Toward precision studies of gluon saturation

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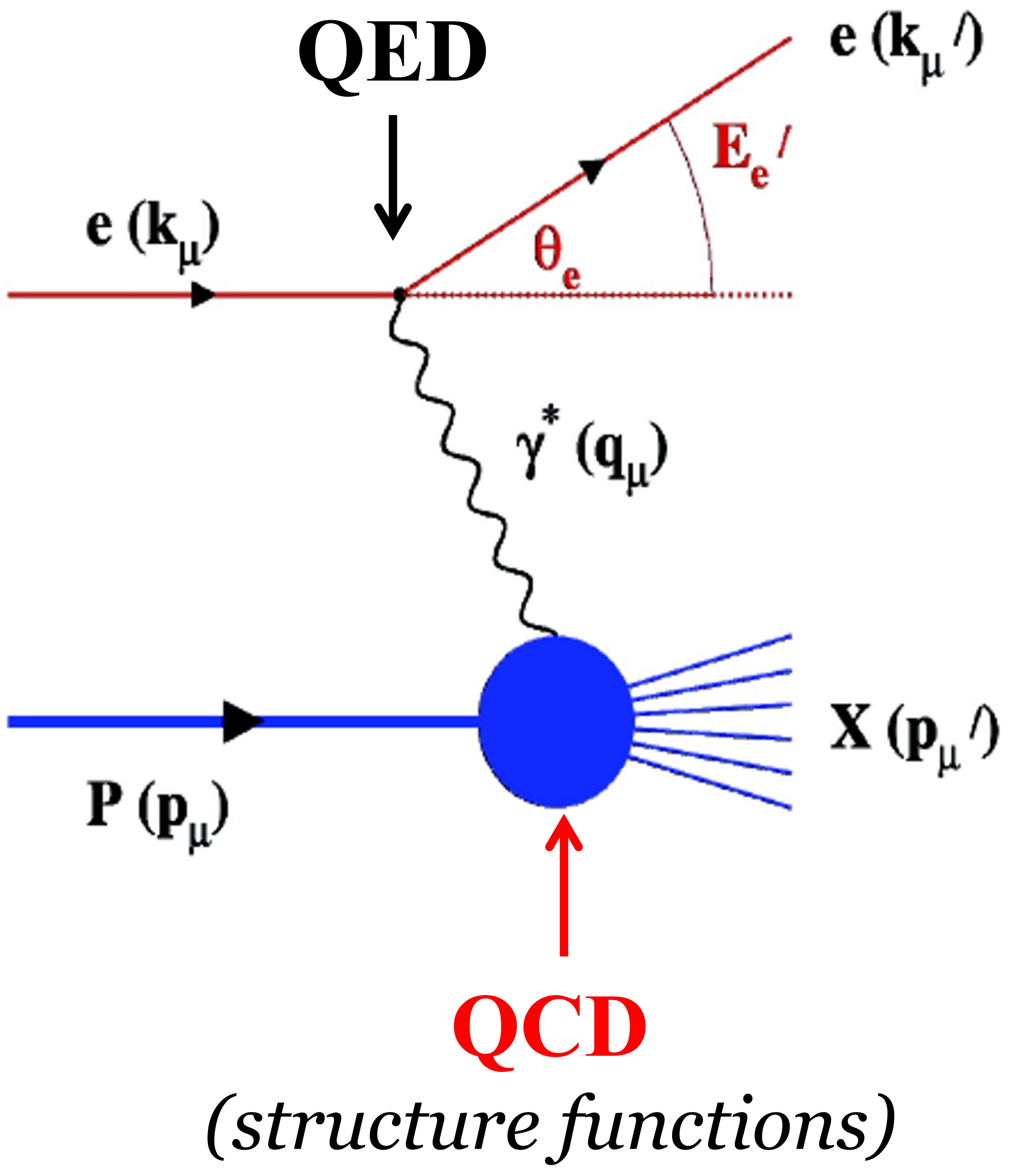
Deep Inelastic Scattering (DIS)

probing hadron structure



Kinematic Invariants

$$Q^2 = -q^2 = -(k_\mu - k'_\mu)^2$$

$$Q^2 = 4E_e E'_e \sin^2\left(\frac{\theta'_e}{2}\right)$$

Measure of resolution power

$$y = \frac{pq}{pk} = 1 - \frac{E'_e}{E_e}\cos^2\left(\frac{\theta'_e}{2}\right)$$

Measure of inelasticity

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

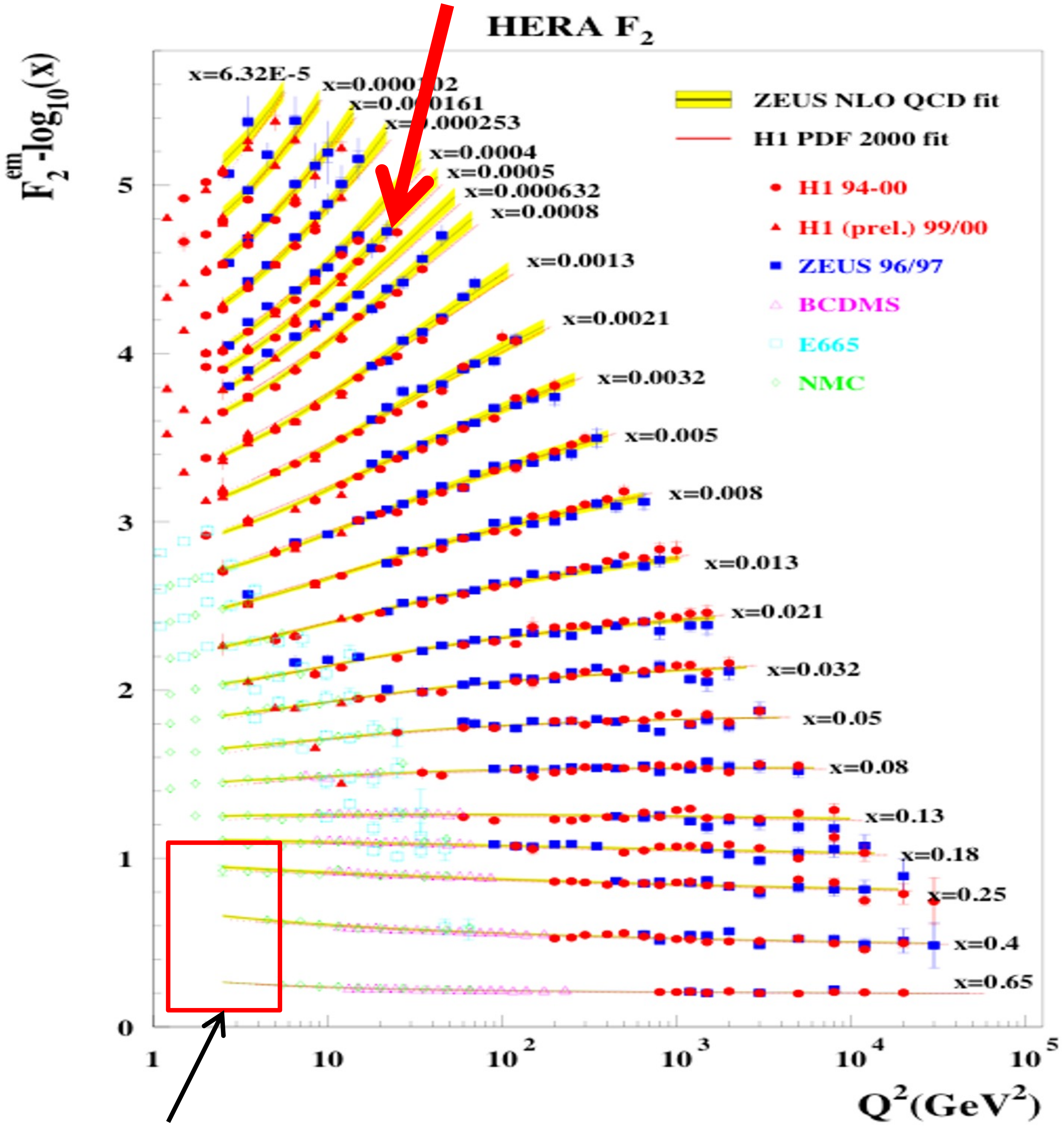
Measure of momentum fraction of struck quark

$$\mathbf{s} \equiv (\mathbf{p} + \mathbf{k})^2$$

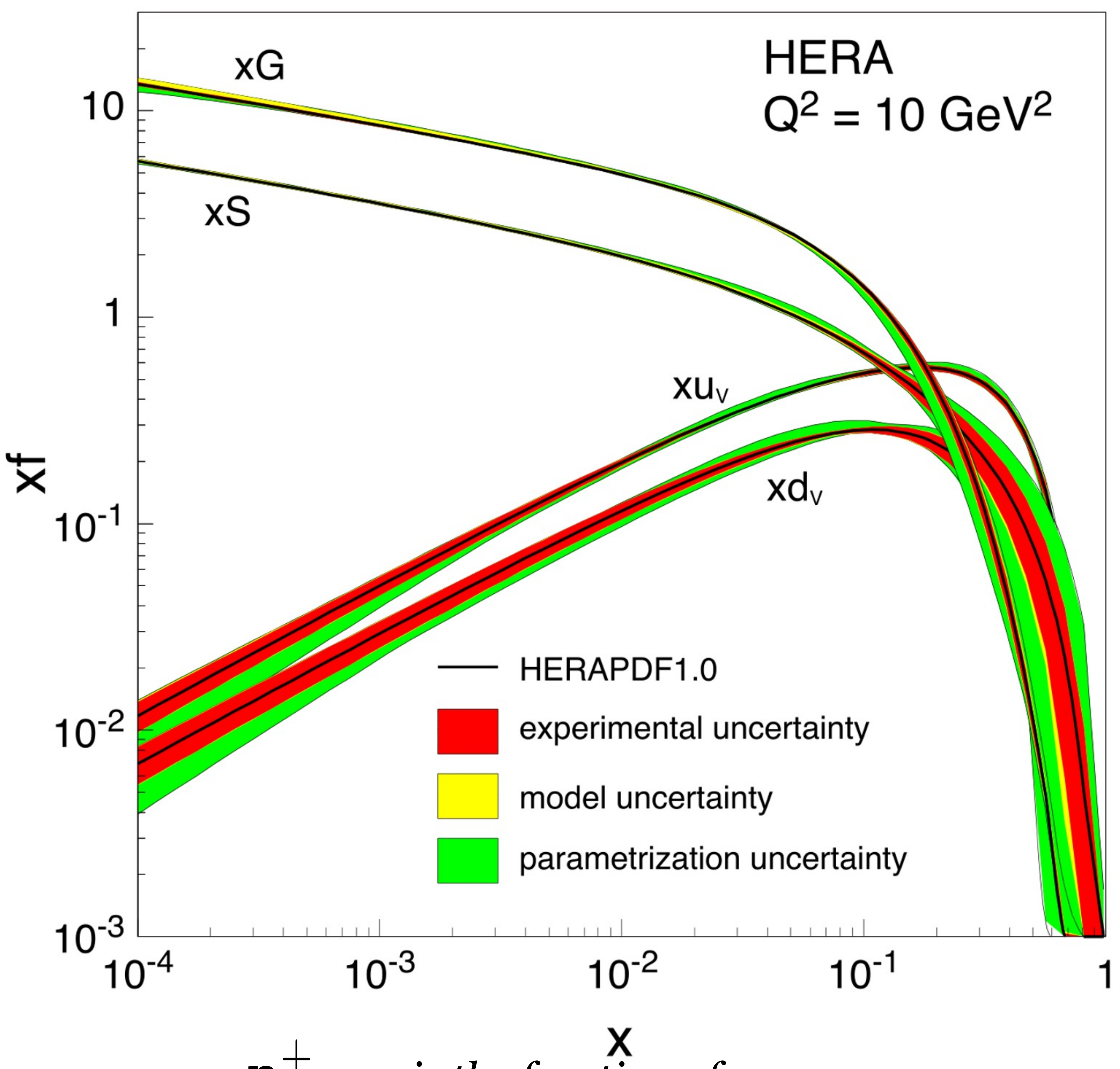
Deep Inelastic Scattering

$$F_2 \equiv \sum_{f=q,\bar{q}} e_f^2 \, xq(x, Q^2)$$

QCD: scaling violations



early experiments (SLAC,...):
scale invariance of hadron structure



$$x = \frac{p^+}{P^+}$$

x is the fraction of hadron energy carried by a parton

What drives the growth of parton distributions?

Splitting functions at leading order $O(\alpha_S{}^0)$ $(x \neq 1)$

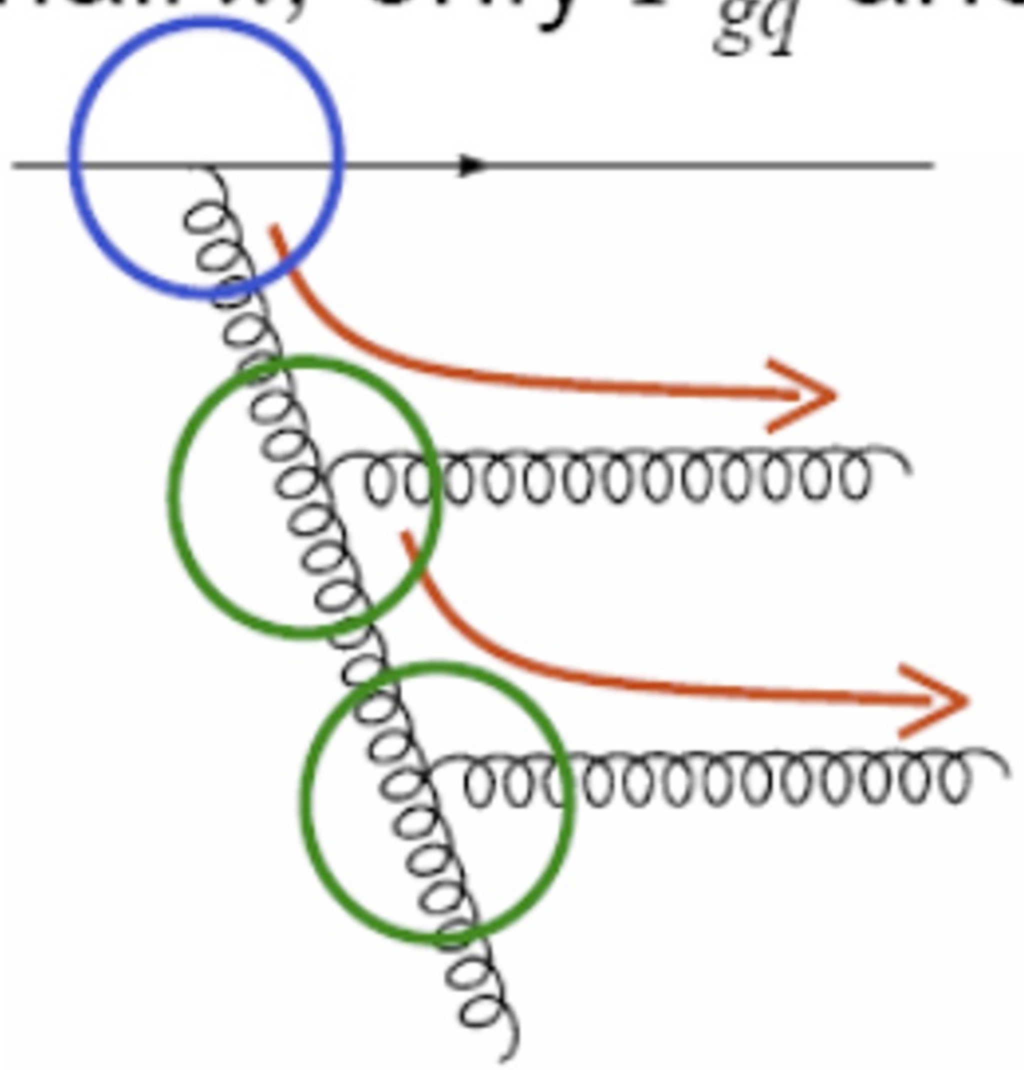
$$P_{qq}^{(0)}(x) = C_F \frac{1+x^2}{1-x}$$

$$P_{qg}^{(0)}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$$

$$P_{gq}^{(0)}(x) = C_F \frac{1+(1-x)^2}{x}$$

$$P_{gg}^{(0)}(x) = 2C_A\left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x)\right]$$

At small x, only P_{gq} and P_{gg} are relevant.



→ Gluon dominant at small *x*!

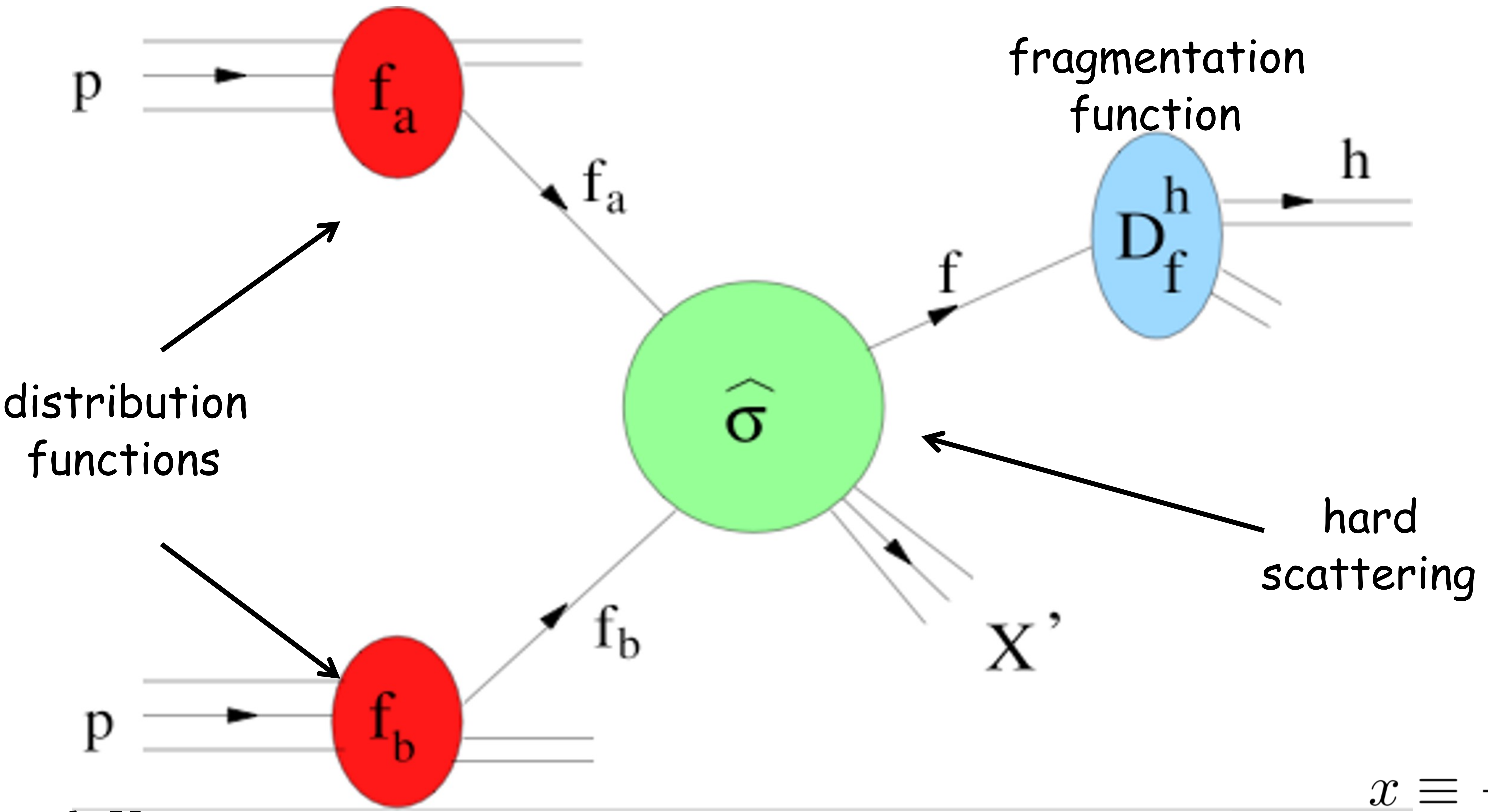
The double log approximation (DLA) of DGLAP is easily solved.

-- increase of gluon distribution at small x

$$\mathbf{xg(x, Q^2) \sim e^{\sqrt{\alpha_s (\log 1/x)(\log Q^2)}}}$$

QCD in proton-proton collisions

collinear factorization: separation of soft (long distance) and hard (short distance)



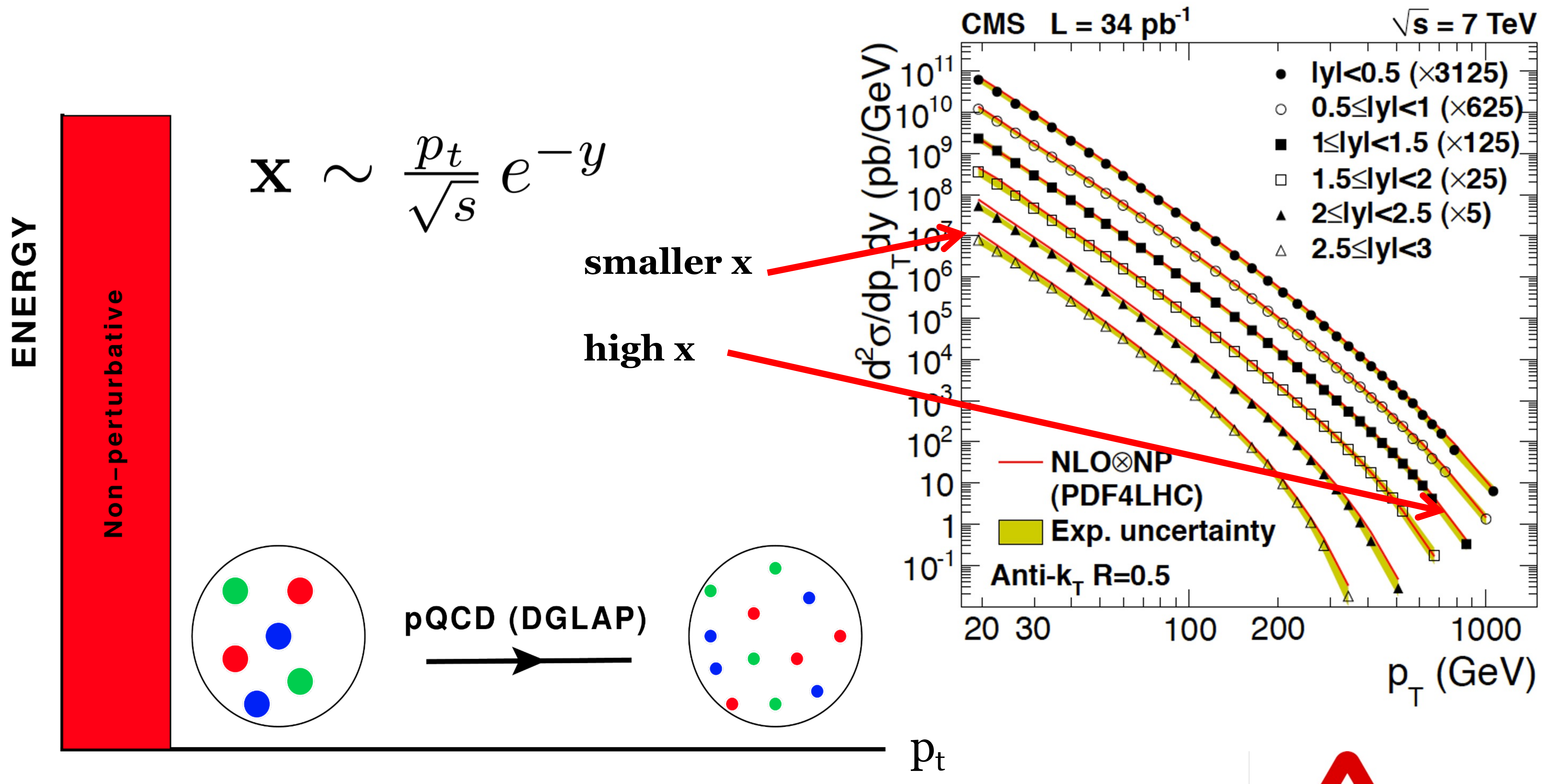
$$x \equiv \frac{p^+}{P^+}$$

$$\frac{d\sigma^{pp\to h\,X}}{d^2p_t\,dy} \sim f_a(x_1)\otimes f_b(x_2)\otimes\hat{\sigma}\otimes D_f^h(z) + \cdots\cdots$$

power corrections

pQCD: the standard paradigm

$$\mathbf{x} \sim \frac{p_t}{\sqrt{s}}\, e^{-y}$$



bulk of QCD phenomena happens at low p_t (<u>small x</u>)

QCD in the Regge-Gribov limit

recall $x_{Bj} \equiv \frac{Q^2}{S}$ $\quad$ $S \to \infty\,,\ Q^2$ fixed $:\ x_{Bj} \to 0$

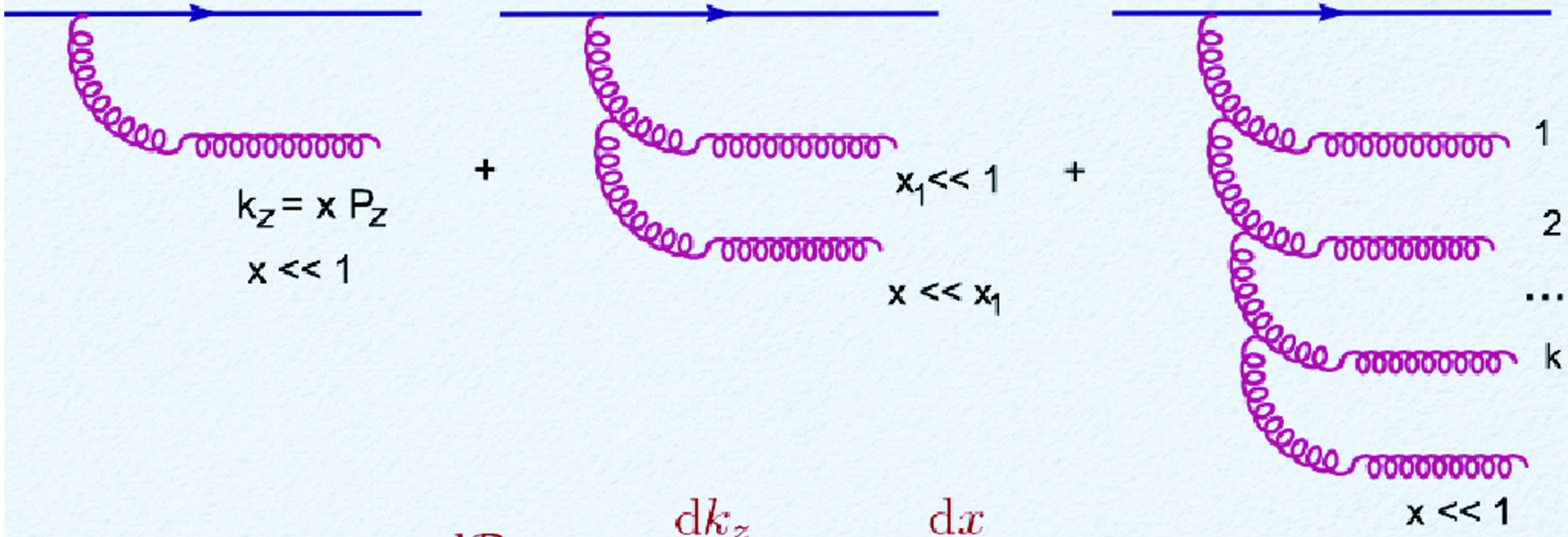
Regge

Gribov

gluon radiation at small x :pQCD

The infrared sensitivity of bremsstrahlung favors the emission of 'soft' (= small–x) gluons

$$\mathbf{P_{gg}(x) \sim \frac{1}{x} \text{ for } x \to 0}$$



$$d\mathcal{P} \propto \alpha_s \frac{dk_z}{k_z} = \alpha_s \frac{dx}{x}$$

The 'price' of an additional gluon:

$$\mathcal{P}(1) \propto \alpha_s \int_x^1 \frac{dx_1}{x_1} = \alpha_s \ln\frac{1}{x}$$

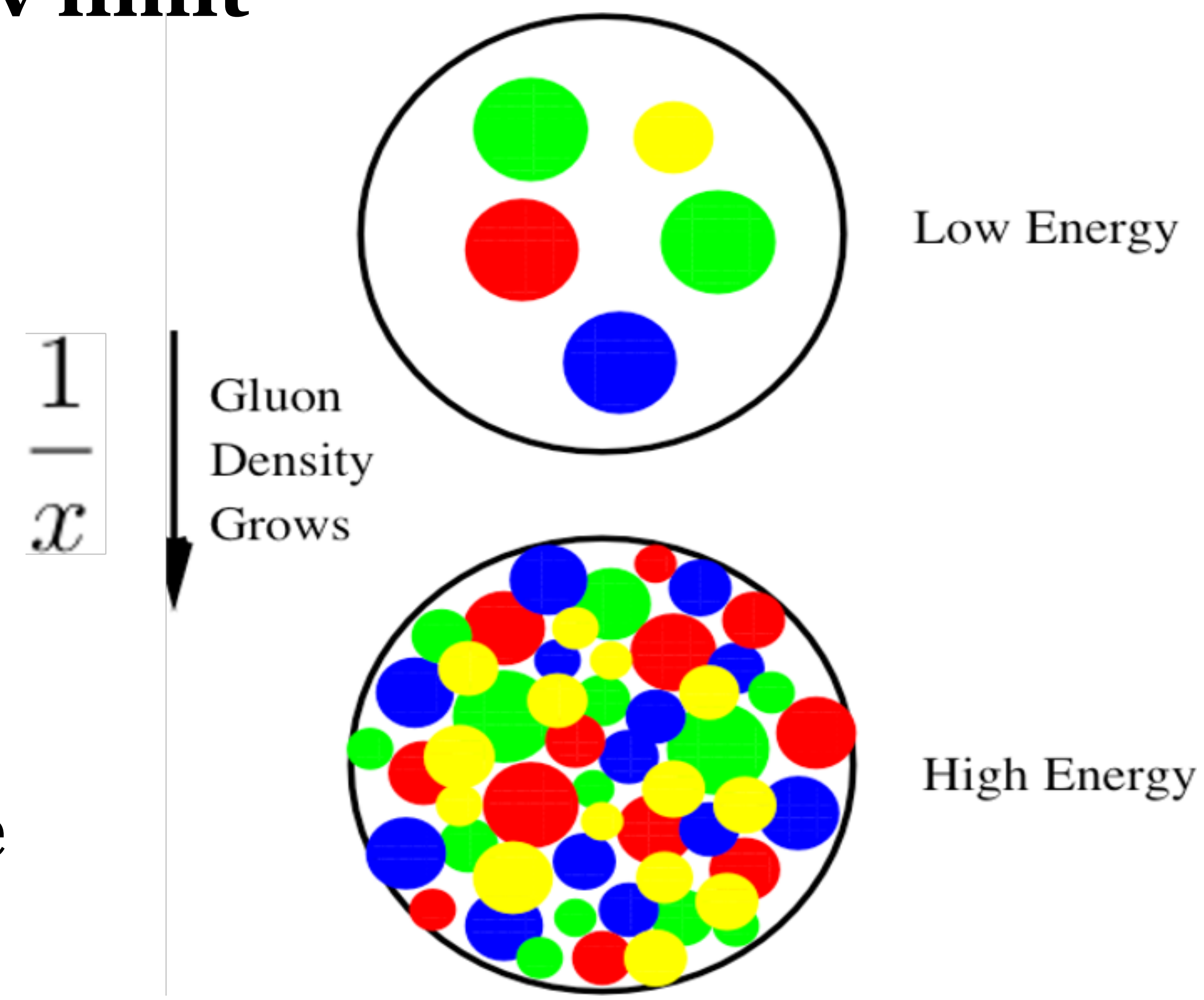
number of gluons grows fast

$$n \sim e^{\alpha_s \, ln 1/x}$$

Resolving the nucleus/hadron: Regge-Gribov limit

Gribov-Levin-Ryskin

Q^2 fixed and $\sqrt{S} \to \infty \quad (x \equiv \frac{Q^2}{S} \to 0)$



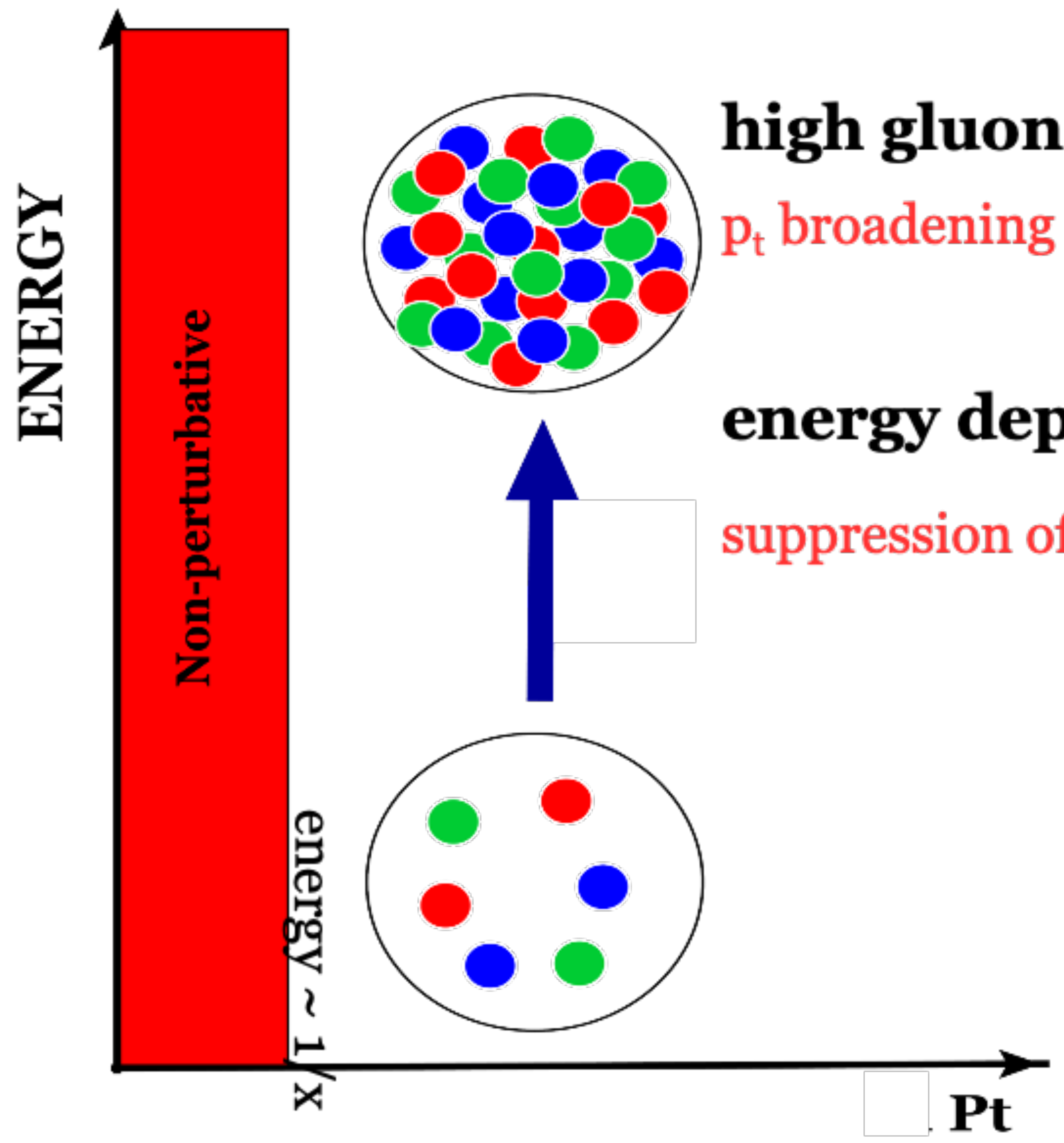
gluons are radiated into fixed resolved area

number of gluons increases due to increased longitudinal phase space

hadron/nucleus becomes a dense state of gluons (CGC) $\frac{\alpha_s}{Q^2} \frac{xG(x, Q^2)}{S_\perp} \sim 1$ $\quad Q_s^2(x, A, b_\perp) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$

possible universal properties of QCD observables ?

QCD at high energy: gluon saturation



high gluon density: Eikonal multiple scattering

p_t broadening

energy dependence: x-evolution via JIMWLK/BK

suppression of spectra/away side peaks

$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3}\,(\frac{1}{x})^{0.3}}$$

$$Q_s^2(x = 3\times 10^{-4}) \sim 1\,GeV^2$$

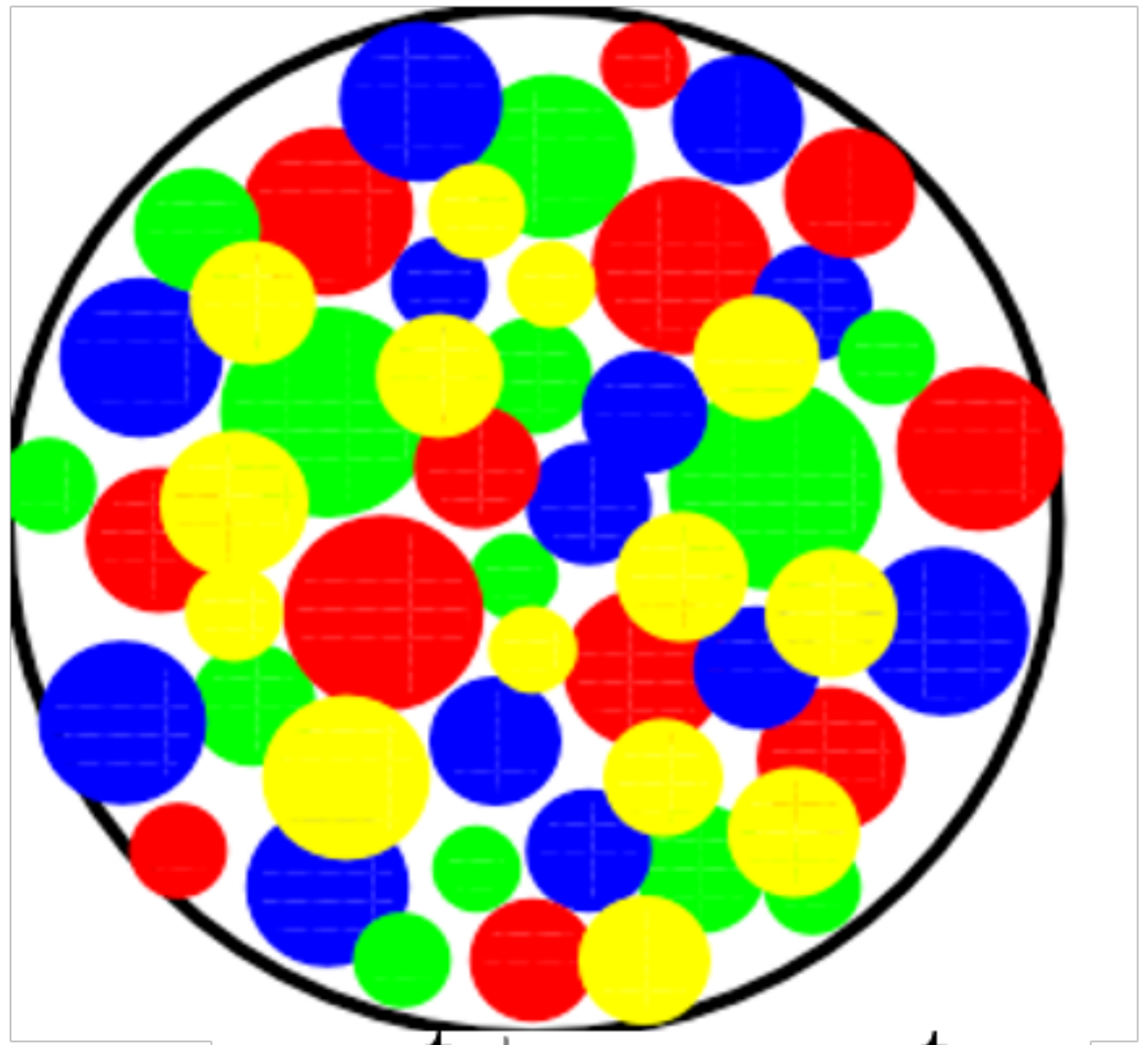
for a proton target (quarks)

a framework for multi-particle production in QCD at small x/low p_t

Shadowing/Nuclear modification factor
Azimuthal angular correlations (dihadrons/dijets,...)
Long range rapidity correlations (ridge,...)
Connections to TMDs,...

$$\mathbf{x \leq 0.01}$$

A very large nucleus at high energy: MV model

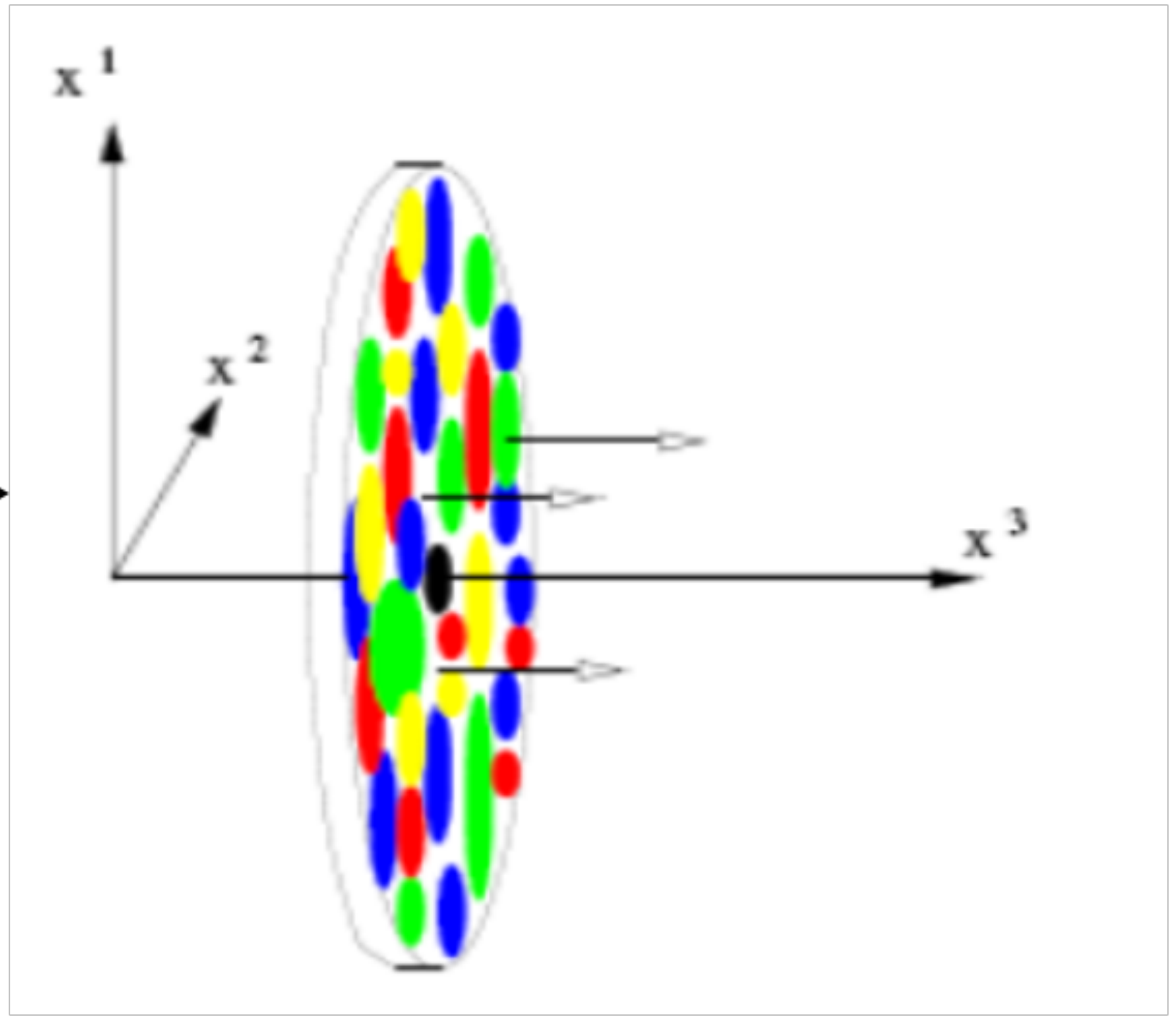


boost $\longrightarrow$

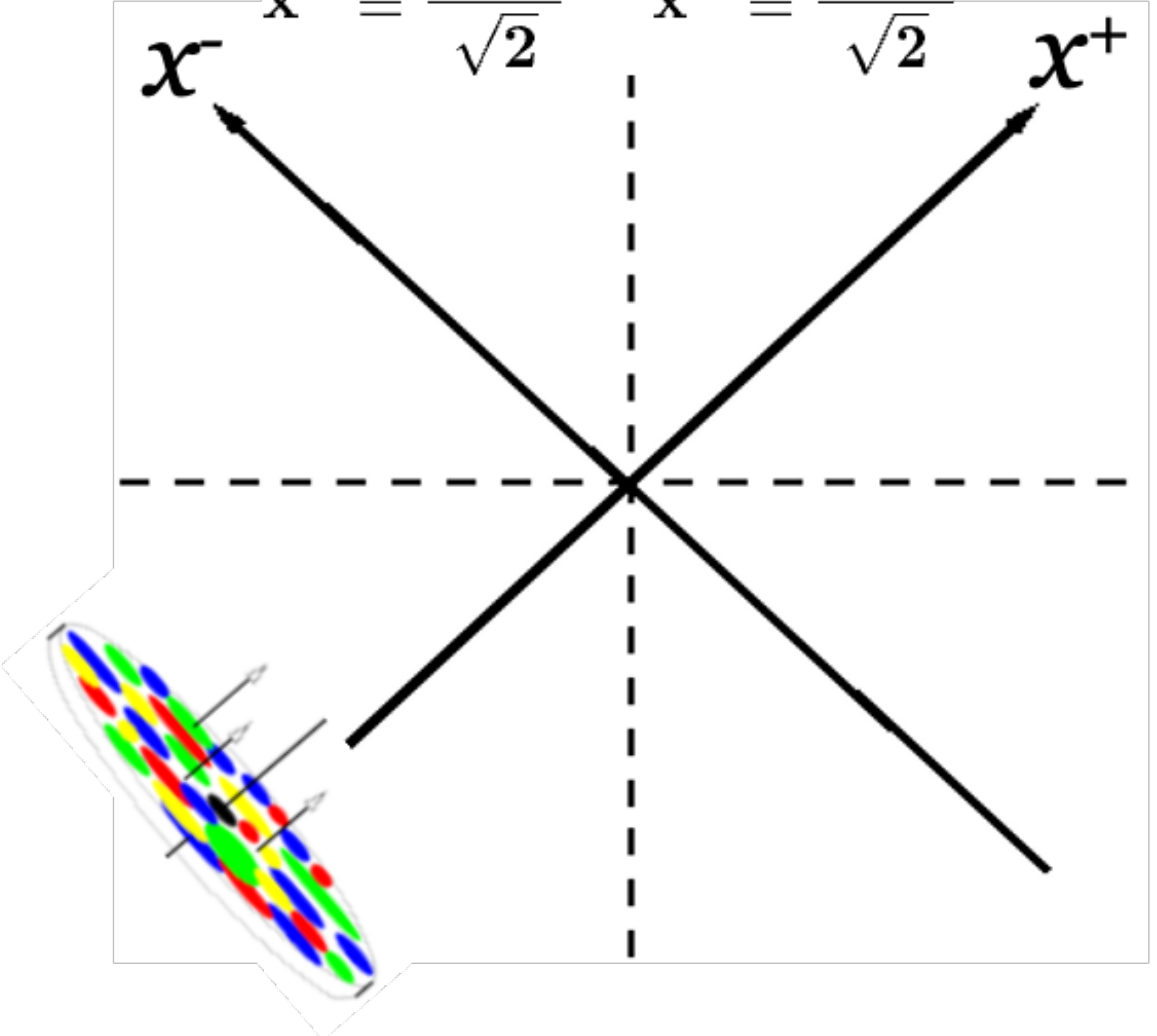
$$R \to \frac{R}{\gamma}$$

$$\gamma \sim 100 \quad \text{RHIC}$$

$$\gamma \sim 2500 \quad \text{LHC}$$



$$\mathbf{x}^+ \equiv \frac{\mathbf{t}+\mathbf{z}}{\sqrt{2}} \qquad \mathbf{x}^- \equiv \frac{\mathbf{t}-\mathbf{z}}{\sqrt{2}}$$



sheet of color charge moving along x⁺ and sitting at x⁻ = 0

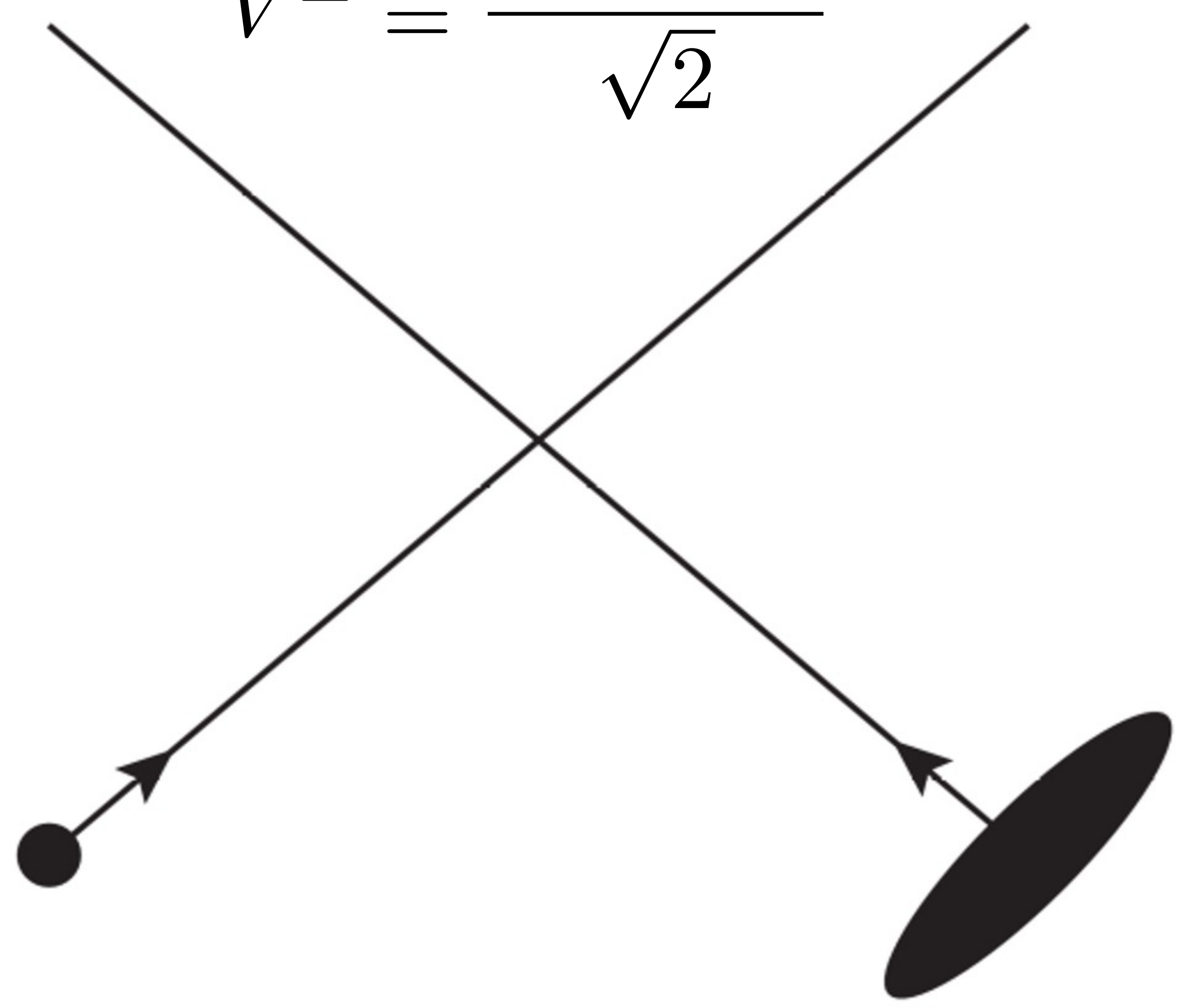
$$\mathbf{J}^\mu_\mathbf{a}(\mathbf{x}) \equiv \delta^{\mu +}\,\delta(\mathbf{x}^-)\,\rho_\mathbf{a}(\mathbf{x_t})$$

color current *color charge*

$$\mathbf{A}^\mathbf{a}_\mathbf{i}(\mathbf{x}^-, \mathbf{x_t}) = \theta(\mathbf{x}^-)\,\alpha^\mathbf{a}_\mathbf{i}(\mathbf{x_t})$$

with $\partial_i\,\alpha^a_i = g\,\rho^a$

Eikonal approximation



$$J_a^\mu \simeq \delta^{\mu -} \rho_a$$

$$D_\mu J^\mu = D_- J^- = 0$$

$$\partial_- J^- = 0 \qquad \text{(in } A^+ = 0 \text{ gauge)}$$

does not depend on x^-

solution to classical EOM:

$$A_a^-(x^+, x_t) \equiv n^- \, S_a(x^+, x_t)$$

with

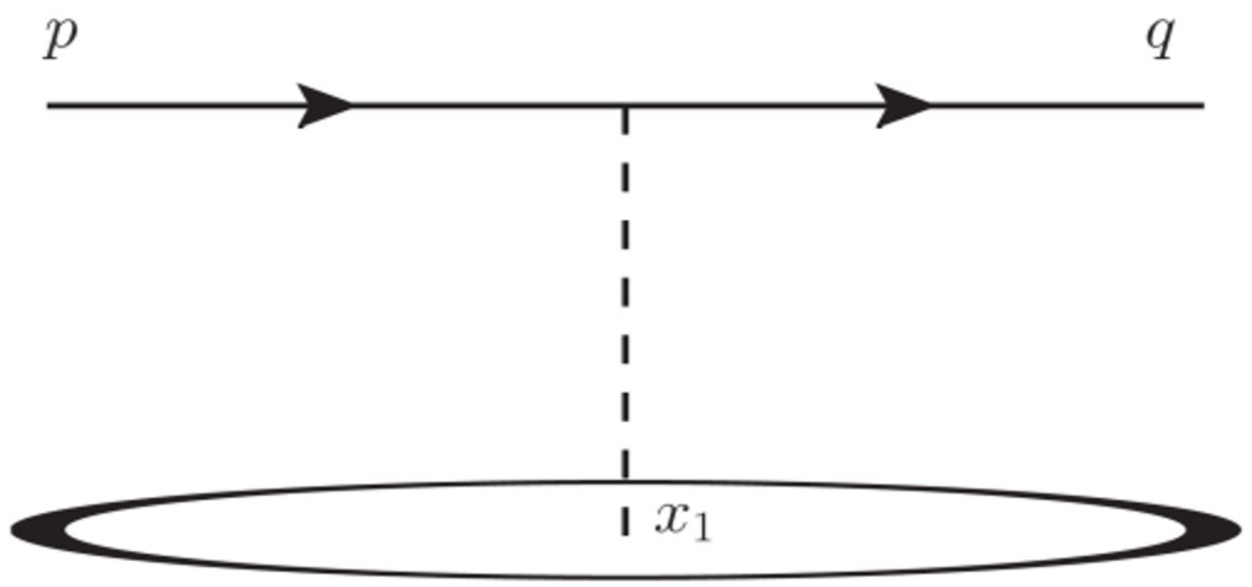
$$n^\mu = (n^+ = 0, n^- = 1, n_\perp = 0)$$
$$n^2 = 2\, n^+ n^- - n_\perp^2 = 0$$

recall (eikonal limit):

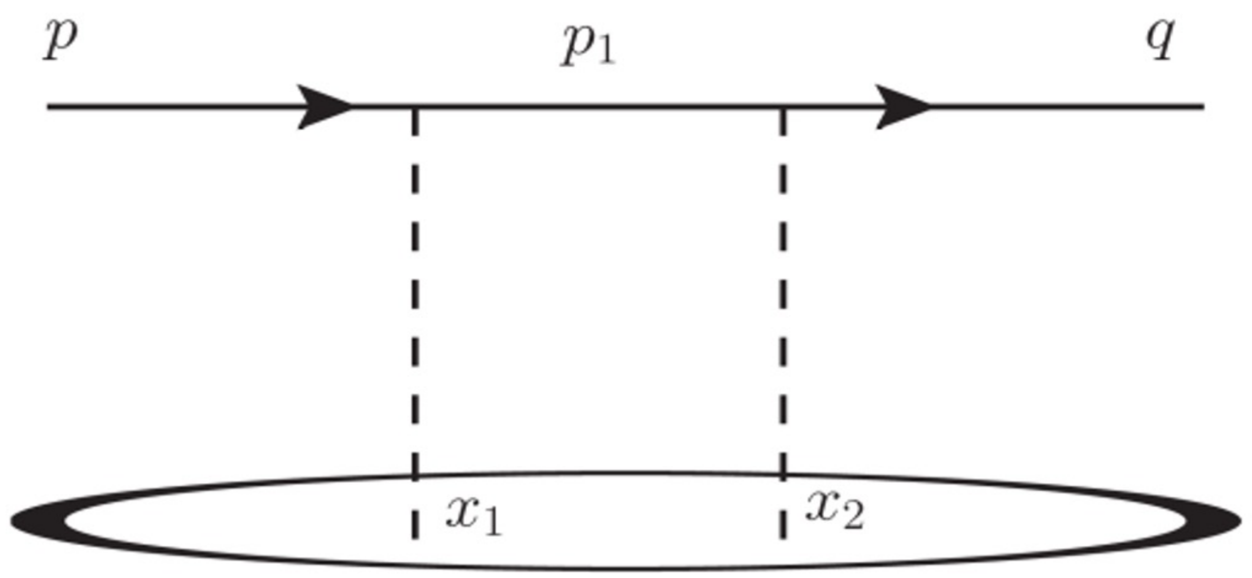
$$\bar{u}(q)\gamma^\mu u(p) \to \bar{u}(p)\gamma^\mu u(p) \sim p^\mu$$
$$\bar{u}(q) \not{A}\, u(p) \to p \cdot A \sim p^+ A^-$$

scattering of a quark from background color field $A_a^-(x^+, x_t)$

$$
\begin{aligned}
i\mathcal{M}_1 &= (ig)\int d^4x_1\, e^{i(q-p)x_1}\, \bar{u}(q)\, \left[\not{n}\, S(x_1)\right]\, u(p) \\
&= (ig)(2\pi)\delta(p^+ - q^+)\int d^2x_{1t}\, dx_1^+\, e^{i(q^- - p^-)x_1^+}\, e^{-i(q_t - p_t)x_{1t}} \\
&\quad \bar{u}(q)\left[\not{n}\, S(x_1^+, x_{1t})\right] u(p)
\end{aligned}
$$



$$
\begin{aligned}
i\mathcal{M}_2 &= (ig)^2 \int d^4x_1\, d^4x_2 \int \frac{d^4p_1}{(2\pi)^4}\, e^{i(p_1 - p)x_1}\, e^{i(q-p_1)x_2} \\
&\quad \bar{u}(q)\, \left[\not{n}\, S(x_2)\, \frac{i\not{p}_1}{p_1^2 + i\epsilon}\, \not{n}\, S(x_1)\right]\, u(p)
\end{aligned}
$$



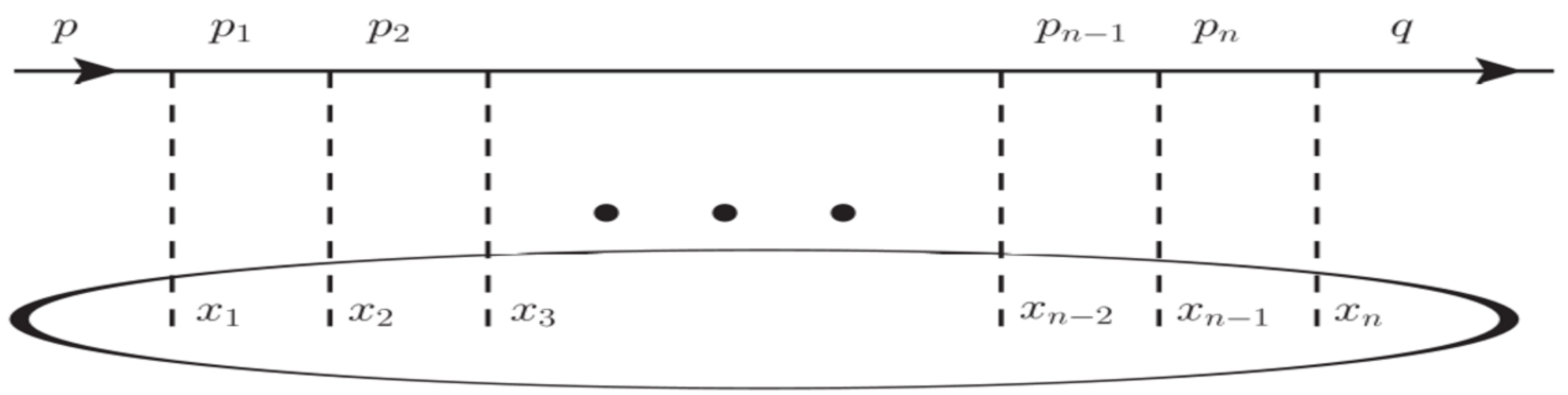
$$
\int \frac{dp_1^-}{(2\pi)} \frac{e^{ip_1^-(x_1^+ - x_2^+)}}{2p^+\left[p_1^- - \frac{p_{1t}^2 - i\epsilon}{2p^+}\right]} = \frac{-i}{2p^+}\, \theta(x_2^+ - x_1^+)\, e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x_2^+)}
$$

contour integration over the pole leads to path ordering of scattering

ignore all terms: $O(\frac{p_t}{p^+}, \frac{q_t}{q^+})$ and use $\not{n}\, \frac{\not{p}_1}{2n\cdot p}\, \not{n} = \not{n}$

$$
\begin{aligned}
i\mathcal{M}_2 &= (ig)^2\, (-i)(i)\, 2\pi\delta(p^+ - q^+) \int dx_1^+\, dx_2^+\, \theta(x_2^+ - x_1^+) \int d^2x_{1t}\, e^{-i(q_t - p_t)\cdot x_{1t}} \\
&\quad \bar{u}(q)\, \left[S(x_2^+, x_{1t})\, \not{n}\, S(x_1^+, x_{1t})\right]\, u(p)
\end{aligned}
$$

Eikonal scattering from a dense target (proton/nucleus)



$$i\mathcal{M}_n = 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\,\not{n}\int d^2x_t\, e^{-i(q_t-p_t)\cdot x_t}$$
$$\Big\{(ig)^n\,(-i)^n(i)^n\int dx_1^+\,dx_2^+\cdots dx_n^+\,\theta(x_n^+ - x_{n-1}^+)\cdots\theta(x_2^+ - x_1^+)$$
$$[S(x_n^+,x_t)\,S(x_{n-1}^+,x_t)\,\cdots S(x_2^+,x_t)S(x_1^+,x_t)]\Big\}\,u(p)$$

sum all multiple scatterings $i\mathcal{M}(p,q) = 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\,\not{n}\int d^2x_t\, e^{-i(q_t-p_t)\cdot x_t}\,[V(x_t) - 1]\,u(p)$

with $V(x_t) \equiv \hat{P}\,\exp\left\{ig\int_{-\infty}^{+\infty} dx^+\,n^-\,S_a(x^+,x_t)\,t_a\right\}$ Wilson lines: effective degree of freedom

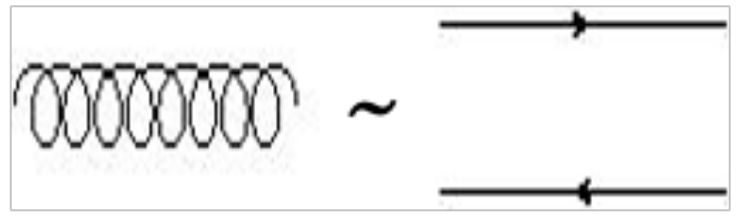
$$\frac{d\,\sigma^{q\,T\to q\,X}}{d^2p_t\,dy} \sim |i\mathcal{M}|^2 \sim F.T. < Tr\,V(x_t)\,V^\dagger(y_t) >$$

dipole

One-loop corrections: BK-JIMWLK eq.

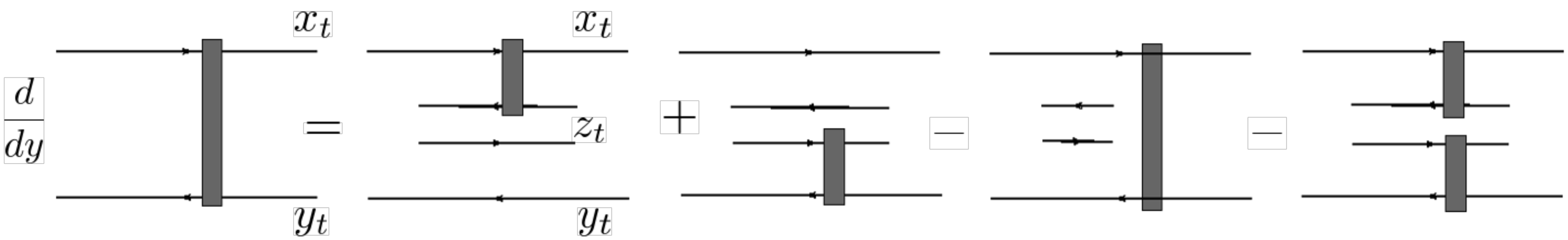
at large N_c

$3 \otimes \bar{3} = 8 \oplus 1 \simeq 8$



$$\frac{d}{dy} T(x_t, y_t) = \frac{N_c \alpha_s}{2\pi^2} \int d^2 z_t \, \frac{(x_t - y_t)^2}{(x_t - z_t)^2 (y_t - z_t)^2} \left[T(x_t, z_t) + T(z_t, y_t) - T(x_t, y_t) - T(x_t, z_t) T(z_t, y_t) \right]$$

$T \equiv 1 - S$



$$\tilde{T}(p_t) \sim \frac{1}{p_t^2} \left[\frac{Q_s^2}{p_t^2} \right] \qquad Q_s^2 \ll p_t^2$$

$$\tilde{T}(p_t) \sim \log \left[\frac{Q_s^2}{p_t^2} \right] \qquad Q_s^2 \gg p_t^2$$

$$\tilde{T}(p_t) \sim \frac{1}{p_t^2} \left[\frac{Q_s^2}{p_t^2} \right]^{\gamma} \qquad Q_s^2 < p_t^2$$

nuclear modification factor

$$R_{pA} \equiv \frac{\frac{d\sigma^{pA}}{d^2 p_t \, dy}}{A^{1/3} \frac{d\sigma^{pp}}{d^2 p_t \, dy}}$$

nuclear shadowing

suppression of p_t spectra

disappearance of back to back peaks

....

initial condition

McLerran-Venugopalan (93) $$< \mathbf{O}(\rho) > \equiv \int \mathbf{D}[\rho]\, \mathbf{O}(\rho)\, \mathbf{W}[\rho]$$

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int \mathbf{d^2 x_t} \frac{\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{a}}(\mathbf{x_t})}{\mathbf{2}\,\mu^{\mathbf{2}}}} \qquad \mu^{\mathbf{2}} \equiv \frac{\mathbf{g^2\, A}}{\mathbf{S}_\perp}$$

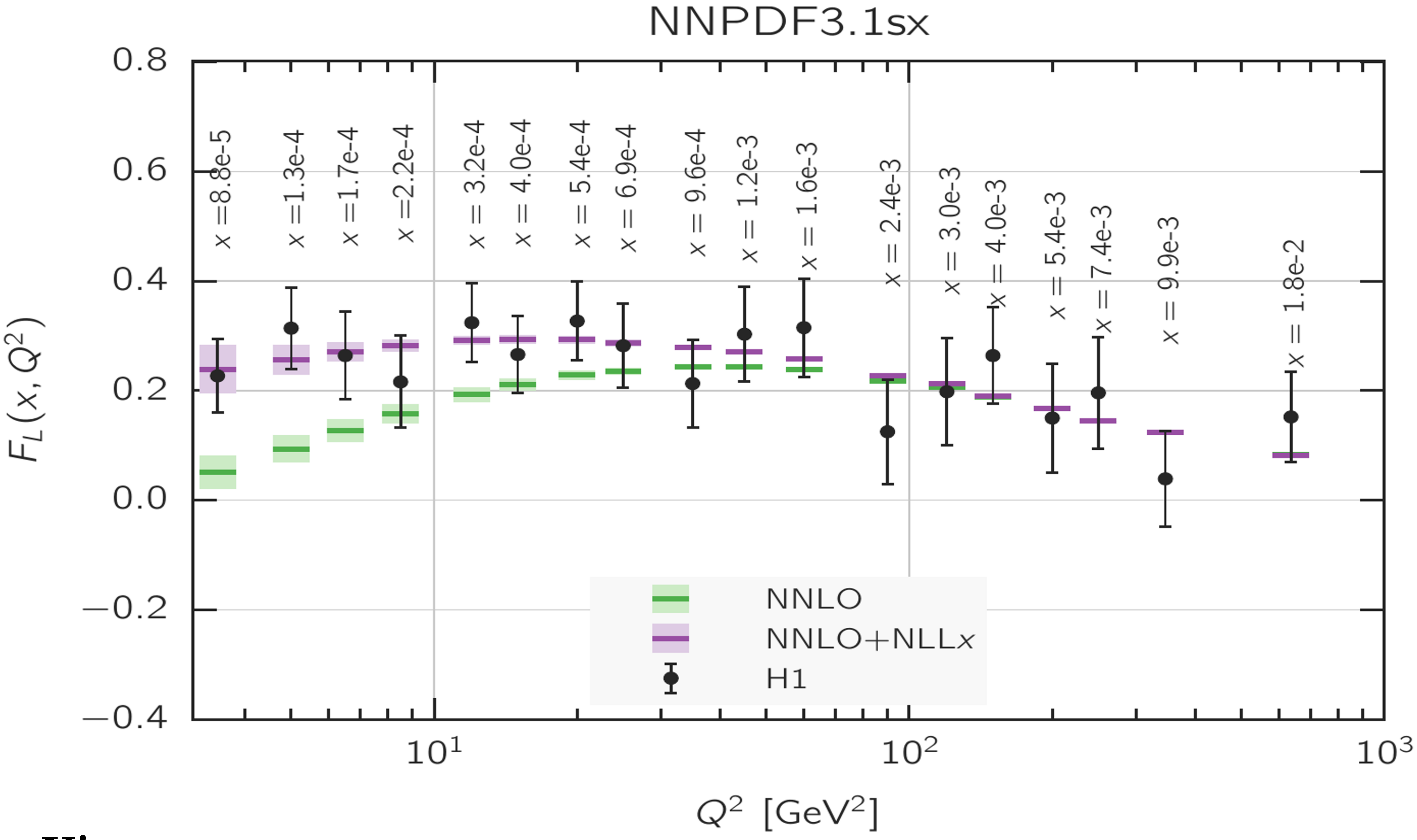
$$\mathbf{T(r_t)} \equiv \frac{\mathbf{1}}{\mathbf{N_c}} < \mathbf{Tr}\,[\mathbf{1} - \mathbf{V(r_t)}^\dagger\, \mathbf{V(0)}] > \sim \mathbf{1} - \mathbf{e}^{-[\mathbf{r_t^2\, Q_s^2}]\, \mathbf{log}(\frac{\mathbf{1}}{\mathbf{r_t}\,\Lambda_{\mathbf{QCD}}})}$$

$$\mathbf{r_t} \equiv \mathbf{x_t} - \mathbf{y_t}$$

$r_t \ll \frac{1}{Q_s}$ $\quad T(r_t) \longrightarrow r_t^2\, Q_s^2\, log(\frac{1}{r_t\, \Lambda_{QCD}})$ *color transparency*

$r_t \gg \frac{1}{Q_s}$ $\quad T(r_t) \longrightarrow 1$ *perturbative unitarization*

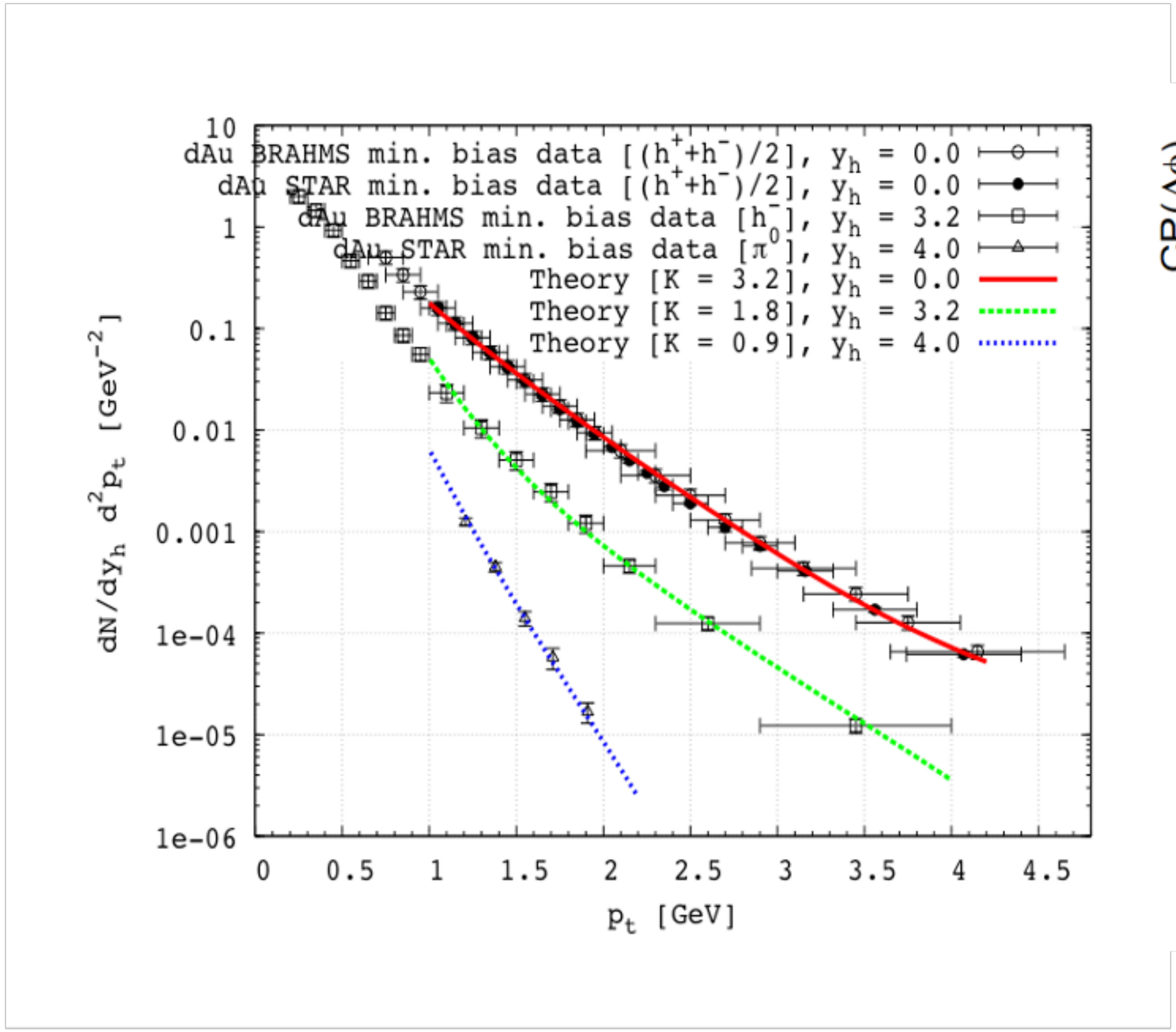
F_L at HERA

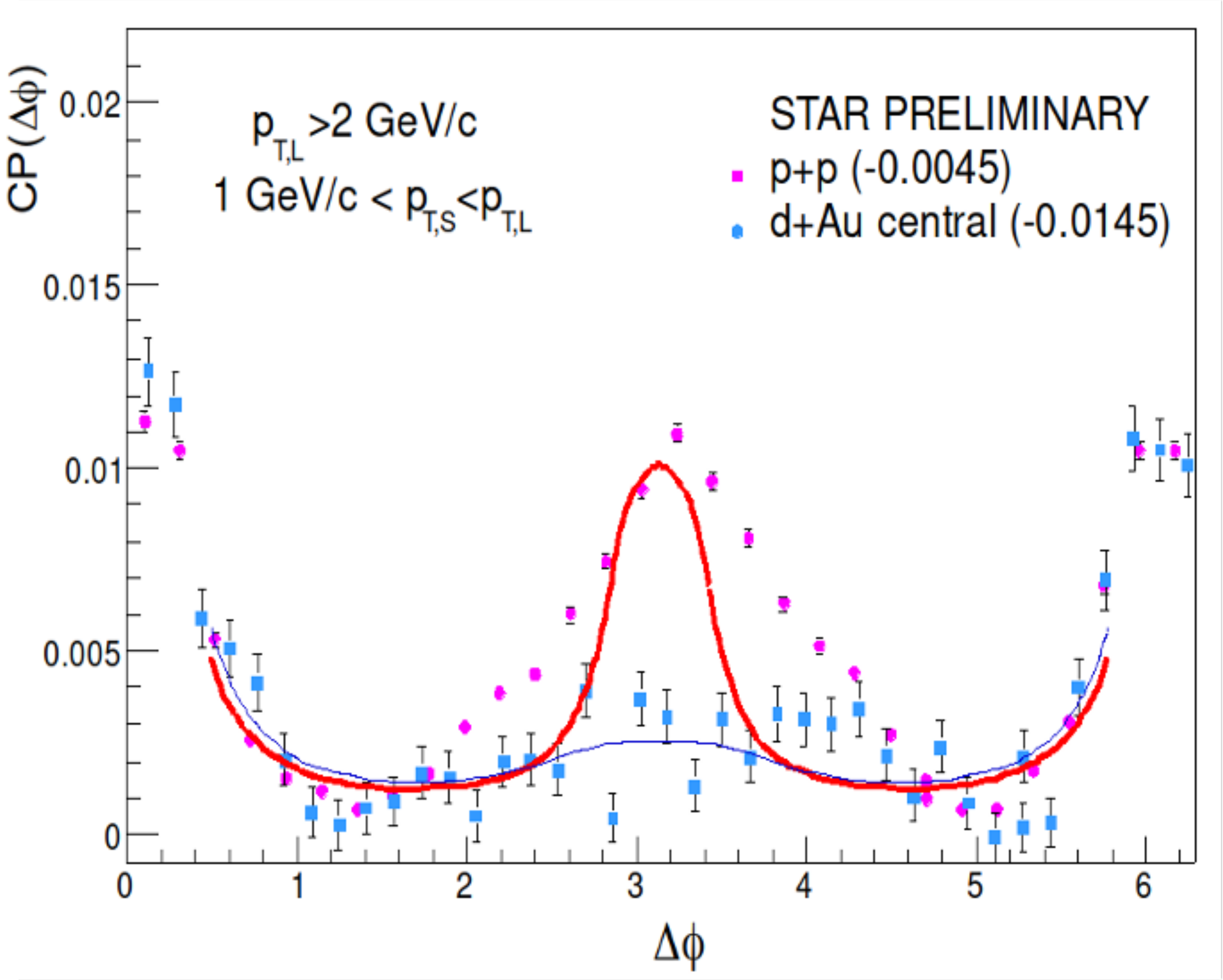


arXiv:1710.05935

CGC at RHIC

Single and double inclusive hadron production in dA collisions





Dumitru,Hayashigaki, JJM, NPA770 (2006) 57

Albacete, Marquet, PRL105 (2010) 162301

Toward precision CGC: inclusive DIS

NLO BK/JIMWLK evolution equations

Kovner, Lublinsky, Mulian (2013)

Balitsky, Chirilli (2007)

NLO corrections to structure functions

Beuf, Lappi, Paatelainen (2022), Beuf (2017)

NLO corrections to SIDIS (+)

Altinoluk, JJM, Marquet (2024)

Bergabo, JJM (2023, 2024)

Caucal, Ferrand, Salazar (2024)

NLO corrections to dihadron/dijets (+)

Bergabo, JJM (2022, 2023)

Iancu, Mulian (2023)

Caucal, Salazar, Schenke, Stebel, Venugopalan (2023), Caucal, Salazar, Schenke, Venugopalan (2022)

Taels, Altinoluk, Beuf, Marquet (2022), Taels (2023)

Caucal, Salazar, Venugopalan (2021)

Ayala, Hentschinski, JJM, Tejeda-Yeomans (2016,2017),…………

Toward precision CGC: exclusive/diffractive DIS

NLO corrections to diffractive structure functions

Beuf, Hanninen, Lappi, Mulian, Mantyssari (2022)

……………………..

NLO corrections to diffractive dihadron/dijets (+)

Boussarie, Grabovsky, Szymanowski, Wallon (2016)

Iancu, Mueller, Triantafyllopoulos (2021, 2022)

Fucilla, Grabovsky, Li, Szymanowski, Wallon (2023)

…………………….

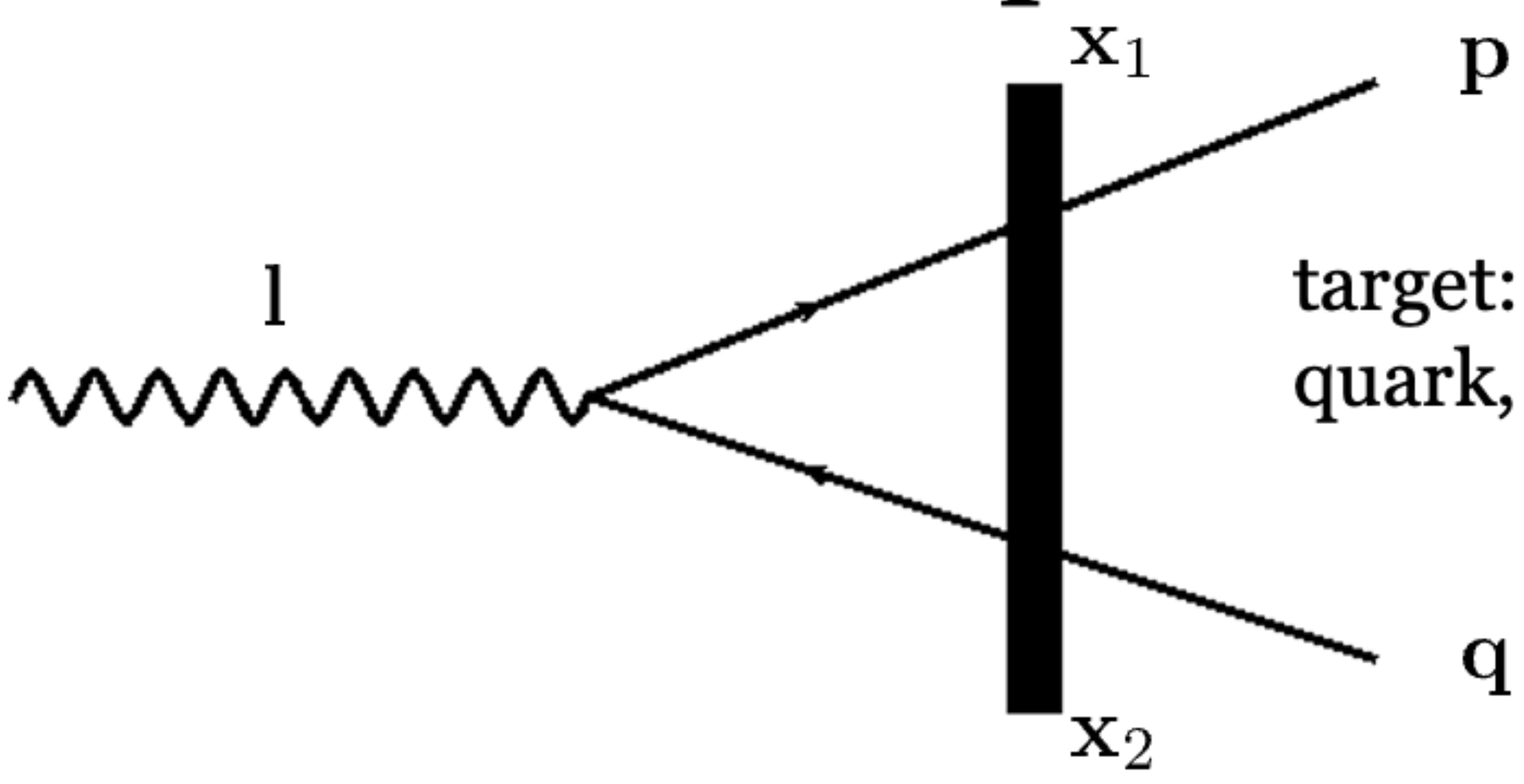
NLO corrections to exclusive light/heavy vector meson production (+)

Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2016)

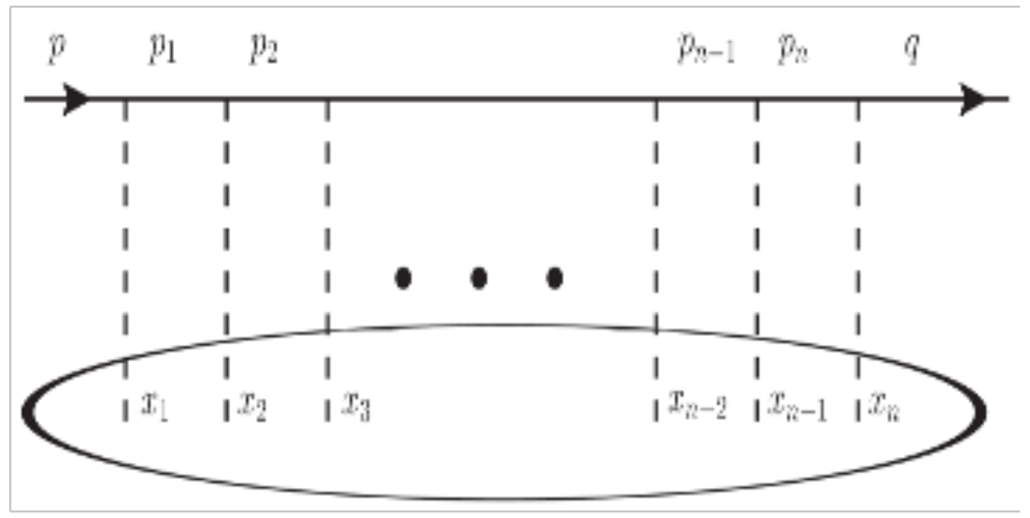
Mantyssari, Penttala (2021, 2022)

…………………….

Inclusive dihadron production in forward rapidity: LO



target: a classical color field
quark, antiquark multiply scatter on the target



$$\frac{d\sigma^{\gamma^* A\to q\bar{q}X}}{d^2p\, d^2q\, dy_1\, dy_2} = \frac{e^2Q^2(z_1z_2)^2N_c}{(2\pi)^7}\delta(1-z_1-z_2)$$

$$\int d^8x_\perp e^{ip\cdot(x_1'-x_1)}e^{iq\cdot(x_2'-x_2)}\left[S_{122'1'} - S_{12} - S_{1'2'} + 1\right]$$

$$\Big\{4z_1z_2K_0(|x_{12}|Q_1)K_0(|x_{1'2'}|Q_1)+$$

$$(z_1^2+z_2^2)\,\frac{x_{12}\cdot x_{1'2'}}{|x_{12}||x_{1'2'}|}\,K_1(|x_{12}|Q_1)K_1(|x_{1'2'}|Q_1)\Big\}$$

with

dipole $\mathbf{S}_{12} \equiv \frac{1}{N_c}Tr\, V(x_1)\, V^\dagger(x_2)$

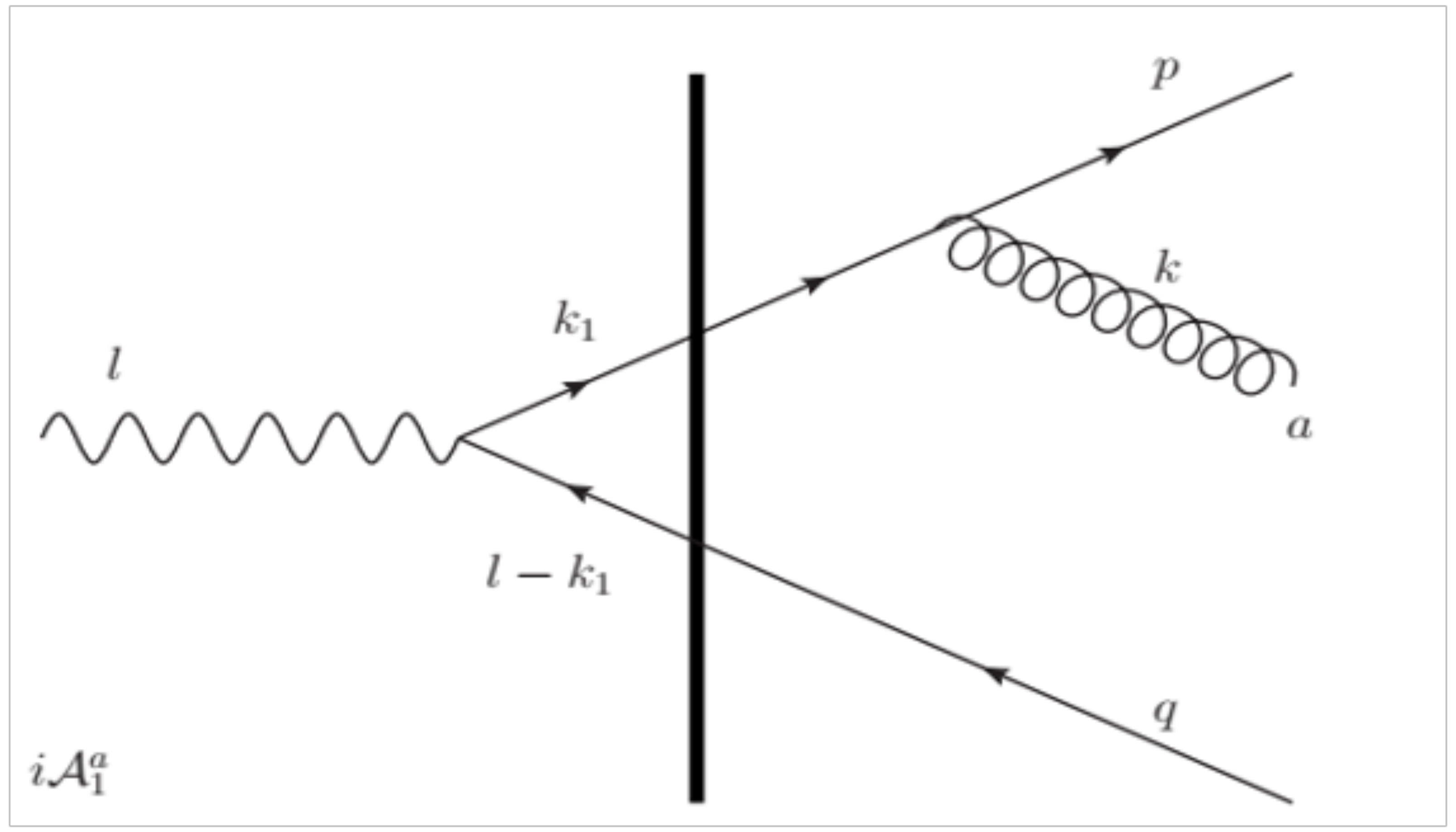
$$\mathbf{x}_{12} \equiv \mathbf{x}_1 - \mathbf{x}_2$$

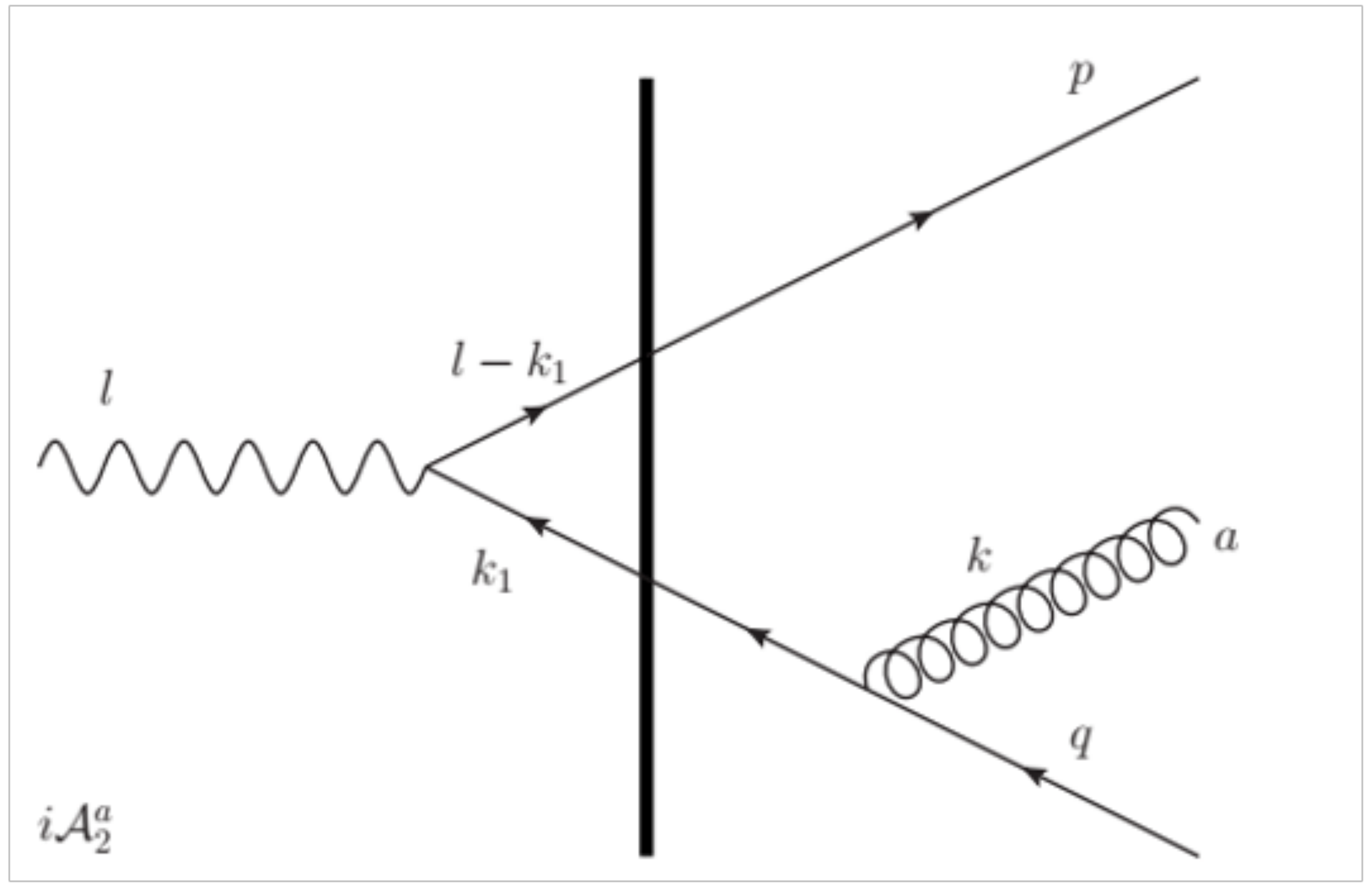
quadrupole

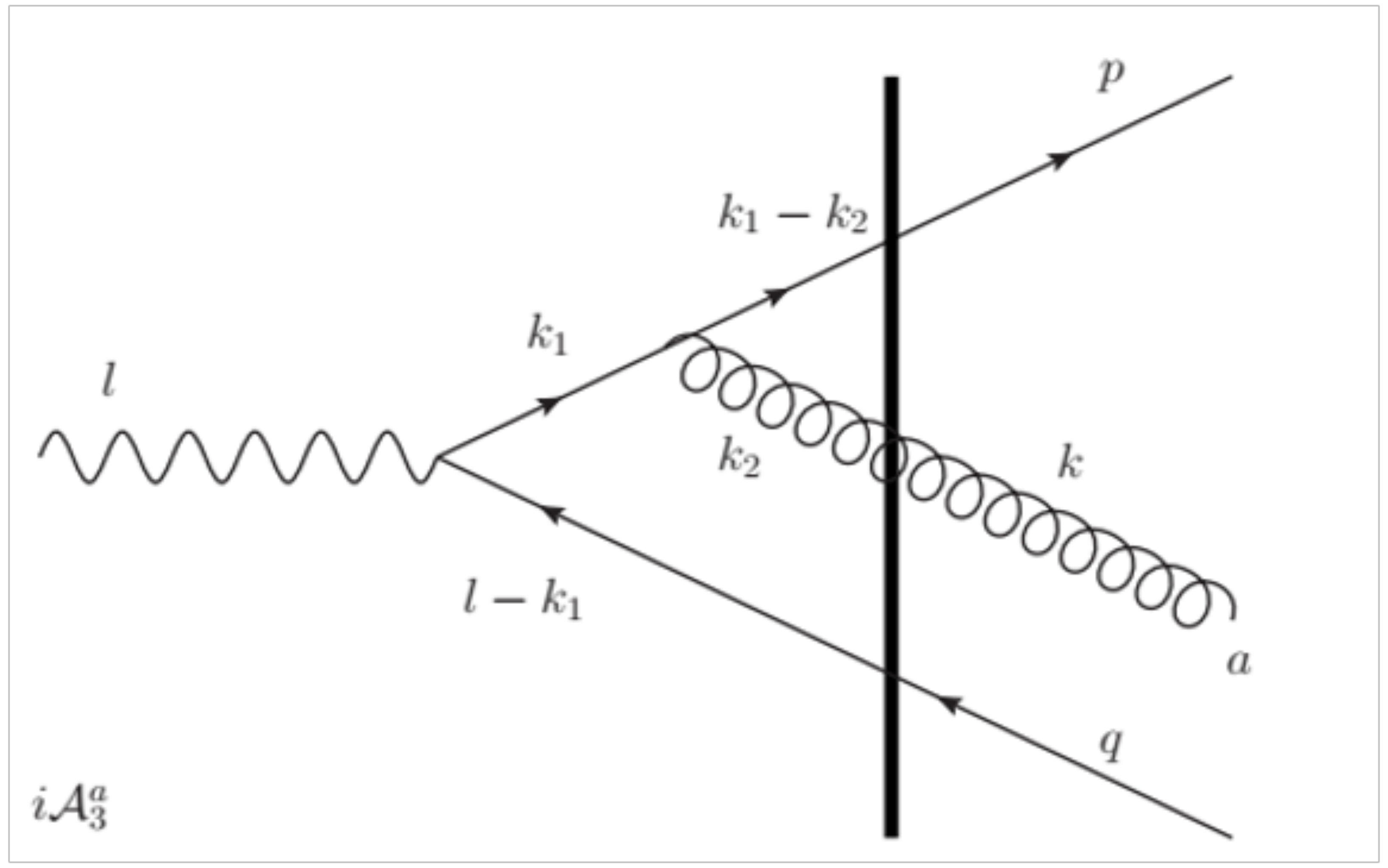
$$\mathrm{S}_{122'1'} \equiv \frac{1}{N_c}Tr\, V(\mathbf{x}_1)\, V^\dagger(\mathbf{x}_2)\, V(\mathbf{x}_{2'})\, V^\dagger(\mathbf{x}_{1'})$$

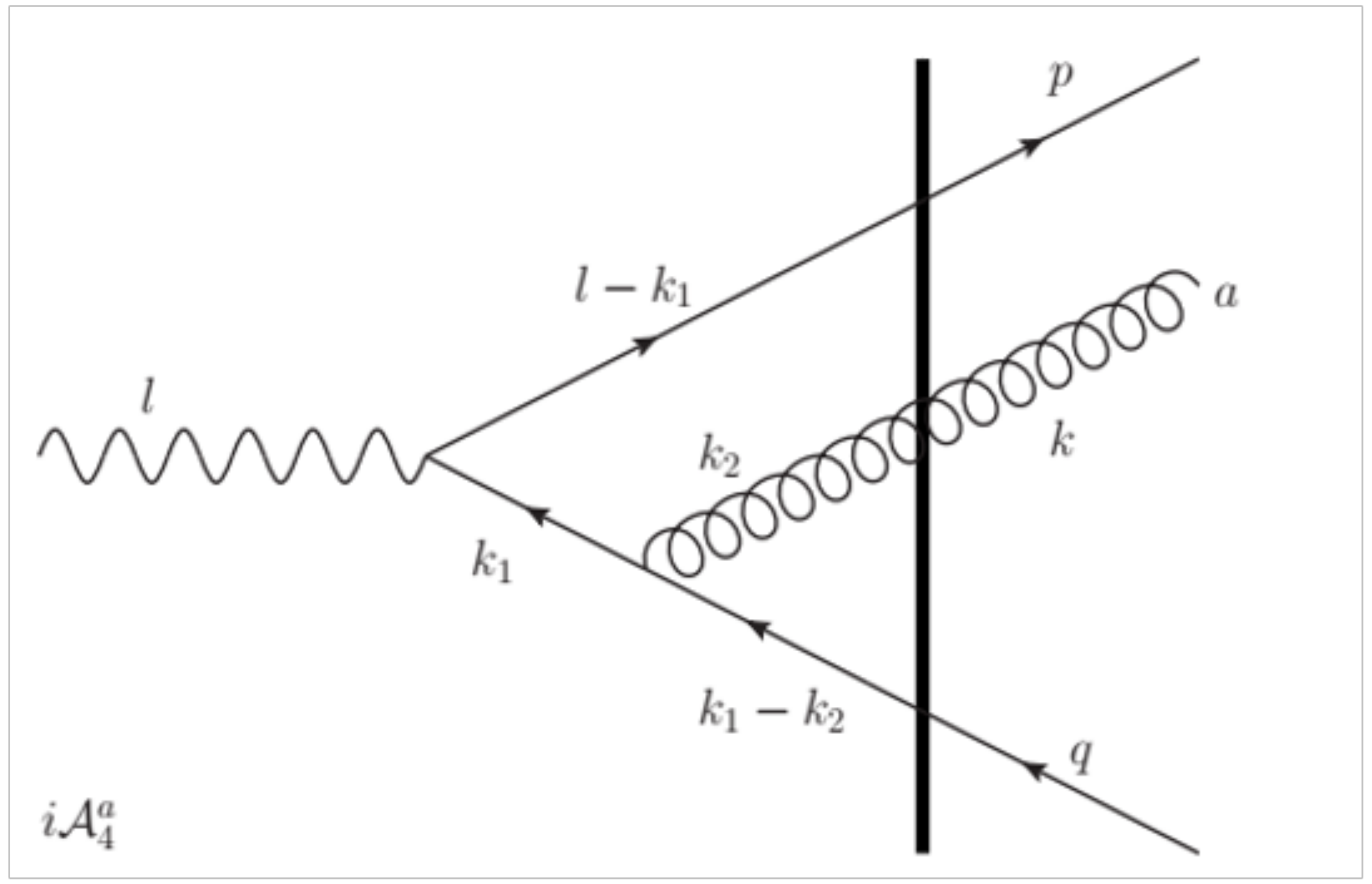
Only dipoles and quadrupoles contribute: DMXY, PRD 83 (2011) 105005

One loop corrections - real diagrams









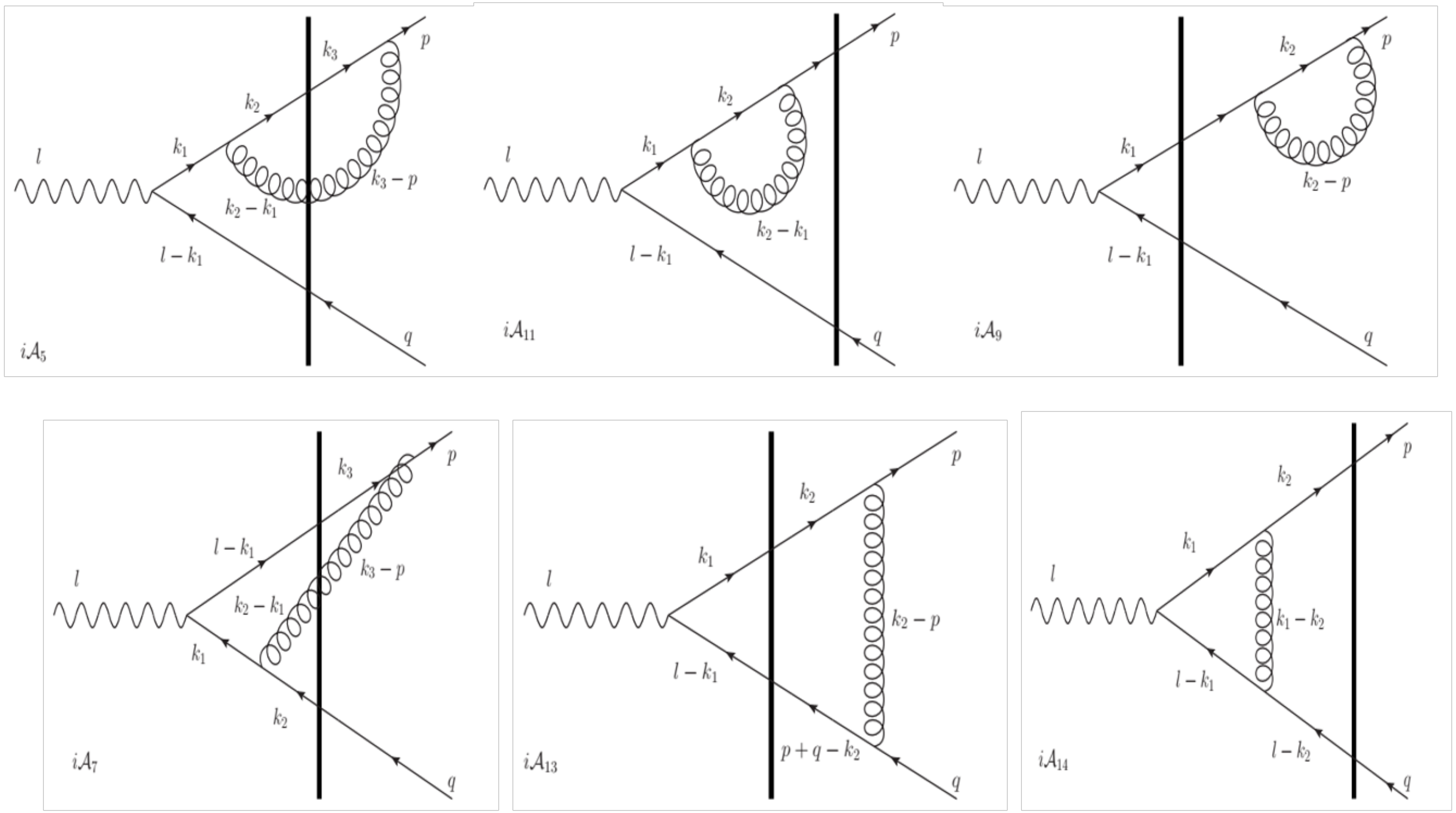
3-parton production: Ayala, Hentschinski, JJM, Tejeda-Yeomans
PLB 761 (2016) 229 and NPB 920 (2017) 232

Spinor helicity formalism: helicity amplitudes

Numerator	$\lambda_\gamma; \lambda_q, \lambda_g$	$N_i^{\lambda_\gamma;\lambda_q,\lambda_g}$
N_1	$L;+,+$	$-Q(z_1 z_2)^{3/2}(1-z_2)\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}$
	$L;+,-$	$-Q(z_2)^{3/2}\sqrt{z_1}(1-z_2)^2\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}$
	$+;+,+$	$-(z_1)^{3/2}\sqrt{z_2}(1-z_2)\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}\,(\mathbf{k}_1\cdot\epsilon)$
	$+;+,-$	$-\sqrt{z_1 z_2}(1-z_2)^2\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon^{*}]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}\,(\mathbf{k}_1\cdot\epsilon)$
	$+;-,+$	$(z_2)^{3/2}\sqrt{z_1}(1-z_2)\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}\,(\mathbf{k}_1\cdot\epsilon)$
	$+,-,-$	$(z_1 z_2)^{3/2}\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon^{*}]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}\,(\mathbf{k}_1\cdot\epsilon)$
N_2	$L;+,+$	$Q(z_1)^{3/2}\sqrt{z_2}(1-z_1)^2\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}$
	$L;+,-$	$Q(z_1 z_2)^{3/2}(1-z_1)\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon^{*}]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}$
	$+;+,+$	$-(z_1)^{3/2}\sqrt{z_2}(1-z_1)\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}\,(\mathbf{k}_1\cdot\epsilon)$
	$+;+,-$	$-(z_1 z_2)^{3/2}\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon^{*}]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}\,(\mathbf{k}_1\cdot\epsilon)$
	$+;-,+$	$(z_2)^{3/2}\sqrt{z_1}(1-z_1)\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}\,(\mathbf{k}_1\cdot\epsilon)$
	$+,-,-$	$\sqrt{z_1 z_2}(1-z_1)^2\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon^{*}]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}\,(\mathbf{k}_1\cdot\epsilon)$
N_3	$L;+,+$	$Q(z_1 z_2)^{3/2}(1-z_2)\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_2}\right)$
	$L;+,-$	$Q(z_2)^{3/2}\sqrt{z_1}(1-z_2)^2\left(\frac{\mathbf{k}_2\cdot\epsilon^{*}}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^{*}}{1-z_2}\right)$
	$+;+,+$	$(z_1)^{3/2}\sqrt{z_2}(1-z_2)\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_2}\right)\mathbf{k}_1\cdot\epsilon$
	$+;+,-$	$\sqrt{z_1 z_2}(1-z_2)^2\left(\frac{\mathbf{k}_2\cdot\epsilon^{*}}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^{*}}{1-z_2}\right)\mathbf{k}_1\cdot\epsilon$
	$+;-,+$	$-(z_2)^{3/2}\sqrt{z_1}(1-z_2)\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_2}\right)\mathbf{k}_1\cdot\epsilon$
	$+,-,-$	$-(z_1 z_2)^{3/2}\left[\left(\frac{\mathbf{k}_2\cdot\epsilon^{*}}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^{*}}{(1-z_2)}\right)\mathbf{k}_1\cdot\epsilon+\frac{\mathbf{k}_1^2+z_2(1-z_2)Q^2}{2z_2(1-z_2)}\right]$
N_4	$L;+,+$	$-Q(z_1)^{3/2}\sqrt{z_2}(1-z_1)^2\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_1}\right)$
	$L;+,-$	$-Q(z_1 z_2)^{3/2}(1-z_1)\left(\frac{\mathbf{k}_2\cdot\epsilon^{*}}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^{*}}{1-z_1}\right)$
	$+;+,+$	$(z_1)^{3/2}\sqrt{z_2}(1-z_1)\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_1}\right)\mathbf{k}_1\cdot\epsilon$
	$+;+,-$	$(z_1 z_2)^{3/2}\left[\left(\frac{\mathbf{k}_2\cdot\epsilon^{*}}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^{*}}{(1-z_1)}\right)\mathbf{k}_1\cdot\epsilon+\frac{\mathbf{k}_1^2+z_1(1-z_1)Q^2}{2z_1(1-z_1)}\right]$
	$+;-,+$	$-(z_2)^{3/2}\sqrt{z_1}(1-z_1)\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_1}\right)\mathbf{k}_1\cdot\epsilon$
	$+,-,-$	$-\sqrt{z_1 z_2}(1-z_1)^2\left(\frac{\mathbf{k}_2\cdot\epsilon^{*}}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^{*}}{1-z_1}\right)\mathbf{k}_1\cdot\epsilon$

$$
\begin{aligned}
\frac{\sigma^{\mathrm{Real},L}_{1-1}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} &= \frac{2e^2g^2Q^2N_c^2z_2^3(1-z_2)^2(z_1^2+(1-z_2)^2)}{(2\pi)^{10}z_1}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(|\mathbf{x}_{12}|Q_2)K_0(|\mathbf{x}_{1'2'}|Q_2)\Delta^{(3)}_{11'}\\
&\quad [S_{122'1'}-S_{12}-S_{1'2'}+1]\,e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}e^{i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}\delta(1-z_1-z_2-z).\\
\frac{\sigma^{\mathrm{Real},L}_{2-2}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} &= \frac{2e^2g^2Q^2N_c^2z_1^3(1-z_1)^2(z_2^2+(1-z_1)^2)}{(2\pi)^{10}z_2}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(|\mathbf{x}_{12}|Q_1)K_0(|\mathbf{x}_{1'2'}|Q_1)\Delta^{(3)}_{22'}\\
&\quad [S_{122'1'}-S_{12}-S_{1'2'}+1]e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}\delta(1-z_1-z_2-z).\\
\frac{\sigma^{\mathrm{Real},L}_{1-2}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} &= \frac{-2e^2g^2Q^2N_c^2z_1z_2(1-z_1)(1-z_2)(z_1(1-z_1)+z_2(1-z_2))}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(|\mathbf{x}_{12}|Q_2)K_0(|\mathbf{x}_{1'2'}|Q_1)\\
&\quad \Delta^{(3)}_{12'}[S_{12}S_{1'2'}-S_{12}-S_{1'2'}+1]e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}e^{i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}'_2-\mathbf{x}_3)}e^{i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}_3-\mathbf{x}_1)}\delta(1-z_1-z_2-z).\\
\frac{\sigma^{\mathrm{Real},L}_{3-3}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} &= \frac{2e^2g^2Q^2N_c^2z_1z_2^3(z_1^2+(1-z_2)^2)}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(QX)K_0(QX')\Delta^{(3)}_{11'}\\
&\quad [S_{11'}S_{22'}-S_{13}S_{23}-S_{1'3}S_{2'3}+1]e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}\delta(1-z_1-z_2-z).\\
\frac{\sigma^{\mathrm{Real},L}_{4-4}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} &= \frac{2e^2g^2Q^2N_c^2z_1^3z_2(z_2^2+(1-z_1)^2)}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(QX)K_0(QX')\Delta^{(3)}_{22'}\\
&\quad [S_{11'}S_{22'}-S_{13}S_{23}-S_{1'3}S_{2'3}+1]e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}\delta(1-z_1-z_2-z).\\
\frac{\sigma^{\mathrm{Real},L}_{3-4}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} &= \frac{-2e^2g^2Q^2N_c^2z_1^2z_2^2(z_1(1-z_1)+z_2(1-z_2))}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(QX)K_0(QX')\Delta^{(3)}_{12'}\\
&\quad [S_{11'}S_{22'}-S_{13}S_{23}-S_{1'3}S_{2'3}+1]e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}\delta(1-z_1-z_2-z).\\
\frac{\sigma^{\mathrm{Real},L}_{1-3}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} &= \frac{-2e^2g^2Q^2N_c^2z_2^3(1-z_2)(z_1^2+(1-z_2)^2)}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(|\mathbf{x}_{12}|Q_2)K_0(QX')\Delta^{(3)}_{11'}\\
&\quad [S_{122'3}S_{1'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}e^{i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}_3-\mathbf{x}_1)}\delta(1-z_1-z_2-z).\\
\frac{\sigma^{\mathrm{Real},L}_{1-4}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} &= \frac{2e^2g^2Q^2N_c^2z_1z_2^2(1-z_2)(z_1(1-z_1)+z_2(1-z_2))}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}K_0(|\mathbf{x}_{12}|Q_2)K_0(QX')\Delta^{(3)}_{12'}\\
&\quad [S_{122'3}S_{1'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}e^{i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}_3-\mathbf{x}_1)}\delta(1-z_1-z_2-z).\\
\frac{\sigma^{\mathrm{Real},L}_{2-3}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} &= \frac{2e^2g^2Q^2N_c^2z_1^2z_2(1-z_1)(z_1(1-z_1)+z_2(1-z_2))}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}K_0(|\mathbf{x}_{12}|Q_1)K_0(QX')\Delta^{(3)}_{21'}\\
&\quad [S_{1231'}S_{2'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}e^{i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}_3-\mathbf{x}_2)}\delta(1-z_1-z_2-z).\\
\frac{\sigma^{\mathrm{Real},L}_{2-4}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} &= \frac{-2e^2g^2Q^2N_c^2z_1^3(1-z_1)(z_2^2+(1-z_1)^2)}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}K_0(|\mathbf{x}_{12}|Q_1)K_0(QX')\Delta^{(3)}_{22'}\\
&\quad [S_{1231'}S_{2'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}e^{i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}_3-\mathbf{x}_2)}\delta(1-z_1-z_2-z).
\end{aligned}
$$

One loop corrections – virtual diagrams



F. Bergabo and JJM, dihadrons, 2207.03606
P. Taels et al., dijets, 2204.11650
P. Caucal et al., dijets, 2108.06347

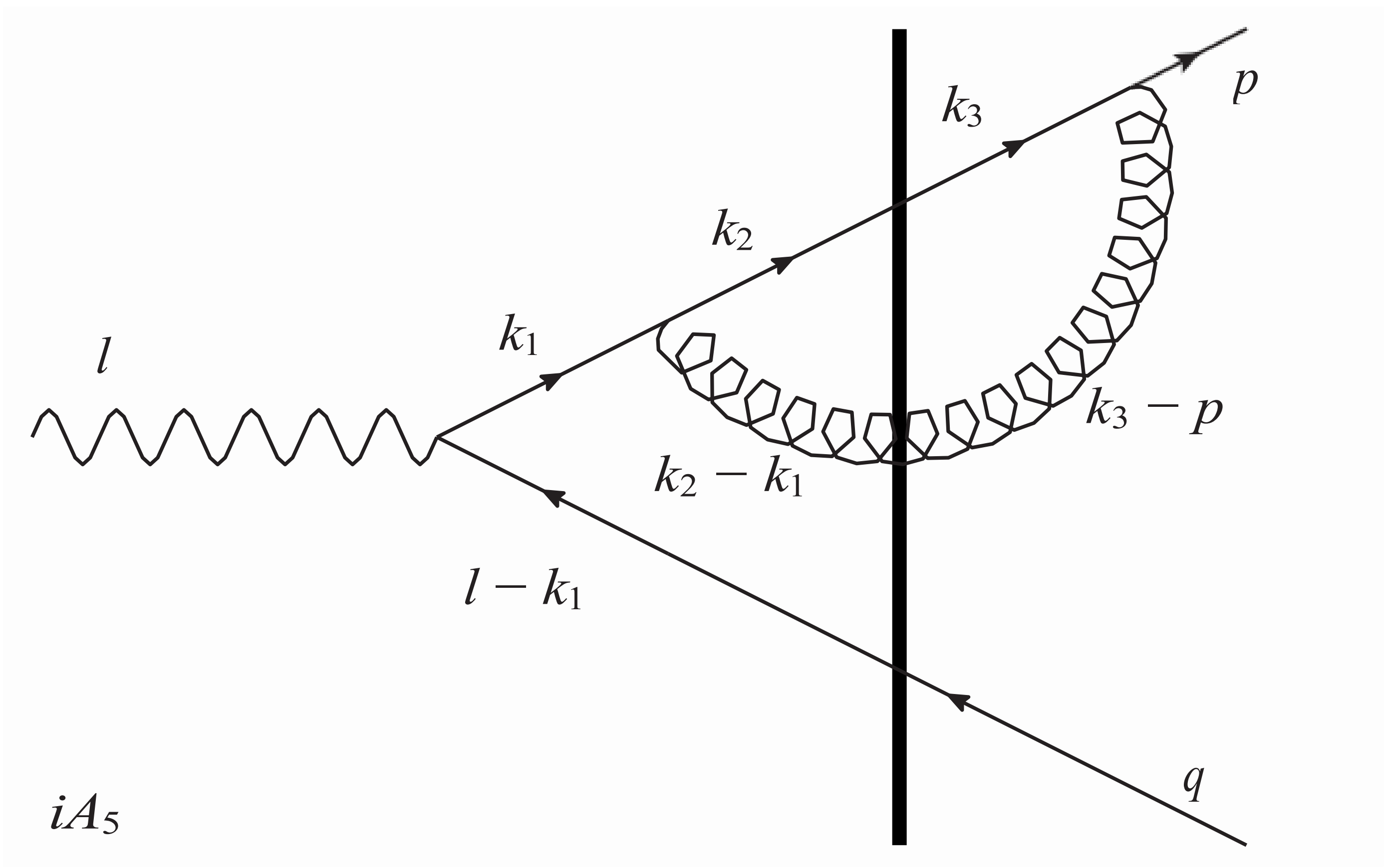
divergences

- ***Ultraviolet:***

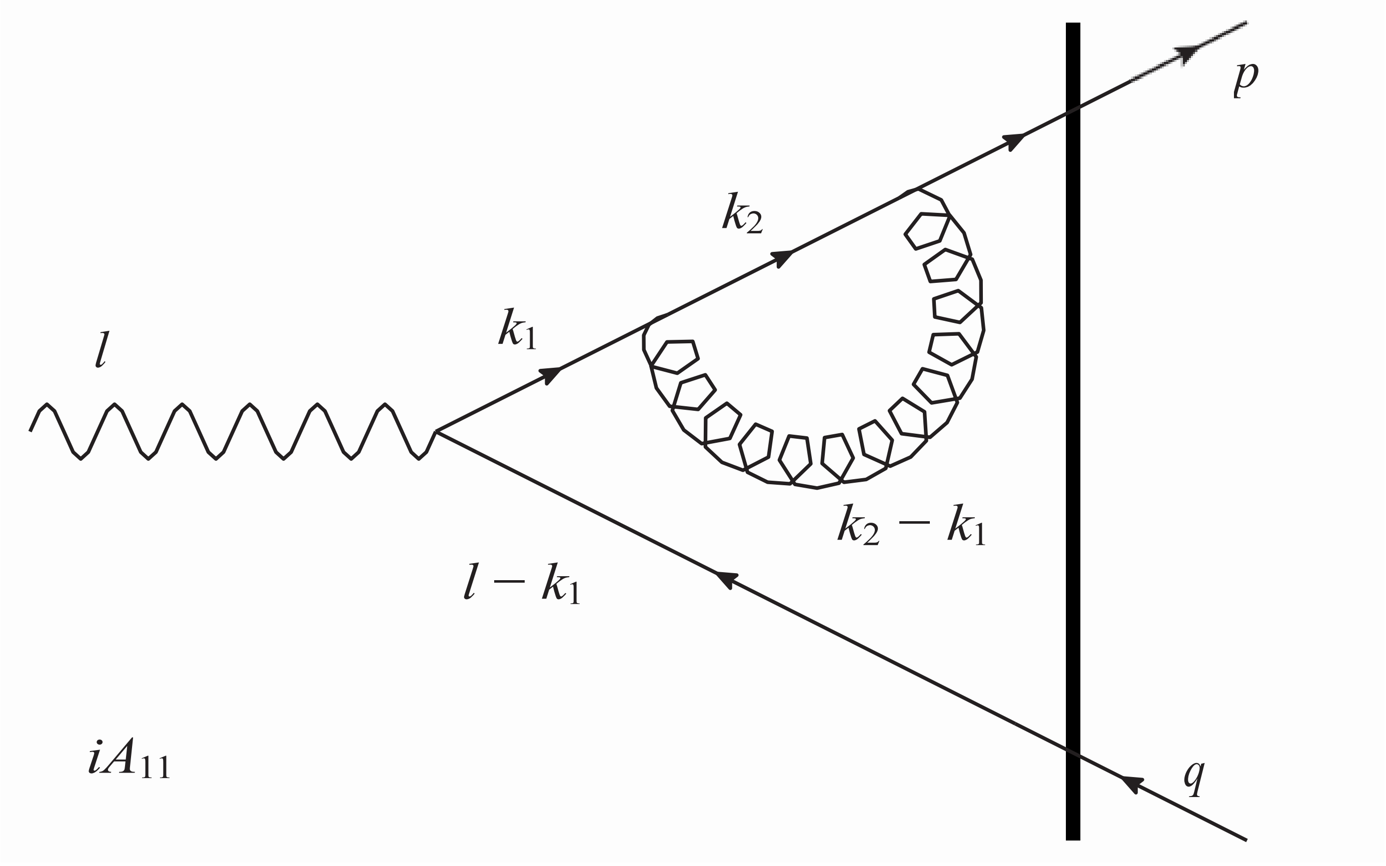
Real corrections are UV finite

UV divergences cancel among virtual corrections $\mathbf{k} \to \infty$ **or** $\mathbf{x_3} \to \mathbf{x_i}$

$$\left(d\sigma_5 + d\sigma_{11}\right)_{UV} = 0$$
$$\left(d\sigma_6 + d\sigma_{12}\right)_{UV} = 0$$
$$\left(d\sigma_9 + d\sigma_{10} + d\sigma_{14(1)} + d\sigma_{14(2)}\right)_{UV} = 0$$



$+$



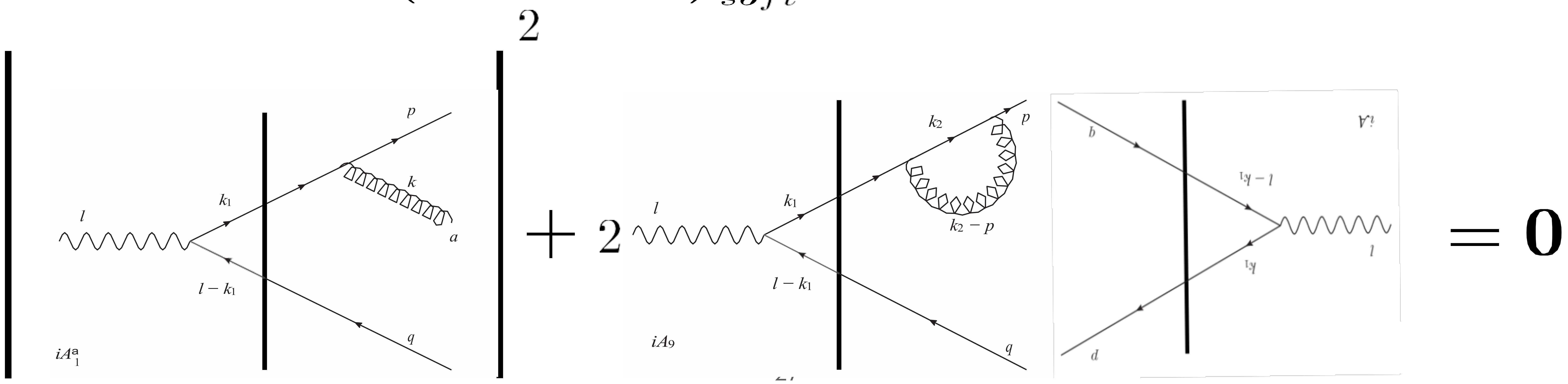
$= 0$

divergences

- ***Soft:*** $\mathbf{k}^{\mu} \to \mathbf{0}\ \ (\mathbf{x_3} \to \infty\ \ \mathbf{AND}\ \ \mathbf{z} \to \mathbf{0})$

Soft divergences cancel between real and virtual corrections

$$(d\sigma_{1-1} + d\sigma_9)_{soft} = 0,$$
$$\left(d\sigma_{1-2} + d\sigma_{13}^{(1)} + d\sigma_{13}^{(2)}\right)_{soft} = 0$$
$$(d\sigma_{3-3} + d\sigma_{4-4} + d\sigma_{3-4})_{soft} = 0$$
$$(d\sigma_{1-3} + d\sigma_{1-4})_{soft} = 0$$
$$(d\sigma_{2-3} + d\sigma_{2-4})_{soft} = 0$$
$$(d\sigma_5 + d\sigma_7)_{soft} = 0$$
$$\left(d\sigma_{11} + d\sigma_{14}^{(1)}\right)_{soft} = 0$$



divergences

- ***Rapidity:*** $\mathbf{z \to 0}$ ***, but finite*** $\boldsymbol{k_t}$

$$\int_0^1 \frac{dz}{z} = \int_0^{z_f} \frac{dz}{z} + \int_{z_f}^1 \frac{dz}{z}$$

rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

$$\frac{d\sigma^L_{\rm NLO}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, y_2} = \frac{2e^2 g^2 Q^2 N_c^2 (z_1 z_2)^3}{(2\pi)^{10}} \delta(1-z_1-z_2) \int_0^{z_f} \frac{dz}{z} \int d^{10}\mathbf{x}\, K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{1'2'}|Q_1)$$

$$e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}} e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \Big\{ \left(\tilde\Delta_{12} + \tilde\Delta_{22'} - \tilde\Delta_{12'}\right) S_{132'1'} S_{23} + \left(\tilde\Delta_{1'2'} + \tilde\Delta_{22'} - \tilde\Delta_{21'}\right) S_{1'321} S_{2'3}$$

$$+ \left(\tilde\Delta_{12} + \tilde\Delta_{11'} - \tilde\Delta_{21'}\right) S_{322'1'} S_{13} + \left(\tilde\Delta_{1'2'} + \tilde\Delta_{11'} - \tilde\Delta_{12'}\right) S_{32'21} S_{1'3}$$

$$- \left(\tilde\Delta_{11'} + \tilde\Delta_{22'} + \tilde\Delta_{12} + \tilde\Delta_{1'2'}\right) S_{122'1'} - \left(\tilde\Delta_{12} + \tilde\Delta_{1'2'} - \tilde\Delta_{12'} - \tilde\Delta_{21'}\right) S_{12} S_{1'2'}$$

$$- \left(\tilde\Delta_{11'} + \tilde\Delta_{22'} - \tilde\Delta_{12'} - \tilde\Delta_{21'}\right) S_{11'} S_{22'} - 2\tilde\Delta_{12}\left(S_{13}S_{23} - S_{12}\right) - 2\tilde\Delta_{1'2'}\left(S_{1'3}S_{2'3} - S_{1'2'}\right) \Big\}$$

JIMWLK evolution of quadrupoles

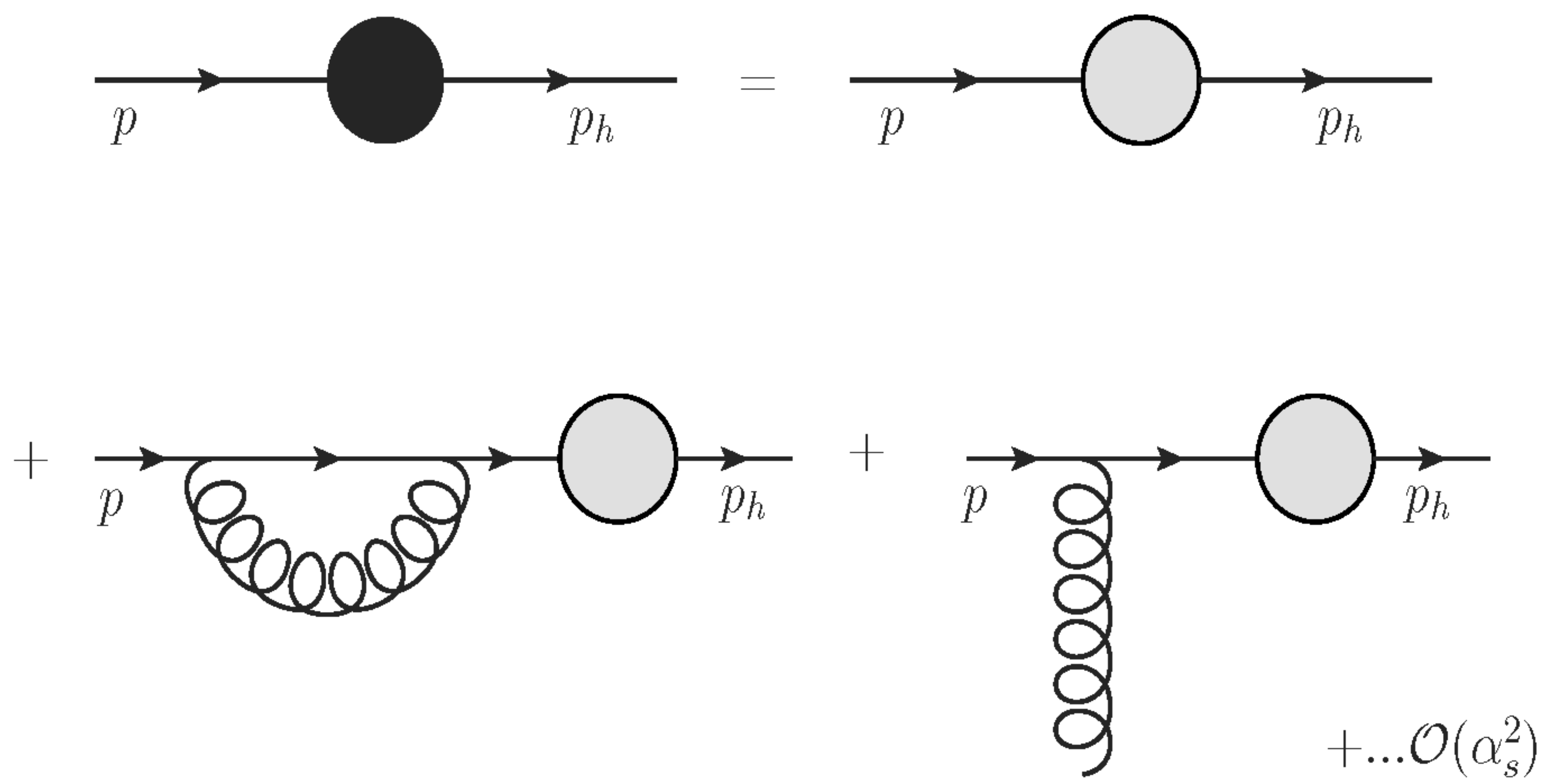
JIMWLK evolution of dipoles

$$\tilde{\boldsymbol{\Delta}}_{\mathbf{12}} \equiv \frac{(\mathbf{x_1} - \mathbf{x_2})^2}{(\mathbf{x_1} - \mathbf{x_3})^2 (\mathbf{x_2} - \mathbf{x_3})^2}$$

divergences

- ***Collinear:*** $$\frac{1}{(p+k)^2} = \frac{1}{|\vec{p}||\vec{k}|(1-\cos\theta)} \longrightarrow \infty \quad as \quad \theta \to 0$$

Collinear divergences are absorbed into evolution of parton-hadron fragmentation functions



collinear divergences

real corrections

$$\frac{d\sigma_{LO+1-1}^{\gamma^* A\to h_1 h_2 X}}{d^2\mathbf{p}_h\, d^2\mathbf{q}_h\, dy_1\, dy_2} = \int_0^1 dz_{h_1} \int_0^1 dz_{h_2} \frac{4e^2Q^2N_c(z_1z_2)^3}{(2\pi)^7(z_{h_1}z_{h_2})^2} H\left(\mathbf{p},\mathbf{q},z_2\right) D^0_{h_1/q}(z_{h_1}) D^0_{h_2/\bar{q}}(z_{h_2})$$

$$\int \frac{d\xi_1}{\xi_1^3}\delta(1-z_2-z_1/\xi_1)\left[\delta(1-\xi_1)+2\alpha_s\, P_{qq}(\xi_1)\int d^2\mathbf{k}\frac{e^{i\mathbf{k}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}}{(\xi_1\mathbf{k}-(1-\xi_1)\mathbf{p})^2}\right]$$

with $P_{qq}(\xi_1) = C_F\frac{(1+\xi_1^2)}{(1-\xi_1)}$

virtual corrections

$$\frac{d\sigma_9^{\gamma^* A\to h_1 h_2 X}}{d^2\mathbf{p}_h\, d^2\mathbf{q}_h\, dy_1\, dy_2} = -\int_0^1 dz_{h_1} \int_0^1 dz_{h_2} \frac{4e^2Q^2(z_1z_2)^3N_c}{(2\pi)^7(z_{h_1}z_{h_2})^2} H(\mathbf{p},\mathbf{q},z_2) D^0_{h_1/q}(z_{h_1}) D^0_{h_2/\bar{q}}(z_{h_2})$$

$$\times\,\alpha_s\int_0^1 d\xi\, P_{qq}(\xi)\int d^2\mathbf{k}\frac{1}{(\mathbf{k}-(1-\xi)\mathbf{p})^2}\delta(1-z_1-z_2)$$

these are combined into DGLAP evolution of fragmentation functions

$$D_{h_1/q}(z_{h1},\mu^2) = \int_{z_{h1}}^1 \frac{d\xi}{\xi} D^0_{h_1/q}\left(\frac{z_{h1}}{\xi}\right)\left[\delta(1-\xi)+\frac{\alpha_s}{2\pi}P_{qq}(\xi)\,\log\left(\frac{\mu^2}{\Lambda^2}\right)\right]$$

Divergences

- Ultraviolet

  real corrections are UV finite

  UV divergences cancel among virtual diagrams

- Soft

  soft divergences cancel Soft

  soft divergences cancel between real and virtual diagrams real and virtual diagrams

- Collinear

  collinear divergences are absorbed into fragmentation functions

- Rapidity

  Rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

$$\sigma^{\gamma^* A\rightarrow h_1 h_2 X} = \sigma_{LO} \otimes \text{JIMWLK} + \sigma_{LO} \otimes D_{h/q}(z_h, \mu^2) \otimes D^{(0)}_{h/q}(z_h) + \sigma^{\text{finite}}_{NLO}$$

Back to back limit: deep connections to physics of TMDs, Sudakov effect,....

Dihadron/dijets kinematics at EIC

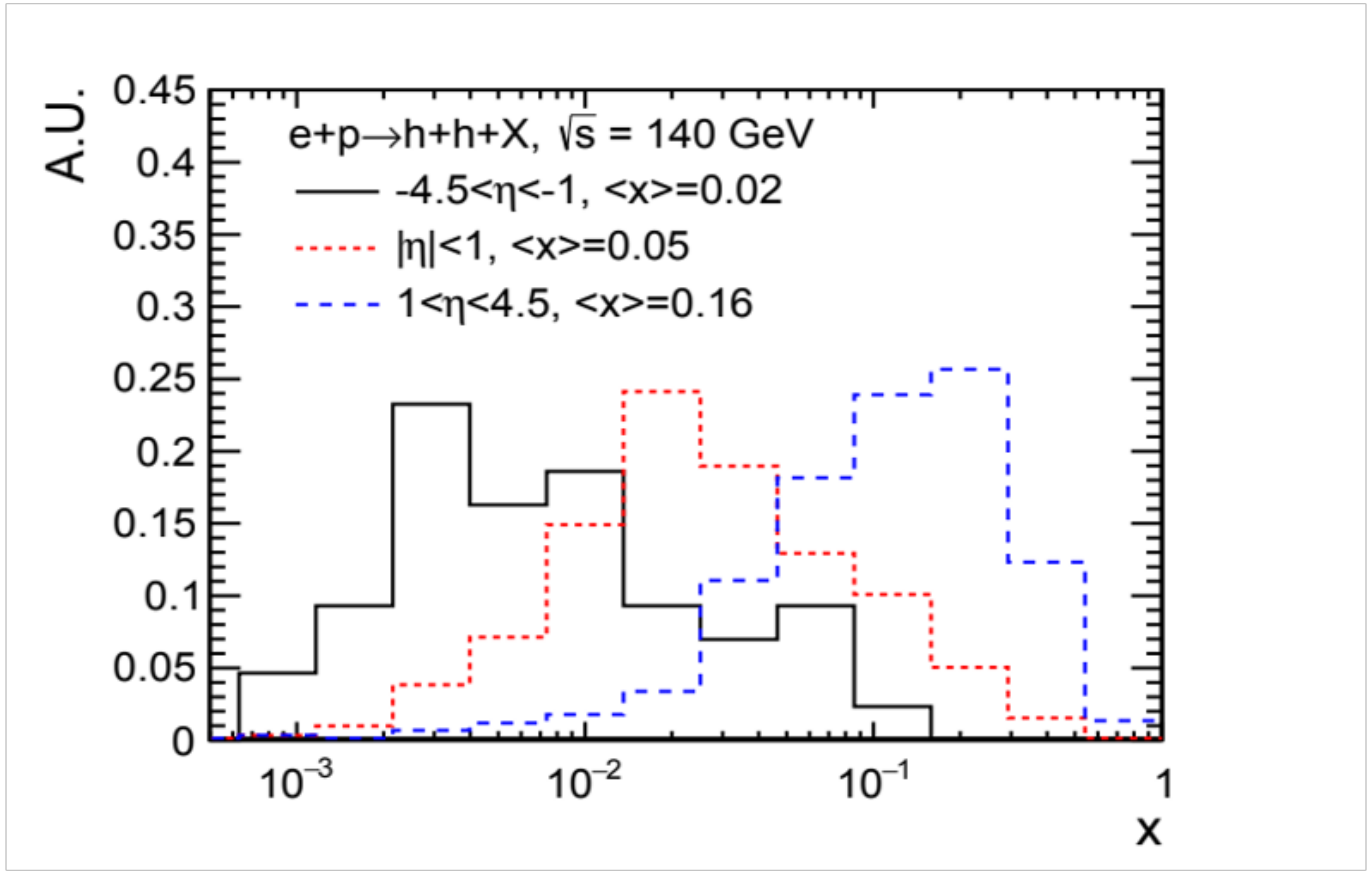
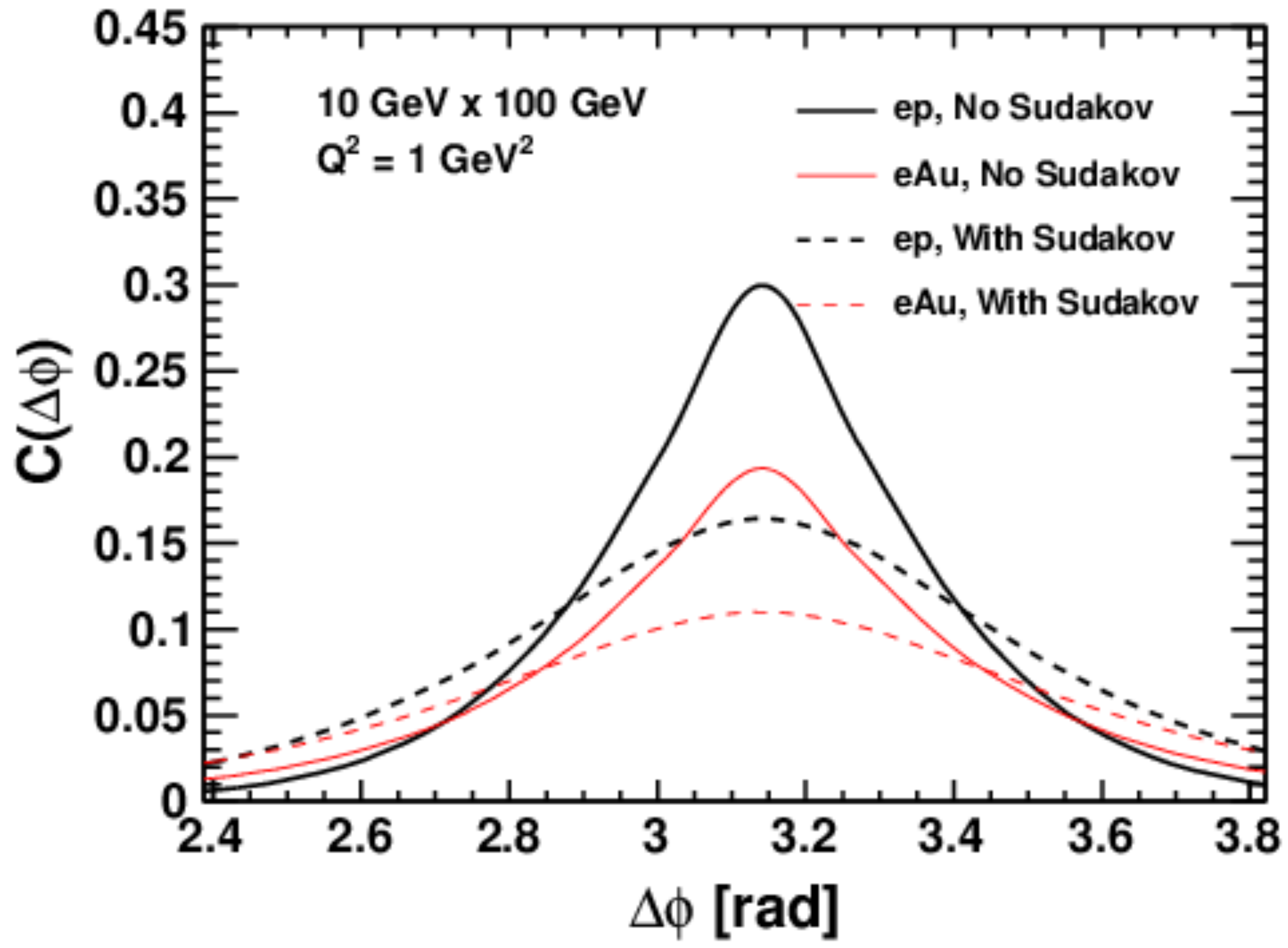


Fig courtesy of Xiaoxuan Chu



Zheng, Aschenauer,Lee,Xiao, arXiv:1403.2413

SIDIS a better process (?)

larger kinematic phase space than dihadrons

Sudakov effect can be avoided

SIDIS at small x: NLO corrections

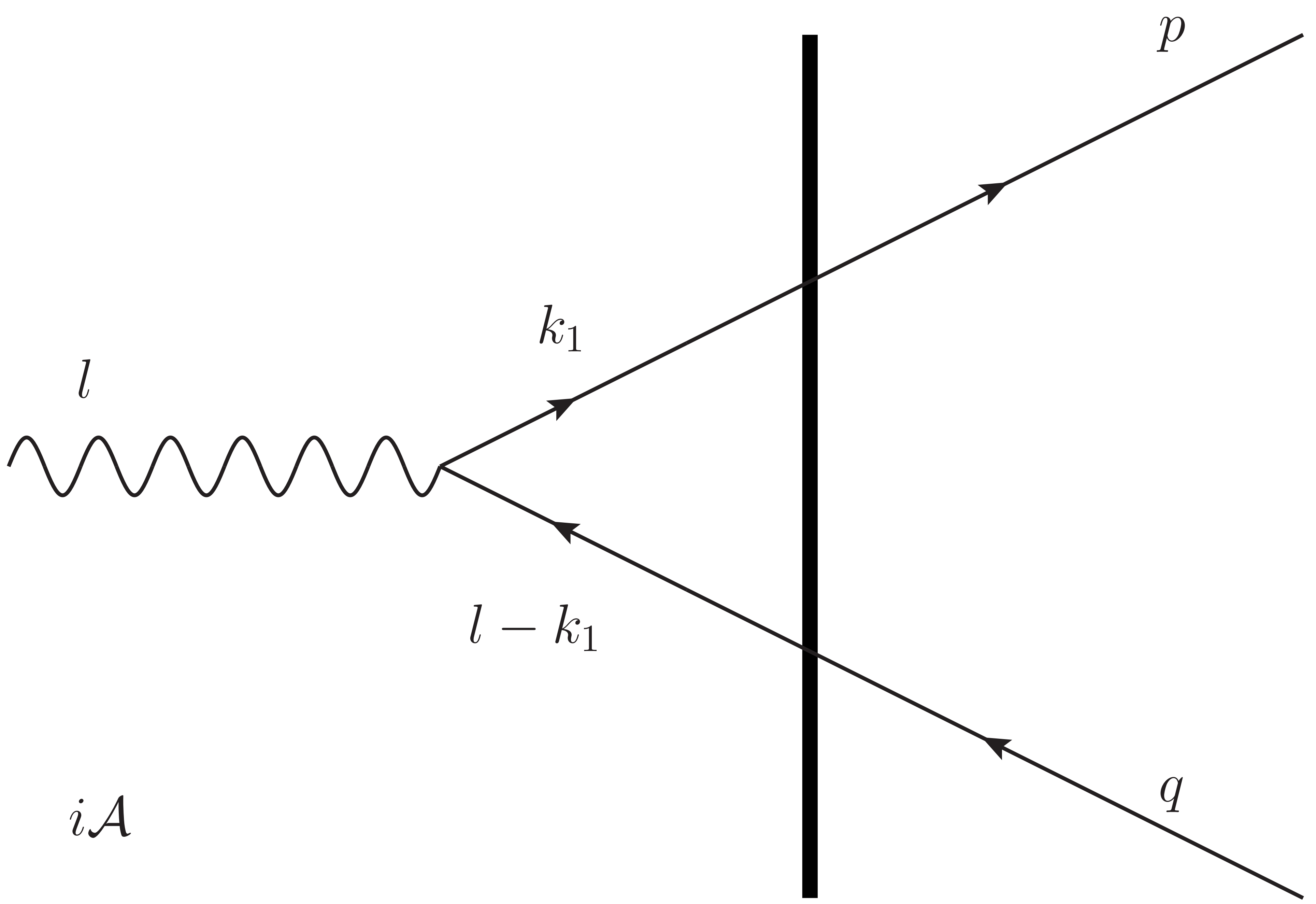
F. Bergabo, JJM, JHEP 01 (2023) 095, and arXiv:2401.06259

Caucal, Ferrand, Salazar, arXiv:2401.01934

Forward rapidity

LO:

integrate over final state antiquark

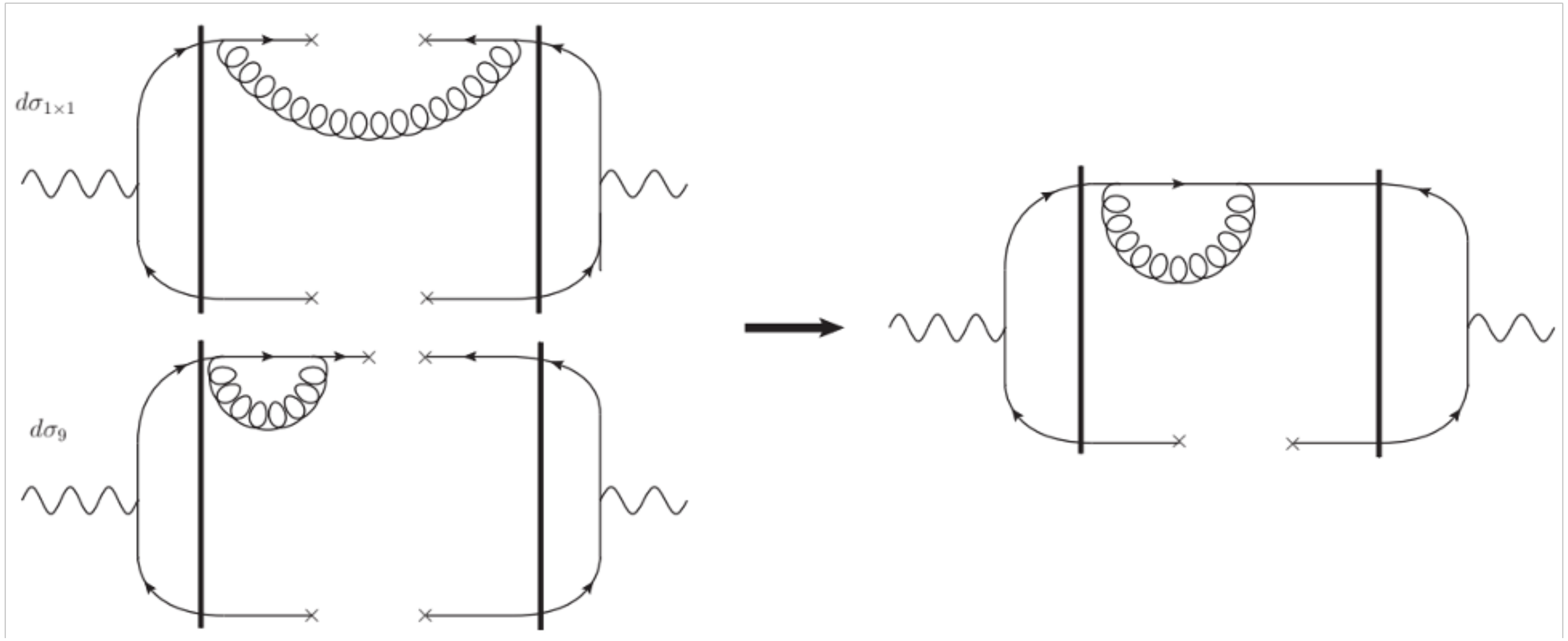


$$\frac{\mathrm{d}\sigma^{\gamma^* p/A \to q(\mathbf{p}, y_1) X}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = \frac{e^2 Q^2 N_c}{(2\pi)^5} \int \mathrm{d}z_2\, \delta(1 - z_1 - z_2)\,(z_1^2\, z_2) \int \mathrm{d}^6\mathbf{x}\,[S_{11'} - S_{12} - S_{1'2} + 1]\, e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}$$
$$\left\{ 4 z_1 z_2 K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{1'2}|Q_1) + (z_1^2 + z_2^2)\, \frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\, K_1(|\mathbf{x}_{12}|Q_1) K_1(|\mathbf{x}_{1'2}|Q_1) \right\}$$

Single inclusive hadron production in DIS at small x: NLO

start with NLO corrections to dihadron production and integrate out quark

cancellations among diagrams



some of the contributions are

$$\frac{\mathrm{d}\sigma^T_{1\times 1}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = \frac{e^2g^2Q^2}{(2\pi)^8}\int_0^{1-z_1}\frac{\mathrm{d}z}{z}\frac{(1-z-z_1)(z+z_1)}{z_1}\left[z_1^2(1-z-z_1)^2+\left(z_1^2+(1-z-z_1)^2\right)(z+z_1)^2+(z+z_1)^4\right]$$
$$\int \mathrm{d}^8\mathbf{x}\,K_1(|\mathbf{x}_{12}|Q_{1z})K_1(|\mathbf{x}_{1'2}|Q_{1z})N_cC_F\left[S_{11'}-S_{12}-S_{21'}+1\right]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\,\Delta^{(3)}_{11'}\,e^{i\frac{z_1+z}{z_1}\mathbf{p}\cdot\mathbf{x}_{1'1}}$$

$$\frac{\mathrm{d}\sigma^T_{9}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = -\frac{e^2g^2Q^2}{2(2\pi)^6}\int_0^{z_1}\frac{\mathrm{d}z}{z}(1-z_1)(z_1^2+(1-z_1)^2)\left[z_1^2+(z_1-z)^2\right]\int \mathrm{d}^6\mathbf{x}\,K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2}|Q_1)e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}$$
$$N_cC_F\left[S_{11'}-S_{12}-S_{21'}+1\right]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\int\frac{\mathrm{d}^2\mathbf{x}_3}{(2\pi)^2}\frac{1}{\mathbf{x}_{31}^2}$$

with $\Delta^{(3)}_{ij} = \dfrac{\mathbf{x}_{3i}\cdot\mathbf{x}_{3j}}{\mathbf{x}_{3i}^2\mathbf{x}_{3j}^2}$ **and**

$$Q_1^2 \equiv z_1(1-z_1)Q^2$$
$$Q_{1z}^2 \equiv (1-z-z_1)(z+z_1)Q^2$$

SIDIS at small x: NLO corrections

all quadrupole contributions cancel: dipoles only at leading N_c

cancelation of UV/soft divergences

rapidity/collinear divergences renormalize the dipoles/fragmentation functions

$$\sigma^{\gamma^\star \mathbf{A} \longrightarrow \mathbf{h\,X}} = \sigma_{\mathbf{LO}} \otimes \mathbf{JIMWLK} + \sigma_{\mathbf{LO}} \otimes \mathbf{D_{h/q}}(\mathbf{z_h},\, \mu^{\mathbf{2}}) + \sigma_{\mathbf{NLO}}^{\mathbf{finite}}$$

there are ***no*** Sudakov double logs in SIDIS ***unlike*** inclusive dihadron production (wrong sign)

SIDIS at small x: including Sudakov double logs

EIC will have a reasonably large window in $\mathbf{Q^2}$ where $\mathbf{Q^2 >> p_{t,h}^2}$ so that $\alpha_\mathbf{s} \log\left(\frac{\mathbf{Q^2}}{\mathbf{p_{t,h}^2}}\right) \sim \mathbf{1}$

avoid large Sudakov logs: $\mathbf{p_{t,h}^2 \sim Q^2}$ so that $\log\left(\frac{\mathbf{Q^2}}{\mathbf{p_{t,h}^2}}\right) \simeq \mathbf{0}$

to get Sudakov logs one must introduce a kinematic constraint

needed for self-consistency of evolution equations (avoid negative cross sections)

at small x we require $\mathbf{k^+ < z_f\, l^+}$, this is sufficient at LL accuracy

at NLO accuracy we also need to impose a condition on lifetimes of fluctuations

introduce a cutoff on - component of momenta $\mathbf{k^- > \tilde{z}_f\, l^-}$ with $\mathbf{z_f\, \tilde{z}_f = 1}$

Altinoluk, JJM, Marquet, arXiv:2406.08277

SIDIS at small x: including Sudakov double logs

taking the high $\mathbf{Q^2}$ is tricky! $\mathbf{Q_1^2 \equiv z_1(1-z_1)Q^2 \longrightarrow 0}$ inside the Bessel functions

introduce the delta function

$$Q_1^{2n}\, K_{(0,1)}(|\mathbf{x}_{12}|Q_1)\, K_{(0,1)}(|\mathbf{x}_{1'2}|Q_1) = \int_0^{Q^2/4} d\bar{Q}^2\ (\bar{Q}^2)^n K_{(0,1)}(|\mathbf{x}_{12}|\bar{Q}) K_{(0,1)}(|\mathbf{x}_{1'2}|\bar{Q})\ \delta\left[\bar{Q}^2 - z_1(1-z_1)Q^2\right]$$

$$\delta\left[\bar{Q}^2 - z_1(1-z_1)Q^2\right] = \frac{\delta(z_1-z_+)}{Q^2|1-2z_+|} + \frac{\delta(z_1-z_-)}{Q^2|1-2z_-|} \quad \text{with} \quad z_\pm = \frac{1}{2}\left(1 \pm \sqrt{1-4\bar{Q}^2/Q^2}\right)$$

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}_{LO+NLO}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = \frac{\pi e^2}{Q^2}\frac{1}{z_h}\int \frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\, e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\, x\tilde{q}(x,\mathbf{x}_{11'})$$
$$\times\left\{D_{h/q}(z_h) + \left[\frac{\alpha_s C_F}{2\pi^2}\left(\int_0^{1-z_h}\frac{\mathrm{d}z}{z}\frac{1+(1-z)^2}{1-z}D_{h/q}(\frac{z_h}{1-z}) - \int_0^1\frac{\mathrm{d}z}{z}[1+(1-z)^2]D_{h/q}(z_h)\right)\right]\int_{|\mathbf{x}_3|}\right.$$
$$\left. + \left[\frac{\alpha_s C_F}{2\pi^2}\int_0^{1-z_h}\frac{\mathrm{d}z}{z}\frac{\left[1+(1-z)^2\right]}{1-z}\left(\int \mathrm{d}^2\mathbf{x}_3\,\Delta^{(3)}_{1'1} - \int_{|\mathbf{x}_3|\mu>1}\frac{\mathrm{d}^2\mathbf{x}_3}{\mathbf{x}_3^2}\right)\right]D_{h/q}(\frac{z_h}{1-z})\right\}$$

SIDIS at small x: including Sudakov double logs

introduce the longitudinal factorization z_f

$$\int_0^{1-z_h} dz = \int_0^{z_f} dz + \int_{z_f}^{1-z_h} dz$$

add and subtract the kinematic constraint

$$\Theta(\text{kin.const.}) = \Theta\left(z_f \frac{p^2}{Q^2} - z\right)$$

$$\int_0^{z_f} \frac{dz}{z}\left[1 - \Theta\left(z_f \frac{p^2}{Q^2} - z\right)\right] = \int_0^{z_f} \frac{dz}{z}\Theta\left(z - z_f \frac{p^2}{Q^2}\right) = \Theta\left(\frac{Q^2}{p^2}\right)\ln\left(\frac{Q^2}{p^2}\right)$$

and dipoles satisfy constrained JIMWLK evolution

$$\left.\frac{d\sigma_{LO+NLO}^{\gamma^* A \to h(\mathbf{p}_h, y_h)X}}{d^2\mathbf{p}_h\, dy_h}\right|_{LP} = d\sigma_{LO}(z_f) \otimes D_{h/q}(z_h, Q^2) + d\sigma_{NLO-rap-finite} +$$

$$\frac{\pi e^2}{Q^2} \frac{D_{h/q}(z_h)}{z_h} \int \frac{d^2\mathbf{x}_{11'}}{(2\pi)^2} e^{-i\frac{\mathbf{p}_h}{z_h}\cdot \mathbf{x}_{11'}} x\tilde{q}(x, \mathbf{x}_{11'})$$

$$\times \left\{ \frac{\alpha_s C_F}{\pi^2} \int^{Q^2} \frac{d^2\mathbf{p}}{\mathbf{p}^2} \left(e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}} - 1\right) \ln\left(\frac{Q^2}{\mathbf{p}^2}\right) \right\}$$

Sudakov double logs in SIDIS

using

$$\int^{Q^2} \frac{\mathrm{d}^2\mathbf{p}}{\mathbf{p}^2}\left[e^{-i\mathbf{p}\cdot\mathbf{x}_{11'}}-1\right]\ln\left(\frac{Q^2}{\mathbf{p}^2}\right) = 4\pi\int_0^{Q|\mathbf{x}_{11'}|}\frac{\mathrm{d}\tau}{\tau}\left[J_0(\tau)-1\right]\ln\left(\frac{Q|\mathbf{x}_{11'}|}{\tau}\right)$$

$$= -\frac{\pi}{2}\ln^2\left(Q^2\mathbf{x}_{11'}^2/c_0^2\right) + O\left(\frac{1}{\sqrt{Q|\mathbf{x}_{11'}|}}\right)$$

we get

$$\left.\frac{\mathrm{d}\sigma_{LO+NLO}^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = d\sigma_{NLO-rap-finite}$$

$$+\frac{\pi e^2}{Q^2}\frac{D_{h/q}(z_h,Q^2)}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\,e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'})\,e^{-S_{sud}(\mathbf{x}_{11'})}$$

with

$$S_{sud}(\mathbf{x}_{11'}) \equiv \frac{\alpha_s C_F}{2\pi}\ln^2\left(Q^2\mathbf{x}_{11'}^2/c_0^2\right)$$ and transverse momentum dependent distribution (TMD)

$$xq(x,\mathbf{p}) = \frac{2N_c}{(2\pi)^6}\int \mathrm{d}^6\mathbf{x}\; e^{-i\mathbf{p}\cdot\mathbf{x}_{11'}}\;\left[S_{11'}-S_{12}-S_{1'2}+1\right]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\int_0^\infty d\bar{Q}^2\bar{Q}^2\,K_1(|\mathbf{x}_{12}|\bar{Q})K_1(|\mathbf{x}_{1'2}|\bar{Q})$$

Summary I

QCD at high energy

dense hadron/nucleus: gluon saturation, strong color fields - CGC

strong hints from RHIC, LHC,..., to be probed precisely at EIC

toward precision: NLO, sub-eikonal corrections, ...

CGC is limited to small x (low p_t)

How good is eikonal approximation?

SUMMARY

pQCD and collinear factorization at high p_t

breaks down at small x (low p_t)

CGC is a systematic approach to high energy collisions

strong hints from RHIC, LHC,...

to be probed extensively at EIC

toward precision: NLO, sub-eikonal corrections, ...

CGC breaks down at large x (high p_t)

a significant part of EIC/RHIC/LHC phase space is at large x

transition from large x physics (pQCD) to small x (CGC)

Toward inclusion of large x physics:

spin asymmetries

beam rapidity loss

particle production in both small and large p_t kinematics

two-particle correlations: from forward-forward to forward-backward

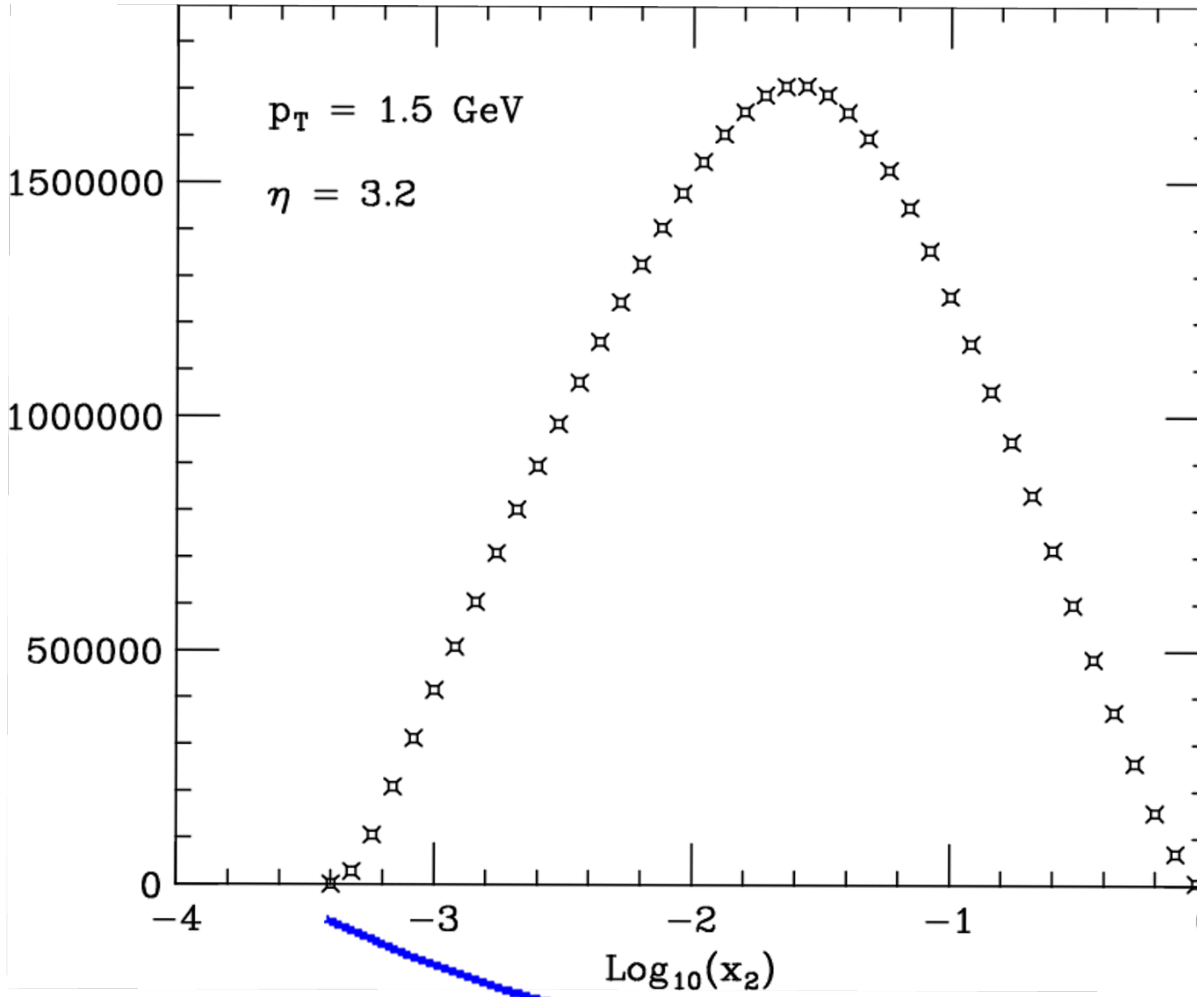
one-loop correction: both collinear and CGC factorization limits

need to clarify/understand: gauge invariance, initial conditions, ...

Single inclusive pion production in pp at RHIC

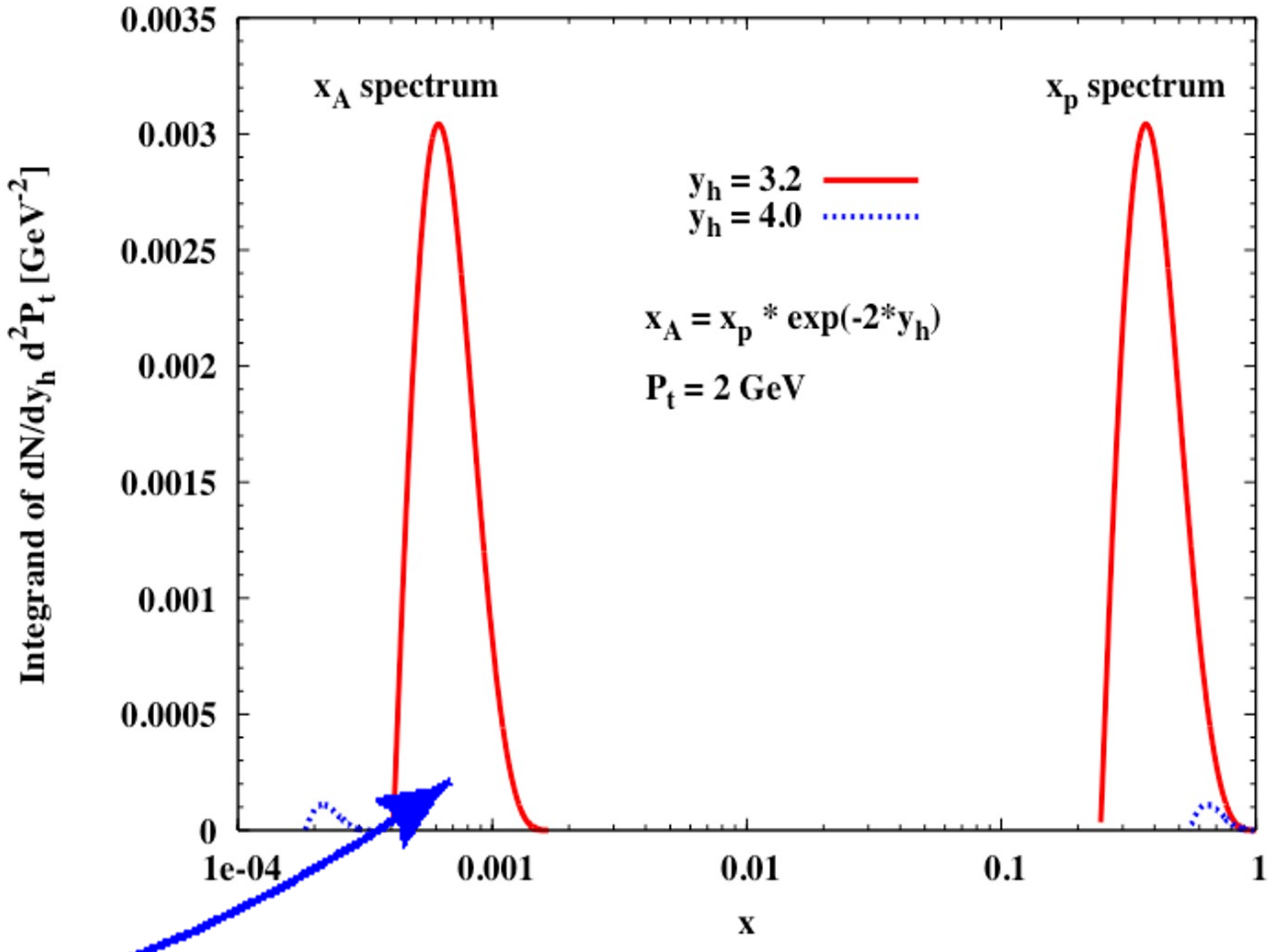
collinear factorization

GSV, PLB603 (2004) 173-183



CGC

DHJ, NPA765 (2006) 57-70



$$\int_{x_{min}}^{1} dx\, xG(x, Q^2) \cdots\cdots \longrightarrow x_{min} G(x_{min}, Q^2) \cdots$$

which kinematics are we in?

toward unifying small and large x (multiple scattering)

JJM, 1708.07533, 1809.04625, 1912.08878

scattering from small x modes of the target field $\mathbf{A^- = n^- \ S}$ involves only

small transverse momenta exchange (small angle deflection)

$$p^\mu = (p^+ \sim \sqrt{s},\, p^- = 0, p_t = 0)$$
$$S = S(p^+ \sim 0, p^-/P^- \ll 1, p_t)$$

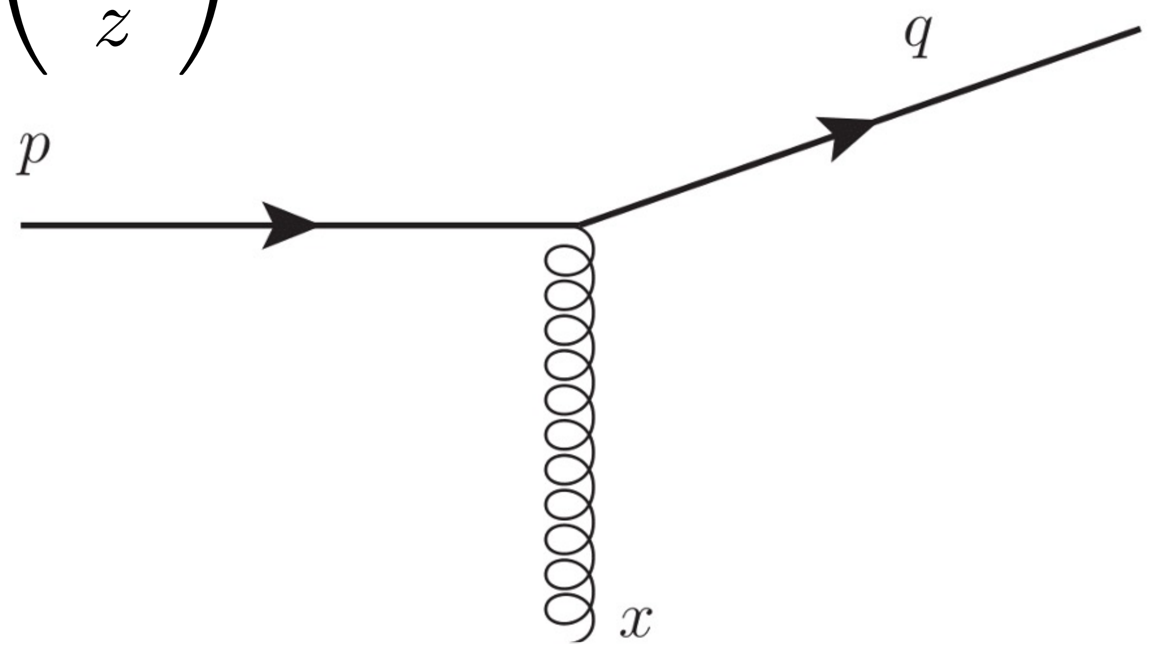
allow hard scattering by including one all x field during which there is large momenta exchanged and quark can get deflected by a large angle.

$$A^\mu_a(x^+, x^-, x_t)$$

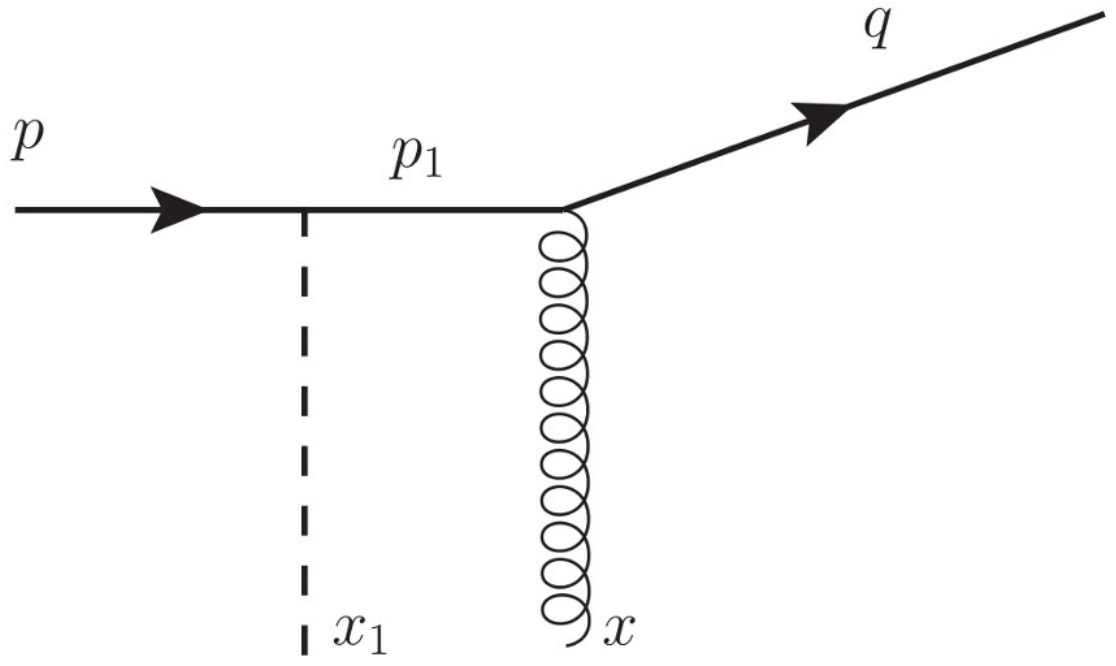
include eikonal multiple scattering before and after (along a different direction) the hard scattering

hard scattering: large deflection
scattered quark travels in the new “z” direction: $\bar{z}$

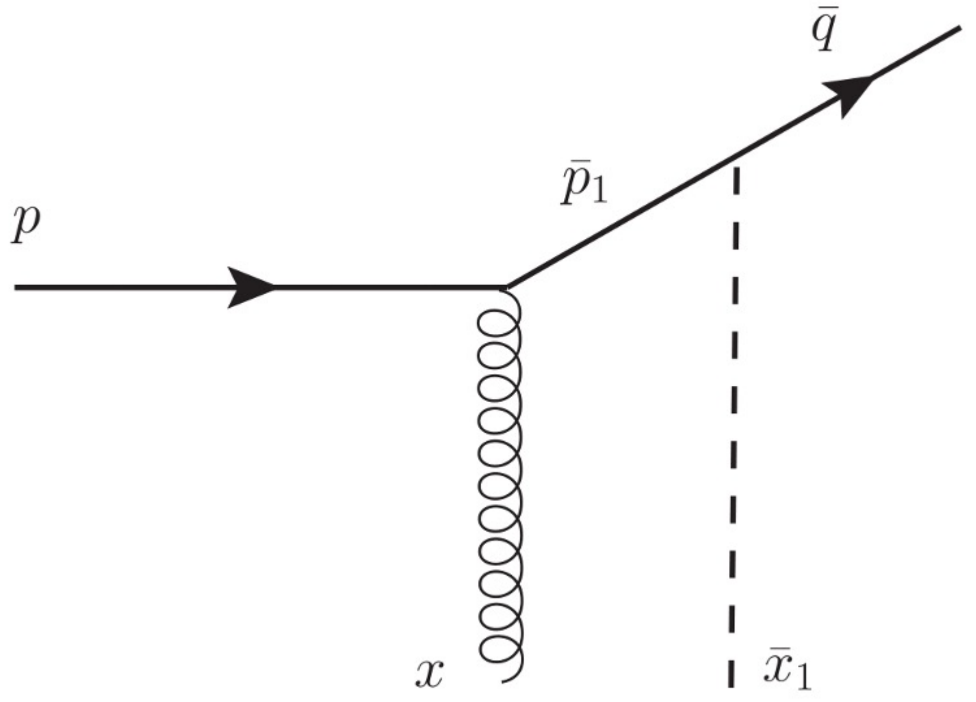
$$\begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \mathcal{O} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



$$i\mathcal{M}_1 = (ig) \int d^4x \, e^{i(\bar{q}-p)x} \, \bar{u}(\bar{q}) \, \left[\not{A}(x) \right] \, u(p)$$

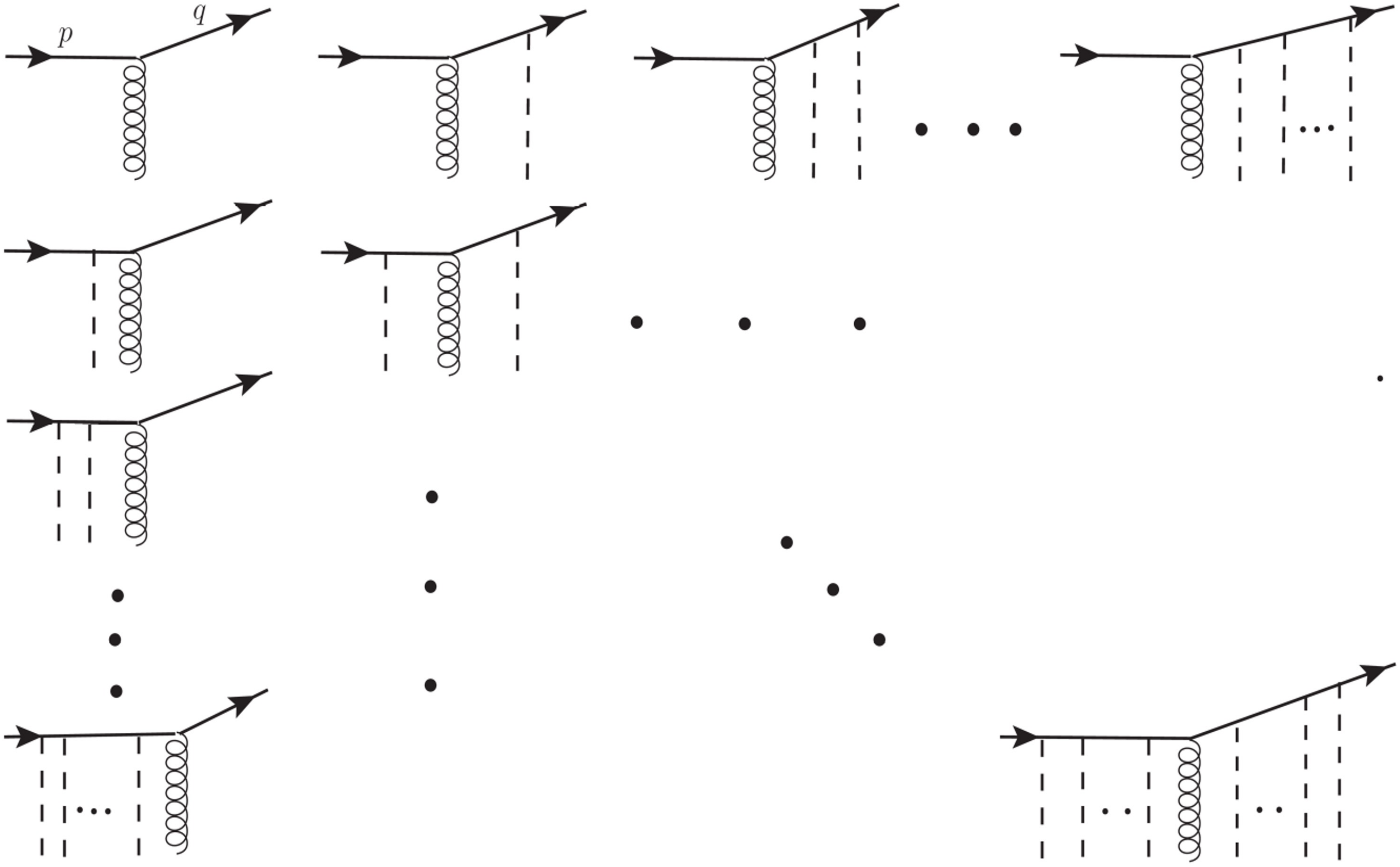


$$i\mathcal{M}_2 = (ig)^2 \int d^4x \, d^4x_1 \int \frac{d^4p_1}{(2\pi)^4} \, e^{i(p_1-p)x_1} \, e^{i(\bar{q}-p_1)x} \, \bar{u}(\bar{q}) \left[\not{A}(x) \, \frac{i\not{p}_1}{p_1^2 + i\epsilon} \, \not{n} \, S(x_1) \right] u(p)$$



$$i\mathcal{M}_2 = (ig)^2 \int d^4x \, d^4\bar{x}_1 \int \frac{d^4\bar{p}_1}{(2\pi)^4} \, e^{i(\bar{p}_1-p)x} \, e^{i(\bar{q}-\bar{p}_1)\bar{x}_1} \, \bar{u}(\bar{q}) \left[\not{\bar{n}} \, \bar{S}(\bar{x}_1) \, \frac{i\not{\bar{p}}_1}{\bar{p}_1^2 + i\epsilon} \not{A}(x) \right] u(p)$$

with $\bar{v}^\mu = \Lambda^\mu_\nu \, v^\nu$


p
q

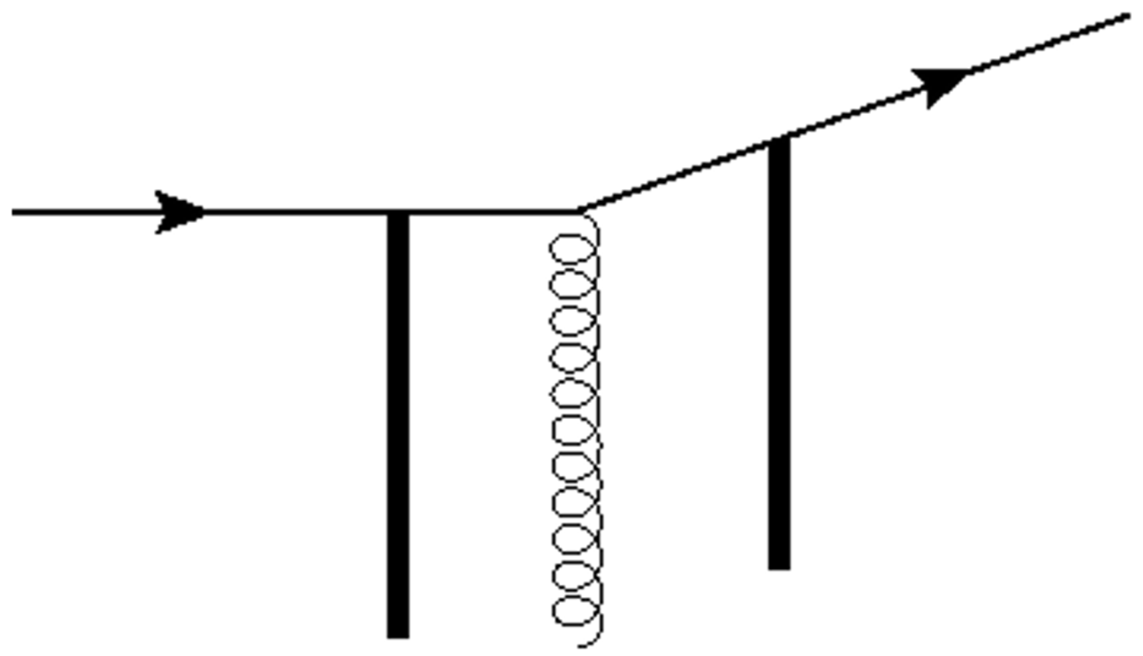
summing all the terms gives:

$$i\mathcal{M}_1 \quad = \quad \int d^4x\, d^2z_t\, d^2\bar{z}_t \int \frac{d^2k_t}{(2\pi)^2}\,\frac{d^2\bar{k}_t}{(2\pi)^2}\, e^{i(\bar{k}-k)x}\, e^{-i(\bar{q}_t-\bar{k}_t)\cdot\bar{z}_t}\, e^{-i(k_t-p_t)\cdot z_t}$$

$$\bar{u}(\bar{q})\,\left[\overline{V}_{AP}(x^+,\bar{z}_t)\, \not{n}\, \frac{\bar{\not{k}}}{2\bar{k}^+}\, \left[ig\not{A}(x)\right]\, \frac{\not{k}}{2k^+}\, \not{n}\, V_{AP}(z_t,x^+)\right]\, u(p)$$

with

$$\overline{V}_{AP}(x^+,\bar{z}_t) \equiv \hat{P}\, \exp\left\{ig\int_{x^+}^{+\infty} d\bar{z}^+\, \bar{S}_a^-(\bar{z}_t,\bar{z}^+)\, t_a\right\}$$



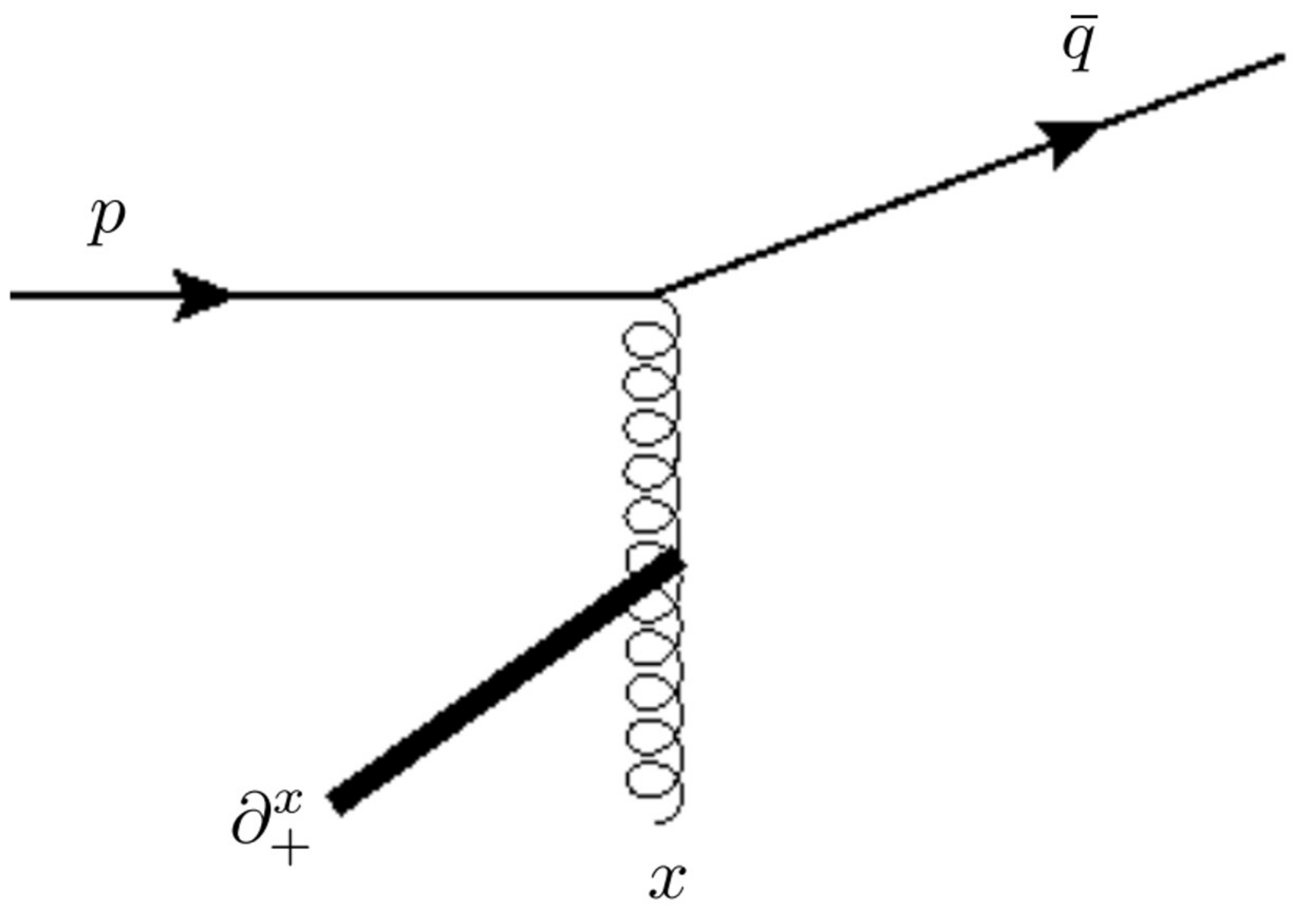
$$V_{AP}(z_t,x^+) \equiv \hat{P}\, \exp\left\{ig\int_{-\infty}^{x^+} dz^+\, S_a^-(z_t,z^+)\, t_a\right\}$$

this is the building block for DIS structure functions, single inclusive particle production in pA,....

$$
\begin{aligned}
i\mathcal{M}_2 \quad = \quad & \frac{2\,i}{(p-\bar{q})^2} \int d^4x\, e^{i(\bar{q}-p)x}\, \bar{u}(\bar{q}) \Bigg[(ig\, t^a) \; \Big[\partial_{x^+} U^{\dagger}_{AP}(x_t, x^+)\Big]^{ab} \\
& \Big[n \cdot (p-\bar{q}) \not{A}_b(x) - (p-\bar{q}) \cdot A_b(x) \not{n} \Big] \Bigg] u(p)
\end{aligned}
$$

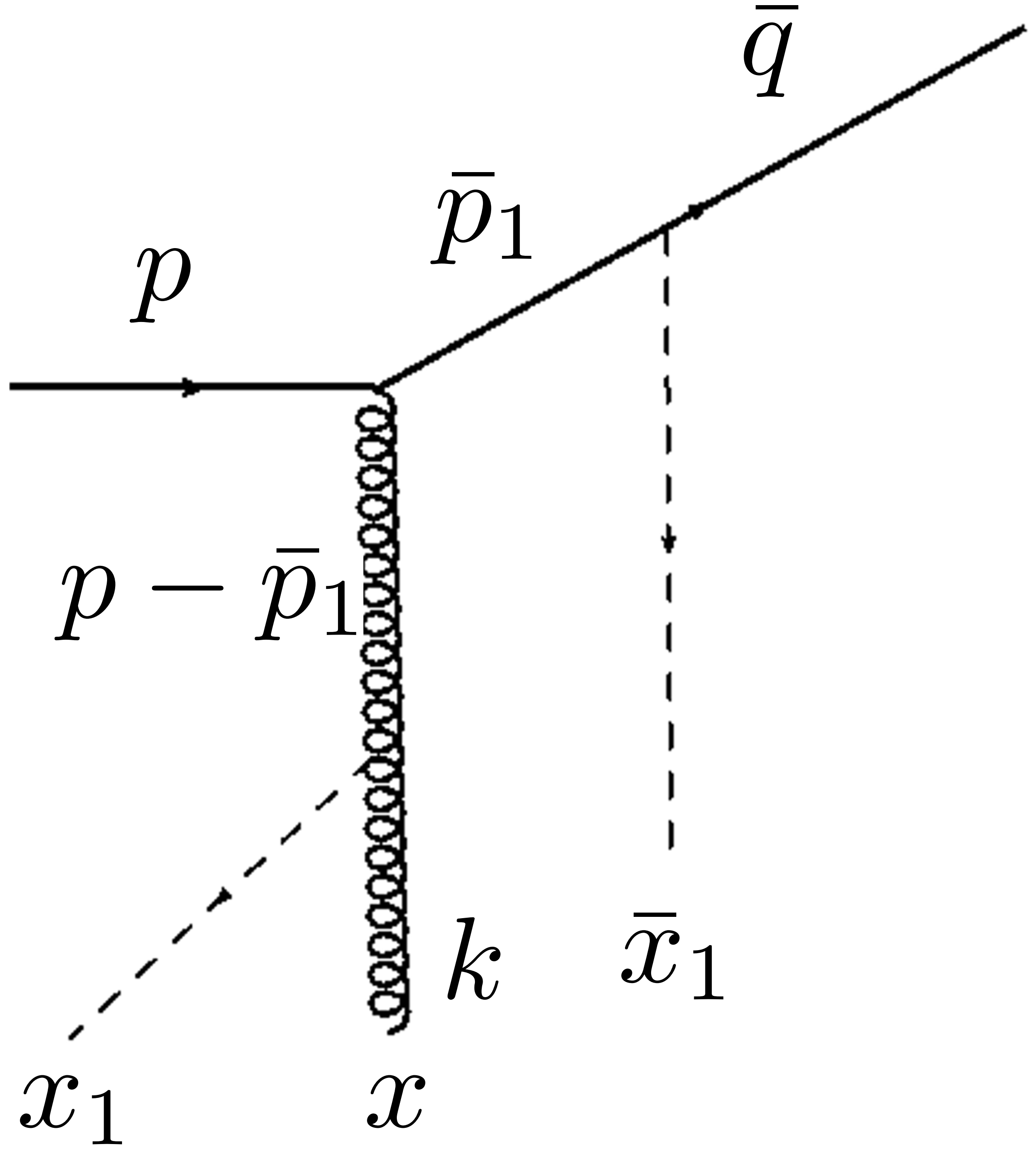
with

$$
U_{AP}(x_t, x^+) \equiv \hat{P}\, exp \left\{ ig \int_{-\infty}^{x^+} dz^+\, S^-_a(z^+, x_t)\, T_a \right\}
$$



but there is more!

how about the final state quark interactions?



integration over $\bar{p}_1^-$

$$\int \frac{d\bar{p}_1^-}{2\pi} \frac{e^{i\bar{p}_1^-(\bar{x}_1^+ - x^+)}}{[\bar{p}_1^2 + i\epsilon]\,[(p_1 - \bar{p}_1)^2 + i\epsilon]}$$

now the poles are on the opposite side of the real axis, we get both ordering

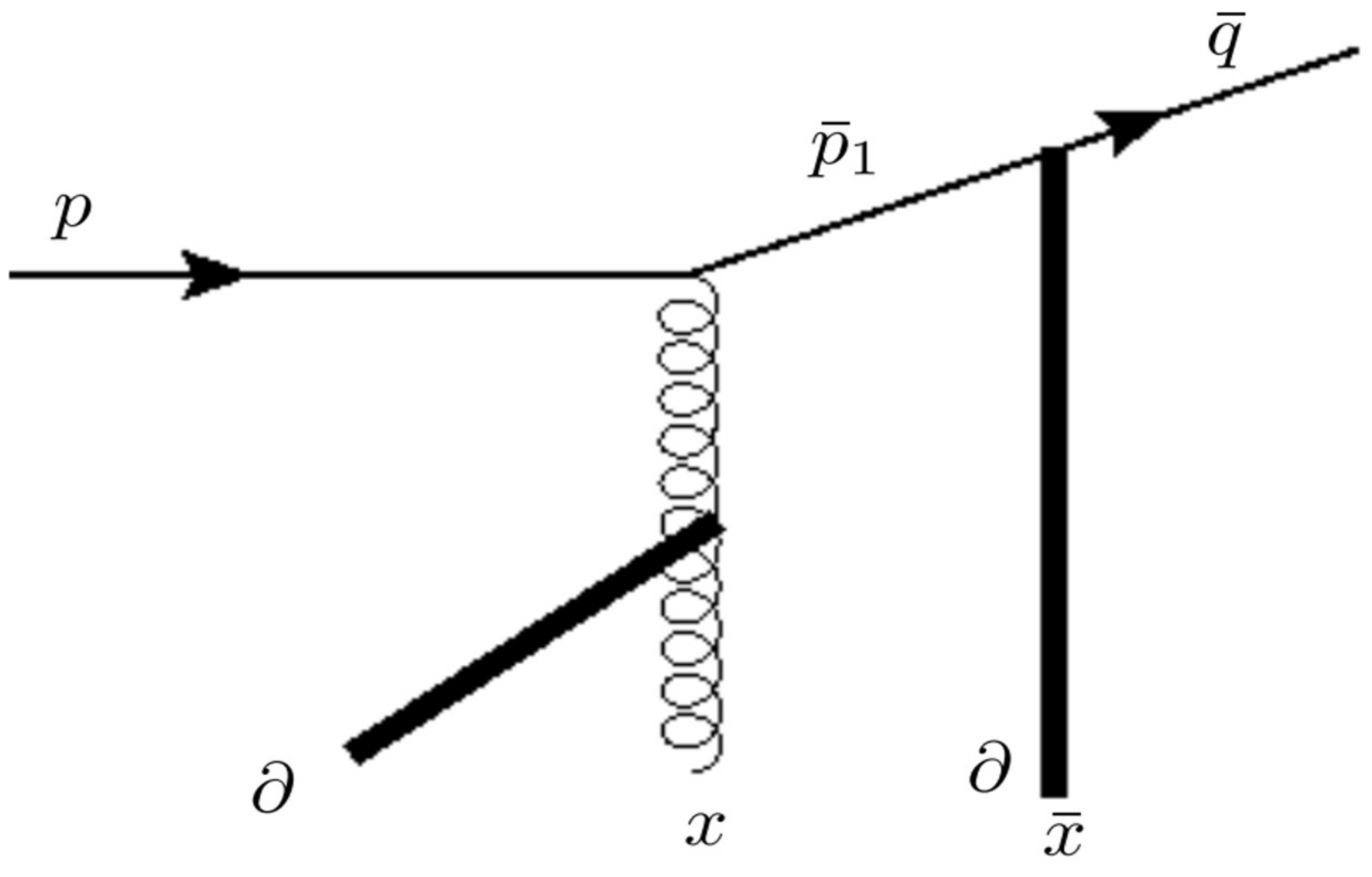
$\theta(x^+ - \bar{x}_1^+)$ and $\theta(\bar{x}_1^+ - x^+)$

ignoring the phases the contribution of the two poles add!

path ordering is lost!

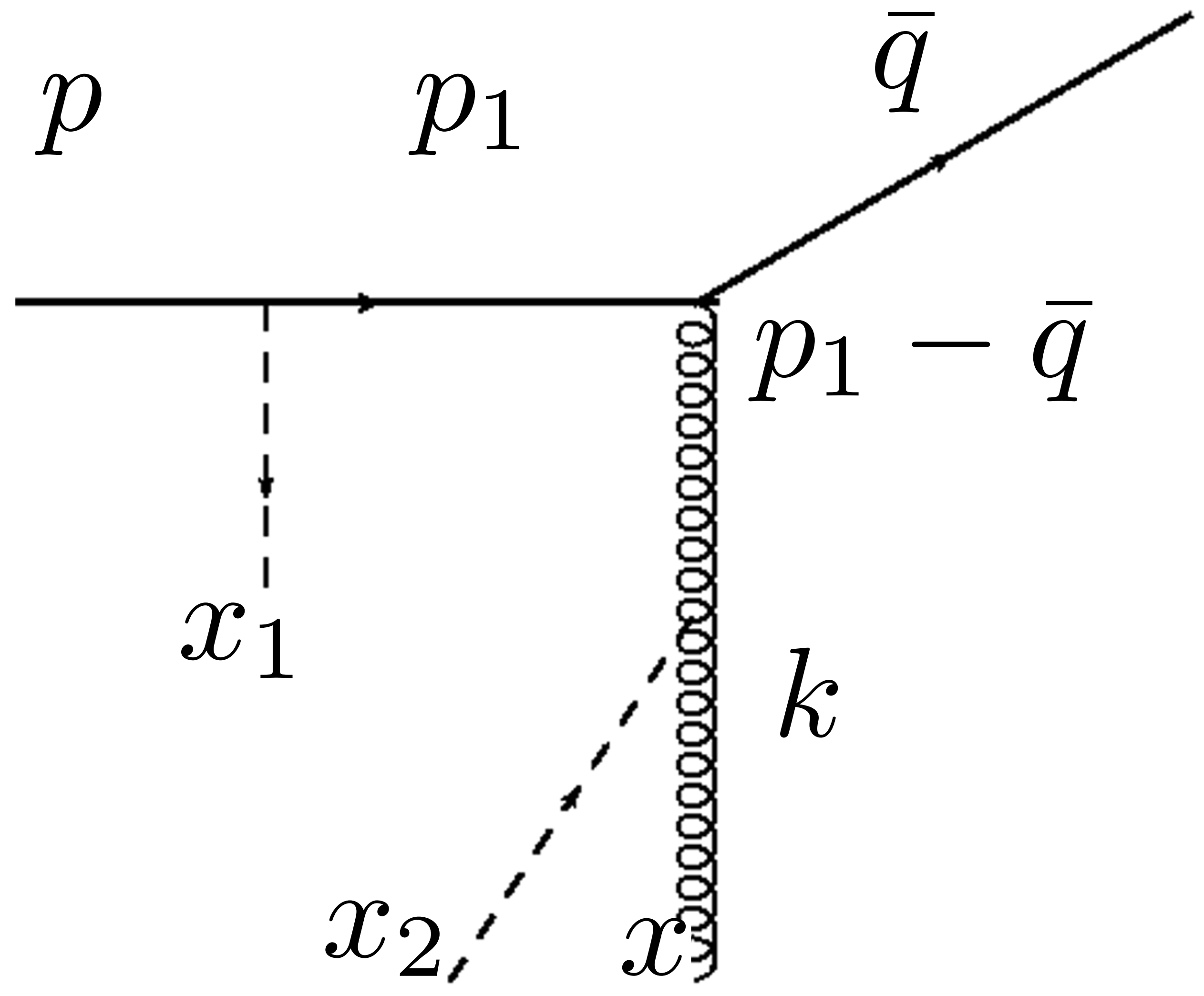
however further rescatterings are still path-ordered before/after $\mathbf{x_1^+}, \bar{\mathbf{x}}_1^+$

rescatterings of hard
gluon and final state quark resum to



$$
\begin{aligned}
i\mathcal{M}_3 \;=\; & -2\,i \int d^4x\, d^2\bar{x}_t\, d\bar{x}^+ \,\frac{d^2\bar{p}_{1t}}{(2\pi)^2}\, e^{i(\bar{q}^+ - p^+)x^-}\; e^{-i(\bar{p}_{1t}-p_t)\cdot x_t}\; e^{-i(\bar{q}_t - \bar{p}_{1t})\cdot \bar{x}_t} \\
& \bar{u}(\bar{q}) \Bigg[\left[\partial_{\bar{x}^+}\, \overline{V}_{AP}(\bar{x}^+, \bar{x}_t)\right] \not{n}\, \not{\bar{p}}_1\, (igt^a) \left[\partial_{x^+}\, U^\dagger_{AP}(x_t, x^+)\right]^{ab} \\
& \frac{\left[n\cdot(p-\bar{q})\not{A}^b(x) - (p-\bar{p}_1)\cdot A^b(x)\,\not{n}\right]}{\left[2n\cdot\bar{q}\, 2n\cdot(p-\bar{q})\, p^- - 2n\cdot(p-\bar{q})\,\bar{p}_{1t}^2 - 2n\cdot\bar{q}\,(\bar{p}_{1t}-p_t)^2\right]} \Bigg] u(p)
\end{aligned}
$$

both initial state quark and hard gluon interacting:



integration over p_1^-

$$\int \frac{dp_1^-}{2\pi} \frac{e^{ip_1^-(x_1^+ - x^+)}}{[p_1^2 + i\epsilon]\,[(p_1 - \bar{q})^2 + i\epsilon]}$$

both poles are below the real axis, we get

$$\frac{e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x^+)}}{\left[\frac{p_{1t}^2}{2p^+} - \bar{q}^- - \frac{(p_{1t}-\bar{q}_t)^2}{2(p^+-\bar{q}^+)}\right]} + \frac{e^{i\left[\bar{q}^- + \frac{(p_{1t}-\bar{q}_t)^2}{2(p^+-\bar{q}^+)}\right](x_1^+ - x^+)}}{\left[\bar{q}^- + \frac{(p_{1t}-\bar{q}_t)^2}{2(p^+-\bar{q}^+)} - \frac{p_{1t}^2}{2p^+}\right]}$$

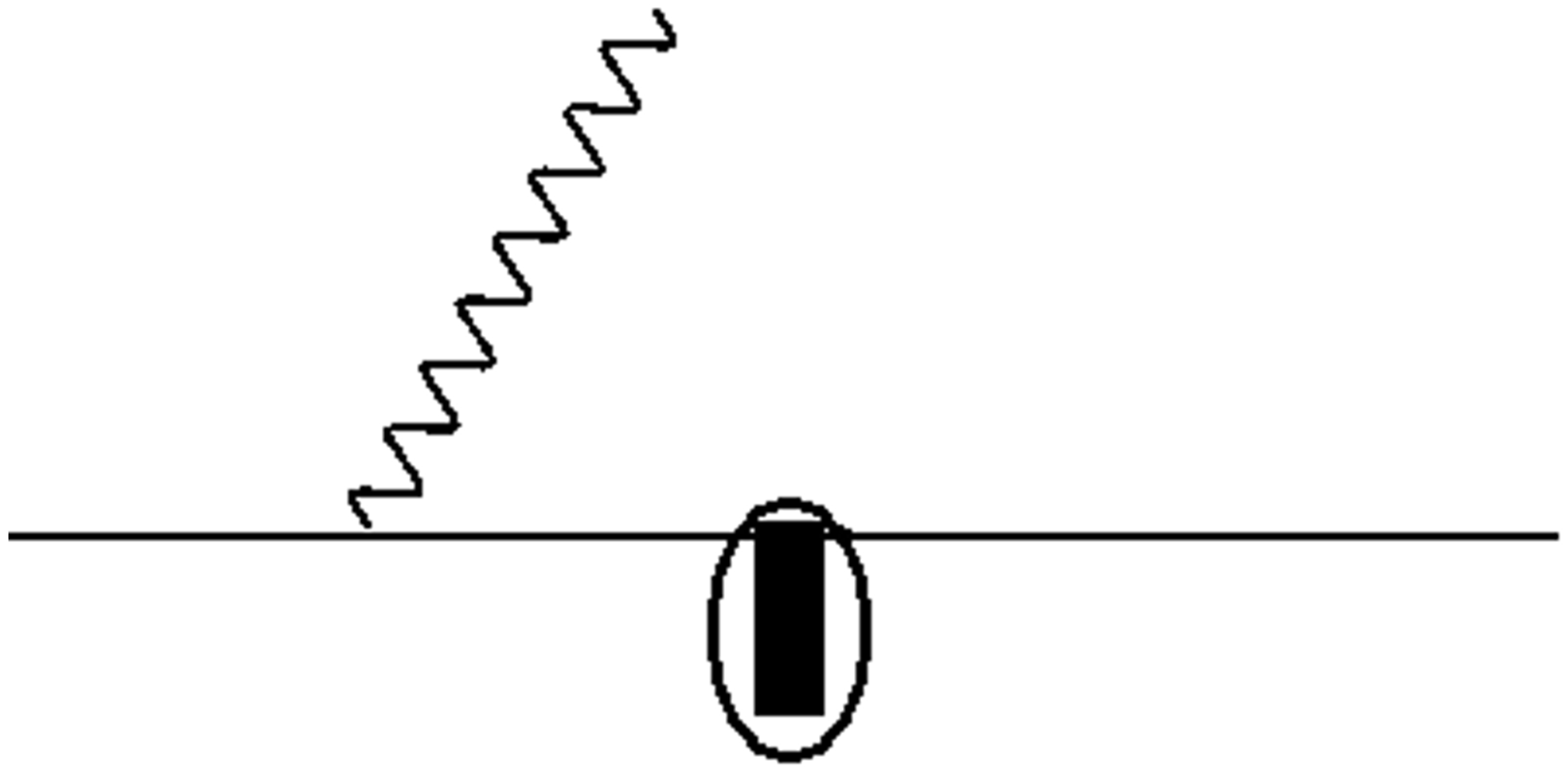
ignoring phases we get a cancellation!

this can be shown to hold to all orders whenever both initial state quark and hard gluon scatter from the soft fields!

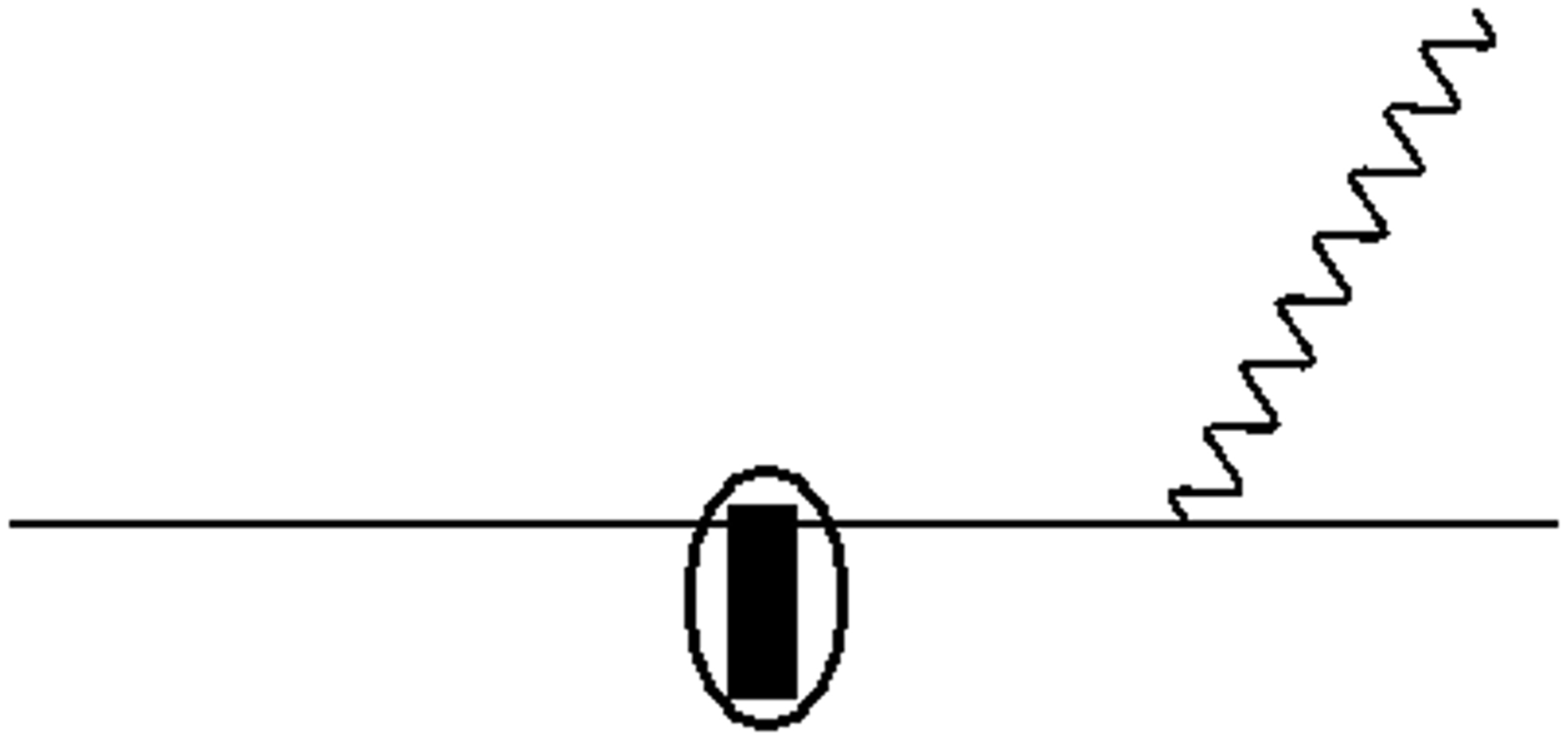
$$
\begin{aligned}
i\mathcal{M}_{eik}(p,q) &= 2\pi\delta(p^+ - q^+) \int d^2x_t\, e^{-i(q_t-p_t)\cdot x_t}\ [V(x_t)-1]\ \mathcal{N}_{eik} \\
i\mathcal{M}_1(p,q) &= \int d^4x\, d^2z_t\, d^2\bar{z}_t \int \frac{d^2k_t}{(2\pi)^2}\, \frac{d^2\bar{k}_t}{(2\pi)^2}\, e^{i(\bar{k}-k)x}\, e^{-i(\bar{q}_t-\bar{k}_t)\cdot\bar{z}_t}\, e^{-i(k_t-p_t)\cdot z_t} \\
&\quad \overline{V}(x^+,\bar{z}_t)\,(ig\,t^b)\,V(z_t,x^+)\,\mathcal{N}_1^b \\
i\mathcal{M}_2(p,q) &= 2\,i \int d^4x\, e^{i(\bar{q}^+-p^+)x^- - i(\bar{q}_t-p_t)\cdot x_t}\,(ig\,t^a)\ \left[\partial_{x^+} U^\dagger(x_t,x^+)\right]^{ab}\ \mathcal{N}_2^b \\
i\mathcal{M}_3(p,q) &= -2\,i \int d^4x\, d^2\bar{x}_t\, d\bar{x}^+\, \frac{d^2\bar{p}_{1t}}{(2\pi)^2}\, e^{i(\bar{p}_1^+-p^+)x^-}\, e^{-i(\bar{p}_{1t}-p_t)\cdot x_t}\, e^{-i(\bar{q}_t-\bar{p}_{1t})\cdot\bar{x}_t} \\
&\quad \left[\partial_{\bar{x}^+}\overline{V}(\bar{x}^+,\bar{x}_t)\right]\ (ig\,t^a)\ \left[\partial_{x^+} U^\dagger(x_t,x^+)\right]^{ab}\ \mathcal{N}_3^b
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{N}_{eik} &= \bar{u}(q)\,\not{n}\,u(p) \\
\mathcal{N}_1^b &= \frac{1}{2k^+}\,\frac{1}{2\bar{k}^+}\,\bar{u}(\bar{q})\ \left[\not{\bar{n}}\,\not{\bar{k}}\not{A}^b(x)\,\not{k}\,\not{n}\right]\ u(p) \\
\mathcal{N}_2^b &= \frac{1}{(p-\bar{q})^2}\bar{u}(\bar{q})\ \left[n\cdot(p-\bar{q})\not{A}^b(x) - (p-\bar{q})\cdot A^b(x)\,\not{n}\right]\ \Big]\,u(p) \\
\mathcal{N}_3^b &= \frac{1}{2\bar{n}\cdot\bar{p}_1(p-\bar{p}_1)^2}\bar{u}(\bar{q})\left[\not{\bar{n}}\,\not{\bar{p}}_1\left(n\cdot(p-\bar{p}_1)\not{A}^b(x) - (p-\bar{p}_1)\cdot A^b(x)\,\not{n}\right)\right]u(p)
\end{aligned}
$$

photon production: small x



before quark scatters on the target



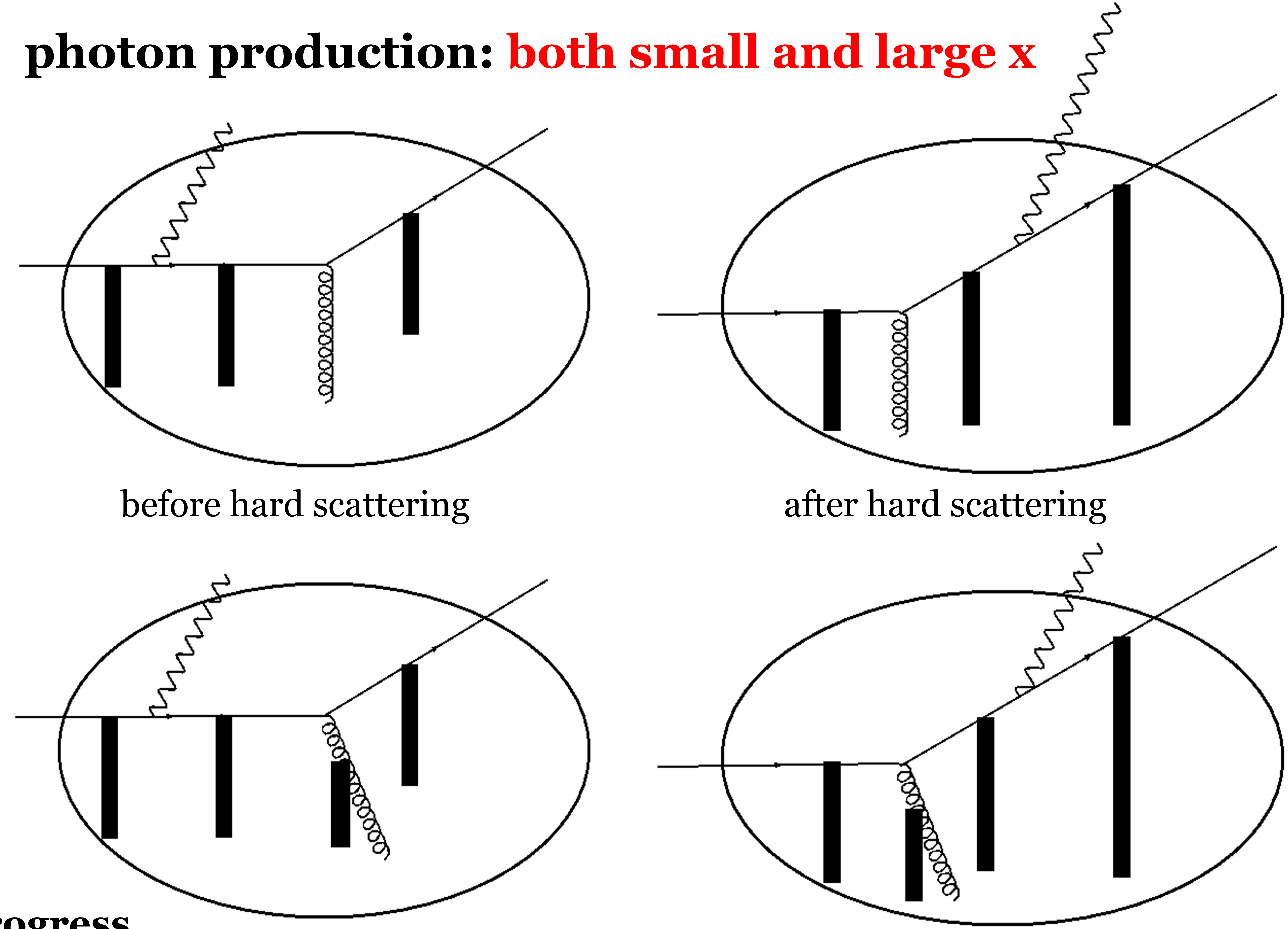
after quark scatters on the target

No radiation inside the target

FG-JJM, PRD66 (2002) 014021
JJM, EPJC61 (2009) 789
JJM-AR, arXiv:1204.1319

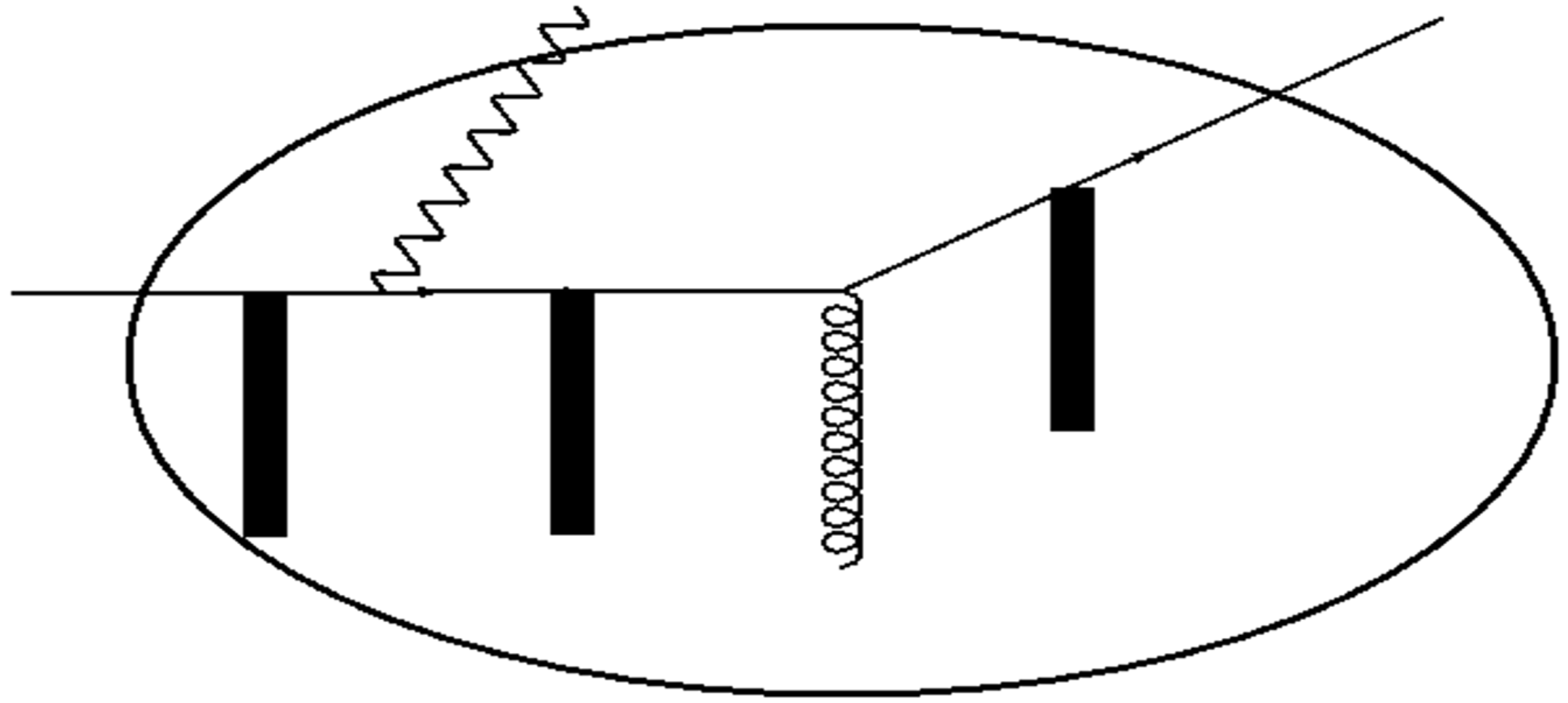
Kopeliovich et al.,
Rezaeian 2010

photon production: both small and large x



JJM, in progress

photon radiation: helicity amplitudes



$$i\mathcal{M}_1(p,q,l) =$$

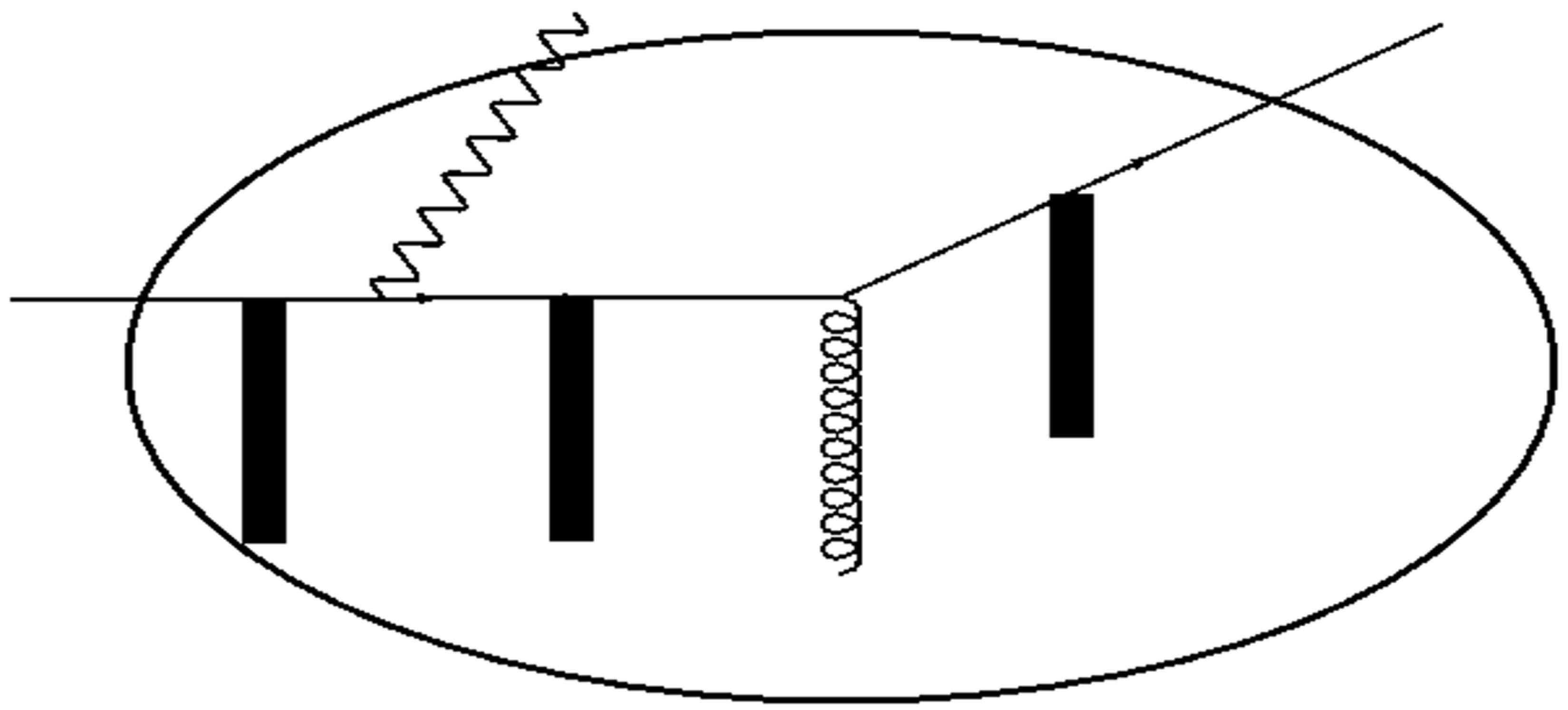
$$eg \int \frac{d^2k_{2t}}{(2\pi)^2}\frac{d^2k_{3t}}{(2\pi)^2}\frac{d^2\bar{k}_{1t}}{(2\pi)^2}\int d^4x\, d^2y_{1t}\, d^2y_{2t}\, d^2\bar{y}_{1t}\, dz^+\,\theta(x^+ - z^+)\, e^{i(l^+ + \bar{q}^+ - p^+)x^-}$$

$$e^{-i(\bar{q}_t - \bar{k}_{1t})\cdot\bar{y}_{1t}}\, e^{-i(\bar{k}_{1t} - k_{3t})\cdot x_t}\, e^{-i(k_{3t} - k_{2t})\cdot y_{2t}}\, e^{-i(l_t + k_{2t} - p_t)\cdot y_{1t}}\, \bar{u}(\bar{q})\,\overline{V}(\bar{y}_{1t}; x^+, \infty)\,\frac{\not{\bar{n}}\,\not{\bar{k}}_1}{2\bar{n}\cdot\bar{q}}$$

$$\not{A}(x)\left[\frac{\not{k}_3}{2n\cdot(p-l)}\,V(y_{2t}; z^+, x^+)\,\frac{\not{n}\,\not{k}_2}{2n\cdot(p-l)} + i\,\frac{\delta(x^+ - z^+)}{2n\cdot(p-l)}\not{n}\right]$$

$$\not{\epsilon}(l)\,\frac{\not{k}_1}{2n\cdot p}\,V(y_{1t}; -\infty, z^+)\,\not{n}\,u(p)$$

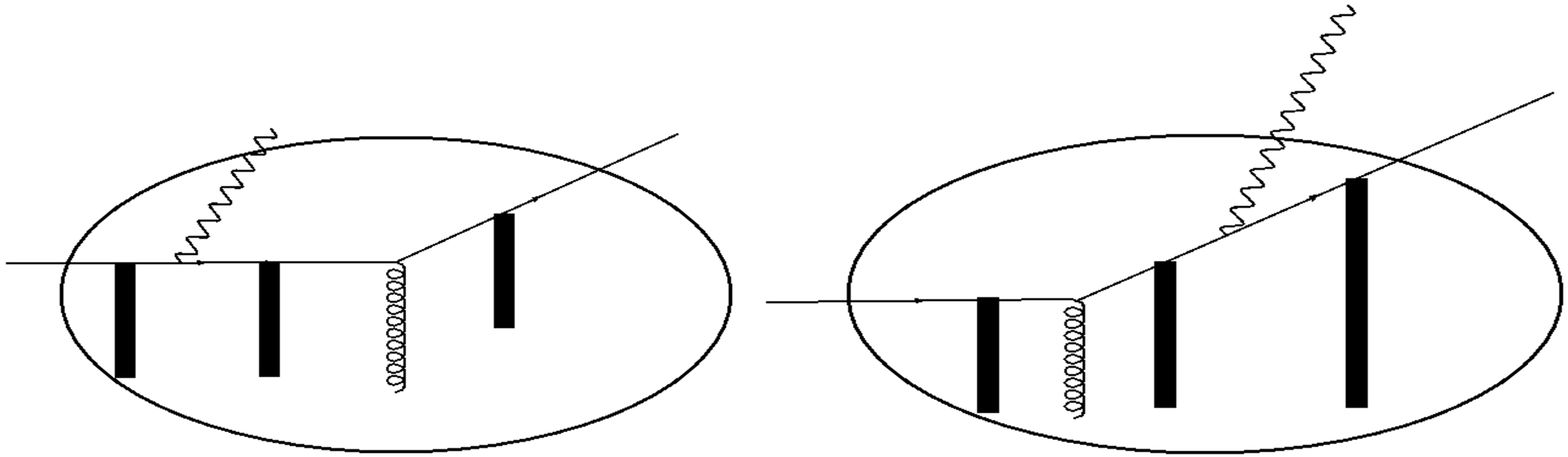
photon production: both small and large x



$$
\begin{aligned}
\mathcal{N}_{1-1} &= \bar{u}(\bar{q}) \frac{\not{\bar{n}}\, \not{\bar{k}}_1}{2\bar{n}\cdot\bar{q}} \not{A}(x) \frac{\not{k}_3 \not{n} \not{k}_2 \not{\epsilon}(l) \not{k}_1 \not{n}}{2n\cdot p\, 2n\cdot(p-l)\, 2n\cdot(p-l)} u(p) \\
\mathcal{N}_{1-2} &= \bar{u}(\bar{q}) \frac{\not{\bar{n}}\, \not{\bar{k}}_1}{2\bar{n}\cdot\bar{q}} \not{A}(x) \frac{\not{n} \not{\epsilon}(l) \not{k}_1 \not{n}}{2n\cdot p\, 2n\cdot(p-l)} u(p)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{N}^{++}_{1-1} &= (\mathcal{N}^{--}_{1-1})^\star = -\sqrt{\frac{n\cdot p}{n\cdot(p-l)}} \frac{[n\cdot l\, k_{2\perp}\cdot\epsilon^\star_\perp - n\cdot(p-l)\, l_\perp\cdot\epsilon^\star_\perp]}{n\cdot l\, n\cdot(p-l)} \langle \bar{k}^+_1 | \not{A}(x) | k^+_3 \rangle \\
\mathcal{N}^{++}_{1-2} &= (\mathcal{N}^{--}_{1-2})^\star = -\sqrt{\frac{n\cdot p}{n\cdot(p-l)}} \langle \bar{k}^+_1 | \not{A}(x) | n^+ \rangle \\
\mathcal{N}^{+-}_{1-1} &= (\mathcal{N}^{-+}_{1-1})^\star = -\sqrt{\frac{n\cdot p}{n\cdot(p-l)}} \frac{[n\cdot p\, l_\perp\cdot\epsilon_\perp - n\cdot l\, k_{1\perp}\cdot\epsilon_\perp]}{n\cdot p\, n\cdot l} \langle \bar{k}^+_1 | \not{A}(x) | k^+_3 \rangle \\
\mathcal{N}^{+-}_{1-2} &= \mathcal{N}^{-+}_{1-2} = 0
\end{aligned}
$$

pQCD limit (large x: gluon PDF X partonic cross section):

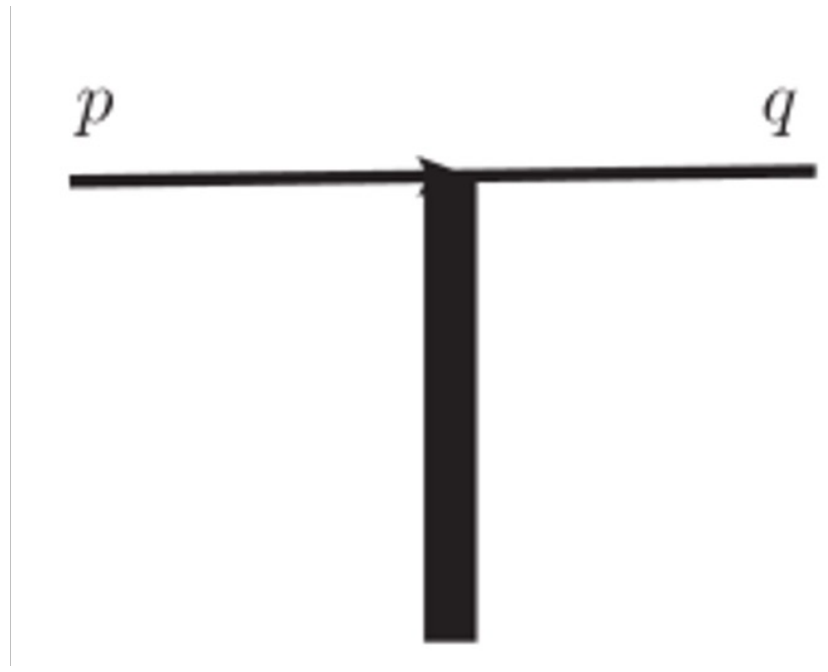


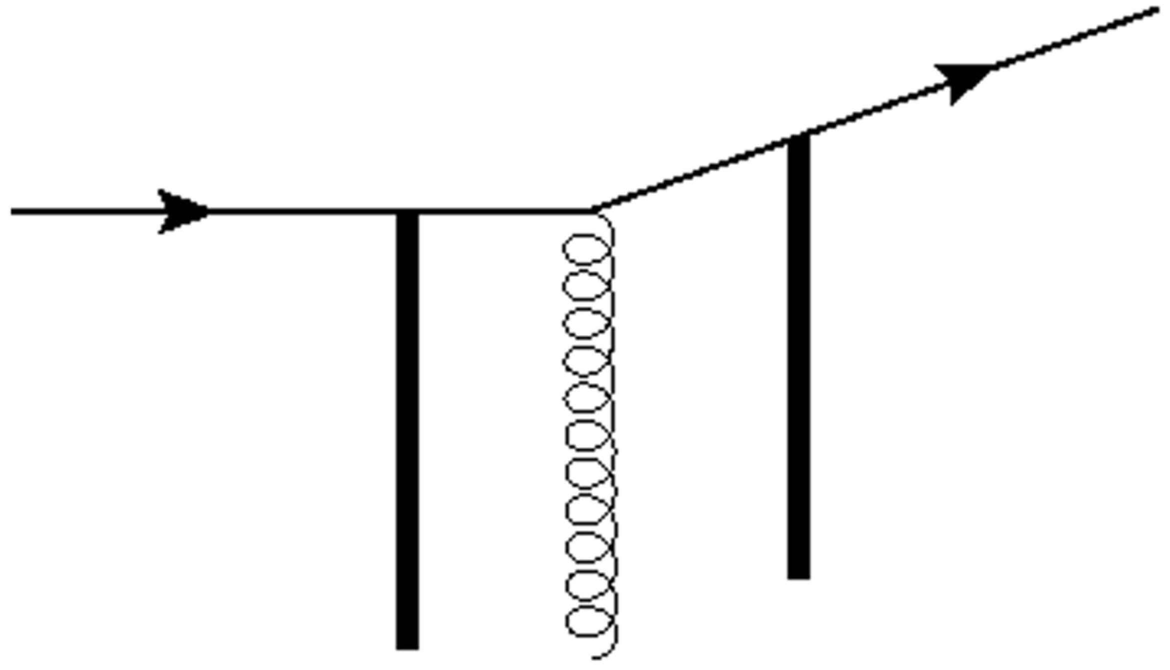
V = U = 1

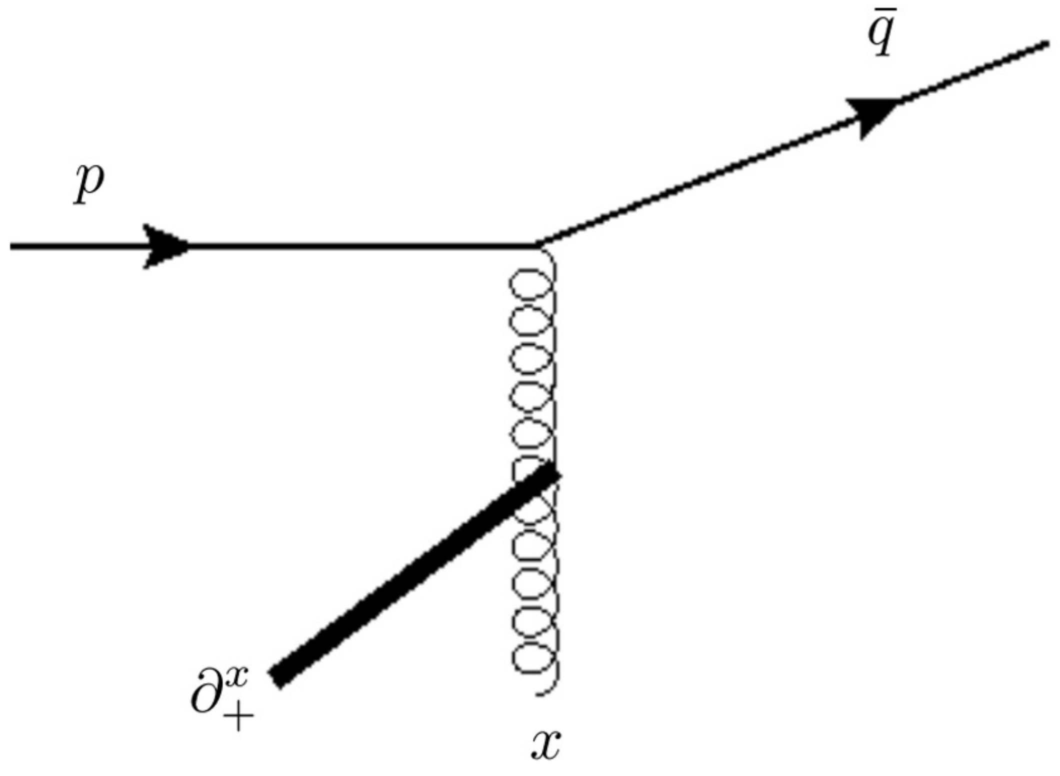
Summary II

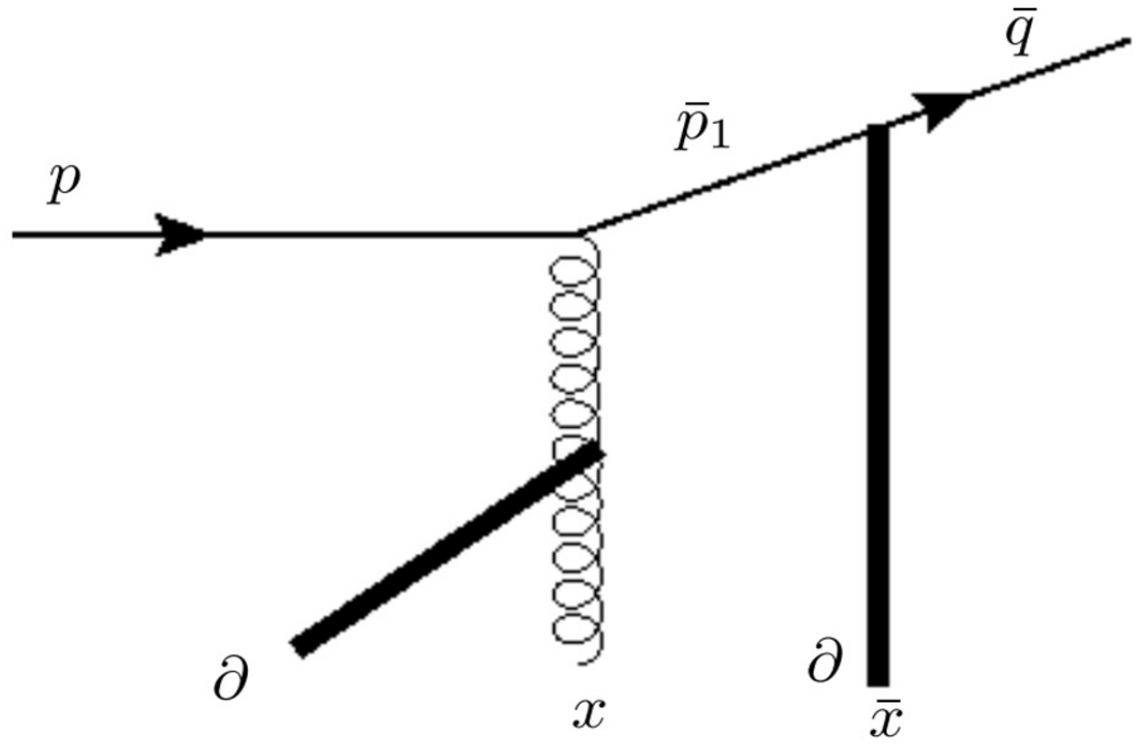
- **EIC will have a large arm in x**
- **EIC will be able to probe the small x - large x transition region**
- **significant progress in relating various approaches**
- **need a unifying framework!**

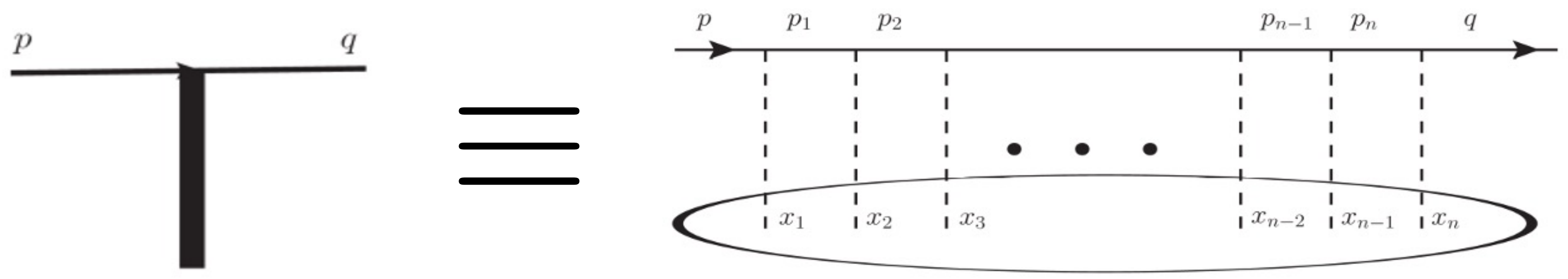
full amplitude: $\mathbf{i\mathcal{M} = i\mathcal{M}_{eik} + i\mathcal{M}_1 + i\mathcal{M}_2 + i\mathcal{M}_3}$









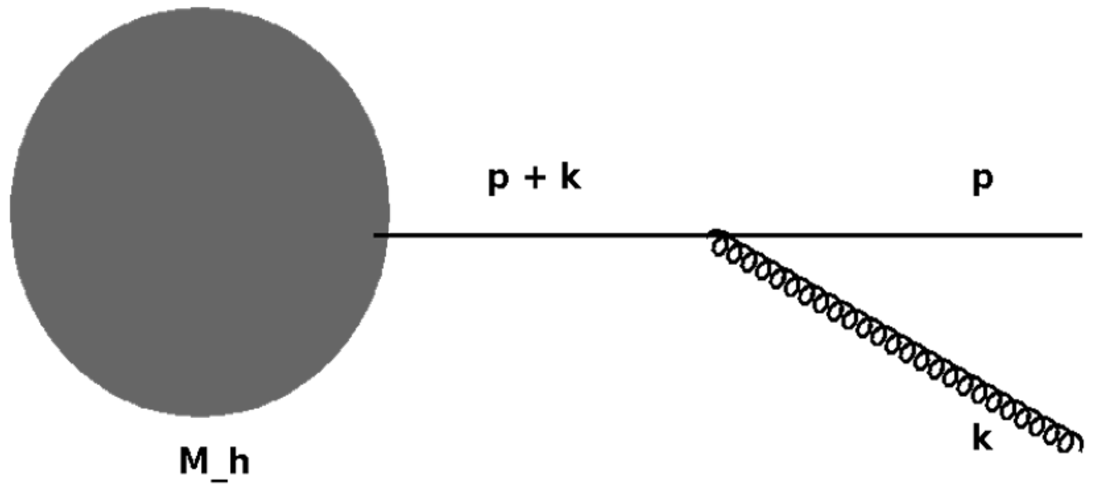


soft (eikonal) limit: $A^\mu(x) \to n^- \, S(x^+, x_t) \quad n \cdot \bar{q} \to n \cdot p$

$$i\mathcal{M} \longrightarrow i\mathcal{M}_{eik}$$

<u>One-loop</u> corrections: soft gluon radiation (k << p)

consider a very energetic quark emerging from a hard process M_h



$$\begin{aligned} M &= \bar{u}(p)\, igt^a \not{\epsilon}^\lambda(k)\, \frac{i(\not{p}+\not{k})}{(p+k)^2+i\epsilon} M_h \\ &\simeq -2gt^a \bar{u}(p)\, \epsilon^\lambda(k)\cdot p\, \frac{1}{2p\cdot k} M_h \\ &\simeq -2gt^a \frac{\vec{k}_\perp \cdot \vec{\epsilon}^{\,\lambda}_\perp}{k_\perp^2}\, \bar{u}(p)\, M_h \end{aligned}$$

using

$$\not{\epsilon}\not{p} = -\not{p}\not{\epsilon} + 2\epsilon\cdot p$$

$$\bar{u}(p)\not{p} = 0$$

$$\epsilon^\lambda_\mu(k) = \left(0, \frac{\vec{k}_\perp\cdot\vec{\epsilon}^{\,\lambda}_\perp}{k^+}, \vec{\epsilon}^{\,\lambda}_\perp\right)$$

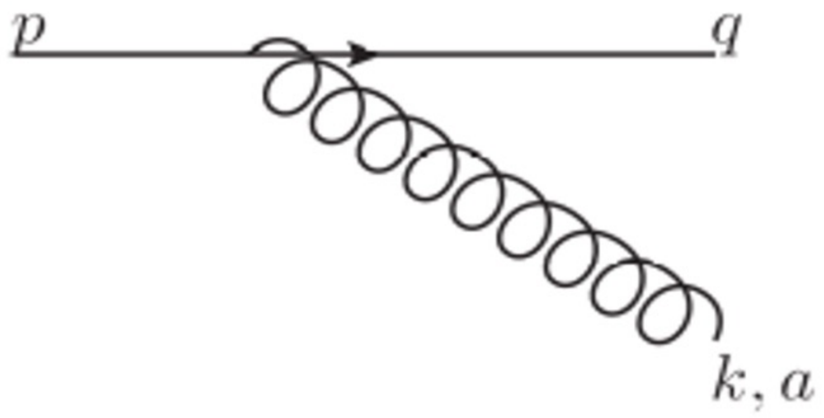
in coordinate space

$$\int \frac{d^2k_\perp}{(2\pi)^2} e^{ik_\perp\cdot(x_\perp - z_\perp)} \frac{\epsilon^\lambda_\perp \cdot k_\perp}{k_\perp^2} = \frac{i}{2\pi} \frac{\epsilon^\lambda_\perp\cdot(x_\perp - z_\perp)}{(x_\perp - z_\perp)^2}$$

$\mathbf{x}_\perp$, $\mathbf{z}_\perp$ are coordinates of quark and gluon

1-loop correction: energy dependence

basic ingredient: soft radiation vertex (LC gauge)



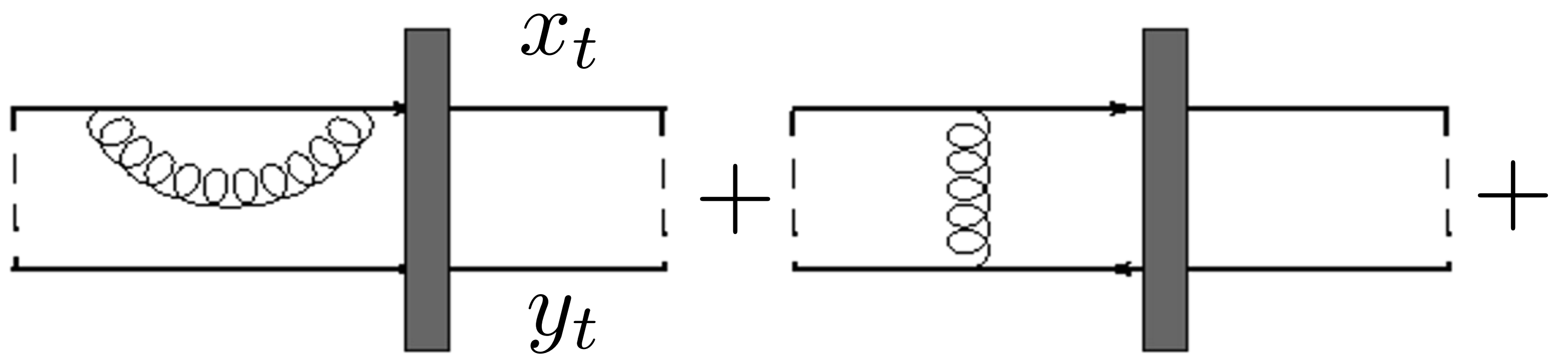
$$g\,\bar{u}(q)\,t^a\,\gamma_\mu\,u(p)\,\epsilon^{\mu}_{(\lambda)}(k) \longrightarrow 2\,g\,t^a\,\frac{\epsilon_{(\lambda)}\cdot k_t}{k_t^2}$$

coordinate space:

$$\int \frac{d^2k_t}{(2\pi)^2} e^{ik_t\cdot(x_t-z_t)} 2\,g\,t^a\,\frac{\epsilon_{(\lambda)}\cdot k_t}{k_t^2} = \frac{2\,i\,g}{2\pi}\,t^a\,\frac{\epsilon_{(\lambda)}\cdot(x_t-z_t)}{(x_t-z_t)^2}$$

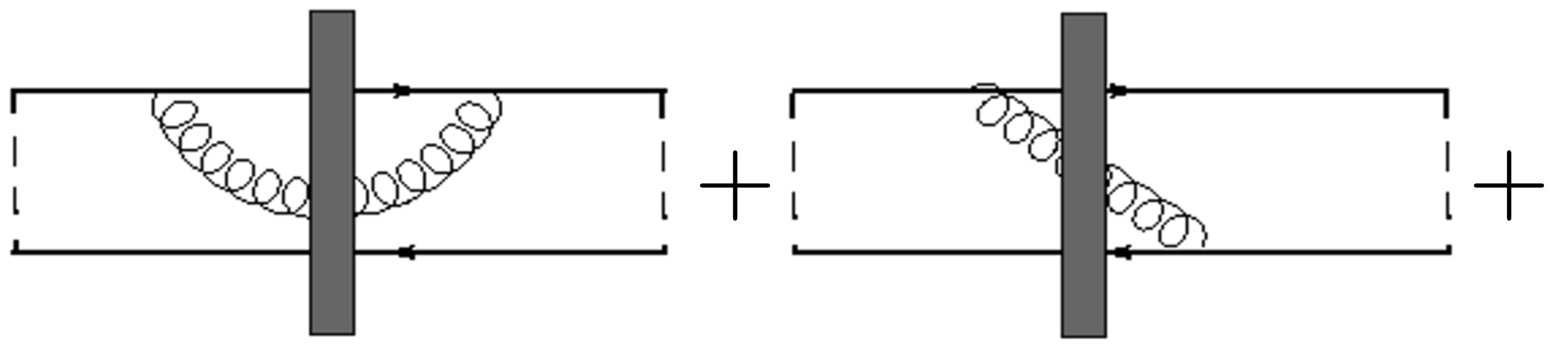
x_t, z_t are transverse coordinates of the quark and gluon

virtual corrections:



$$\longrightarrow Tr\,V(x_t)\,V^\dagger(y_t)$$ *a dipole*

real corrections:



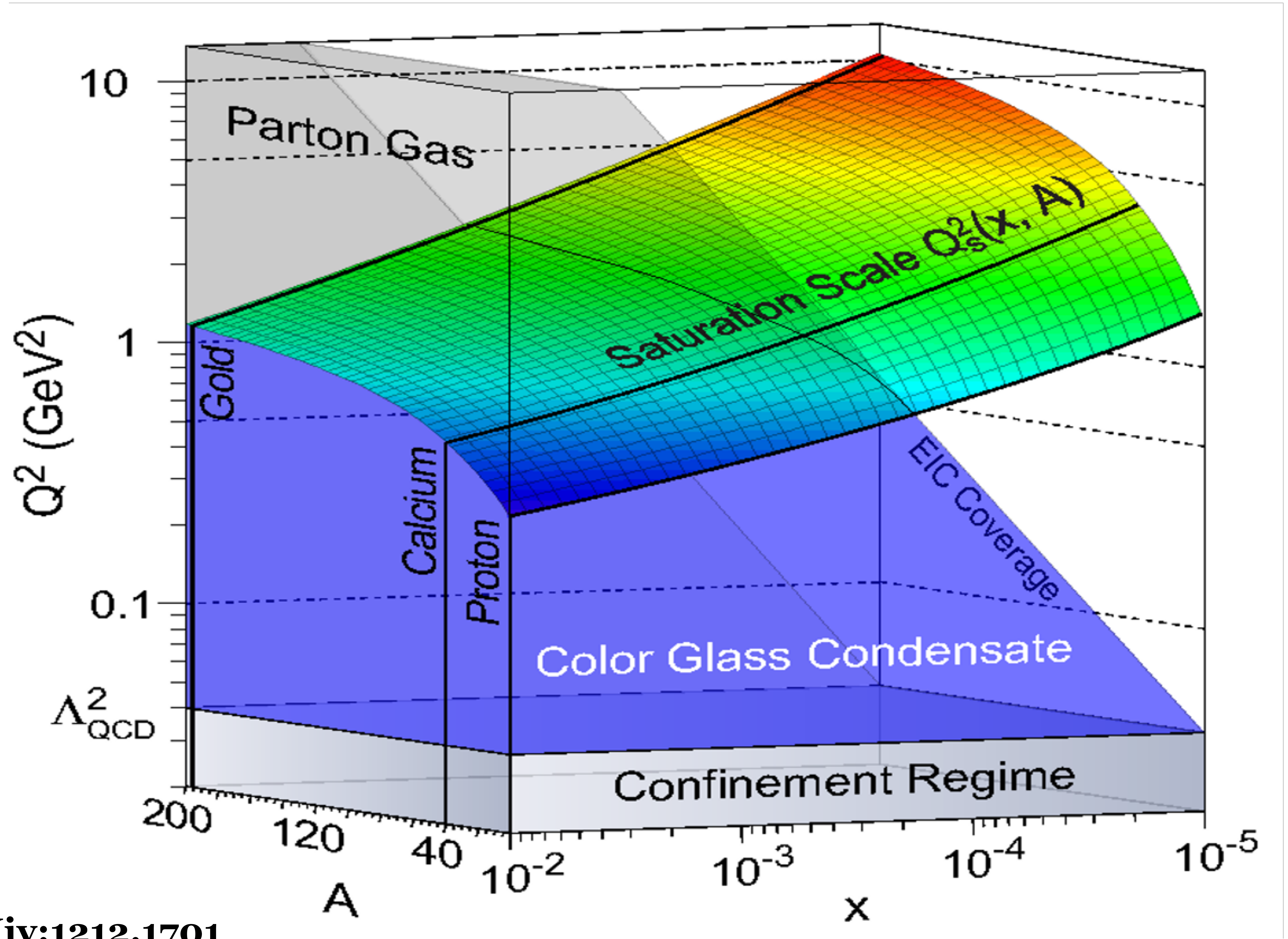
$$\longrightarrow Tr\,V(x_t)\,V^\dagger(z_t)\,Tr\,V(z_t)\,V^\dagger(y_t)$$

$$\frac{1}{(x_t-z_t)^2} \qquad \frac{(x_t-z_t)\cdot(y_t-z_t)}{(x_t-z_t)^2(y_t-z_t)^2}$$

the S matrix

$$S(x_t,y_t) \equiv \frac{1}{N_c} Tr\,V(x_t)\,V^\dagger(y_t)$$

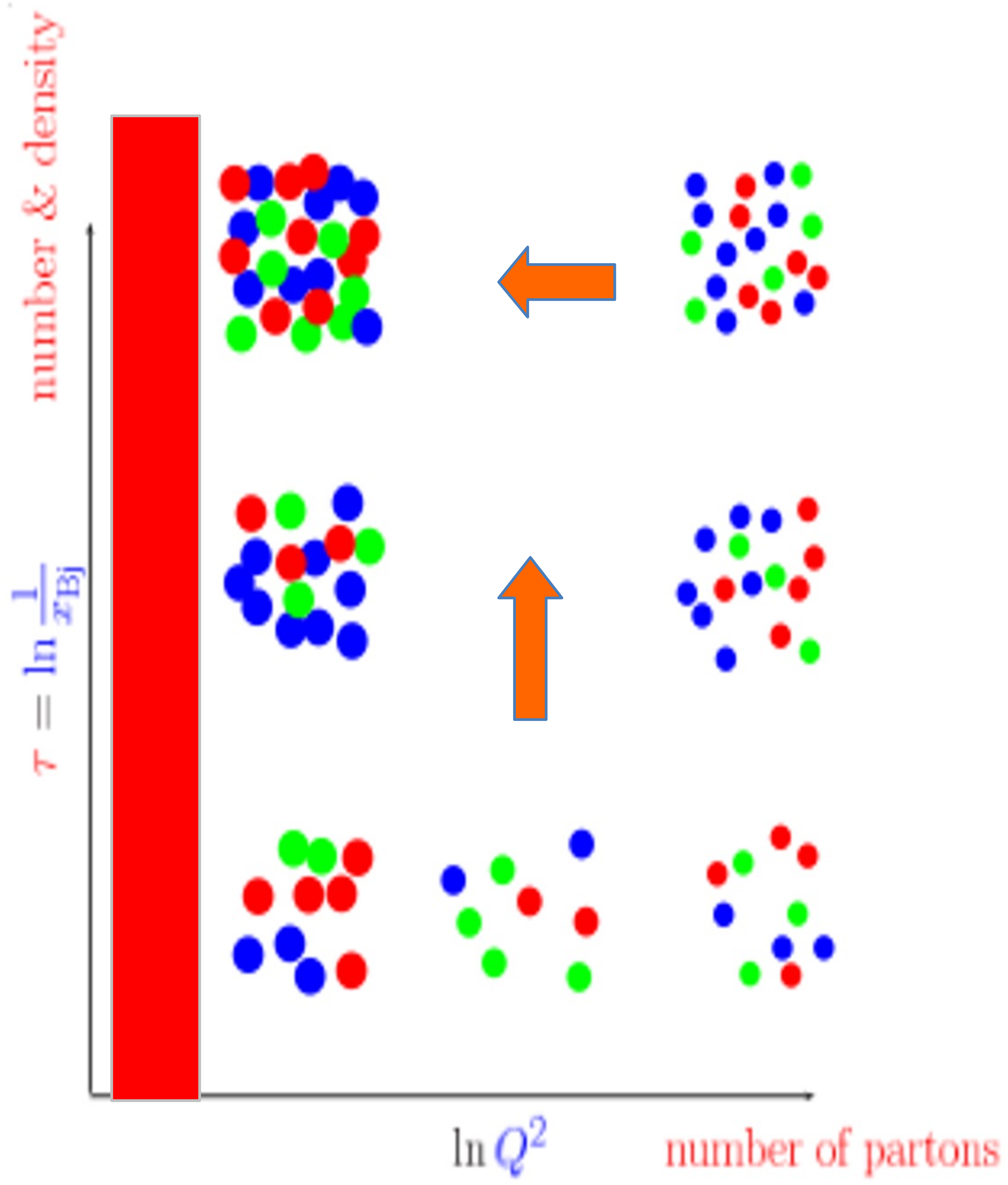
The saturation scale



x 9/4 for gluons

A. Accardi et al., arXiv:1212.1701

QCD at small x:
many-body dynamics of universal gluonic matter (CGC)



How does this happen ?

How do correlation functions evolve ?

Are there scaling laws ?

Can CGC explain aspects of HIC ?

Initial conditions for hydro?
Thermalization ?

Back to back hadron production in pA collisions: forward rapidity

STAR collaboration(2021)
arXiv:2111.10396