QCD thermodynamics and finite temperature spectroscopy with two flavours of Wilson fermions

Bastian Brandt

ITP Goethe University Frankfurt

In collaboration with:

Anthony Francis (York University),

Harvey Meyer, Daniel Robaina, Hartmut Wittig (University ofMainz)

Owe Philipsen (Goethe University Frankfurt)

Benjamin Jäger (Swansea University)

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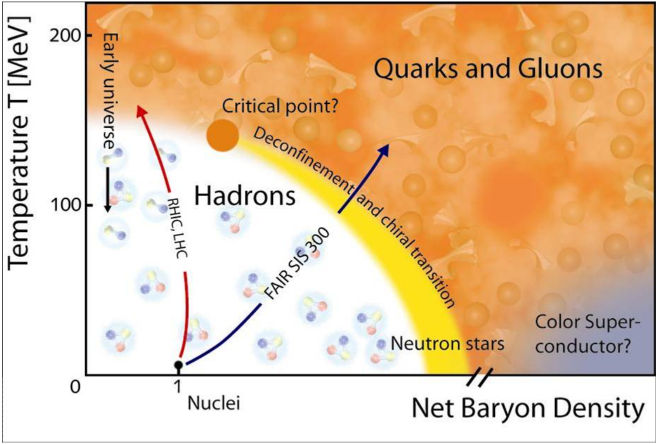
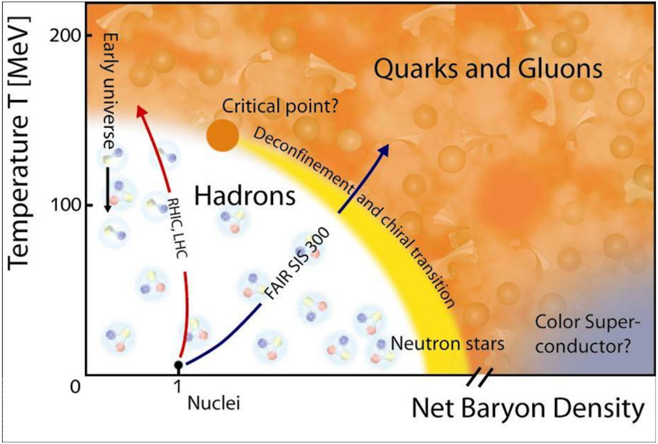
The conjectured QCD phase diagram



[Homepage of the CBM experiment, GSI (FAIR)]

The conjectured QCD phase diagram

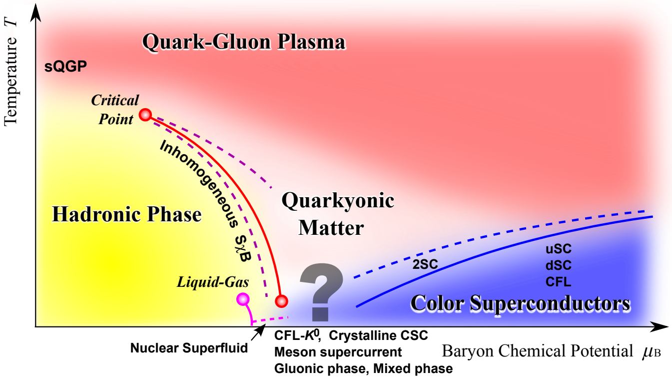
However, the details of the phase diagram ...



[Homepage of the CBM experiment, GSI (FAIR)]

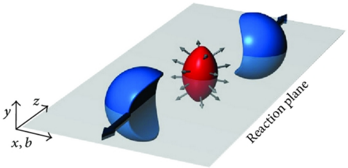
The conjectured QCD phase diagram

... might change, depending on whom you ask.



[Fukushima, Hatsuda, RPP74 (2011)]

Experiment: Heavy Ion Collisions



0 sec — Initial collision

$\sim 10^{-25}$ sec — Thermalisation

$\sim 10^{-24}$ sec — Thermalised QGP?

$\sim 10^{-23}$ sec — Hadronisation

Freezeout

- In some phase the plasma might be well described by hydrodynamics.
- The analysis of experiment in many parts relies on models
- Many effects can occur. (e.g. chiral magnetic effect)
  But: impact depends on plasma properties of QCD.

$\Rightarrow$ First principles measurements of plasma properties are mandatory!

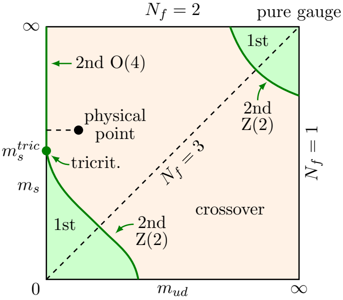
Lattice QCD is the prefered tool!

Contents

1. Exploring the transition in the chiral limit at $N_f = 2$

Directly accessible: Zero density ($\mu = 0$)

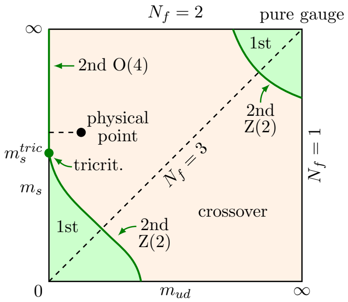
Enlarged parameter space relevant for the QCD phase diagram:



- The charm quark is to heavy to influence the transition properties. (might affect plasma properties above T_C)
- Isospin breaking effects also won't effect the order much.

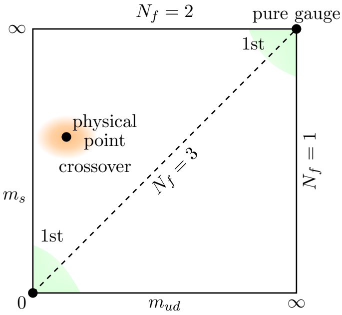
Directly accessible: Zero density ($\mu = 0$)

Enlarged parameter space relevant for the QCD phase diagram:



- Common believe:
  The features of the Columbia plot are well known!
- However: This is not entirely true!

Known facts



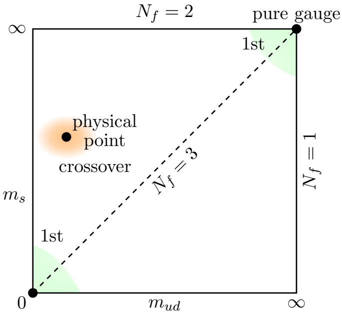
SU(3) chiral limit:

- First order phase transition
  [Pisarski, Wilczek, PRD 29, 338 (1984)]
- Order parameter:
  Chiral condensate
- Associated broken symmetry:
  $SU_V(3) \times SU_A(3)$

Pure gauge theory:

- First order phase transition
  [Yaffe, Svetitsky, PRD 26, 963 (1982)]
- Order parameter:
  Polyakov loop
- Associated broken symmetry:
  Center symmetry

Known facts



Physical point: Crossover

[Aoki *et al*, Nature 444, 675 (2006)]

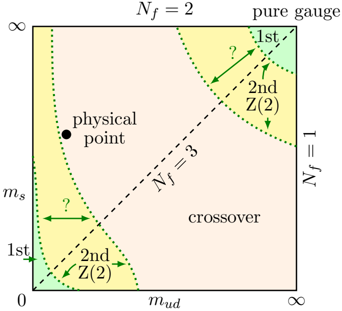
- Staggered fermions:
  (continuum limit)
  $T_C \approx 150 - 160$ MeV

  [Borsanyi *et al*, JHEP 1009, 073 (2010)]

  [Bazavov *et al*, PRD 85, 054503 (2012)]
- Domain wall fermions:
  (no continuum limit)
  $T_C \approx 155(1)(8)$ MeV

  [Bhattacharya *et al*, PRL113, 082001 (2014)]

Resulting phase diagram

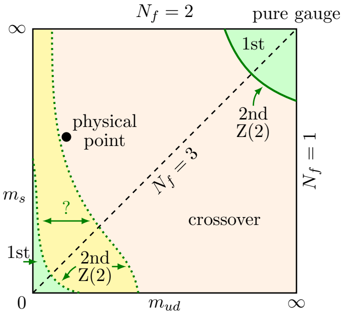


- Border crossover/1st order: Z(2) critical lines
- Positions/Shape: Has to be clarified!
- Particularly relevant: Two possible scenarios for the $N_f = 2$ transition in the chiral limit!

⇒ Let's see what the lattice says!

Here: Focus on the chiral critical line!

Resulting phase diagram



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- Particularly relevant:
  Two possible scenarios for the $N_f = 2$ transition in the chiral limit!

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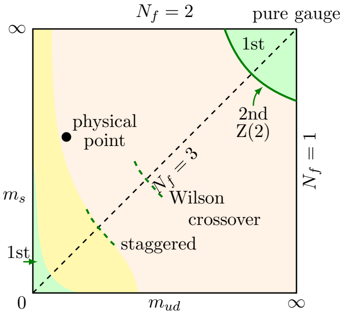
Here: Focus on the chiral critical line!

Chiral critical line: $N_f = 3$ region - $N_t = 4$

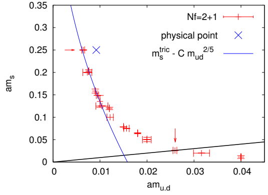


Staggered fermions:

$N_f = 3$ critical point

[Karsch *et al*, PLB 520, 41 (2001)]

[de Forcrand, Philipsen, NPB 673, 170 (2003)]



[de Forcrand, Philipsen, JHEP 0701, 077 (2007)]

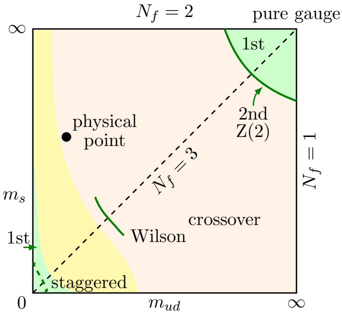
Wilson fermions:

$$m^c_{uds} \approx 36\, m^{\rm phys}_{ud}$$

$N_t = 4$ out of the scaling region!

[Jin, PRD 91, 014508 (2015)]

Chiral critical line: $N_f = 3$ region - $N_t > 4$



Staggered fermions:

1st order region shrinks

[de Forcrand *et al*, PoS LAT 2007, 178]

[Endrődi *et al*, PoS LAT 2007, 182]

$$m^c_{uds}(N_t = 6) \approx m^c_{uds}(N_t = 4)/5$$

Newest upper bound $N_t > 6$:

$$m^c_{uds} < 0.1\, m^{\rm phys}_{ud}$$

[Ding *et al*, PoS LAT 2011, 191]

Wilson fermions:

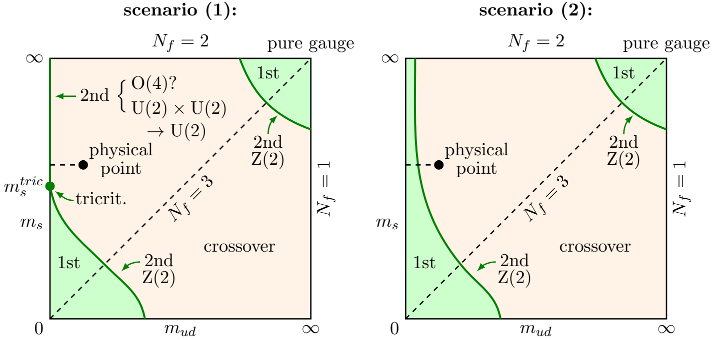
Continuum from $N_t = 6$ and 8

$$\Rightarrow \quad m^c_{uds} \approx 4\, m^{\rm phys}_{ud}$$

[Jin, PRD 91, 014508 (2015)]

Chiral critical line: $m_s > m_{ud}$

Two possible scenarios:

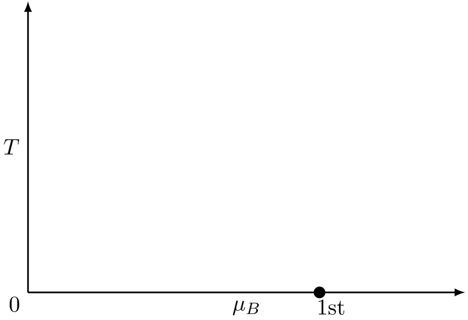


- It has to be true phase transition. [Pisarki, Wilczek, PRD 29, 338 (1984)]
- But it can be of first **or** second order!

[Butti *et al*, JHEP 0308, 029 (2003); Pelisseto, Vicari, 1309.5446]

Impact on finite density scenarios $N_f = 2$

Chiral limit:



- Expect first order transition at $T = 0$? [review: Fukushima, Hatsuda, RPP74 (2011)]

Impact on finite density scenarios $N_f = 2$

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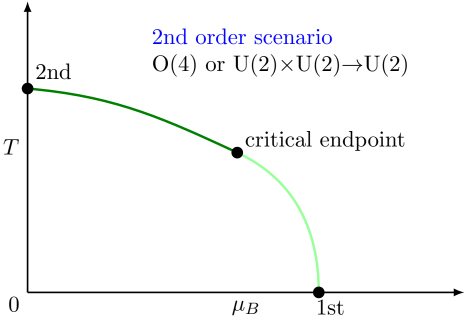

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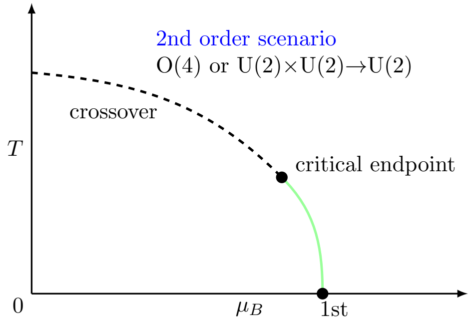
- Critical endpoint exists!

Impact on finite density scenarios $N_f = 2$

Finite m_{ud}:



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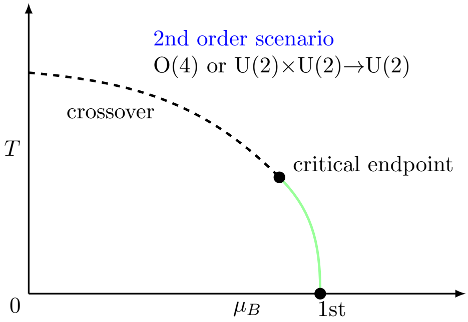
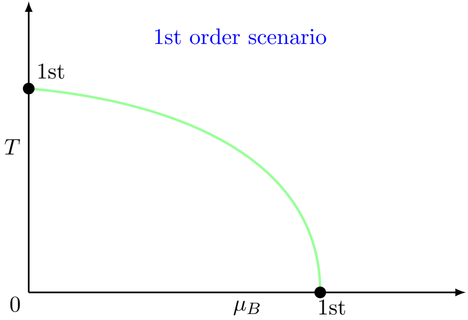


- Expect first order transition at $T = 0$? [review: Fukushima, Hatsuda, RPP74 (2011)]
- Finite strange quark mass is not expected to change much!

Impact on finite density scenarios $N_f = 2$

Chiral limit:



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Impact on finite density scenarios $N_f = 2$

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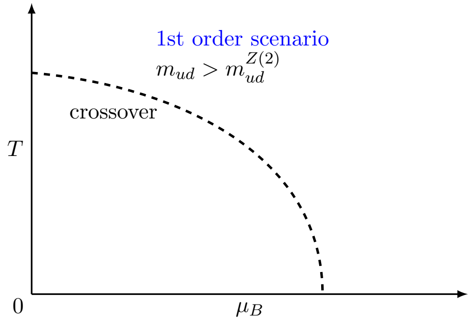
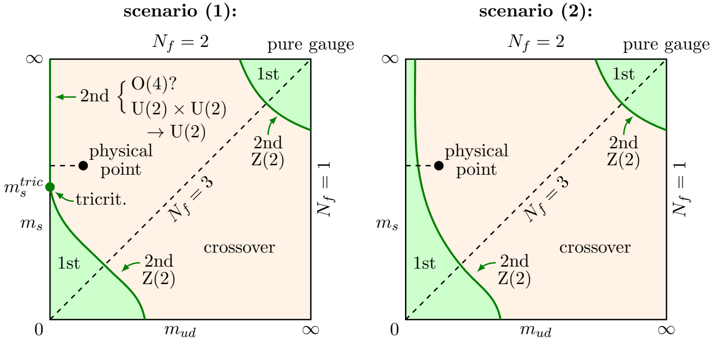


- Expect first order transition at $T = 0$? [review: Fukushima, Hatsuda, RPP74 (2011)]
- No critical endpoint!

Assessing the two scenarios - Our choice

Simulate at $N_f = 2$:



- Simulations are less expensive.
- Can use Wilson fermions on large lattices using the available fast algorithms and the $T = 0$ input from CLS.

Assessing the two scenarios – Scaling

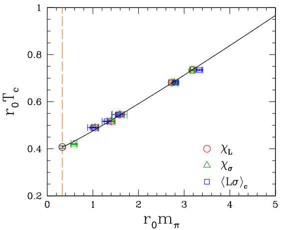
- Cannot simulate directly in (or very close to) the chiral limit.
  Simulate at larger m_{ud}; look for critical scaling for $m_{ud} \to 0$
- What type of scaling can be expected in the two cases?
  - 2nd order: standard $O(4)$ or $U(2) \times U(2) \to U(2)$ scaling
    order parameter: Chiral condensate
    external field: $h = m_{ud}$
  - 1st order: $Z(2)$ scaling
    order parameter: ???
    external feld: ??? maybe $h = m_{ud} - m_{ud}^{\mathrm{cr}}$?
- How close to $m_{ud} = 0$ is necessary?
- Breaking of chiral symmetry due to lattice:
  Scaling laws will only be correct in the continuum limit!
  $\Rightarrow$ Need to be close enough to the continuum (large N_t)!
- There is a number of studies but no conclusive result!
  (contradicting results for staggered; no control over systematics for Wilson)

Scaling of the transition temperature

Scaling of the critical temperature with the external field:

$$T_C(h) = T_C(0)\left[1 + C\, h^{1/(\delta\,\beta)}\right] + \mathrm{sv}\ .$$

Most studies: $O(4)$ scaling

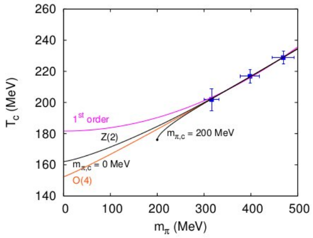


[Bornyakov *et al*, PRD 82, 014502 (2010)]

- Large pion masses
- Control over systematic effects?

Difficult to distinguish:

- O(4): $\delta\,\beta = 1.861$
- U(2): $\delta\,\beta \approx 1.8$
- Z(2): $\delta\,\beta = 1.564$

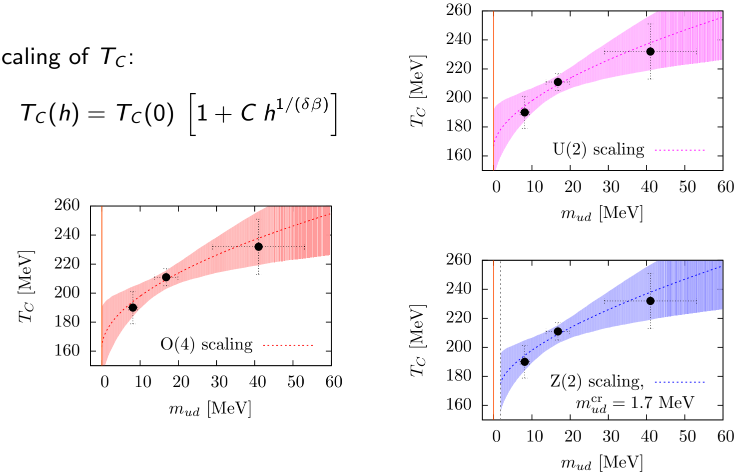


[tmfT collaboration, PRD 87, 074508 (2013)]

Scaling of the transition temperature

Scaling of T_C:

$$T_C(h) = T_C(0)\ \left[1 + C\ h^{1/(\delta\beta)}\right]$$



Even with one order of magnitude smaller error bars no conlusions possible!

Other types of scaling

- Scaling of the order parameter (chiral condensate):

$$\langle\bar{\psi}\psi\rangle \sim h^{1/\delta}\,\Psi(z,\ h) \quad \text{with} \quad z = \frac{\tau}{h^{1/(\delta\,\beta)}}$$

Ψ: universal scaling function

Problem: Ψ is known for O(4) only.

So far simulations show consistency with O(4) scaling.
But $U(2)xU(2) \to U(2)$ and Z(2) scaling might be similar?

- Scaling of the Binder cumulant:

Very powerful tool!

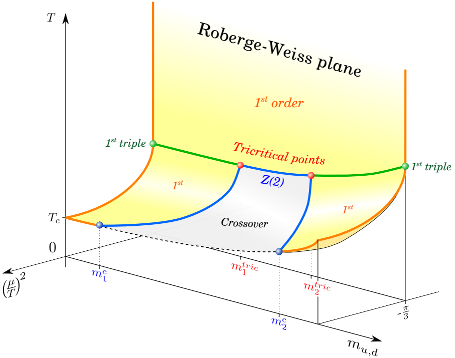
However: So far no one has seen the onset of this scaling!

- Finite size scaling of susceptibility peaks:

Peaks scale like: $\max(\chi) \sim l^{\gamma/\nu}$, $\quad$ width $(\chi) \sim l^{-1/\nu}$, $\quad \Delta T_C(V) \sim l^{-1/\nu}$
Onset only very close to the critical point?

Constraints from imaginary chemical potential

[de Forcrand, Philipsen, PRL105, 152001 (2010); Bonati *et al*, LAT2011; LAT2013]

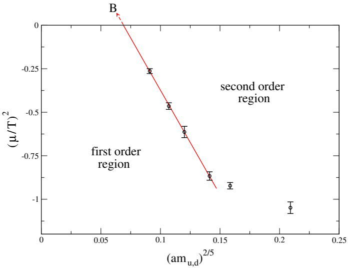


[Cuteri *et al*, PRD 93, 054507 (2016)]

- Follow the Z(2) line from the Roberge-Weiss transition point.
- Use the known tri-critical scaling.

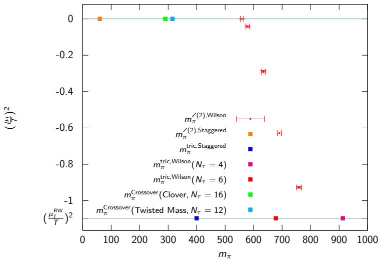
Constraints from imaginary chemical potential

Staggered fermions:



[Bonati *et al*, PRD 90, 074030 (2014)]

Wilson fermions:



[Philipsen, Pinke, arXiv:1602.06129]

- Huge discrepancy $\Rightarrow$ Large cutoff effects!

  $\Rightarrow$ Continuum limit difficult but necessary!
- All-in-all: Strongly favours the 1st order scenario!

Assessing the two scenarios – $U_A(1)$ symmetry

Essential for order of transition:
Strength of the anomalous breaking of the $U_A(1)$ symmetry:

[Pisarki, Wilczek, PRD 29, 338 (1984), Butti *et al*, JHEP 0308, 029 (2003)]

- If the breaking is strong:
  Transition: Second order $SU(2) \times SU(2) \simeq O(4)$ universality
- If the breaking is weak:
  Transition: Second order $U(2) \times U(2) \rightarrow U(2)$ universality
  or first order
- If the symmetry is effectively restored:
  Transition: Likely to be first order.

Possibility for looking at the strength of the breaking:
Look at degeneracies of correlation functions and screening masses in pseudoscalar (P) and scalar channels (S).

$\Rightarrow$ Chiral extrapolation is mandatory!

Screening masses and chiral symmetry

Cleanest method:

Use screening masses to investigate symmetry restoration.

Reason:

Spectral representation of partition function in terms of screening masses.

Channels for screening masses:

scalar (isovector)	–	S	vector	–	V
pseudoscalar	–	P	axial vector	–	A

Interesting symmetries:

- $V \overset{SU_A(2)}{\longleftrightarrow} A$
- $S \overset{U_A(1)}{\longleftrightarrow} P$

Degeneracy signals chiral symmetry restoration!

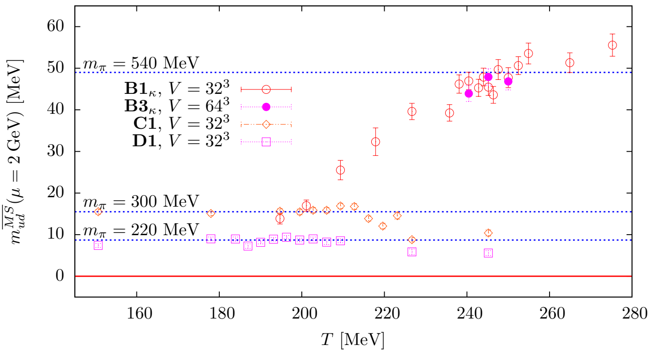
Simulation and temperature scan setup

- Non-perturbatively $\mathcal{O}(a)$-improved Wilson fermions
  Wilson plaquette gauge action
- Algorithms: DD-HMC [Lüscher (2004-2005), e.g. CPC 165, 199 (2005)]
  MP-HMC with DFL-SAP-GCR solver [Marinkovic, Schäfer PoS LAT 2010, 031 (2010)]
  $\Rightarrow$ Good scaling properties with volume and quark masses!
- Scale setting, renormalisation and $T = 0$ subtractions: CLS input

Basic strategy:

- Use $N_t = 16$ for all scans.
- Use 3 different volumes: 32^3, 48^3 and 64^3.
  (enables a finite volume scaling study; control FS effects)
- At least 3 different pion masses below $m_\pi \leq$300 MeV.
  (ideally even below the physical point)
- We scan in β:
  - First attempts: keep κ fixed (m_{ud} changes)
  - Now: Line of constant physics (LCP) (m_{ud} fixed)
    (conceptually much cleaner)

Temperature scans



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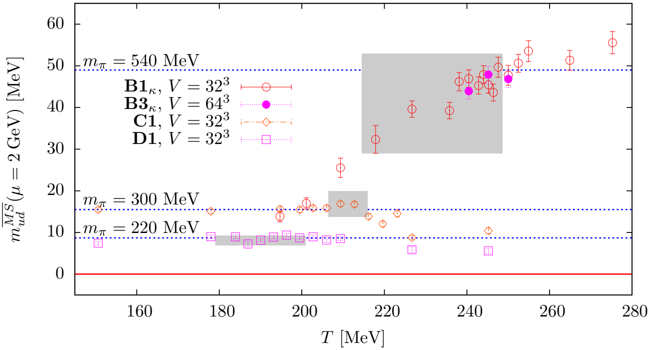


Transition temperatures:

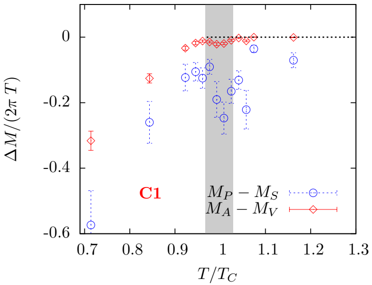
Defined via peak in the susceptibility of the chiral condensate.

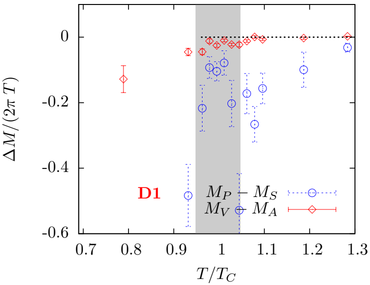
Screening mass differences

[Scan details: BB *et al*, PoS LAT2013]

- C1: 16×32^3 Lattice
  LCP at m_π ≈300 MeV
  ($m_{ud} \approx 16.0$ MeV)
- Statistic: ∼300 configurations
  48 source positions each
  separated by 40 MDUs

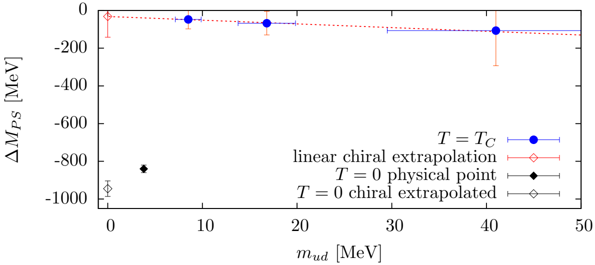
- D1: 16×32^3 Lattice
  LCP at m_π ≈220 MeV
  ($m_{ud} \approx 8.5$ MeV)
- Statistic: ∼700 configurations
  48 source positions each
  separated by 20 MDUs





Breaking of $U_A(1)$ at T_C

Strength of breaking: Need a quantitative estimate!
Possibility: Compare to $(m_P - m_S)$ at $T = 0$. (strong breaking)



- Phenomenological estimate from PDG masses.
  (see details in upcomming paper).
- Effect of breaking at least factor of 5 reduced at $m_{ud} = 0$.
  This is in qualitative agreement with recent results with overlap fermions.

[Cossu *et al*, PRD87 (2013); Chiu *et al*, 1311.6220]

Chiral transition: Conclusions

- The scaling analysis is inclonclusive!
  $\Rightarrow$ Very expensive points at small m_{ud} are needed.

  Moreover: No guarantee for success!
- As expected: $SU_A(2)$ is restored around T_C.
- $U_A(1)$ symmetry effectively restored for $T/T_C \gtrsim 1.25$.
  (agreement with hotQCD domain wall result)

  [Bazavov *et al*, PRD 86, 094503 (2012)]
- Breaking becomes weaker for $m_{ud} \to 0$.

  At $m_{ud} = 0$ breaking at least by factor 5 reduced!

  (might even vanish?)
- Our results speak in favour of a first order transition!
- Need to check possible systematic effects:
  finite volume effects; quark mass difference; ...

2. Finite-T spectroscopy and plasma properties

Anthony Francis and Harvey Meyer

Spectral functions and Euclidean correlators

[Review: Meyer, EPJA 47, 86 (2011)]

Important for hydrodynamical treatment of the plasma: Transport coefficients

They are related to spectral functions (SPFs) via Kubo relations.

Spectral function $\rho(\omega, \mathbf{p}; T)$ in a given channel:

- Directly related to Wightman correlation functions and the retarded correlator. (important for linear response)
- By analytic continuation formally related to the Euclidean correlator

$$\pi\rho(\omega, \mathbf{p}; T) = \mathrm{Im}\Big(G_E(\omega_n \to -i[\omega + i\epsilon]\,, \mathbf{p}; T)\Big)$$

- Formulation in terms of the temporal Euclidean correlator $G_E(\tau, \mathbf{p}; T)$:

$$G_E(\tau, \mathbf{p}; T) = \int_0^\infty d\omega \rho(\omega, \mathbf{p}; T)\; K(\omega, T, \tau)$$

Here: $G_E(\tau, \mathbf{p}; T) = \langle O_1(\tau) O_2(0)\rangle_T$

Spectral functions: physical significance

[Review: Meyer, EPJA 47, 86 (2011)]

- Low frequency region is related to hydrodynamics.
  $\Rightarrow$ Kubo formulas!
  Examples:
  - Shear and bulk viscosity: $(\eta) \leftrightarrow T_{\mu\nu}$ SPFs
  - Electrical conductivity: $(\sigma) \leftrightarrow$ vector channel SPF
- Also includes information about quasiparticles/resonances. Show up as poles/peaks in $\rho(\omega, \mathbf{p}; T)$.

But: Extraction demands finding the solution of

$$G_E(\tau, \mathbf{p}; T) = \int_0^\infty d\omega \rho(\omega, \mathbf{p}; T)\, K(\omega, T, \tau)$$

$\Rightarrow$ Ill-posed problem!

Vector correlator and SPF

Here: Focus on the vector current correlation function (at $\mathbf{p} = 0$).

$$G_{\mu\nu}(\tau, T) = \int_0^\infty \frac{d\omega}{2\pi}\, \rho_{\mu\nu}(\omega, T)\, \frac{\cosh\left[\omega\left(1/(2T) - \tau\right)\right]}{\sinh\left(\omega/2T\right)} \,.$$

Of particular relevance since:

- Is related to the electrical conductivity of the plasma.

  Kubo formula: $\dfrac{\sigma(T)}{T} = \dfrac{C_{\mathrm{em}}}{6} \lim_{\omega\to 0} \dfrac{\rho_{ii}(\omega, T)}{\omega\ T}$.

- It can be used to investigate the behaviour of the ρ-meson when crossing the transition.
- $G_{ii}(x, T)$ can be related to second order hydrodynamical coefficients relevant for screening (or anti-screening) of the electromagnetic forces in the plasma. [BB, Francis, Meyer, PRD 89 3, 034506 (2014)]

Extraction of $\rho_{ii}(\omega, T)$

Two standard options to extract $\rho_{ii}(\omega, T)$:

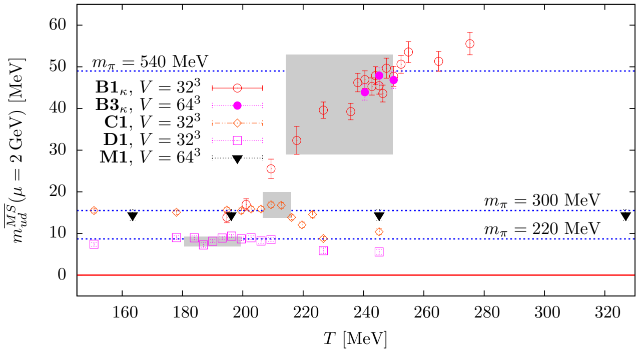
- Maximum entropy method (MEM)
  - Provides an accurate (model independent) answer if data is good enough.
  - Can lead to wrong results if this is not the case.
  - In particular: Demands an exhaustive study of input model dependence.
- Use a phenomenologically motivated ansatz for $\rho(\omega; T)$
  - Inherent model dependence.
    $\Rightarrow$ The ansätze have to be justified!
  - By comparing different ansätze one can get accurate results for certain properties of the spectral function.

Here: We use the second method!

In addition: Backus-Gilbert method (introduced later)

Temperature scans

New scan in the fixed scale approach:



- Fixed β and m_{ud} $\Rightarrow$ Vary temperature via N_t!
- $T = 0$ data available for a 128×64^3 lattice from CLS.

Our ansatz

[BB *et al*, PRD 93, 054510 (2016) (also: JHEP 1303, 100 (2013))]

Simultaneous fit to the "$T = 0$" and $T > 0$ ensembles (note: $\beta_T = aN_t$):

Ansatz $T = 0$:

$$\frac{\rho_{ii}(\omega; T \simeq 0)}{2\pi} = a_V \delta(\omega - m_V) + \frac{3\kappa_0}{4\pi^2}\Theta(\omega - \Omega_0)\omega^2 \tanh\left(\frac{\omega\beta_0}{4}\right)$$

Free parameters: a_V, κ_0
m_V: mass of ρ-meson

Ω_0: Perturbative threshold

Ansatz $T > 0$:

$$\frac{\rho_{ii}(\omega; T)}{2\pi} = \frac{\omega\, A_T\, \Gamma_T}{\pi(\Omega_T^2 + \omega^2)} + a_T\delta(\omega - m_V) + \frac{3\tilde{\kappa}_0}{4\pi^2}\Theta(\omega - \Omega_T)\omega^2 \tanh\left(\frac{\omega\beta_T}{4}\right) + \frac{3\kappa_O}{4\pi^2}\Theta(\omega - \Omega_0)\frac{1}{\omega^2}$$

Γ_T: Breit-Wigner parameter
$\Omega_T = 0$ or Ω_0
$\tilde{\kappa}_0 = \left[\kappa_0 + \kappa_1\left(1 - \tanh\left(\frac{\omega}{\Omega_0\eta}\right)^2\right)\right]$

$a_T = 0$ or free (with $a_T > 0$)
$\kappa_O = 0$ or free
η and $\kappa_1 = 0$ or free

Our ansätze: Example for parameter settings

[BB *et al*, PRD 93, 054510 (2016) (also: JHEP 1303, 100 (2013))]

$$\frac{\rho_{ii}(\omega;T)}{2\pi} = \frac{\omega\, A_T\, \Gamma_T}{\pi(\Omega_T^2+\omega^2)} + a_T\delta(\omega-m_V) + \frac{3\tilde{\kappa}_0}{4\pi^2}\Theta(\omega-\Omega_T)\omega^2\tanh\left(\frac{\omega\beta_T}{4}\right) + \frac{3\kappa_O}{4\pi^2}\Theta(\omega-\Omega_0)\frac{1}{\omega^2}$$

Ansatz	N_τ	T/T_c	a_T	κ_1	Ω_T	κ_O
2b	24	0.80	free	0	Ω_0	free
	20	1.00	free	free	0	0
	16	1.25	free	free	0	0
	12	1.67	free	free	0	0
2c	24	0.80	free	0	Ω_0	free
	20	1.00	free	free	0	0
	16	1.25	0	free	0	0
	12	1.67	0	free	0	0

A_T fixed by the sumrule:

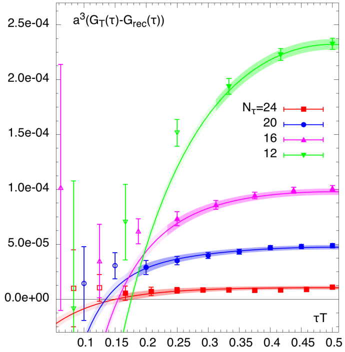
[Bernecker, Meyer, EPJA 47, 148 (2011)]

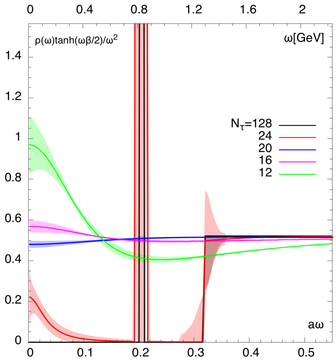
$$\int_{-\infty}^{\infty}\frac{d\omega}{\omega}\left(\rho_{ii}(\omega;T)-\rho_{ii}(\omega;0)\right)=0$$

Spectral function from fits

[BB *et al*, PRD 93, 054510 (2016)]

Fit 2c

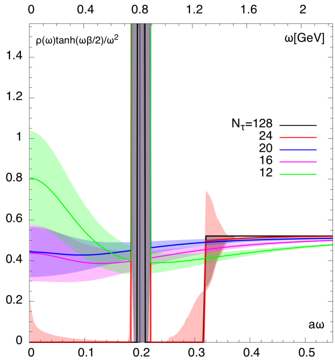




Spectral function from fits

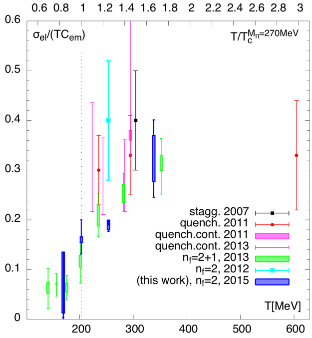
[BB *et al*, PRD 93, 054510 (2016)]

Fit 2b



Results for ρ-meson and conductivity

[BB *et al*, PRD 93, 054510 (2016)]



- ρ-meson:

  Contribution of ρ-meson significantly lowered at $T \gtrsim T_C$.

  $\Rightarrow$ ρ-meson dissociates rapidly in the transion region!

- At the same time:

  Increase of spectral weight of the transport contribution.

- From intercept at $\omega = 0$:

  See an increase of the electrical conductivity!

The Backus-Gilbert method

[BB *et al*, PRD 92, 094510 (2015); PRD 93, 054510 (2016)]

Aim: Try to get as much local constraints on ρ as possible from correlator.

BGM: Provides this via filtered spectral function.

$$\hat{\rho}(\omega) = f(\omega/T) \int_0^\infty d\omega' \delta(\omega, \omega') \frac{\rho(\omega')}{f(\omega'/T)}$$

$\delta(\omega, \omega')$: resolution function

Using a linear ansatz for $\hat{\rho}$:

$$\hat{\rho}(\omega) = f(\omega/T) \sum_{i=1}^{N_t} g_i(\omega) G(\tau_i)$$

$g_i(\omega)$: coefficients for the desired frequency

$$\Rightarrow \quad \delta(\omega, \omega') = \sum_{i=1}^{N_t} g_i(\omega) K(\tau_i, \omega')$$

$K(\tau_i, \omega) = f(\omega/T) \frac{\cosh(\omega(\beta/2-\tau))}{\sinh(\omega\beta/2)}$ rescaled kernel

The Backus-Gilbert method

[BB *et al*, PRD 92, 094510 (2015); PRD 93, 054510 (2016)]

- Backus and Gilbert: [Backus, Gilbert, Geophys.J.R.Astron.Soc. 16, 169 (1968)]

  Determine the coefficients $g_i(\omega)$ via minimisation of the "width"

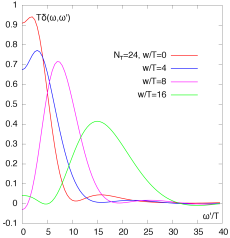
$$\Gamma_\omega = \int_0^\infty d\omega' (\omega - \omega')^2 \delta(\omega, \omega') \, .$$

- $f(\omega/T)$ chosen such that ρ/f as flat as possible.

  Suitable choice here: $f(x) = x^2/(\tanh(x/2))$

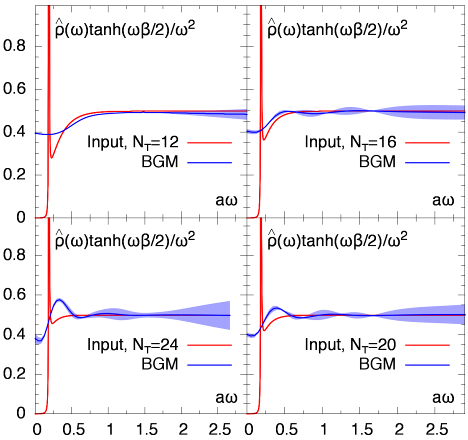
- Resulting filtered spectral function $\hat{\rho}$:

  Averaged over region of support of $\delta(\omega, \omega')$.



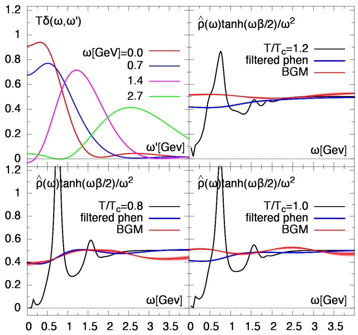
The Backus-Gilbert method [BB *et al*, PRD 92, 094510 (2015); PRD 93, 054510 (2016)]

Example with Mock data:



Comparison to SPFs from Hohler and Rapp

[BB *et al*, PRD 93, 054510 (2016)]



Can be applied to the lattice data.

Most interestingly:
Provides a direct way to compare to phenomenological SPFs.
(model independenty)

Filtered SPF:

$$\hat{\rho} = \int_0^\infty d\omega' \delta(\omega, \omega') \rho(\omega')$$

Here the SPFs from Hohler and Rapp

[Hohler, Rapp, PLB 731, 103 (2014)]

(obtained from QCD and Weinberg sum rules)

Antiscreening of electromagnetic currents

[Anthony Francis and Harvey Meyer]

At second order of hydrodynamical treatment additional coefficients appear:

$$\kappa_\ell \text{ and } \kappa_t$$

Interpretation:

Coulomb potential in plasma (QED or QCD) for static leptons (at long distance):

$$V_C(R) = e^2 \left(1 + e^2 \kappa_\ell\right) \frac{Q_1 Q_2}{4\pi R} e^{-M_{\rm el} R}$$

(last term: Debye screening; $M_{\rm el}$: electromagn. screening mass)

Ampere force in plasma:

$$F_A(R) = e^2 \left(1 + e^2 \kappa_t\right) \frac{I_1 I_2}{4\pi R}$$

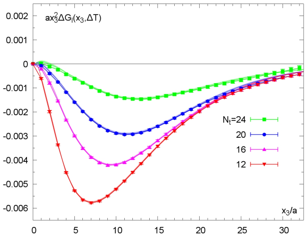
$\Rightarrow$ Ampere force is enhanced in medium!

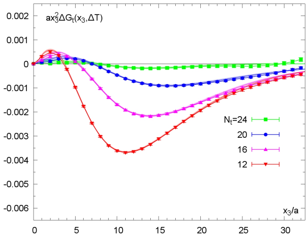
Constitutive em-current equation in hydrodynamics (2nd order):

$$(1 + \tau_J \partial_t) e\mathbf{j} = -eD\nabla\rho + \sigma\mathbf{E} + \kappa_t e^2 \nabla \times \mathbf{B}$$

Extraction of κ_t and κ_ℓ from the correlator

[BB, Francis, Meyer, PRD 89 3, 034506 (2014)]





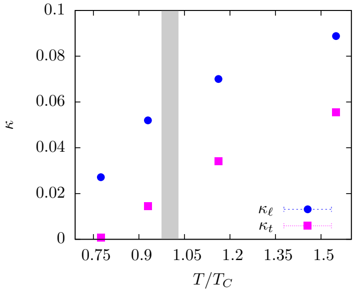
$$\kappa_t = -\int_0^\infty dx\, x^2\, \Delta G_t(x,T) \qquad \Delta G_t(x,T) = G_t(x,T) - G(x,0)$$

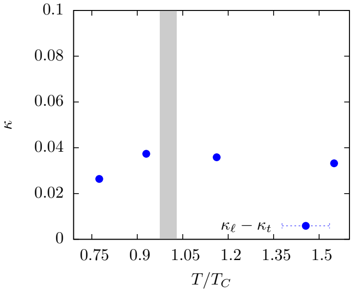
$$\kappa_\ell = -\int_0^\infty dx\, x^2\, \Delta G_\ell(x,T) \qquad \Delta G_\ell(x,T) = G_\ell(x,T) - G(x,0)$$

Difference does not need $T = 0$ input:

$$\kappa_\ell - \kappa_t = -\int_0^\infty dx\, x^2\, \Delta G_{\ell-t}(x,T) \qquad \Delta G_{\ell-t}(x,T) = G_\ell(x,T) - G_t(x,T)$$

Results for κ_t and κ_ℓ





- They are positive ...
  - $\Rightarrow$ Anti-screening of the Ampère force in the plasma!
- ... but rather small in magnitude!
  - $\Rightarrow$ Only tiny effect in the QGP!

Chiral dynamics close to T_C

What happens to the $T = 0$ real-time excitations of QCD at finite T?

Intuitive starting point: What is known about the pion at $T \neq 0$?

- Chiral perturbation theory around $(T = 0, m_{ud} = 0)$:
  - Pion quasiparticle persists.

    [Schenk, NPB 363, 97 (1991); PRD 47, 5138 (1993); Toublan, PRD 56, 5629 (1997)]
  - Question: Up to which temperatures is this expansion applicable?
- Goldstone theorem: $m_\pi = 0$ at $m_{ud} = 0$ for the chirally broken phase.

$\Rightarrow$ Can perform a chiral expansion around $(T, m_{ud} = 0)$.

[Son, Stephanov, PRL 88, 202302 (2002); PRD 66, 076011 (2002)]

  - Quasiparticle persists (pole in retarded propagator at $\mathbf{p} = 0$).
  - Modified dispersion relation: $\omega_{\mathbf{p}}^2 = u^2(M_P^2 + \mathbf{p}^2)$
    u: 'Pion velocity'

Can we somehow test this in lattice QCD?

Exploring the pion dispersion relation

[Daniel Robaina]

Two options:

- Extract the pseudoscalar screening mass M_P and the Matsubara frequency $\omega_{\mathbf{p}}$ from the x and t correlation functions of the pion.

  $\Rightarrow$ Means that we need to extract again a spectral function of the t-correlator in P and/or A channels.

- Can use a relation which connects u to static quantities:

  Lattice estimates for u: [BB *et al*, PRD 90, 054509 (2014)]

$$u_m^2 = -\frac{4m_{PCAC}^2}{M_P^2} \left.\frac{G_P(x_0,\mathbf{0})}{G_A(x_0,\mathbf{0})}\right|_{x_0=N_t/2} \qquad u_f \sinh\left(u_f \frac{M_P N_t}{2}\right) = \frac{f_P^2 M_P}{2G_A(N_t/2,\mathbf{0})}$$

  Here: N_t temporal lattice extent; f_P the $T \neq 0$ analogue of f_π

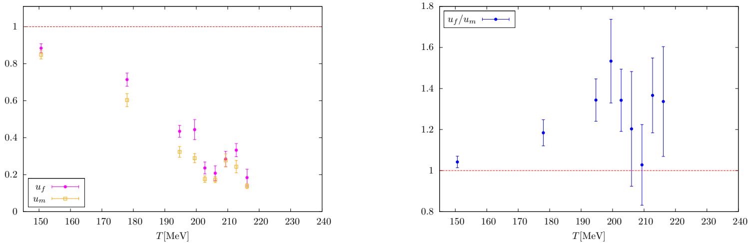
  Prediction from χPT: $u_f/u_m = 1 \quad \Leftarrow \quad$ Can be used as a check.

Note: All quantities well defined for all T.

But: Interpretation depends on the reliablity of χPT.

Results for the pion velocity

Measurements on scan **C1**:



- Only one temperature (~ 150 MeV) indicates validity of χPT.
- At that point $u < 1$ indicating a significant modification of the pion dispersion relation.
  $\Rightarrow$ Implies a violation of boost (Lorentz) invariance.
- Interpretation relies on the existence of a pion quasiparticle.
  $\Rightarrow$ We have checked this via the SPF using MEM.

Consistency with the pion pole at finite momentum

[BB, Francis, Meyer, Robaina, PRD 92, 094510 (2015)]

Best sensitivity for pion contribution in $G^A_{00}(\tau, \mathbf{p})$.

Using an ansatz for the associated spectral function:

$$\rho^A(\omega, \mathbf{p}) = A_1(\mathbf{p}) \sinh(\omega\beta/2)\delta(\omega - \omega_{\mathbf{p}}) + A_2(\mathbf{p}) \frac{1}{8\pi^2} \left(1 - \exp(-\omega\beta)\right) \Theta(\omega - c)$$

Free parameters: A_1, A_2 and c (Perturbative threshold)
set: $\omega_{\mathbf{p}} = u\sqrt{M_P^2 + \mathbf{p}^2}$ with measured values.

- Indeed: For small momenta very good description of data!
- Note: picture depends on validity of χPT.
  $\Rightarrow$ Consistency checks have to be applied.
  They show very good agreement with χPT!
- Can also test this with MEM or the BGM:
  Also very good agrement with these fits!

Summary

Exploring the transition in the chiral limit at $N_f = 2$:

- The order of the transition in the $m_{ud} = 0$ limit is the remaining completely open question concerning the phase diagram at $\mu = 0$.
- Our analysis of the strength of the breaking of the $U_A(1)$ symmetry indicates a weak breaking in the chiral limit.
  $\Rightarrow$ In favour of a first order phase transition!

Finite-T spectroscopy and plasma properties:

- Have been the first to resolve the dssociation of a light hadron across the crossover from first principles.
- Have measured the electrical conductivity and second order hydrodynamic coefficients κ_t and κ_ℓ.
  (First computations with dynamical fermions.)
- Studied the fate of the pion and the reliability of χPT close to T_C.
  Pion quasiparticle persists at least up to $\approx 0.75\,T_C$.
  Dispersion relation is significantly modified by the medium.
  ($\Rightarrow$ breaking of Lorentz invariance).

Perspectives

Exploring the transition in the chiral limit at $N_f = 2$:

- Simulate at lighter pion masses.
  Physical pion mass (at a larger volume) is in preparation!
- Simulate the additional volumes.
  (First results are already available.)
- Long term: Continuum limit?!

Finite-T spectroscopy and plasma properties:

- Gain better understanding of quasiparticles in the transition region.
- Extand the studies to smaller (physical) quark masses.
- Compute more complicated quantities.
  (shear viscosity; bulk viscosity ...)
- Extend the study to $N_f = 2 + 1$.

Thank you for your attention!