Temperature dependence of shear viscosity of SU(3) gluodynamics within lattice simulation

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Outline:

- Introduction
- Details of the calculation
- Shear viscocity
  - Fitting of the data
  - Backus-Gilbert method
- Bulk viscosity (preliminary results)
- Conclusion

Relativistic Hydrodynamics

- $T^{\mu\nu} = (e+p)u^\mu u^\nu + pg^{\mu\nu} + (\eta\nabla^{\langle\mu}u^{\nu\rangle} + \zeta\Delta^{\mu\nu}\nabla_\alpha u^\alpha) + \ldots$
  $\nabla^\alpha = \Delta^{\alpha\nu}\partial_\nu, \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$
  $\nabla^{\langle\mu}u^{\nu\rangle} = \nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha$

- EOM $\partial_\mu T^{\mu\nu} = 0$

- Non-relativistic limit ($u^\mu = (1, \vec{v})$)

  - *Continuity equation:* $\partial_t\rho + \rho(\vec{\partial}\vec{v}) + \vec{v}\vec{\partial}\rho = 0$
  - *Navier–Stokes equation:* $\frac{\partial v^i}{\partial t} + v^k\frac{\partial v^i}{\partial x^k} = -\frac{1}{\rho}\frac{\partial p}{\partial x^i} - \frac{1}{\rho}\frac{\partial \Pi^{ki}}{\partial x^k}$
  - *Viscous stress tensor:* $\Pi^{ik} = -\eta\left(\frac{\partial v^i}{\partial x^k} + \frac{\partial v^k}{\partial x^i} - \frac{2}{3}\delta^{ik}\frac{\partial v^l}{\partial x^l}\right) - \zeta\delta^{ik}\frac{\partial v^l}{\partial x^l}$

- η–shear viscosity, ζ–bulk viscosity

Calculation of transport coefficients:

- Shear viscosity

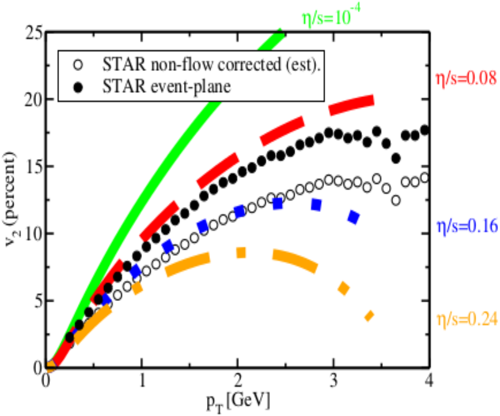
$$\eta = \lim_{\omega\to 0} \frac{1}{\omega} \int_0^\infty dt \int d^3x \; e^{i\omega t} \langle [T_{12}(x), \, T_{12}(0)] \rangle$$
$$\rho_{12,12}(\omega) = -\frac{1}{\pi} Im G_R^{12,12}(\omega, \vec{k}=0)$$
$$\eta = \pi \lim_{\omega\to 0} \frac{1}{\omega} \rho_{12,12}(\omega)$$

- Bulk viscosity

$$\zeta = \frac{1}{9} \lim_{\omega\to 0} \frac{1}{\omega} \int_0^\infty dt \int d^3x \; e^{i\omega t} \langle [T_\mu^\mu(x), \, T_\nu^\nu(0)] \rangle$$
$$\rho_{\mu\mu,\nu\nu}(\omega) = -\frac{1}{\pi} Im G_R^{\mu\mu,\nu\nu}(\omega, \vec{k}=0)$$
$$\zeta = \frac{\pi}{9} \lim_{\omega\to 0} \frac{1}{\omega} \rho_{\mu\mu,\nu\nu}(\omega)$$

- Analytic continuation

$$G_E(\omega, \vec{p}) = -G_R(i\omega, \vec{p}), \quad \omega > 0$$
$$C_E(t) = \int_0^\infty d\omega \rho(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega t\right)}{sh\left(\frac{\omega}{2T}\right)}$$

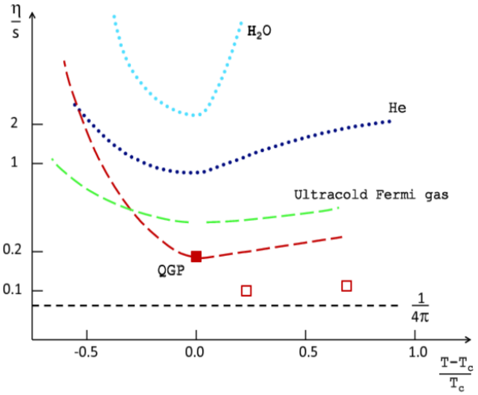


Elliptic flow from STAR experiment (Nucl. Phys. A 757, 102 (2005))

$$\frac{dN}{d\phi} \sim (1 + 2v_1 cos(\phi) + 2v_2 cos^2(\phi))$$

Quark-gluon plasma is close to ideal liquid $(\frac{\eta}{s} = (1-3)\frac{1}{4\pi})$

M. Luzum and P. Romatschke, Phys. Rev. C 78, 034915 (2008)

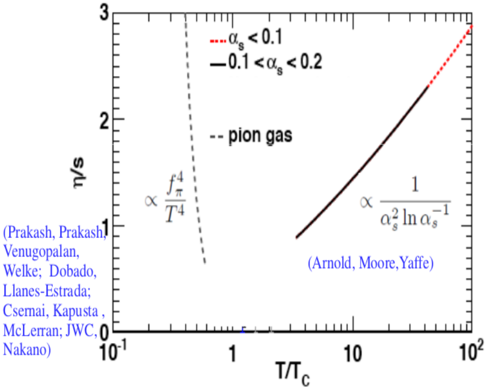


S.Cremonini, U.Gursoy, P.Szepietowski, JHEP 1208 (2012) 167

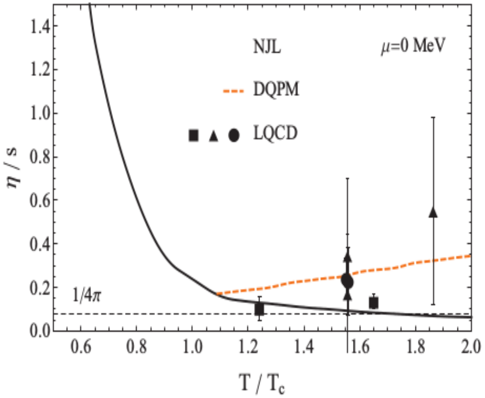
Comparison of different liquids

Quark-gluon plasma is the most ideal liquid

Study of shear viscosity (perturbative calculation + pion gas)



Study of shear viscosity (effective models)



R. Marty, E. Bratkovskaya, W. Cassing, J. Aichelin and H. Berrehrah, Phys. Rev. C 88, 045204 (2013)

Other works (SU(3) gluodynamics):

- Karsch, F. et al. Phys.Rev. D35 (1987)
- A. Nakamura, S. Sakai Phys. Rev. Lett. 94, 072305 (2005)
- H. B. Meyer, Phys.Rev. D76 (2007) 101701
- H. B. Meyer, Nucl.Phys. A830 (2009) 641C-648C

Results:

- $\frac{\eta}{s} = 0.134 \pm 0.033$ $(T/T_c = 1.65, 8 \times 28^3)$
- $\frac{\eta}{s} = 0.102 \pm 0.056$ $(T/T_c = 1.24, 8 \times 28^3)$
- $\frac{\eta}{s} = 0.20 \pm 0.03$ $(T/T_c = 1.58, 16 \times 48^3)$
- $\frac{\eta}{s} = 0.26 \pm 0.03$ $(T/T_c = 2.32, 16 \times 48^3)$

SU(2) gluodynamics:

- $\frac{\eta}{s} = 0.134 \pm 0.057$ $(T/T_c = 1.2, 16 \times 32^3)$

N.Yu. Astrakhantsev, V.V. Braguta, A.Yu. Kotov, JHEP 1509 (2015) 082

Lattice calculation of shear viscocity

The first step:

Measurement of the correlation function:

$$C_E(t) = \langle T_{12}(t) T_{12}(0) \rangle$$

The second step:

Calculation of the spectral function $\rho(\omega)$:

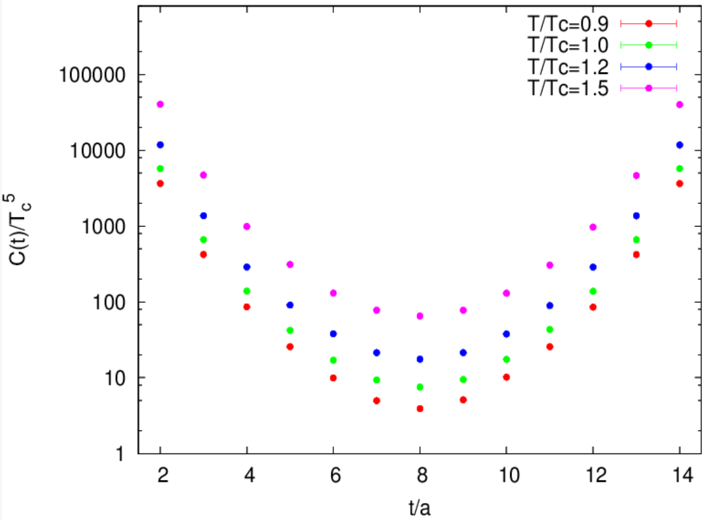
$$C_E(t) = \int_0^\infty d\omega \rho(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega t\right)}{sh\left(\frac{\omega}{2T}\right)}$$

$$\eta = \pi \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

Details of the calculation

- SU(3) gluodynamics
- Two-level algorithm
- Lattice size $32^3 \times 16$
- Temperatures $T/T_c = 0.9, 0.925, 0.95, 1.0, 1.1, 1.2, 1.35, 1.5$
- Accuracy $\sim 2-3\%$ at $t = \frac{1}{2T}$
- $\langle T_{12}(x)T_{12}(y)\rangle \sim (\langle T_{11}(x)T_{11}(y)\rangle - \langle T_{11}(x)T_{22}(y)\rangle)$
- Clover discretization for the $\hat{F}_{\mu\nu}$
- Renormalization of EMT: F. Karsch, Nucl.Phys. B205 (1982) 285-300
- ...

Correlation functions



Spectral function

$$C(t) = \int_0^\infty d\omega \rho(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega t\right)}{sh\left(\frac{\omega}{2T}\right)}$$

Properties of the spectral function:

- $\rho(\omega) \geq 0, \; \rho(-\omega) = -\rho(\omega)$
- Asymptotic freedom: $\rho(\omega)|^{NLO}_{\omega\to\infty} = \frac{1}{10} \frac{d_A}{(4\pi)^2} \omega^4 \left(1 - \frac{5N_c\alpha_s}{9\pi}\right)$

  $\sim$ 90% of the total contribution $t = 1/2/T$
- Hydrodynamics: $\rho(\omega)|_{\omega\to 0} = \frac{\eta}{\pi}\omega$

Spectral function

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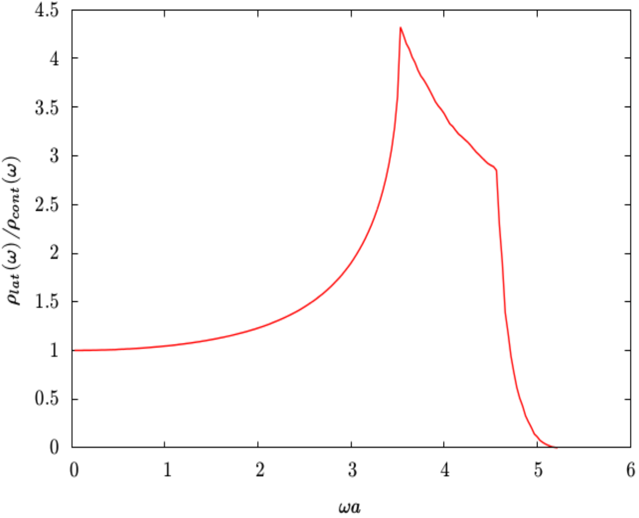
Ansatz for the spectral function (QCD sum rules motivation)

$$\rho(\omega) = \frac{\eta}{\pi}\omega\theta(\omega_0 - \omega) + A\rho_{lat}(\omega)\theta(\omega - \omega_0)$$

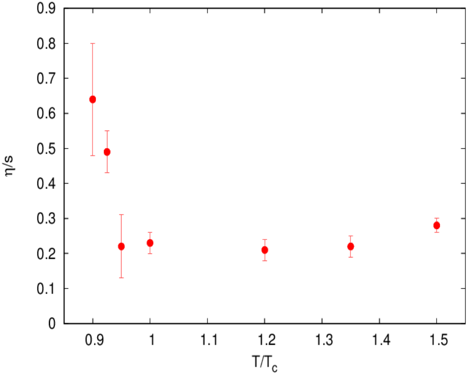
Lattice spectral function ρ_{lat}

Takes into account discretization errors in temporal direction



Spectral function

$$\rho_1(\omega) = \frac{\eta}{\pi}\omega\theta(\omega_0 - \omega) + A\rho_{lat}(\omega)\theta(\omega - \omega_0)$$
$$\chi^2/dof \sim 1, \; A \sim 1, \; \omega_0/T \sim 7-8$$



Two additional ansatzs:

- $\rho_2(\omega) = \frac{1}{2} B\ \omega(1 + \tanh[\gamma(w_0 - w)]) + \frac{1}{2} A\rho_{lat}(\omega)(1 + \tanh[\gamma(w - w_0)])$
- $\rho_3(\omega) = B\ \omega(1 + C\omega^2)\theta(\omega_0 - \omega) + A\rho_{lat}(\omega)\theta(\omega - \omega_0)$

Properties of the spectral function

- Hydrodynamical approximation works well up to $\omega < \pi T \sim 1 GeV$ (H.B. Meyer, arXiv:0809.5202)
- Asymptotic freedom works well from $\omega > 3$ GeV
- Poor knowledge of the spectral function in the region $\omega \in (1, 3)$ GeV
  $\Rightarrow$ Main source of uncertainty in the fitting procedure

Backus-Gilbert method for the spectral function

- Problem: find $f(\omega)$ from the integral equation

$$C(x_i) = \int_0^\infty d\omega f(\omega) K(x_i, \omega), \quad K(x_i, \omega) = \frac{ch\left(\frac{\omega}{2T} - \omega x_i\right)}{sh\left(\frac{\omega}{2T}\right)}$$

- Define an estimator $\tilde{f}(\bar{\omega})$ ($\delta(\bar{\omega}, \omega)$ - resolution function):

$$\tilde{f}(\bar{\omega}) = \int_0^\infty d\omega \hat{\delta}(\bar{\omega}, \omega) f(\omega)$$

- Let us expand $\delta(\bar{\omega}, \omega)$ as

$$\delta(\bar{\omega}, \omega) = \sum_i b_i(\bar{\omega}) K(x_i, \omega) \quad \tilde{f}(\bar{\omega}) = \sum_i b_i(\bar{\omega}) C(x_i)$$

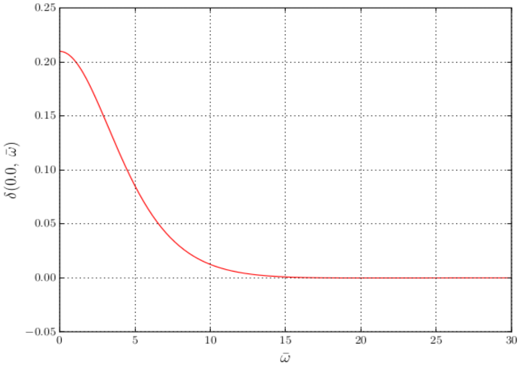
- Goal: minimize the width of the resolution function

$$b_i(\bar{\omega}) = \frac{\sum_j W_{ij}^{-1} R_j}{\sum_{ij} R_i W_{ij}^{-1} R_j},$$
$$W_{ij} = \int d\omega K(x_i, \omega)(\omega - \bar{\omega})^2 K(x_j, \omega), R_i = \int d\omega K(x_i, \omega)$$

- Regularization by the covariance matrix S_{ij}:

$$W_{ij} \to \lambda W_{ij} + (1 - \lambda) S_{ij}, \quad 0 < \lambda < 1$$

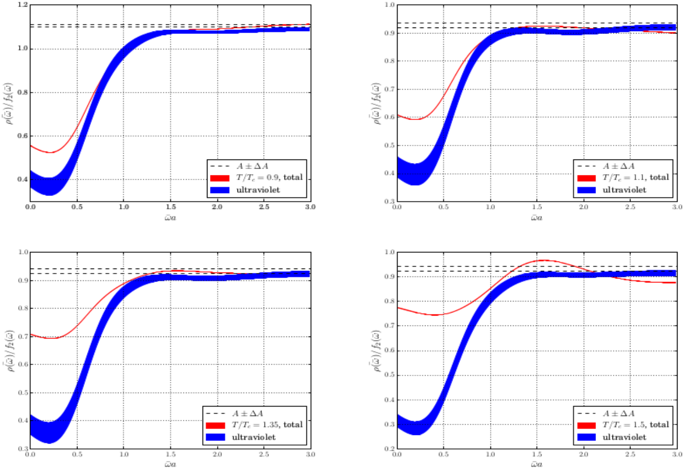
Resolution function $\delta(0,\omega)$ ($T/T_c = 1.2,\ \lambda = 0.001$)



- Width of the resolution function $\omega/T \sim 4$
- Hydrodynamical approximation works up to $\omega/T < \pi$
- Problem: large contribution from ultraviolet tail (more than $\sim 100\%$)

Model for the ultraviolet contibution

$$\rho_{ultr} = A\rho_{lat}(\omega)\theta(\omega - \omega_0)$$

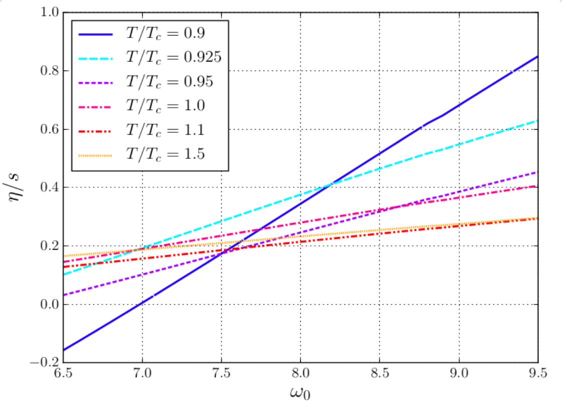


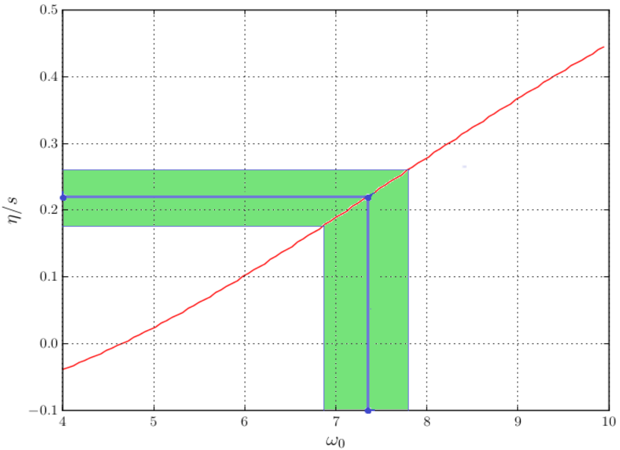
Solution:

- Take ultraviolet contribution in the form:

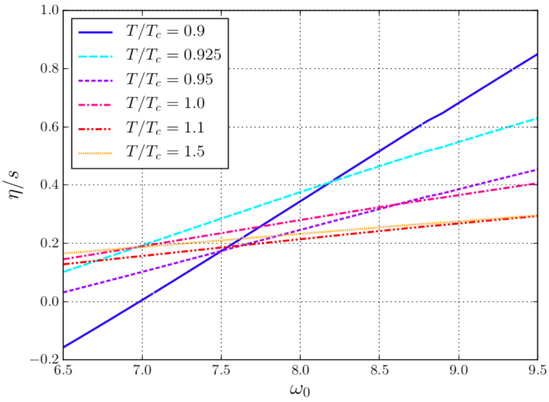
$$\rho_{ultr} = A\rho_{lat}(\omega)\theta(\omega - \omega_0)$$

- Determine the value of the constant A from the BG method
- Subtract ultraviolet contribution and obtain η/s as a function of ω_0



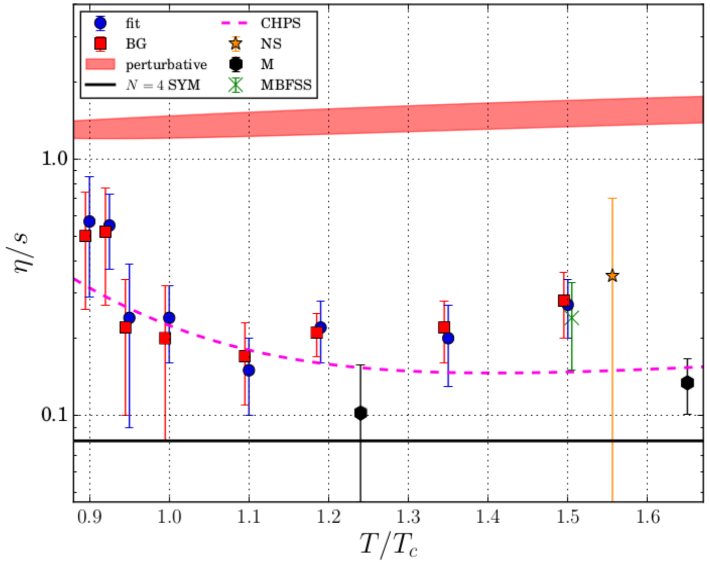


- For $T/T_c = 1.$ $\quad \omega_0/T = 7.33 \pm 0.47$
- $\eta/s = 0.22 \pm 0.04$



$T/T_c = 0.9$
$T/T_c = 0.925$
$T/T_c = 0.95$
$T/T_c = 1.0$
$T/T_c = 1.1$
$T/T_c = 1.5$
η/s
ω_0
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
6.5
7.0
7.5
8.0
8.5
9.0
9.5

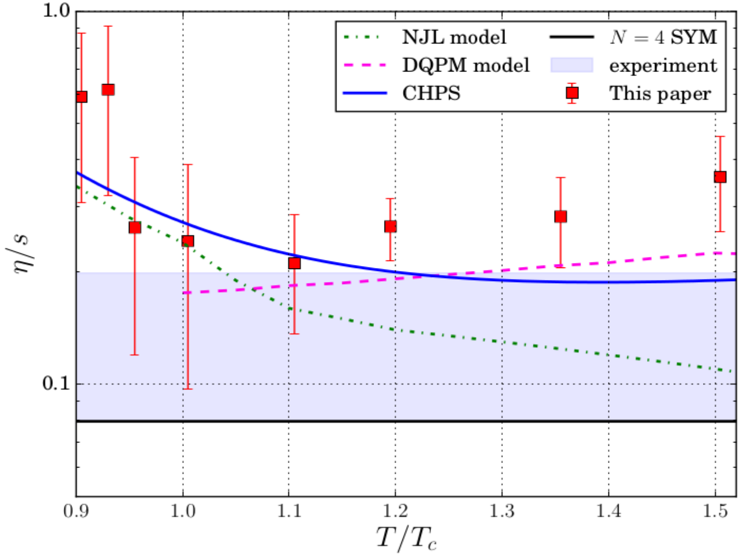
Our results


fit
BG
perturbative
$N = 4$ SYM
CHPS
NS
M
MBFSS
1.0
0.1
η/s
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
T/T_c

Estimation of the η/s for QCD with dynamical quarks ($N_f = 3$):

$$\left(\frac{\eta}{s}\right)_{QCD} = \left(\frac{(\eta/s)_{QCD}}{(\eta/s)_{Gluodynamics}}\right)_{pert} \times \left(\frac{\eta}{s}\right)_{Gluodynamics}$$

Our results


1.0
0.1
η/s
0.9
1.0
1.1
1.2
1.3
1.4
1.5
T/Tc
NJL model
DQPM model
CHPS
N = 4 SYM
experiment
This paper

Bulk viscosity

- $\zeta = \frac{1}{9} \lim_{\omega\to 0} \frac{1}{\omega} \int_0^\infty dt \int d^3x \; e^{i\omega t} \langle [T_\mu^\mu(x), T_\nu^\nu(0)] \rangle$
- $C_E(t) = \int_0^\infty d\omega \rho_{\mu\mu,\nu\nu}(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega t\right)}{sh\left(\frac{\omega}{2T}\right)}$
- $\zeta = \frac{\pi}{9} \lim_{\omega\to 0} \frac{1}{\omega} \rho_{\mu\mu,\nu\nu}(\omega)$

Violation of conformal symmetry in QCD

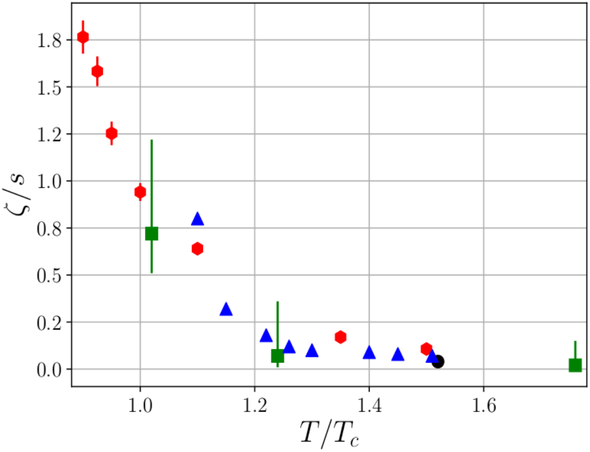
- In conformal theory $T_\mu^\mu = 0$
- Conformal symmetry is violate by quantum corrections
- Trace anomaly: $T_\mu^\mu = \frac{\beta(g)}{2g} Tr\left[G^{\mu\nu} G_{\mu\nu}\right] + \sum_i m_i \bar{q} q$

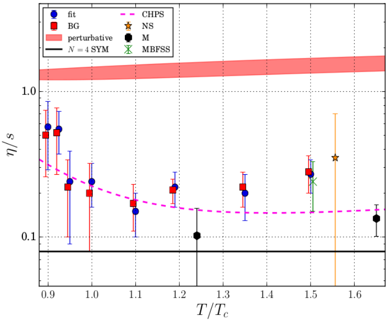
Properties of spectral density:

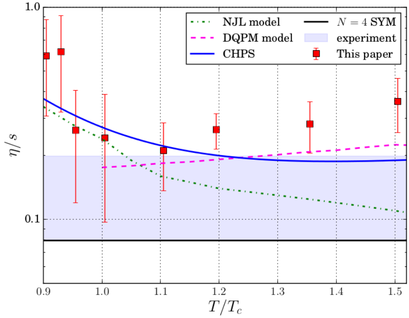
- $\rho_{\mu\mu,\nu\nu}(\omega) \geq 0$, $\rho_{\mu\mu,\nu\nu}(-\omega) = -\rho_{\mu\mu,\nu\nu}(\omega)$
- Asymptotic freedom: $\rho(\omega)|^{NLO}_{\omega\to\infty} = d_A\left(\frac{11\alpha_s}{(4\pi)^2}\right)^2\omega^4$
- Hydrodynamics: $\rho(\omega)|_{\omega\to 0} = 9\frac{\zeta}{\pi}\omega$
- The contribution of asymptotic freedom is suppressed ($\sim \alpha_s^2$)

Bulk viscosity from midpoint

$$C_E(\beta/2) = \int_0^\infty d\omega \rho_{hyd}(\omega) \frac{1}{sh\left(\frac{\omega}{2T}\right)}$$


1.8
1.5
1.2
1.0
0.8
0.5
0.2
0.0
ζ/s
1.0
1.2
1.4
1.6
T/T_c





Conclusion:

- We calculated η/s for set of temperatures $T/T_c \in (0.9, 1.5)$
- Applied fitting procedure and Backus-Gilbert method for the SF
- η/s is close to N=4 SYM and in agreement with experiment
- Large deviation from perturbative results