Gravitational collapse and the quantum

Horizons, Hawking radiation and all that*

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*= don’t expect anything new yet...

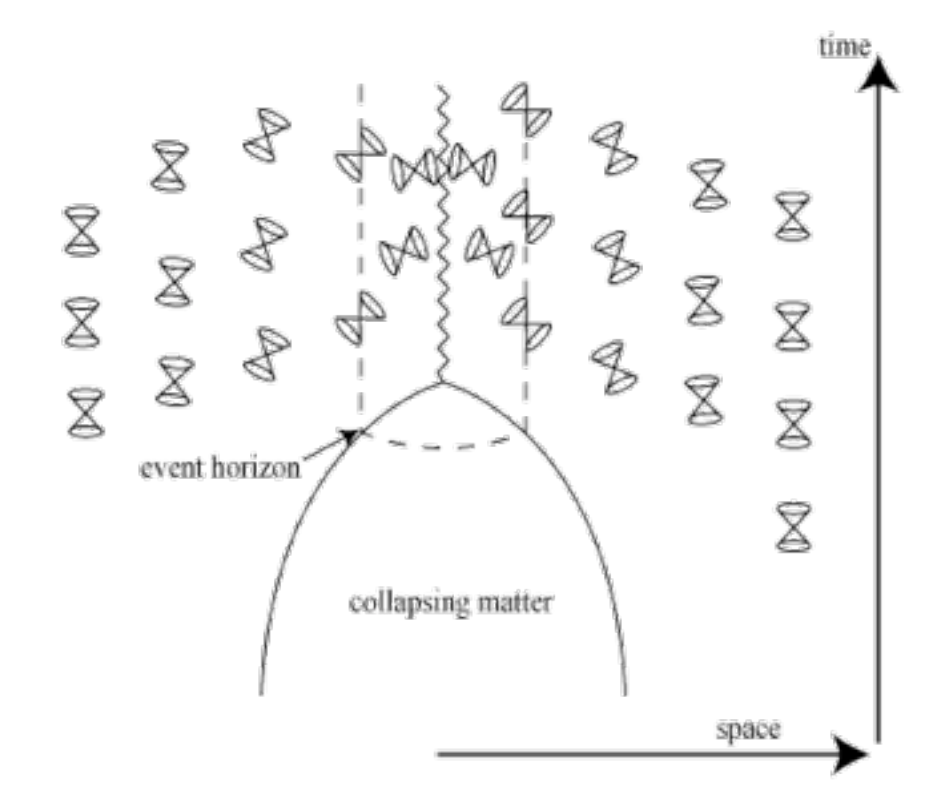
Contents

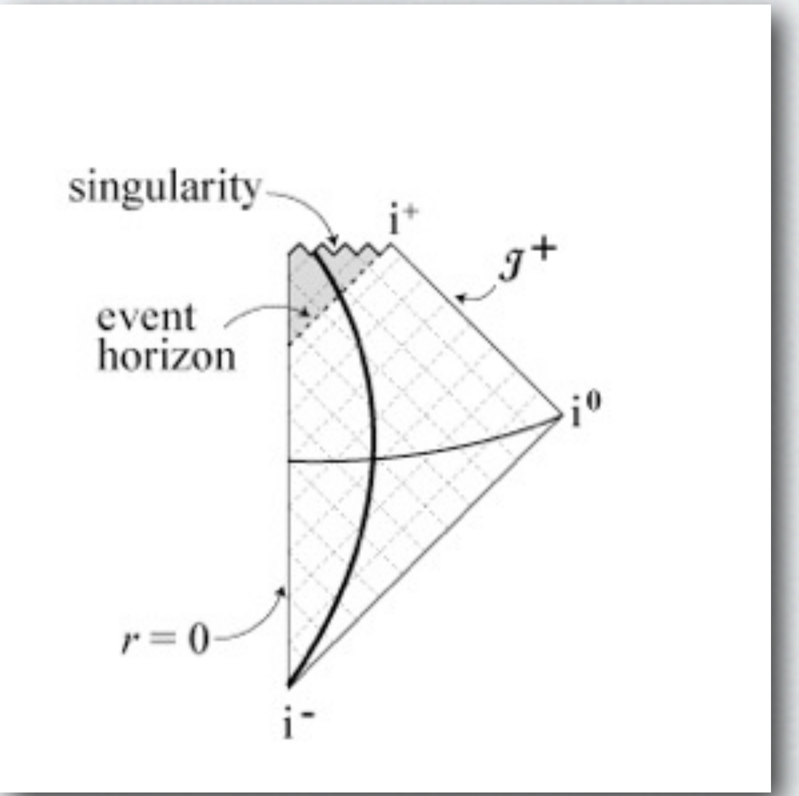
1) Gravitational collapse

Standard semiclassical picture of gravitational collapse

↓

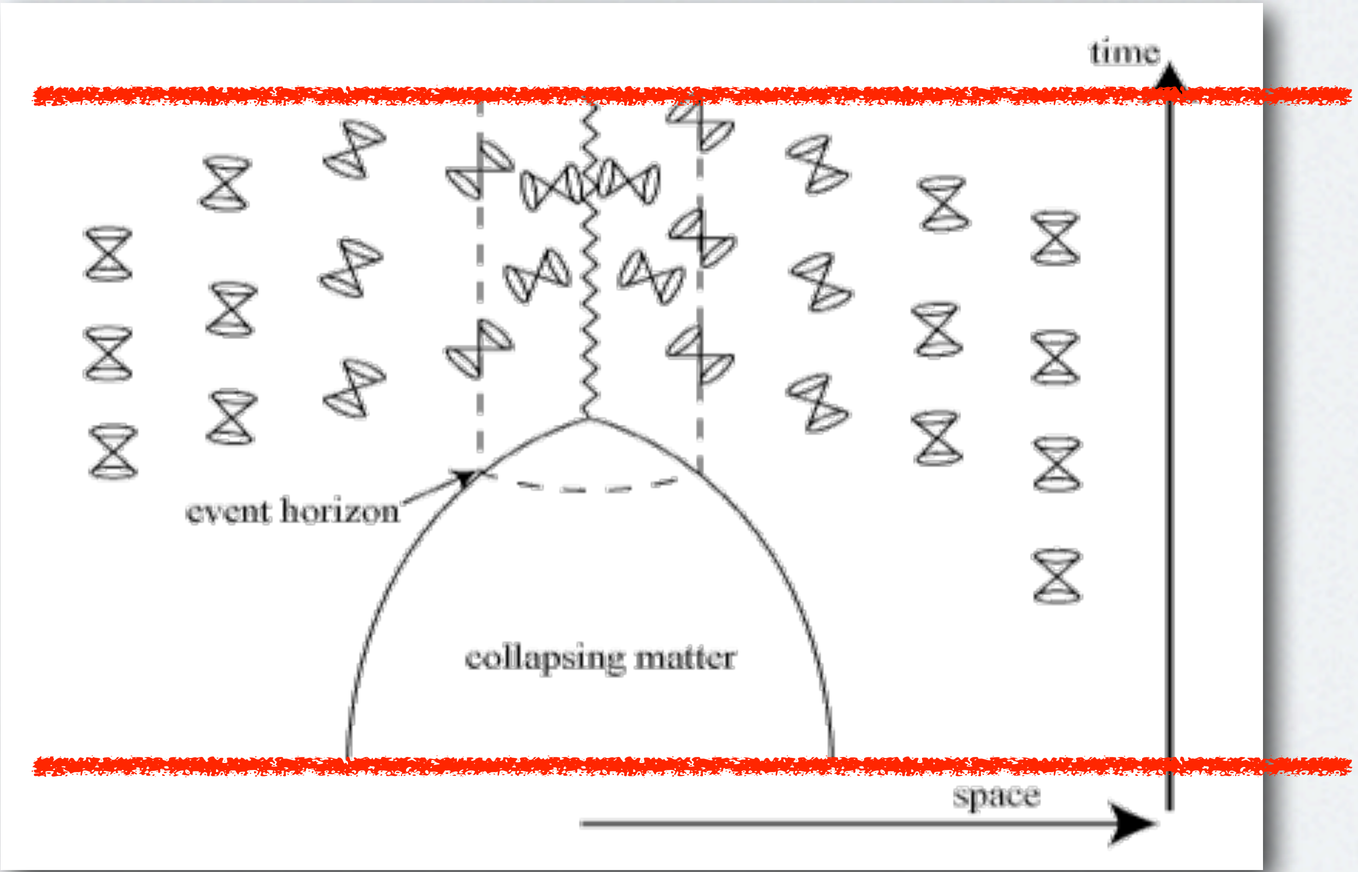
Classical background: classical matter and "geometrical" space-time*





*Prototype background: $ds^2 = -\left(1 - \frac{2\,M}{r}\right) dt^2 + \left(1 - \frac{2\,M}{r}\right)^{-1} dr^2 + r^2\, d\Omega^2$

Quantum foreground: "radiation"



$|0; t = +\infty\rangle$

$e^{-\frac{i}{\hbar} \int \hat{H}\, dt}$

$|0; t = -\infty\rangle$

$$|0; t = +\infty\rangle = \sum \text{excitations} = \text{Hawking radiation}$$

Q1) Background: horizon? → A1) Trapping horizon

Q2) Foreground: particle? → A2) QFT

1) Gravitational collapse

Semiclassical picture: classical background + quantum foreground

(Quantum stress tensor)

$$R_{\mu\nu} + \frac{1}{2} R\, g_{\mu\nu} = 8\pi G \left(T_{\mu\nu} + \langle \hat{T}_{\mu\nu} \rangle \right)$$

In Schwarzschild $\langle 0_{\rm H} | \hat{T}_{\mu\nu} | 0_{\rm H} \rangle \sim \frac{1}{M^2}$ finite and "small" down to horizon*

(in Unruh vacuum = with radiation)

$T_{\mu\nu} \sim 0$ → Small = globally $\int d^3x\, \langle \hat{T}_{\mu\nu} \rangle \ll M$

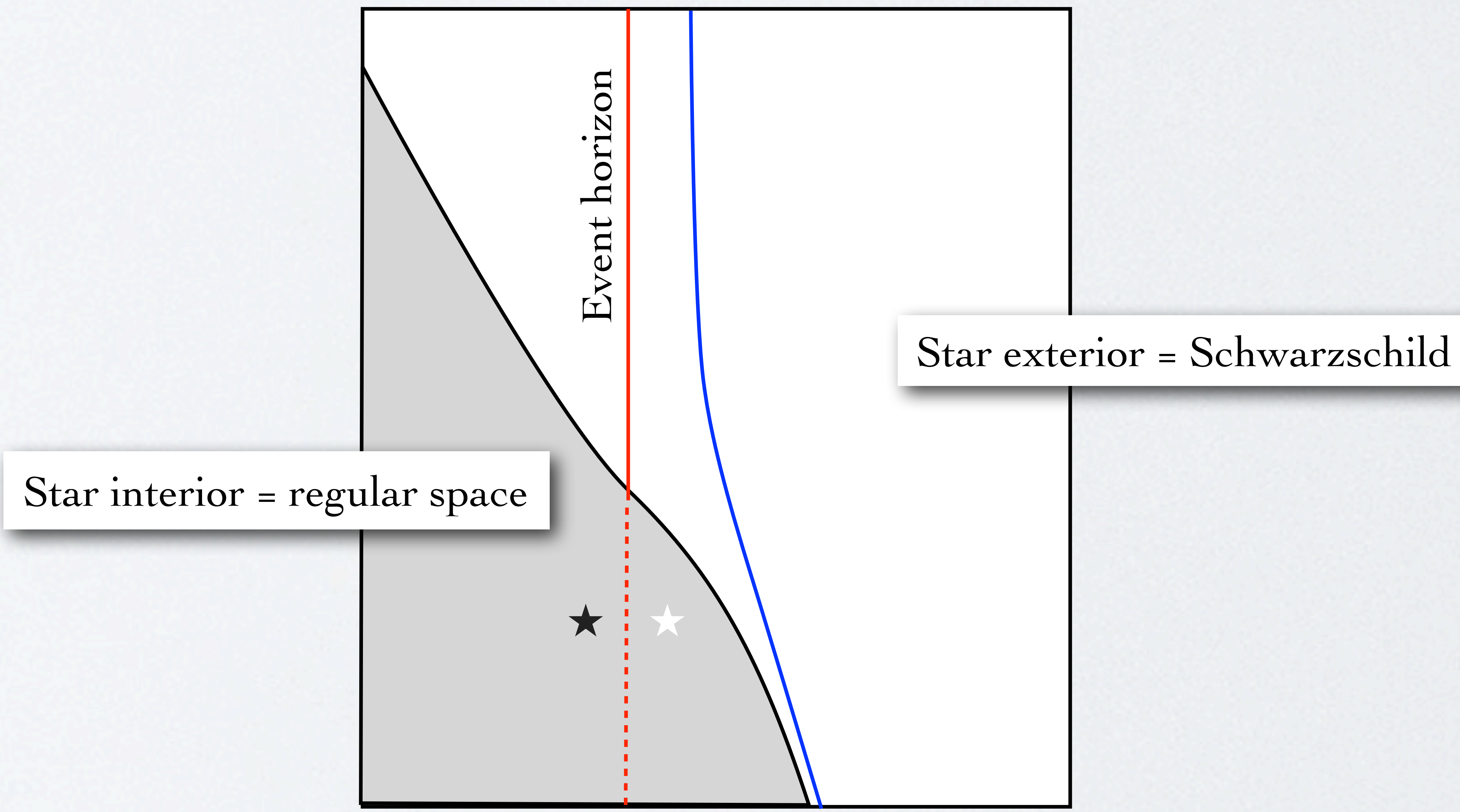
perturbatively $\langle \hat{T}_{\mu\nu} \rangle \ll M_{\rm p}^{-2}$

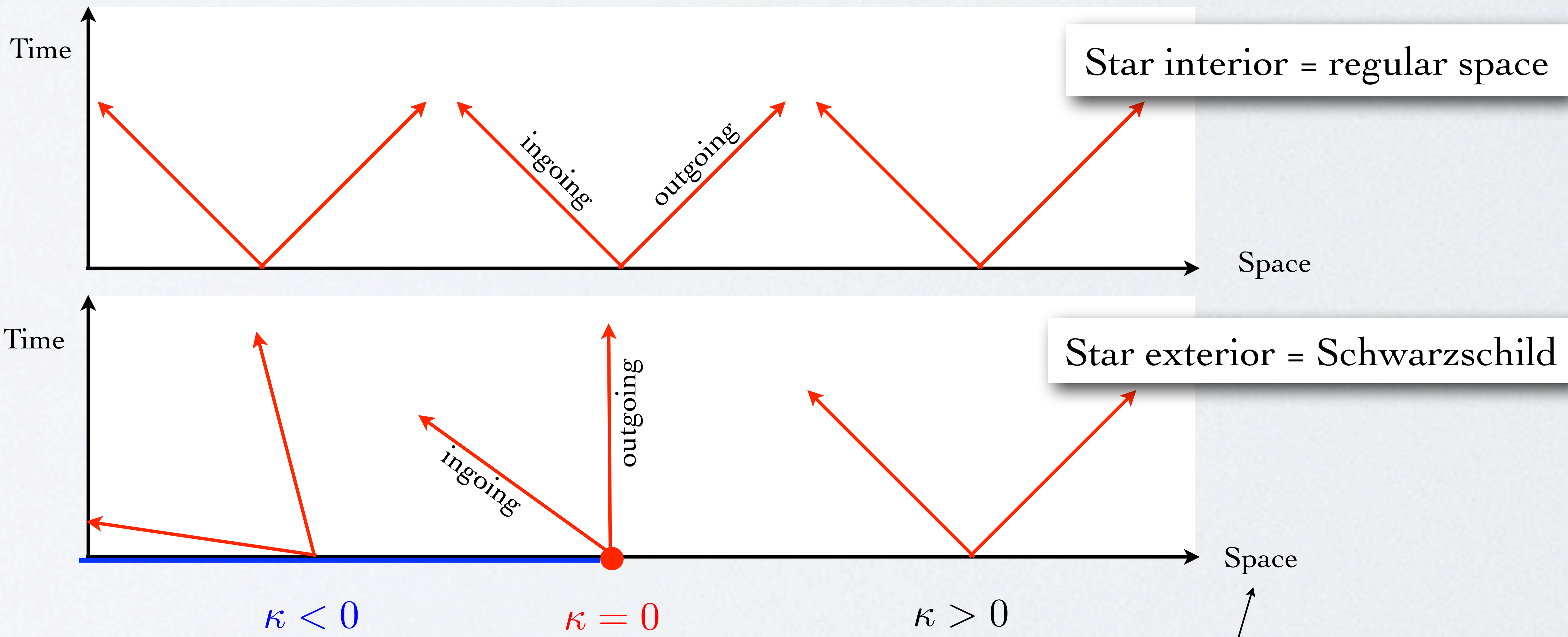
Backreaction large when $M \not\gg M_{\rm p}$ → End of story and talk...

*Around a static star $\langle 0_{\rm B} | \hat{T}_{\mu\nu} | 0_{\rm B} \rangle \sim \frac{1}{r - 2\,M}$ (= radiation better than nothing)

Naive concept: where escape velocity = speed of light
In GR: many definitions (often mathematical and hard to figure...)

1) Event horizon: global (teleological) concept





Trapping surface: local concept = naive definition

$$- \theta_+ \, \theta_- = \kappa = \gamma^{ij} \, \partial_i R \, \partial_j R \qquad (\kappa \sim \dot{R}^2 - 1 \sim R - 2\,M) \qquad (\dot{R} = \partial_\tau R)$$

(Expansion of null geodesics)

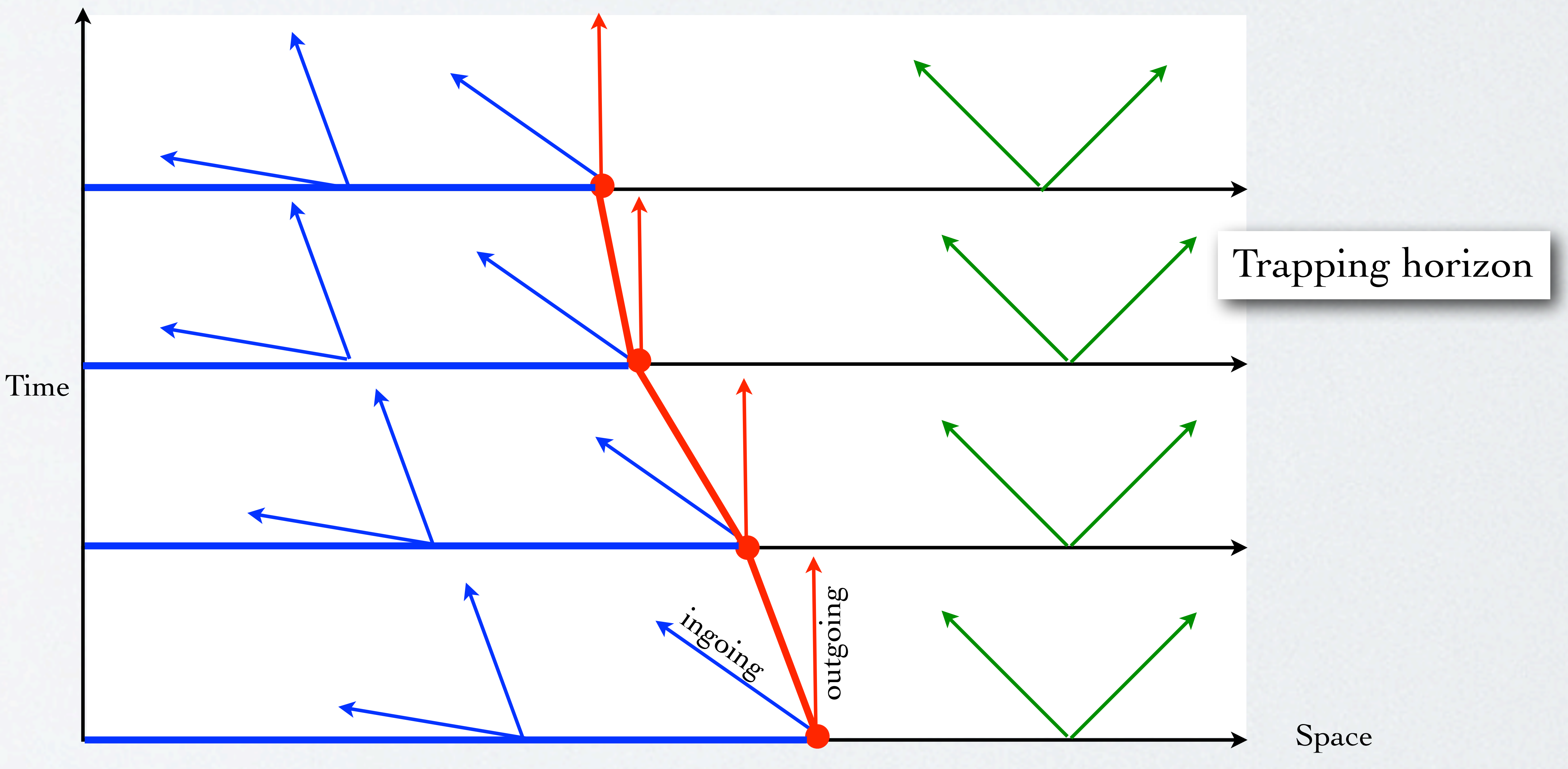
Areal radius

Proper time of freely falling observer

$$ds^2 = \gamma_{ij} \, dx^i \, dx^j + R^2(x^i) \, d\Omega^2$$

2) Trapping horizon: (space-null-time) sequence of trapping surfaces

Dynamical black hole



Quantum field theory in a nutshell

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1) Solve (classical) wave equation:

$$\Box\Phi = 0$$

2) Organize solutions into vector (formal Hilbert) space:

$$\Phi = \sum_{\vec{k}} \left[a_{\vec{k}}\, \phi_{\vec{k}} + a^{\dagger}_{\vec{k}}\, \phi^{*}_{\vec{k}} \right] \qquad (\phi_{\vec{k}} | \phi_{\vec{k}'}) = \delta_{\vec{k}\,\vec{k}'}$$

positive negative

3) Lift (normal mode) solutions (excitations) to operators:

$$a_{\vec{k}} \mapsto \hat{a}_{\vec{k}} \qquad a^{\dagger}_{\vec{k}} \mapsto \hat{a}^{\dagger}_{\vec{k}}$$

4) Build (probabilistic Hilbert) Fock space of quantum states:

$$|\vec{k}; n\rangle \propto \left(\hat{a}^{\dagger}_{\vec{k}} \right)^{n} |\vec{k}, 0\rangle \qquad \hat{a}_{\vec{k}}\, |\vec{k}, 0\rangle = 0$$

positive

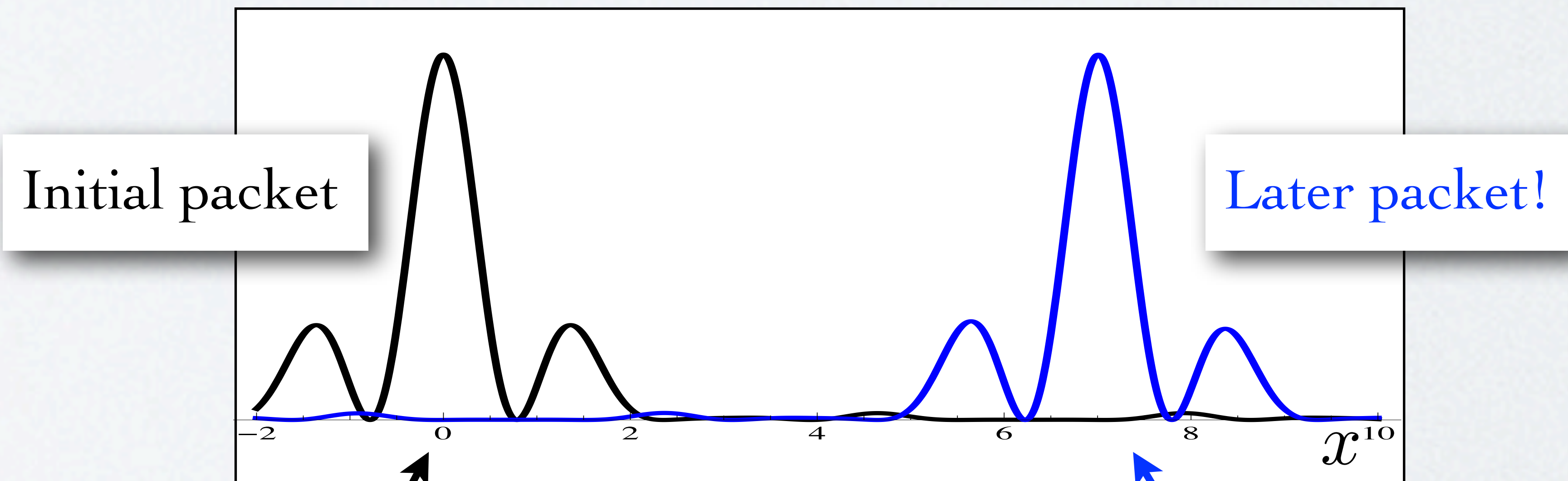
So particles = field excitations or...?

Naive concept: particle = localized object

Example: free scalar field in 1+1 (Fourier transform in "formal solutions")

$$\phi_k = e^{-i\,\omega\,t + i\,k\,x} \qquad \omega^2 = k^2 + m^2$$

Time evolution preserves (physically sensible) "packets":



$$\Phi = \sum_{\text{a few}} \phi_k$$

$t = 0$

localized object

$t = 1$

localized object

The Newton-Wigner operator

1-particle states:

$$|\phi\rangle = \int_{-\infty}^{+\infty} \frac{d^3p}{(2\,\pi)^{3/2}\,\sqrt{2\,\omega}}\, e^{-i\,\omega\,t}\,\phi(\vec{p})\,\hat{a}^\dagger_{\vec{p}}\,|0\rangle$$

$$\omega = \sqrt{\vec{p}\cdot\vec{p} + m^2}$$

"Wave-packet"

$$\langle\phi|\psi\rangle = \int_{-\infty}^{+\infty} \frac{d^3p}{(2\,\pi)^3\,2\,\omega}\,\phi^*(p)\,\psi(p) \equiv (\phi, \psi)$$

Position operator:

$$\hat{Q}_i = i\,\hbar\left(\frac{\partial}{\partial p^i} - \frac{p_i}{2\,\omega^2}\right)$$

$$\left[\hat{Q}_i, \hat{Q}_j\right] = 0$$

$$\left[\hat{Q}_i, \hat{P}_j\right] = i\,\hbar\,\delta_{ij}$$

$$\left[\hat{Q}_i, \hat{J}_j\right] = i\,\epsilon_{ij}{}^k\,\hat{Q}_k$$

$$\frac{d\hat{Q}_i}{dt} = \frac{i}{\hbar}\left[\hat{P}_0, \hat{Q}_i\right] = \frac{p_i}{\omega}$$

SO(3,1)

$$\hat{P}^\mu = p^\mu$$

$$\hat{J}_i = -i\,\epsilon_{ij}{}^k\,p^j\,\frac{\partial}{\partial p^k}$$

$$\hat{K}_i = i\,\omega\,\frac{\partial}{\partial p^i}$$

Example: free scalar field in 1+1

1-particle at rest in x=0: $\phi_0(p) = N\, e^{-\frac{p^2}{2\,\Delta^2}}$ → $(\phi_0, \hat{Q}\,\phi_0) = 0$

1-particle at rest in x=$\bar{x}$: $\phi_{\bar{x}} = e^{-i\,\bar{x}\,\hat{P}}\,\phi_0$ → $(\phi_{\bar{x}}, \hat{Q}\,\phi_{\bar{x}}) = \bar{x}$

1-particle with speed β : $\phi_\beta = e^{+i\,\beta\,\hat{K}}\,\phi_0$ → $(\phi_\beta, \hat{P}\,\phi_\beta) = \beta\,\omega$

"Orthogonality":

$$(\phi_0, \phi_{\bar{x}}) \simeq e^{-\frac{\bar{x}^2}{4\,\ell^2}}\left[1 + \mathcal{O}(\Delta^2/m^2)\right]$$

$$(\phi_0, \phi_\beta) \sim e^{-\beta^2\,\frac{m^2}{\Delta^2}}$$

$\ell = \Delta^{-1}$

$\lambda_m = m^{-1}$

But: what if...? →

Example: free scalar field in 1+1

2-particle states:

$$|\phi,\psi\rangle = \int_{-\infty}^{+\infty} \frac{dp,\,dq}{4\,\pi\,\sqrt{\omega(p)\,\omega(q)}}\, e^{-i\,[\omega(p)+\omega(q)]\,t}\,\phi(p)\,\psi(q)\,\hat{a}_p^\dagger\,\hat{a}_q^\dagger\,|0\rangle$$

$$\langle\phi,\psi|\,\hat{Q}\,|\phi,\psi\rangle = \int_{-\infty}^{+\infty} \frac{dp}{4\,\pi\,\omega(p)}\,\phi^*(p)\,\left[\hat{Q}\,\phi(p)\right] + \int_{-\infty}^{+\infty} \frac{dq}{4\,\pi\,\omega(q)}\,\phi^*(q)\,\psi(q) \times \int_{-\infty}^{+\infty} \frac{dp}{4\,\pi\,\omega(p)}\,\psi^*(p)\,\left[\hat{Q}\,\phi(p)\right]$$

"Should vanish...?"

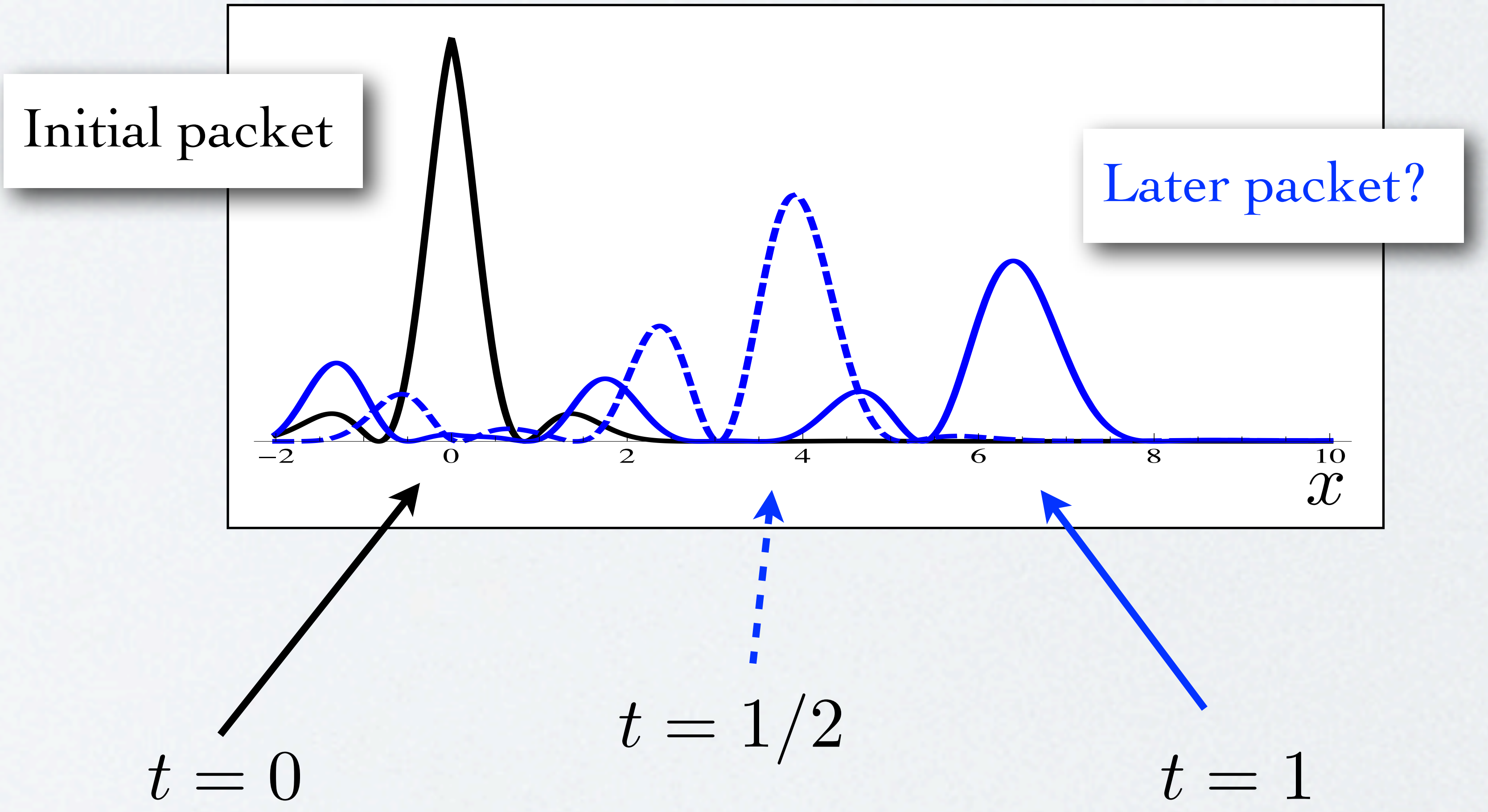
Interacting theory does not preserve particle number...

And: what if...?

Counter-example: toy scalar field in 1+1

$$\phi_k = e^{-i\,\omega\,t + i\,k\,x - k*|x|} \qquad \omega^2 = k^2 + m^2$$

Time evolution does not preserve "packets":



$$\Phi = \sum_{\text{a few}} \phi_k$$

Quantum field theory in curved space-time

↓

5) Usually, normal modes are not plane waves (everywhere)

↓

Packets not preserved
(particles loose identity)

6) Normal modes (excitations) depend on observer

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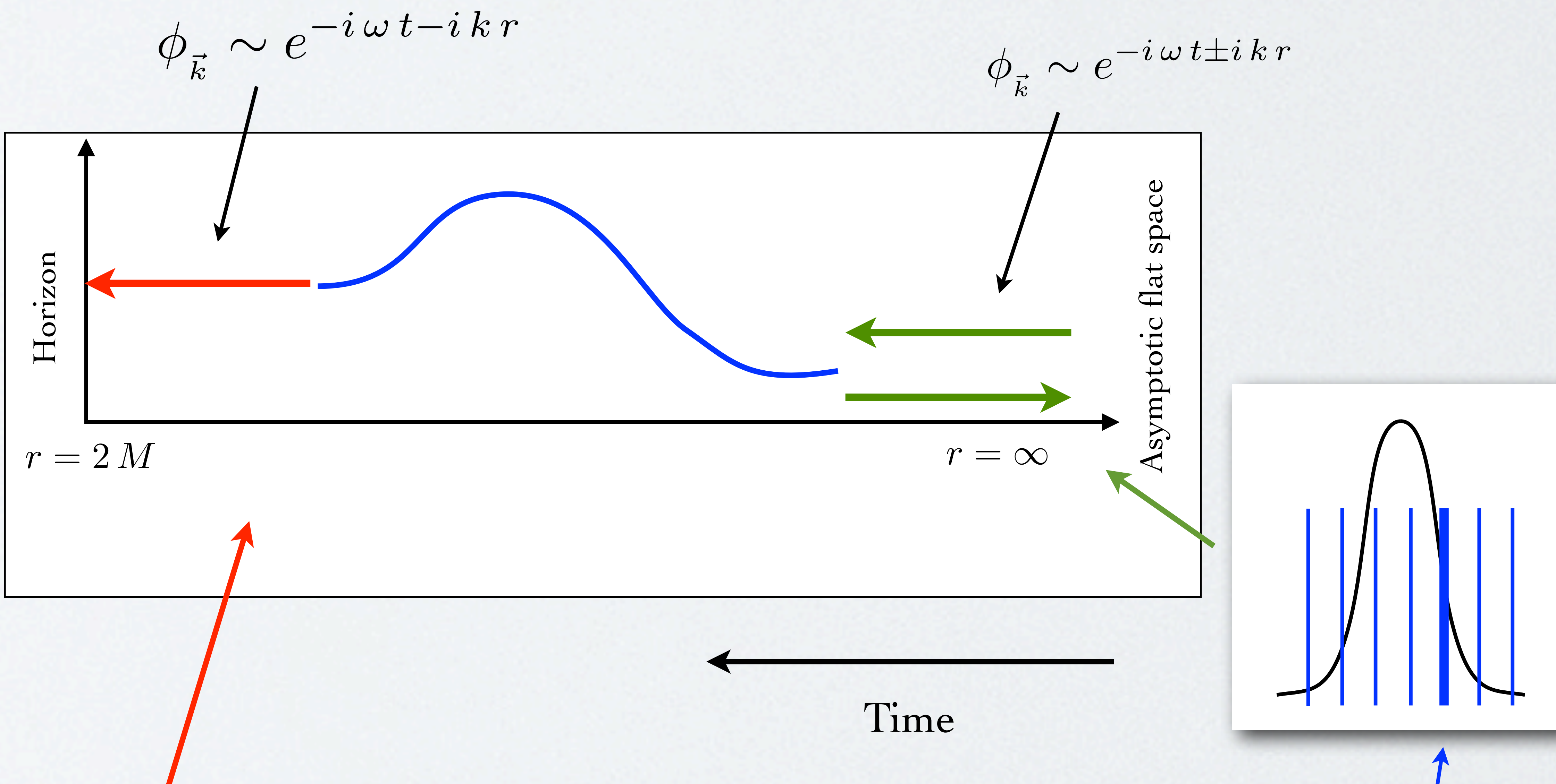
Fock space depends on observer
(observation dependent vacuum)
(+possible non-unitary issue)

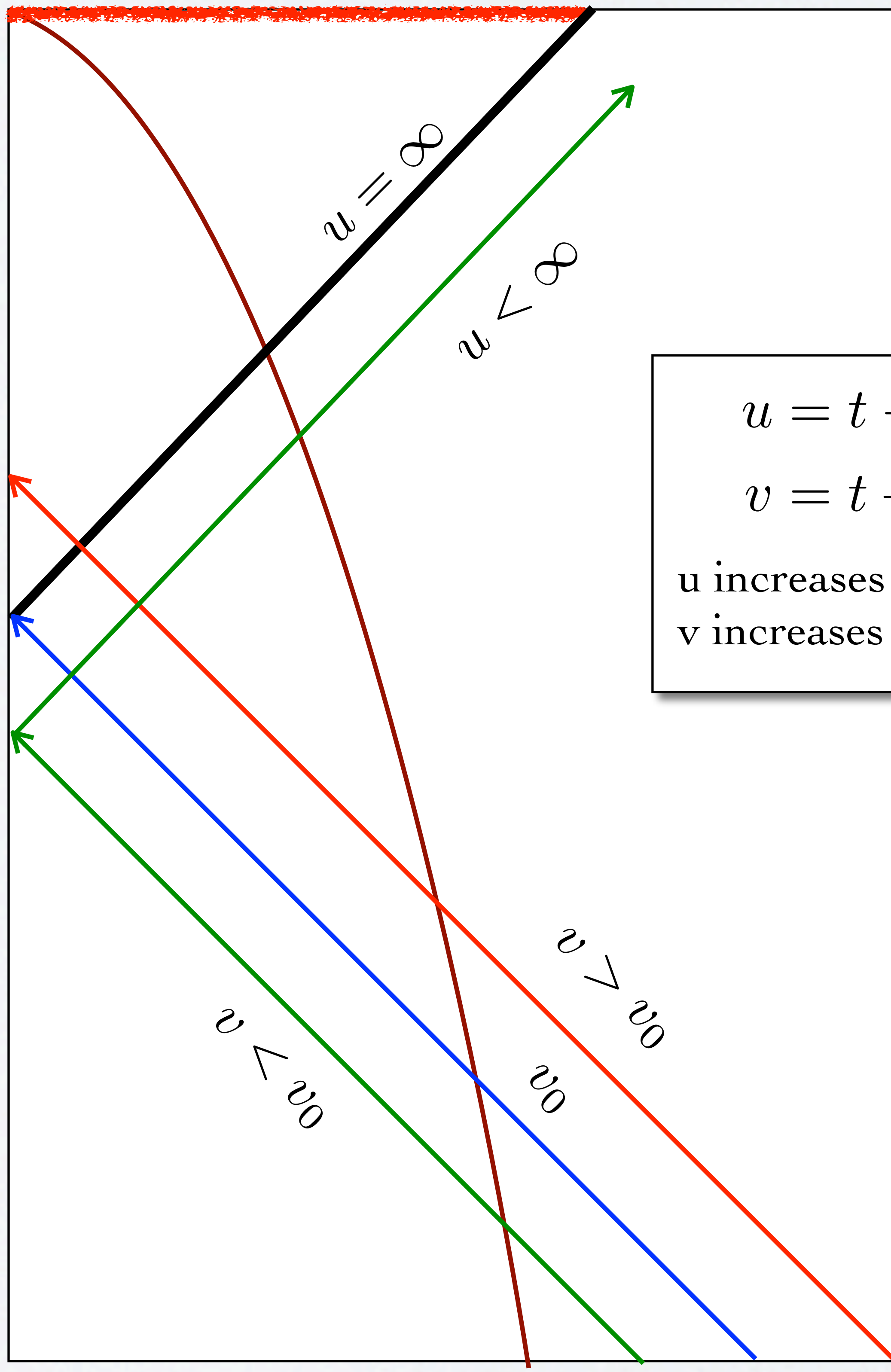
7) Horizon = “boundary condition”: only ingoing modes exist



Tidal forces cause in-falling packets to lose outgoing modes
[W. Unruh, PRD 51 (1985) 2827]

→ “classical analogue” of Hawking radiation

+ occurs to quantum collapsing shells: R.C. et al, PRD 64 (2001) 104012



$u = t - r$

$v = t + r$

u increases BR to TL
v increases BL to TR

OUT Fock space:

Modes pile up (tidal effect)

$$\phi_\omega \sim \begin{cases} e^{4\,M\,i\,\omega\,\ln(v_0 - v)} & v < v_0 \\ 0 & v > v_0 \end{cases} \longrightarrow |0, +\infty\rangle$$

Bogoliubov transformations:

$$\beta_{\omega\,\omega'} \sim \int_{-\infty}^{v_0} \phi_\omega\,\psi_{\omega'}\,dv$$

$$N_\omega = \sum_{\omega'} |\beta_{\omega\,\omega'}|^2 \sim \frac{1}{e^{8\,\pi\,M\,\omega} - 1}$$

IN Fock space:

$$\psi_\omega \sim \begin{cases} e^{-i\,\omega\,u} \\ e^{-i\,\omega\,v} \end{cases} \longrightarrow |0, -\infty\rangle$$

(asymptotic modes + boundary condition) determine "particle" content

a) Trans-Planckian problem:

$$N_\omega = \sum_{\omega'} |\beta_{\omega\,\omega'}|^2 \sim \frac{1}{e^{8\,\pi\,M\,\omega} - 1}$$

$$\omega' \to \infty \gg M_{\mathrm{P}}$$

UV cut-off or modified dispersion relations?

$$n_\omega \leftrightarrow \frac{d\omega}{dk}$$

[R.C., CQG 19 (2002) 2453]

b) Finite frequency blue-shifts to trans-Planckian near horizon:

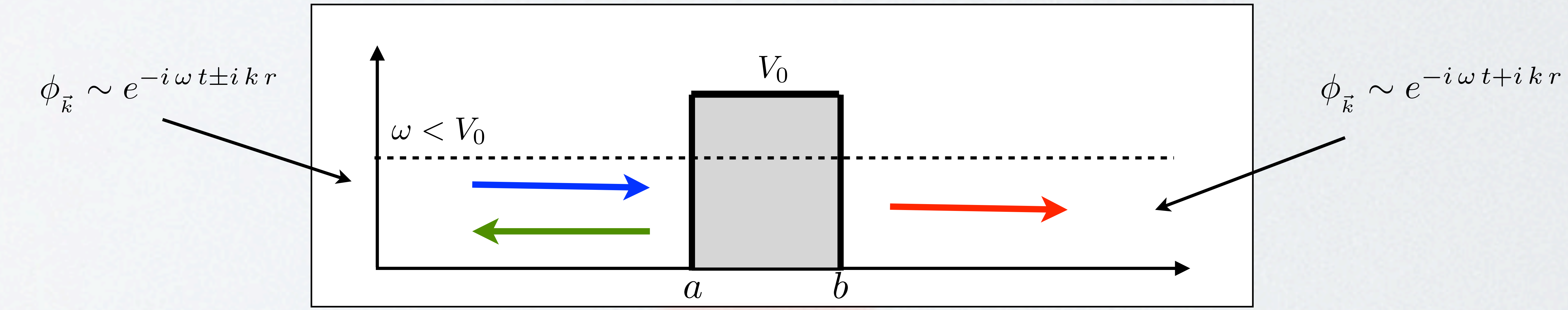
$$\omega_r \sim \frac{\omega_\infty}{\sqrt{r - 2\,M}}$$

Point-like particle

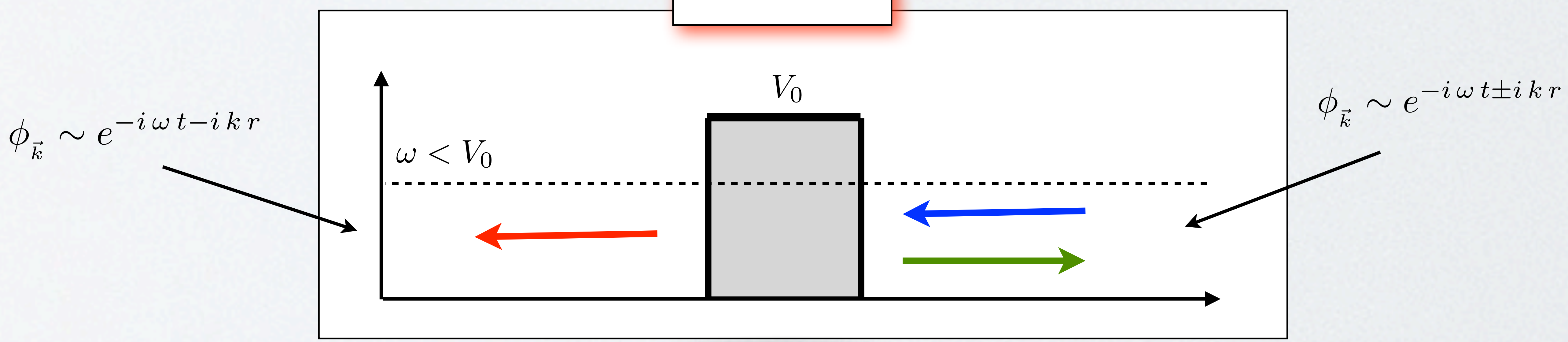
Classically $\omega_{\mathrm{H}} \sim M^{-1}$ spreads over $d_{\mathrm{H}} \sim M$: "averaged blue-shift"?

[R.C., L. Mersini, IJMP A 19 (2004) 1395]

Quantum mechanical tunneling for particles:



$r \to -r$



Transmission amplitude: $\simeq e^{i \int_a^b p\,dr} \simeq e^{-W(a,b)}$ $\quad p = \sqrt{2\,m\,(E - V_0)}$

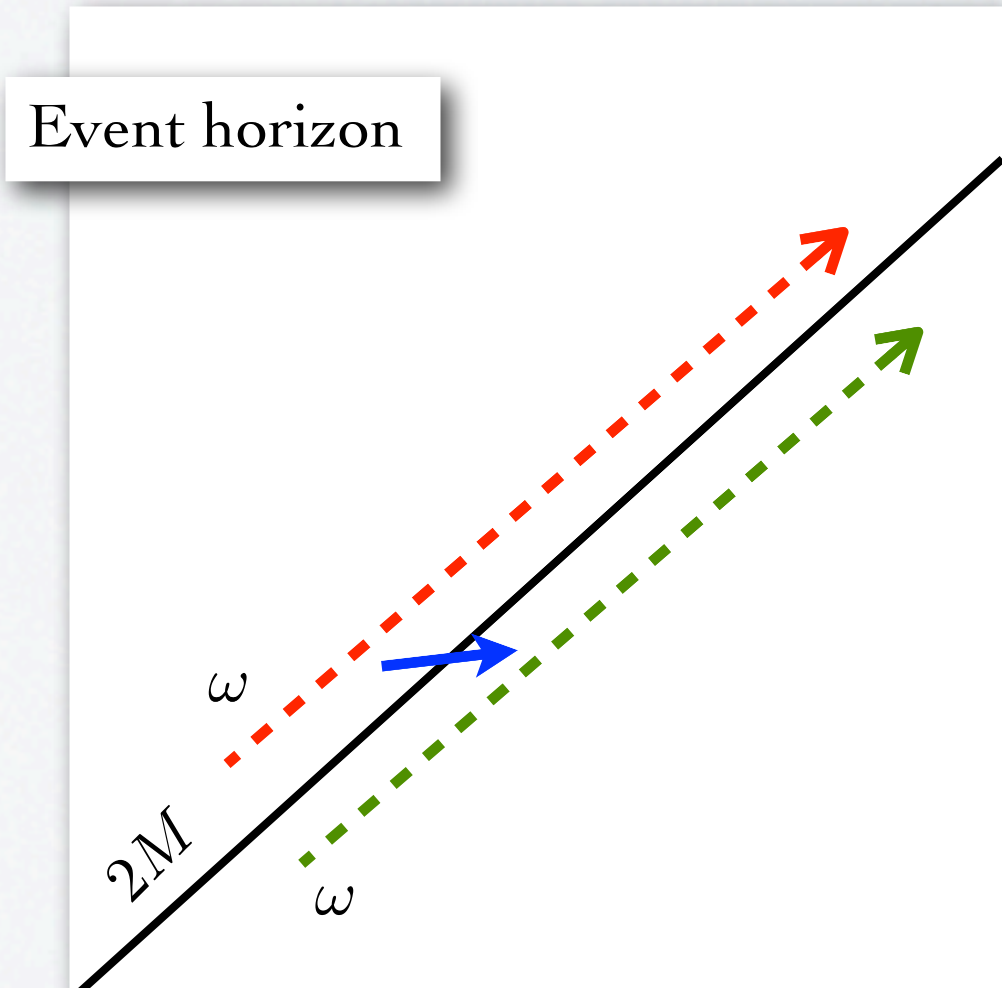
(classically forbidden path)

1.4) Parikh-Wilczek

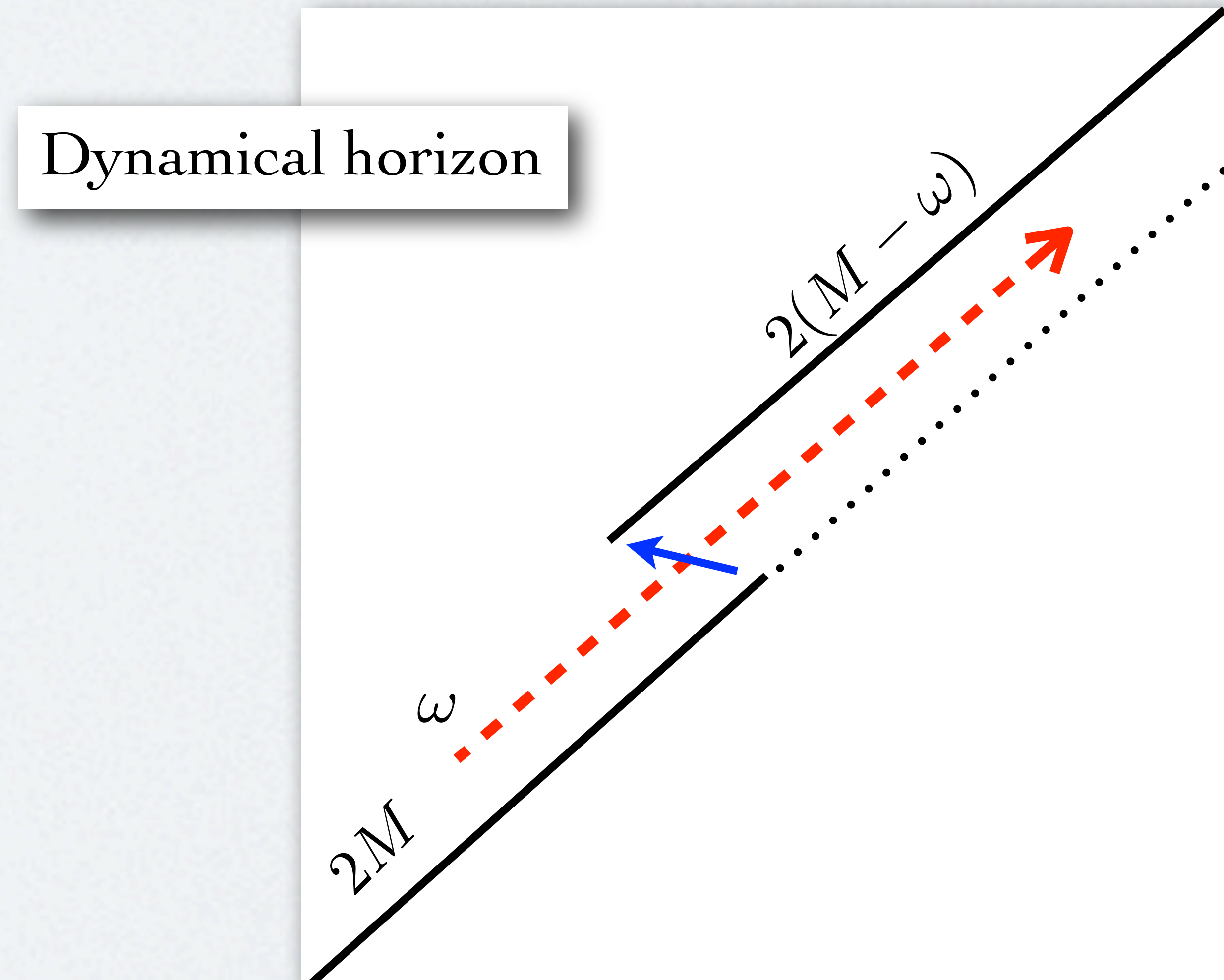
[PRL 85 (2000) 5042]

Quantum mechanical tunneling across horizon does not work

Horizon quantum tunneling (or particle self-tunneling)



Without backreaction



With backreaction

("Particle opens its own exit door")

Transmission probability: $P \sim e^{-\beta_{\mathrm{H}}\,\omega}$ $\qquad \beta_{\mathrm{H}} = 8\,\pi\,M$

Semiclassical gravitational collapse:

1) Trapping horizons Hawking radiate (tidal effect)

2) Hawking radiation = backreaction

↓

Beyond semiclassical gravitational collapse:

1) Collapsing matter is quantum

2) Gravity is quantum?

2.1) Classicalization?

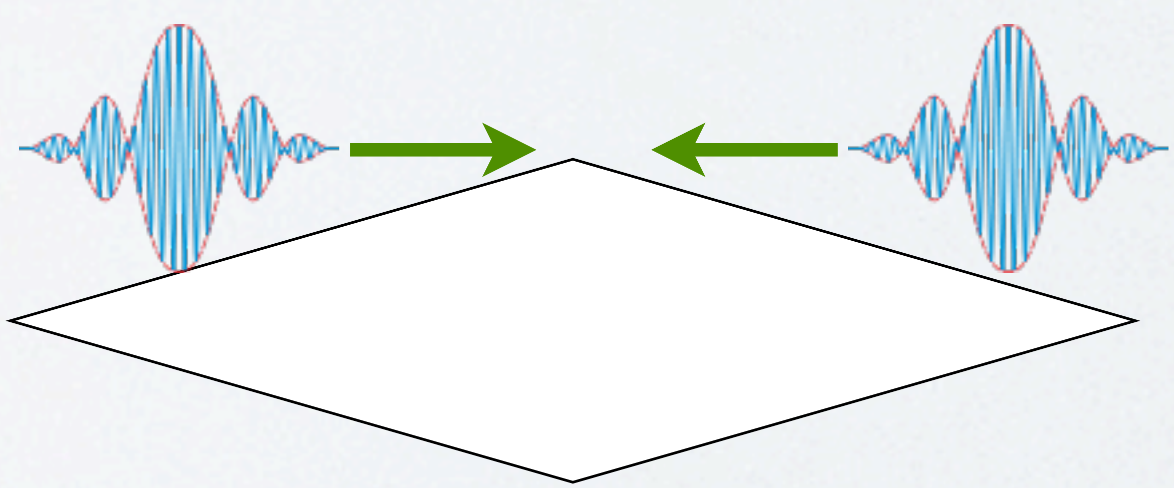
The hoop conjecture (Thorne, 1972):

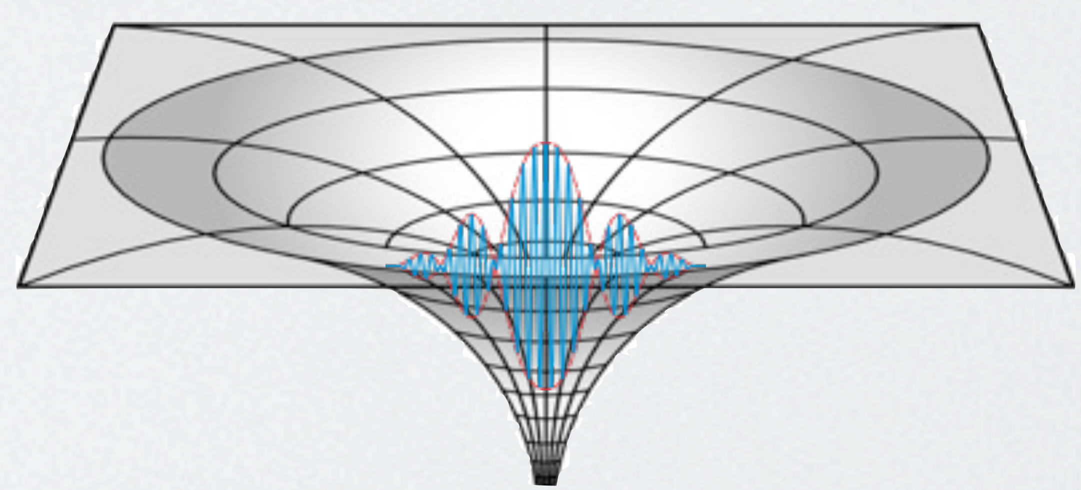
A black hole forms whenever the impact parameter b of two colliding objects (of negligible spatial extension) is shorter than the radius of the would-be-horizon (roughly, the Schwarzschild radius, if angular momentum can be neglected) corresponding to the total energy M of the system, that is for

$$b \lesssim \frac{2\,\ell_{\rm p}\,M}{m_{\rm p}}$$

Classicalization (Dvali, 2010):

At high (~Planckian) energy, quantum particle scatterings lead to formation of "classicalons" and quantum degrees of freedom disappear (*no UV divergences*).
For gravity, "classicalons" = black holes = BEC of gravitons





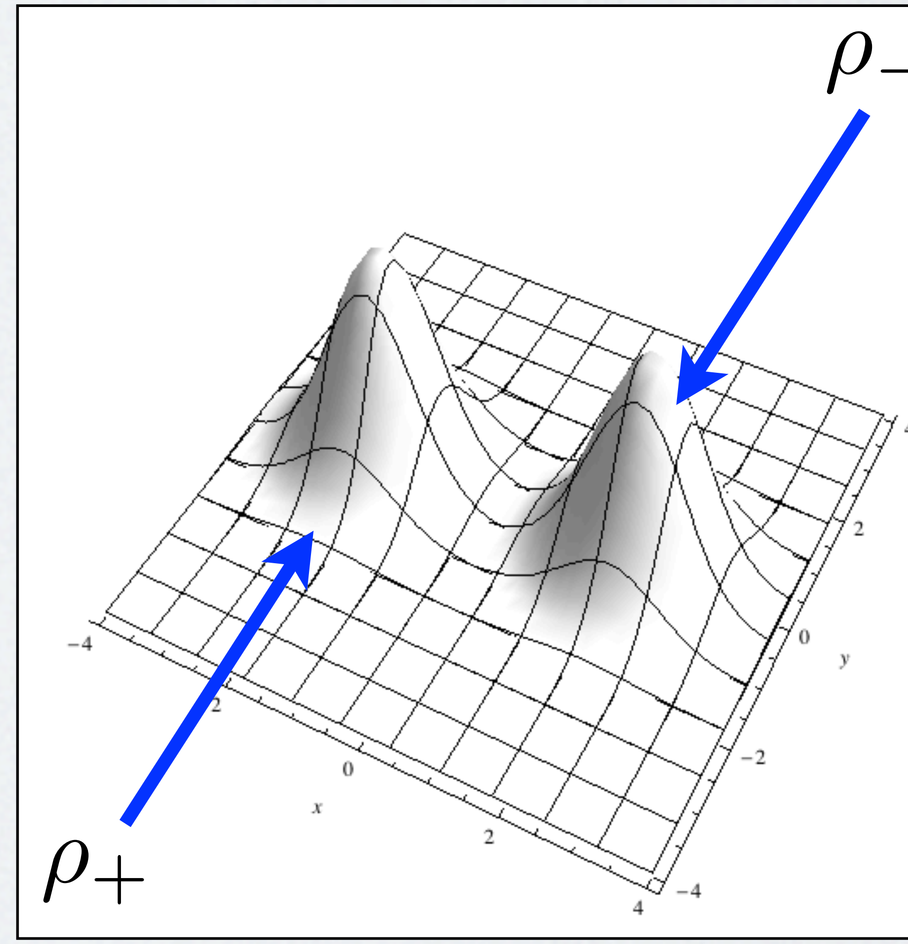
2.1) Classicalization? 2

Example: Gaussian packets

[R.C., O.Micu, A.Orlandi, arXiv:1205.6303, EPJC in press]

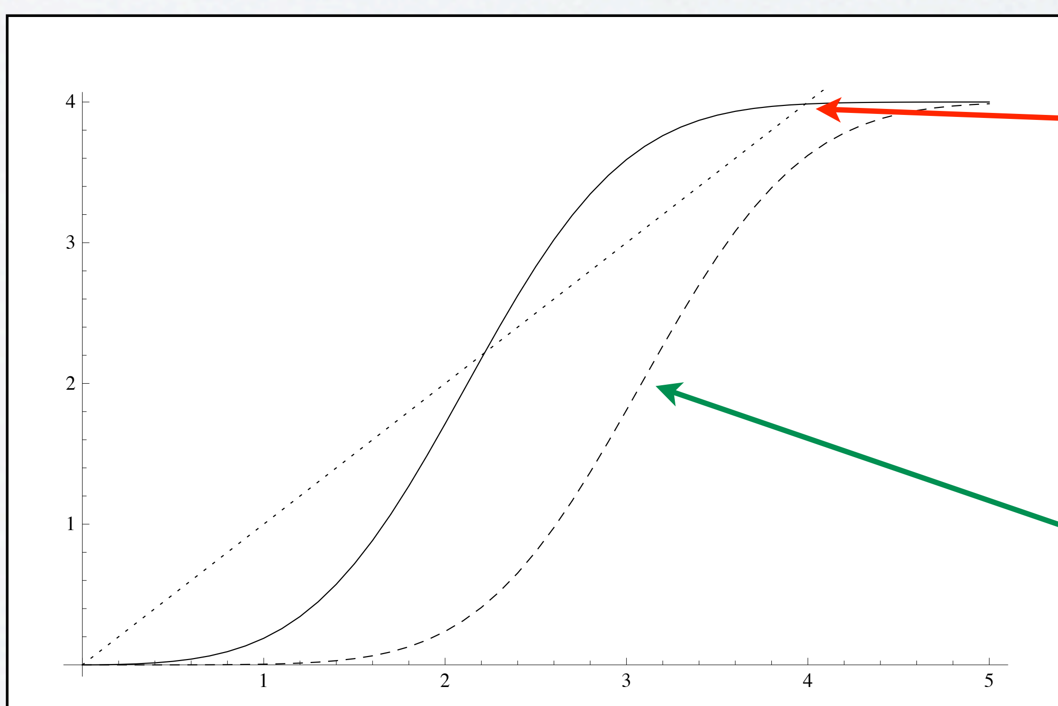
$$\rho_\pm(x,y) = \frac{\rho_0}{\pi\,\ell^2}\,\exp\left\{-\frac{(x\pm b)^2+y^2}{\ell^2}\right\}$$

$$= \frac{\rho_0}{\pi\,\ell^2}\,\exp\left\{-\frac{r^2\pm 2\,b\,r\,\cos(\theta)+b^2}{\ell^2}\right\} = \rho_\pm(r,\theta)$$



(Spherically symmetric) mass function:

$$M(r) = \frac{4\,\pi}{3}\int_0^r \rho(t,\bar{r})\,\bar{r}^2\,d\bar{r}$$



$2\,M(r) = r$

(Outer) horizon!

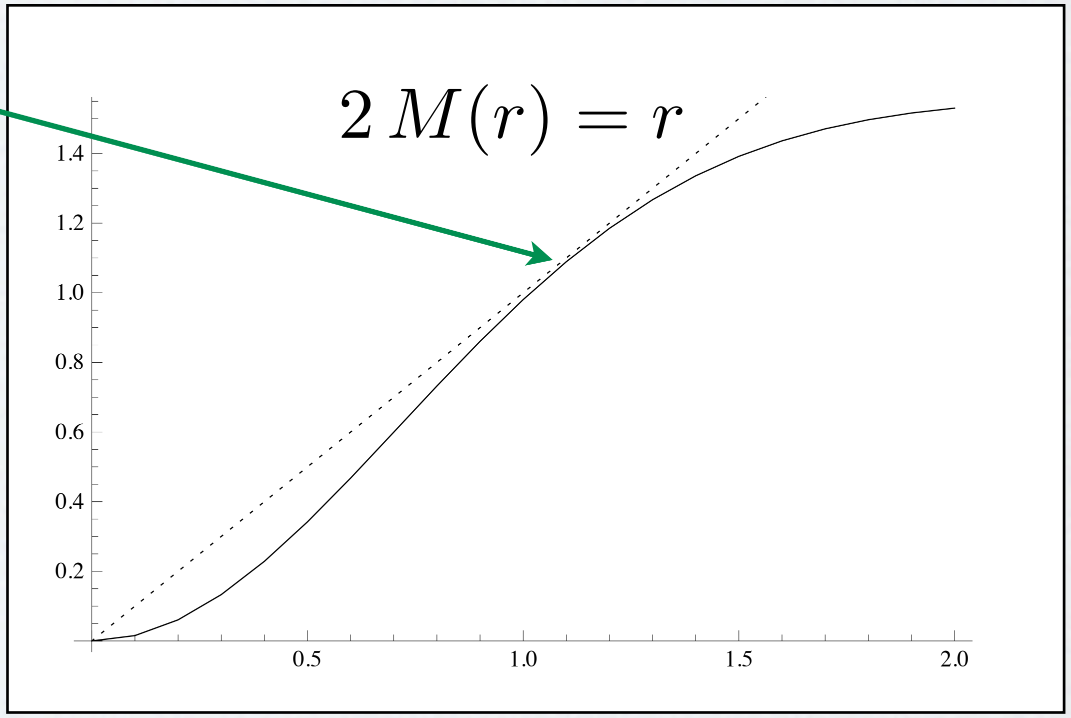
$$r = 2\,\ell_{\rm p}\,\frac{M(r)}{M_{\rm p}}$$

No BH!

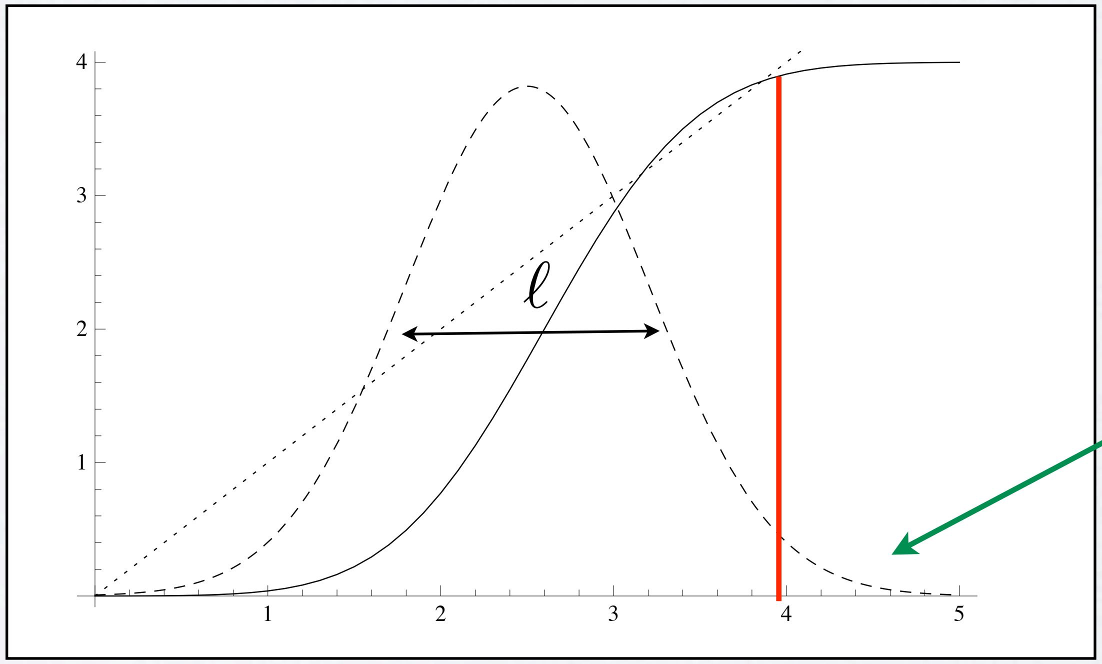
1) Threshold for BH formation:

(Packet width)

$$M_0 \simeq M_{\rm p} \left(\frac{\ell}{\ell_{\rm p}}\right)$$



2) Particle interpretation:



IN or OUT?

(Full dynamics will tell...?)

2.1) Classicalization?

"Classicalization" ↔ Formation of **classical bound** state (gravity is always practically classical)

↕

QFT on a self-consistently evolved classical background

[M. Reuter, PRD 57 (1998) 971]

or

Semiclassical QFT propagators

[R.C. arXiv:0806.0501v3]

or

Space-time non-commutativity

[Nicolini and Spallucci]

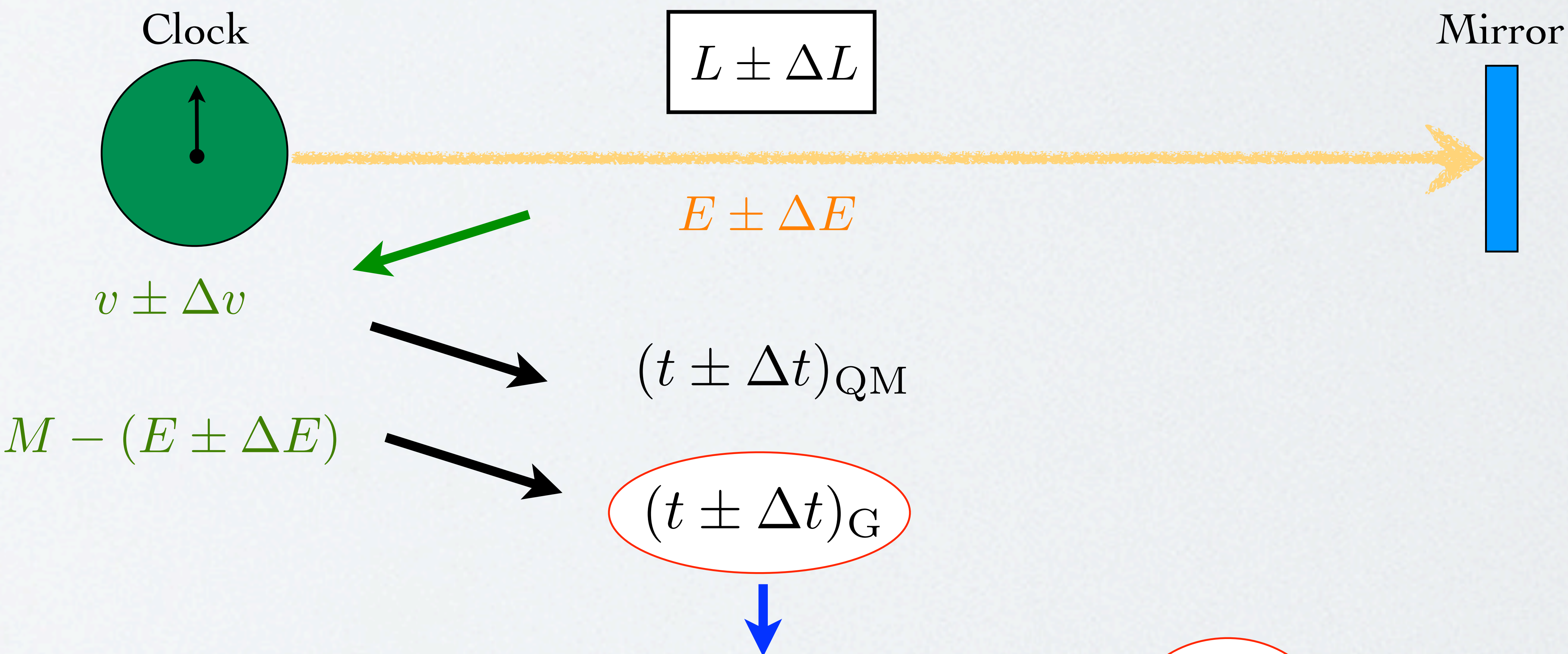
or

Generalized Uncertainty Principle

[F. Scardigli, PLB 452 (1999) 39 and many others]

2.2) GUP in QM 1

"Measuring very short lengths requires much energy: a BH is produced and precision reduces"



$$\Delta L = L_{\rm QM} + L_{\rm G} = 2\left(\ell_{\rm P}\, L\, \frac{m_{\rm P}}{M}\right)^{1/2} + \ell_{\rm P}\, \frac{M}{m_{\rm P}}$$

$$\frac{\Delta L_{\min}}{L} \simeq \left(\frac{\ell_{\rm P}}{L}\right)^{2/3}$$

Minimum length!

F. Scardigli, R.C., Int. J. Mod. Phys. D18 (2009) 319

2.2) GUP in QFT?

$$\Delta x \gtrsim \frac{\ell_{\rm P}\, m_{\rm P}}{\Delta p} + \alpha\, \ell_{\rm P}\, \frac{\Delta p}{m_{\rm P}}$$

Deformed Poincare algebra

[M. Maggiore, PRD 49 (1994) 5182]

$$\hat{X}_i = i\,\hbar\,\cosh\left(\frac{p_0}{2\,\kappa}\right)\left[\frac{\partial}{\partial p_i} - \frac{p_i}{8\,\kappa^2\,\sinh^2(p_0/2\,\kappa)}\right]$$

$$\left[\hat{X}_i, \hat{X}_j\right] = -\frac{\hbar^2}{4\,\kappa^2}\, i\,\epsilon^k{}_{ij}\,\hat{J}_k$$

$$\left[\hat{X}_i, \hat{P}_j\right] = i\,\hbar\,\delta_{ij}\,\cosh\left(\frac{p_0}{2\,\kappa}\right)$$

pro) Modified NW operator

con) What about fields (particle states)?

Modified canonical QFT

[V. Husain et al, arXiv:1208.5761]

$$[\hat{x}, \hat{p}] = i\,\hbar\, f\left(\frac{\hat{p}}{2\,\kappa}\right) \qquad \text{QM}$$

$$\downarrow$$

$$\left[\hat{\phi}_{\vec{k}}, \hat{\pi}_{\vec{k}}\right] = i\,\hbar\, f\left(\frac{\hat{\pi}_{\vec{k}}}{2\,\kappa}\right) \qquad \text{QFT}$$

pro) Workable approach

con) What about localization?

Deformed Hilbert space

[A. Kempf, PRL 92 (2004) 221301]

"Physical fields could be differentiable functions which possess merely a finite density of degrees of freedom."

$$\frac{\Box\psi}{\psi} \sim E^2 - p^2 < \Lambda^2$$

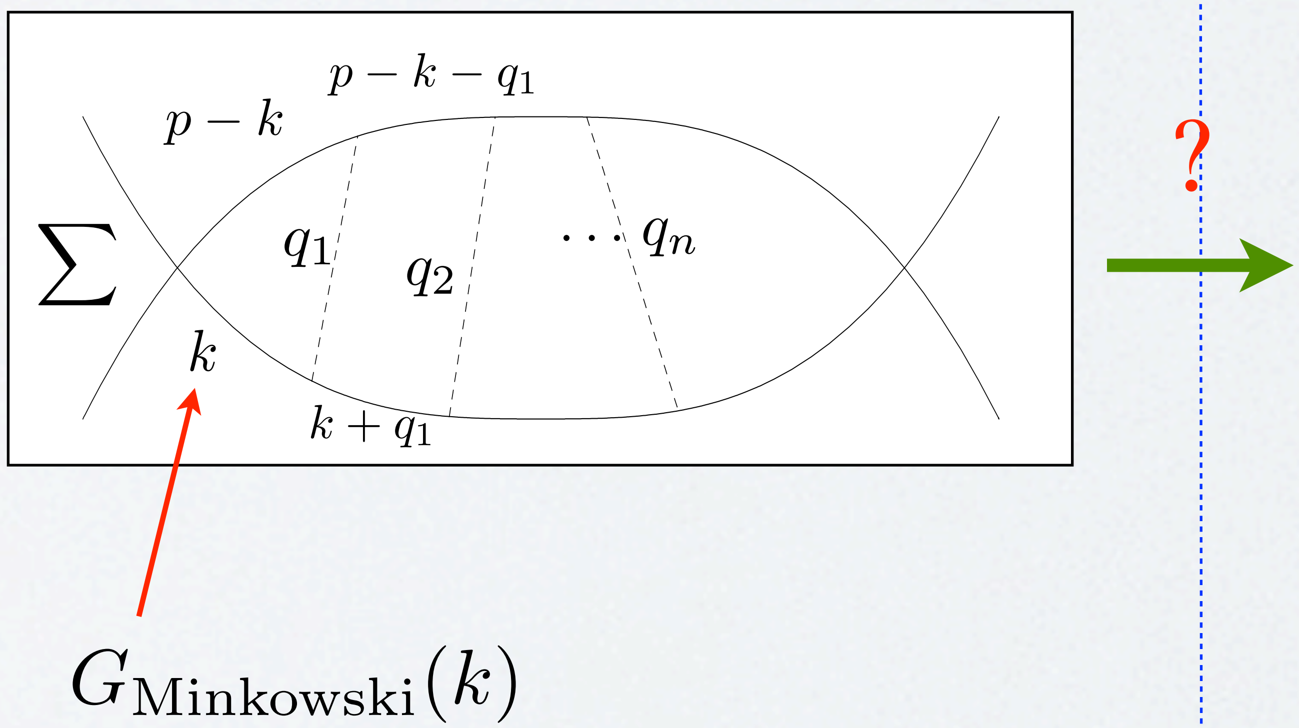
Covariant cut-off*
(virtual states)

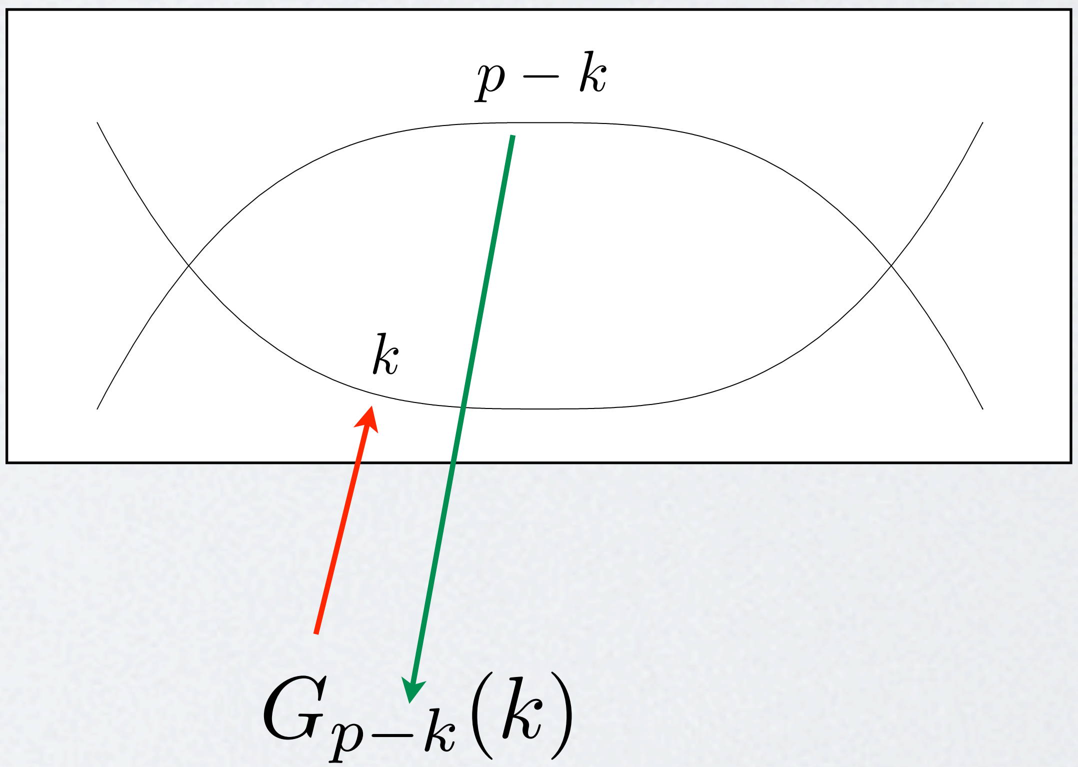
*Trans-Planckian modes do not sample...

2.2) GUP in QFT?

Graviton exchanges
(Sum over graphs)

Propagation on curved space





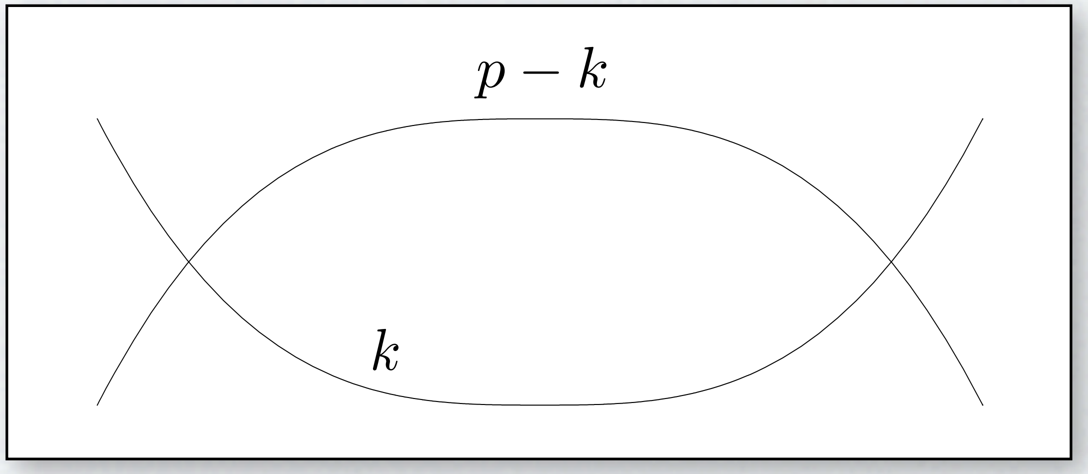
Gravity = quantum spin-2 field

Gravity = classical geometry

Does gravity ever go quantum?

2.2) GUP in QFT?

$$G_k(p-k)$$



$$G_{p-k}(k)$$

$$R_{\mu\nu} - \frac{1}{2}\,R\,g_{\mu\nu} = \frac{1}{M_{\rm p}^2}\,T_{\mu\nu}$$

$$\left(\Box - m^2\right)\phi = 0$$

Does gravity ever go quantum?

3) Outlook

a) we do not yet understand black hole formation at quantum level (scattering of quantum particles)
b) we do not understand what happens with scatterings about the Planck scale
c) we do not understand what happens with black hole evaporation about the Planck scale

d) given a)-c), do we really understand gravitational collapse at all?*

↓

Let's try GUP from QM to QFT as a working tool

*Trapping horizon likely starts forming at Planckian scale from the core and expands...

3) Outlook

Pauli, long ago [around 1930], suggested that gravity could act as a regulator for the UltraViolet divergences* that plague Quantum Field Theory by providing a natural cut-off at the Planck scale. Later on, classical divergences in the self-mass of point-like particles were indeed shown to be cured by gravity [Arnowitt, Deser and Misner, Phys. Rev. Lett. 4 (1960) 375], and the general idea has since then resurfaced in the literature many times.
In spite of that, Pauli's ambition has never been fulfilled.

* same as classicalization

Thanks!