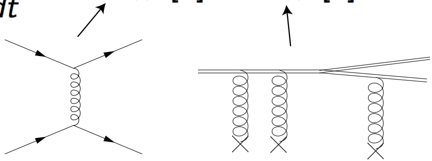


Under construction, AK, Romatschke, van der Schee

Backup sildes

$2 \leftrightarrow 2$ scattering, screening

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$



$$C_{2\leftrightarrow 2}[f] = \int_{k,p',k'} |M|^2 \left[f_p f_k (1 + f_{p'})(1 + f_{k'}) - f_{p'} f_{k'} (1 + f_p)(1 + f_k)) \right]$$

- Naively $|M|^2$ diverges as $1/q^4$. Dynamically regulated by screening

$$\frac{1}{q^4} \Rightarrow \frac{1}{(q^2 + \Pi(\omega, q, m_D))^2} \Rightarrow \frac{1}{(q^2 + \tilde{m}^2)^2}$$

with carefully chosen $\tilde{m}^2 = e^{5/6} 2^{-3/2} m_D$ isotropic case

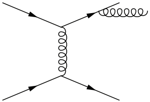
AK, Lu, Moore, York PRD89 (2014) 7, 074036

$1 \leftrightarrow 2$ splitting, soft radiation

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$

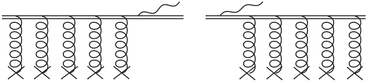
$$C_{1\leftrightarrow 2} \sim \int dp \; \gamma^{p}_{k,p-k} \left[f_p(1+f_k)(1+f_{p-k}) - f_k f_{p-k}(1+f_p)\right]$$

- IR divergence makes soft scattering rate large
- Soft scattering can induce splitting/absorbtion



$$\Gamma_{\text{split}} \sim \alpha_s \Gamma_{\text{soft}}(1+f_{\text{final}}) \gtrsim \Gamma_{\text{hard}}$$

As important for under-, more important for underoccupied



Collinear divergence regulated by interference, formation time

Effective $C_{1\leftrightarrow 2}$ matrix element revisited

$$\gamma^{p'}_{p,k} \sim \underbrace{\frac{p'^4 + p^4 + k^4}{p'^3 p^3 k^3}}_{\text{DGLAP split-kernel}} \int \frac{d^2h}{(2\pi)^2} \mathbf{h} \cdot \mathrm{Re}\mathbf{F}(\mathbf{h}; p', p, k)$$

$$2\mathbf{h} = i\delta E(\mathbf{h})\mathbf{F}(\mathbf{h}) + \frac{g^2 N_c}{2} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \left[T_* \left(\frac{1}{\mathbf{q}^2} - \frac{1}{\mathbf{q}^2 + m^2_{\text{screen}}} \right) \right]$$
$$\times \left(3\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h} - p\mathbf{q}) - \mathbf{F}(\mathbf{h} - k\mathbf{q}) - \mathbf{F}(\mathbf{h} + p\mathbf{q})\right)$$

Where sensitivity to the medium comes from

- δE is the difference of energies of one gluon with momentum p' compared the two with k, p': depends on effective masses
- Dependence on $p/T_*, m/T_*$: In praxis:
  - solve numerically, tabulate
  - Fitting with with correct asymptotics

$1 \leftrightarrow 2$ splitting, soft radiation

Soft limit, parametrically:

- Soft scattering rate $\Gamma_{soft} \sim \lambda T_* \sim \frac{\hat{q}}{m^2}$

$$T_* = \frac{1}{2}\int_{\mathbf{p}} f_p(1+f_p) / \int_{\mathbf{p}} f(p)/p$$

Bose factors enchance, regulated by m^2

- Soft inelastic rate, Bethe-Heitler:

$$\frac{d\Gamma_{\text{BH}}}{dp'} \sim \lambda^2 T_*/p'$$

- Collision kernel related to the rate $\gamma \sim p^2 \frac{d\Gamma}{dp'} \sim \lambda T_* p^2/p'$
- Constant of proportionality analytically:

$$\lim_{p'\to 0} \gamma(p; p', p-p') = \frac{\mathcal{Q}(m^2/m_D^2)}{4(2\pi)^4}\lambda^2 T_* \frac{p^2}{p'}$$

$$\mathcal{Q}(m_\infty^2/m_D^2) \equiv 8\int_{p_\perp, q_\perp}\left[\frac{1}{q_\perp^2} - \frac{1}{q_\perp^2 + m_D^2}\right]\left(\frac{\mathbf{p}_\perp}{m_\infty^2 + p_\perp^2} - \frac{\mathbf{p}_\perp - \mathbf{q}_\perp}{m_\infty^2 + (\mathbf{p}_\perp - \mathbf{q}_\perp)^2}\right.$$

$1 \leftrightarrow 2$ splitting, deep LPM limit

Arnold, Dogan 0804.3359

- Hard collinear radiation suppressed by formation time

$$t_{\text{form}}(p') \sim \sqrt{\frac{p'}{\lambda T_* m^2}}$$

- The rate bounded from above by

$$\gamma \sim p^2 \frac{d\Gamma_{\text{hard}}}{dp'} \sim \lambda p^2 / t_{\text{form}} \sim \lambda^{3/2} p^2 \sqrt{T_* m^2 / p'^3}$$

- Prefactor to NLL by Arnold

log related to the UV div. of $\hat{q}$

$$\gamma(p, p', p - p') = \frac{\sqrt{2}\lambda}{4(2\pi)^5} m^2 \hat{\mu}^2(1, x, 1-x) \frac{1 + x^4 + (1-x)^4}{x^2(1-x)^2}$$

$$\hat{\mu}^2 = \frac{\lambda^{1/2} T_*}{\sqrt{2} m} \left[\frac{1}{\pi} x_1 x_2 x_3 \frac{p}{T_*} \right]^{1/2} \left[\sum_{i=1}^{3} (x_i^2) \ln(\xi \hat{\mu}^2 / x_i^2) \right]^{1/2},$$

With $\xi = 9.09916$

Expanding case: application to HIC

Initial condition from YM:

- In principle, first principle 3+1D calculation in QCD possible for $t < t_{cl}$. Currently not available
- Use the second best thing: 2+1D Lappi Phys.Lett. B703 (2011) 325-330
- Parametrize the initial condition with

$$f(p_z, p_t) = \frac{2}{\lambda} A f_0(p_z \xi / \langle p_T \rangle, p_\perp / \langle p_T \rangle),$$

$$f_0(\hat{p}_z, \hat{p}_\perp) = \frac{1}{\sqrt{\hat{p}_\perp^2 + \hat{p}_z^2}} e^{-2(\hat{p}_\perp^2 + \hat{p}_z^2)/3},$$

  fix parameters keeping by $\epsilon_{YM} = \epsilon_{EKT}$, $\langle p_\perp \rangle_{YM} = \langle p_\perp \rangle_{EKT}$.

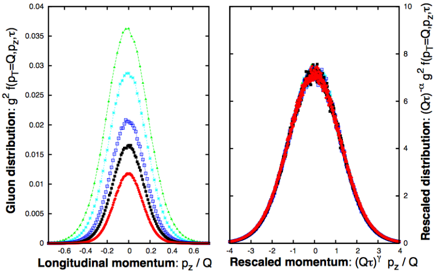
- Difference between 2+1D and 3+1D, $\langle p_z \rangle_{2D} = 0$. Parametrize the effect of instabilities by ξ. Vary to quantify ignorance.

Comparison between CYM and EKT: Expanding

In non-pert classical regime $1 \ll f \ll 1/\alpha_s$

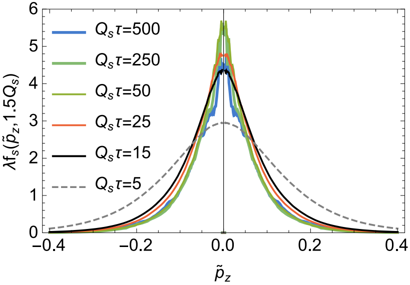
$$f(p_z, p_\perp, \tau) = (Q_s\tau)^{-2/3} f_S((Q_s\tau)^{1/3} p_z, p_\perp),$$

CYM $\alpha_s f \ll 1$ limit but $f \gg 1$



Berges et al. Phys.Rev. D89 (2014) 7, 074011

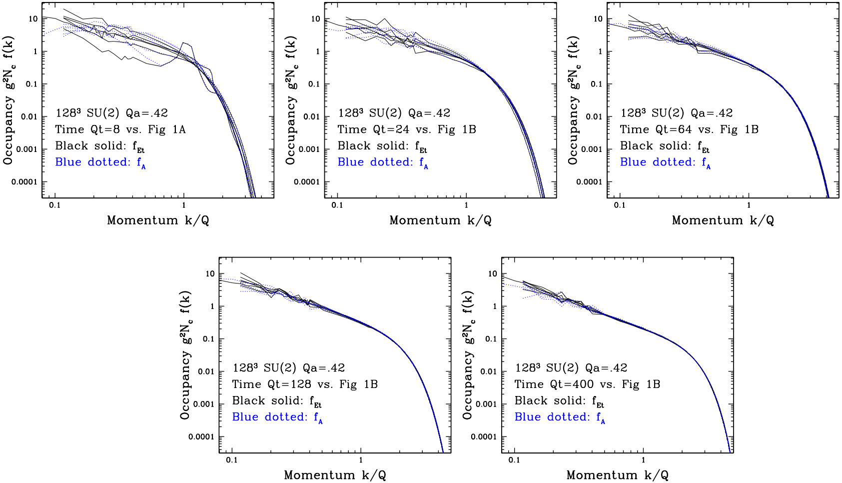
EKT $f \gg 1$ limit but $f \ll 1/\alpha_s$



AK, Zhu 1506.06647

Information on the overoccupied initial condition lost in scattering time of the initial condition

AK, Moore 1207.1663



$$\tau_{\text{init}} \sim \left(\frac{Q}{T}\right)^7 \frac{1}{\alpha^2 T} \ll \frac{1}{\alpha^2 T} \sim \tau_{\text{them}}.$$

Power law from of the cascade

- Low scales have time to thermalize: $1/p$ AK, Moore,1107.5050
- Turbulent kolmogorov cascade $1/p^{4/3}$, (BEC: $1/p^{3/2}$)?

Berges et al 0811.4293