The Constant-Sound-Speed parameterization of the quark matter EoS

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Alford, Han, Prakash, arXiv:1302.4732
Alford, Burgio, Han, Taranto, Zappalà, arXiv:1501.07902
Ranea-Sandoval, Han, Orsaria, Contrera, Weber, Alford, arXiv:1512.09183

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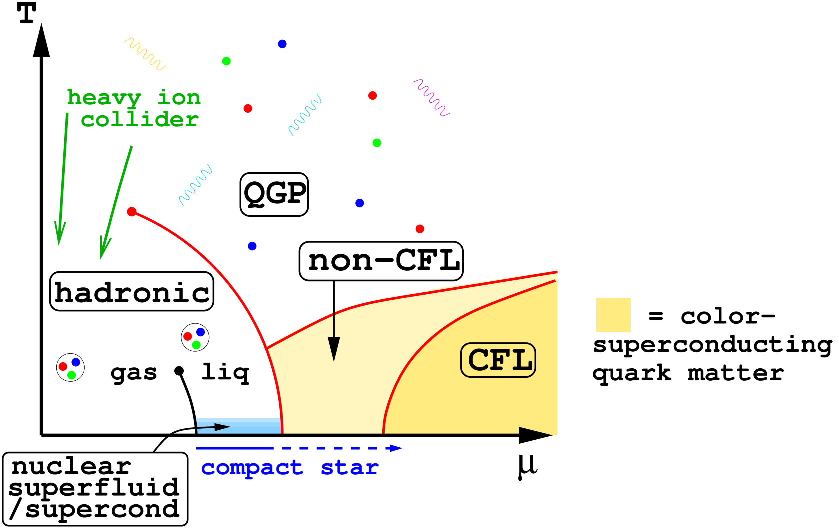
Schematic QCD phase diagram



M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, arXiv:0709.4635 (RMP review)

A. Schmitt, arXiv:1001.3294 (Springer Lecture Notes)

Signatures of quark matter in compact stars

Observable ← Microphysical properties (and neutron star structure) ← Phases of dense matter

	Property	Nuclear phase	Quark phase
mass, radius	eqn of state $\varepsilon(p)$	known up to $\sim n_{\rm sat}$	unknown; many models
spindown (spin freq, age)	bulk viscosity shear viscosity	Depends on phase:	Depends on phase:
		$n\,p\,e$	unpaired
cooling (temp, age)	heat capacity neutrino emissivity thermal cond.	$n\,p\,e,\mu$ $n\,p\,e,\Lambda,\,\Sigma^-$ n superfluid	CFL CFL-K^0 2SC
glitches (superfluid, crystal)	shear modulus vortex pinning energy	p supercond π condensate K condensate	CSL LOFF 1SC ...

Constraining QM EoS by observing $M(R)$

There is lots of literature about specific models of quark matter, e.g.

- MIT Bag Model; (Alford, Braby, Paris, Reddy, nucl-th/0411016)
- NJL models; (Paoli, Menezes, arXiv:1009.2906; Bonanno, Sedrakian, arXiv:1108.0559)
- PNJL models (Blaschke et. al, arXiv:1302.6275; Orsaria et. al.; arXiv:1212.4213)
- hadron-quark NLσ model (Negreiros et. al., arXiv:1006.0380)
- 2-loop perturbation theory (Kurkela et. al., arXiv:1006.4062)
- MIT bag, NJL, CDM, FCM, DSM (Burgio et. al., arXiv:1301.4060)

We need a model-independent parameterization of the quark matter EoS:

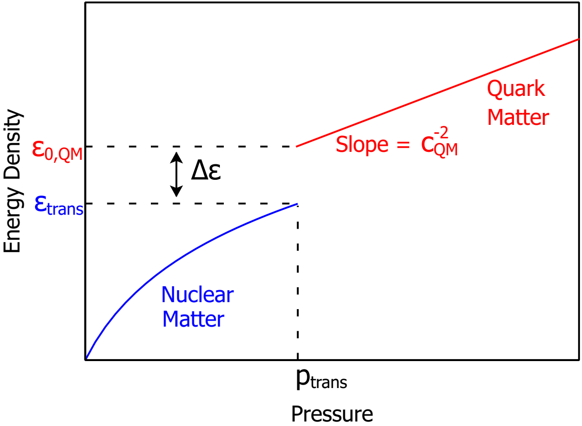
- framework for relating different models to each other
- observational constraints can be expressed in universal terms

CSS: a fairly generic QM EoS

Model-independent parameterization with

- Sharp 1st-order transition
- Constant [density-indp] Speed of Sound (CSS)

$$\varepsilon(p) = \varepsilon_{\mathrm{trans}} + \Delta\varepsilon + c_{\mathrm{QM}}^{-2}(p - p_{\mathrm{trans}})$$



QM EoS params:

$p_{\mathrm{trans}}/\varepsilon_{\mathrm{trans}}$

$\Delta\varepsilon/\varepsilon_{\mathrm{trans}}$

c_{QM}^2

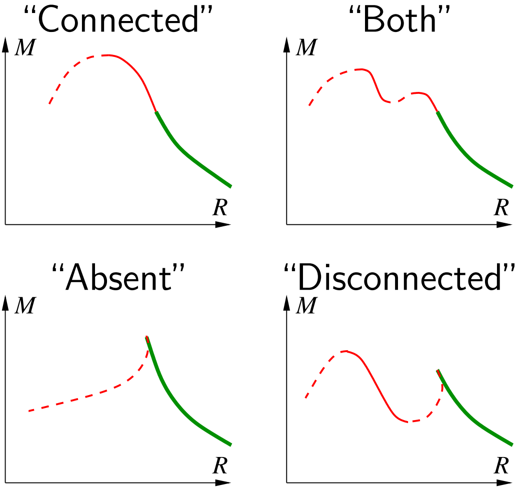
Zdunik, Haensel, `arXiv:1211.1231`; Alford, Han, Prakash, `arXiv:1302.4732`

Hybrid star $M(R)$

Hybrid star branch in $M(R)$ relation has 4 typical forms

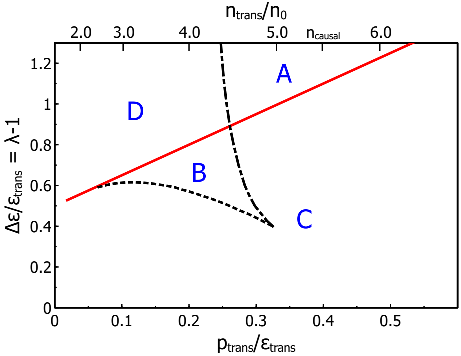
$\Delta\varepsilon < \Delta\varepsilon_{\rm crit}$
small energy density jump at phase transition

$\Delta\varepsilon > \Delta\varepsilon_{\rm crit}$
large energy density jump at phase transition

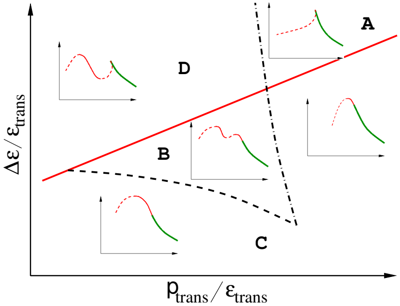


CSS "Phase diagram" of hybrid star $M(R)$

Soft NM + CSS($c^2_{\rm QM}=1$)



Schematic



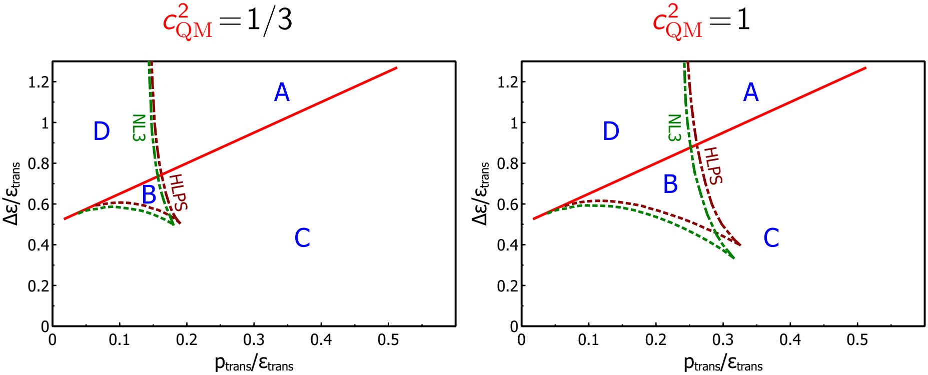
Above the red line ($\Delta\varepsilon > \Delta\varepsilon_{\rm crit}$), connected branch disappears

$$\frac{\Delta\varepsilon_{\rm crit}}{\varepsilon_{\rm trans}} = \frac{1}{2} + \frac{3}{2}\frac{p_{\rm trans}}{\varepsilon_{\rm trans}}$$

(Seidov, 1971; Schaeffer, Zdunik, Haensel, 1983; Lindblom, gr-qc/9802072)

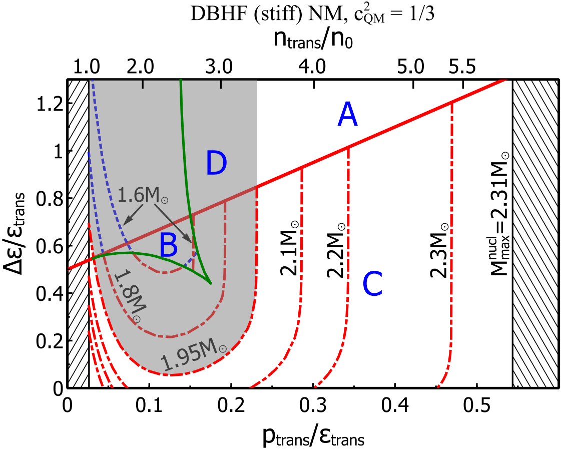
Disconnected branch exists in regions D and B.

Sensitivity to NM EoS and $c^2_{\rm QM}$



- NM EoS (HLPS=soft, NL3=hard) does not make much difference.
- Higher $c^2_{\rm QM}$ favors disconnected branch.

Constraints on QM EoS from $M_{\max}$



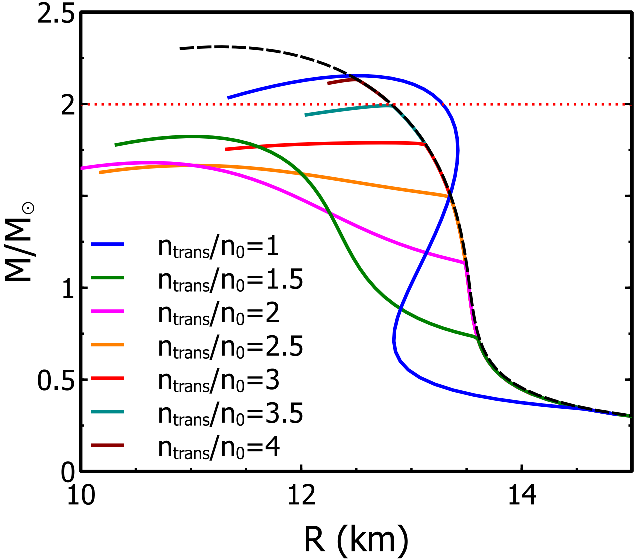
- Increasing $\Delta\varepsilon$ reduces $M_{\max}$
- Increasing $p_{\rm trans}$ at first reduces then increases $M_{\max}$

$2\,M_\odot$ observation allows two scenarios:

- high $p_{\rm trans}$: very small connected branch
- low $p_{\rm trans}$: modest $\Delta\varepsilon$, no disconnected branch.

Low $p_{\rm trans}$ and high $p_{\rm trans}$ windows

DBHF (stiff) NM, $c^2_{\rm QM} = 1/3$, $\Delta\varepsilon/\varepsilon_{\rm trans} = 0.4$



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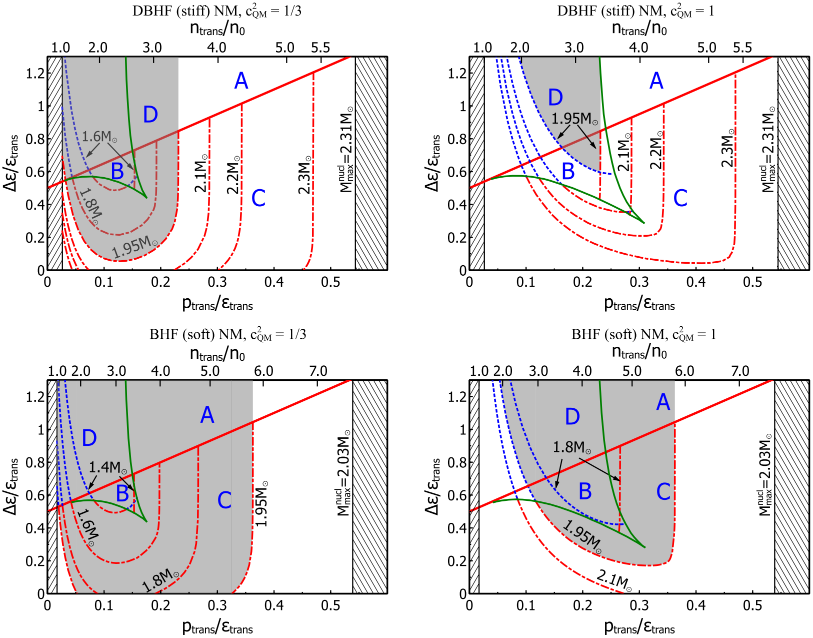
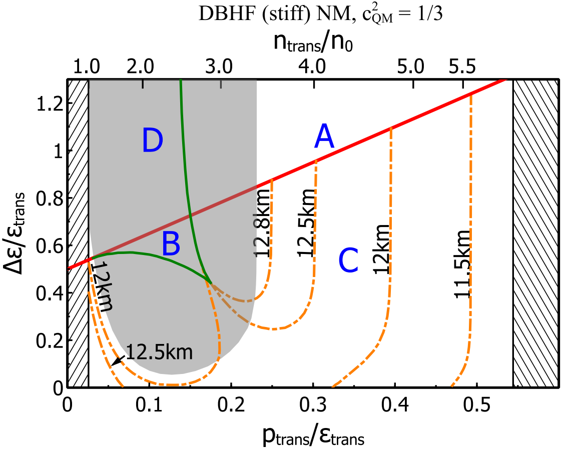


Radius of heaviest star $R_{\mathrm{max}M}$

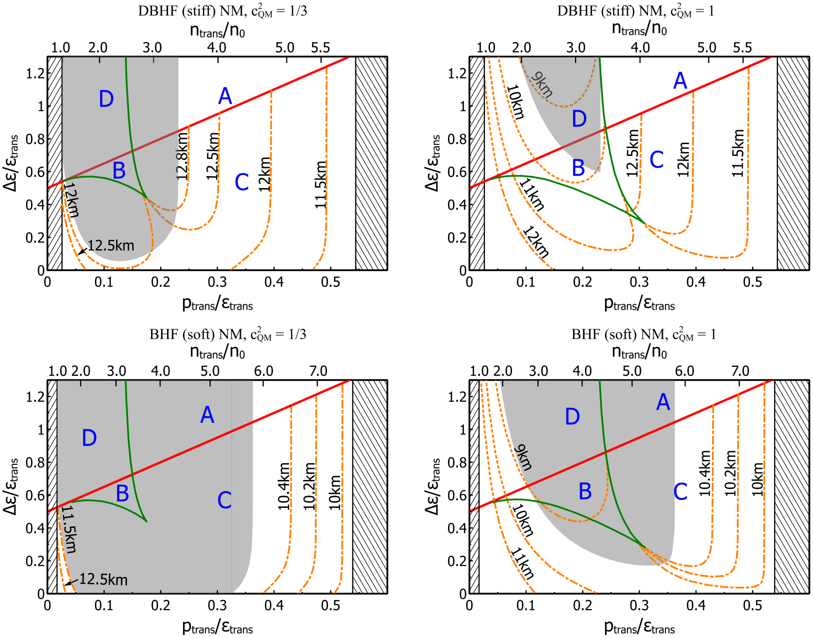


Heaviest star is typically the smallest, so lower limit on $R_{\mathrm{max}M}$ is the minimum radius of compact stars.

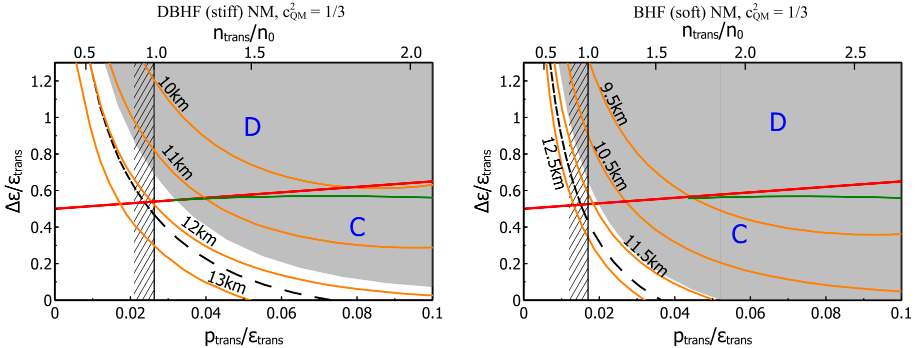
High p_{trans}: very short connected hybrid branch, radius like that of heaviest hadronic star.

Low p_{trans}: need to zoom in.

Constraints on QM EoS from $R_{\max M}$

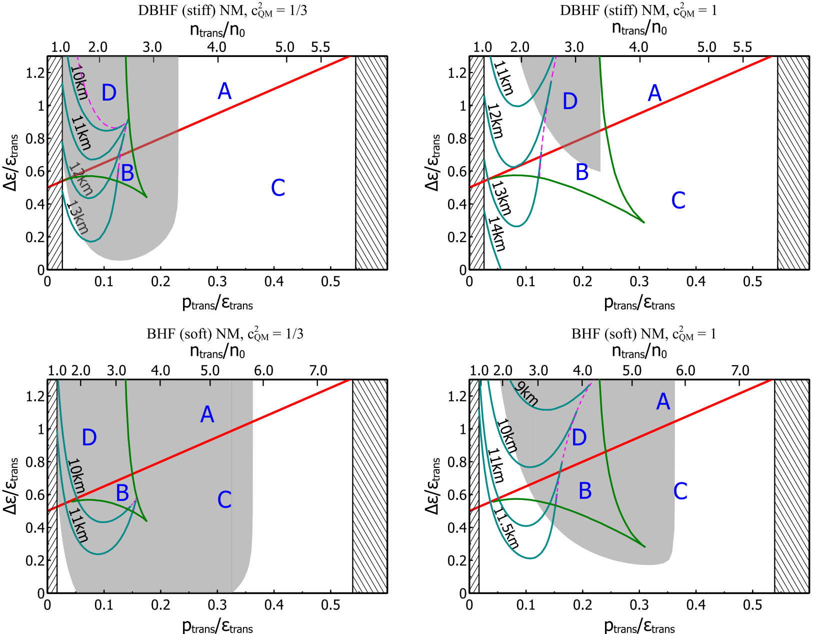


Focus on low $p_{\rm trans}$ and $c^2_{\rm QM} = 1/3$

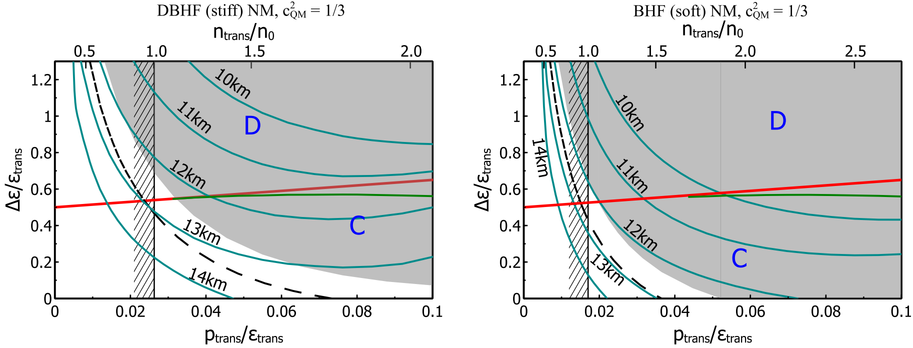


- $R_{\max M}$ contours closely follow mass contours
- $M_{\max} > 1.95\, M_\odot$ requires $R > 11.25\,$km
- dashed line is $M_{\max} = 2.1\, M_\odot$, requires $R > 12.1\,$km
- Observation of a smaller star $\Rightarrow$ high transition pressure or $c^2_{\rm QM} > 1/3$

Constraints on QM EoS from $R_{1.4\,\mathrm{M}_\odot}$



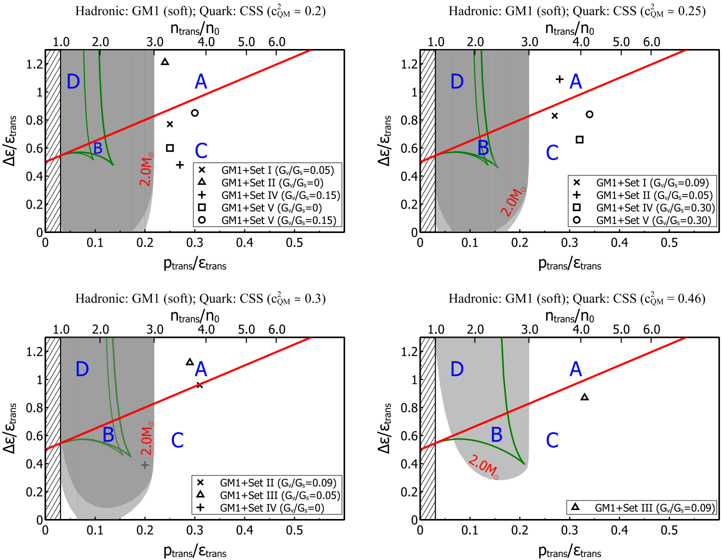
Low transition pressure and $R_{1.4\,\mathrm{M}_\odot}$



- $R_{1.4\,\mathrm{M}_\odot}$ contours roughly follow mass contours
- $M_{\mathrm{max}} > 1.95\, M_\odot$ requires $R_{1.4\,\mathrm{M}_\odot} > 12\,\mathrm{km}$ ($n_{\mathrm{trans}} \approx n_0$), rising with n_{trans}.
- dashed line is $M_{\mathrm{max}} = 2.1\, M_\odot$, requires $R_{1.4\,\mathrm{M}_\odot} > 12.7\,\mathrm{km}$
- Observation of a smaller $1.4\,\mathrm{M}_\odot$ star $\Rightarrow$ $c^2_{\mathrm{QM}} > 1/3$.
- If p_{trans} is high then no hybrid stars have mass $1.4\,\mathrm{M}_\odot$

compare Lattimer `arXiv:1305.3510`: $R > 11\,\mathrm{km}$.

NJL models in CSS space



Summary of CSS

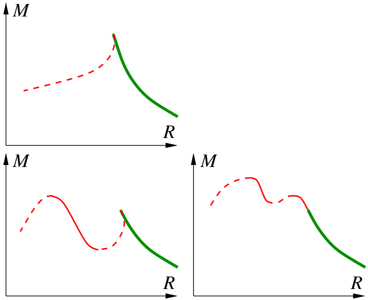
- CSS (Constant Speed of Sound) is a generic parameterization of the EoS close to a sharp first-order transition to quark matter.
- Any specific model of quark matter with such a transition corresponds to particular values of the CSS parameters
  $(p_{\rm trans}/\varepsilon_{\rm trans}, \quad \Delta\varepsilon/\varepsilon_{\rm trans}, \quad c^2_{\rm QM})$.
  Its predictions for hybrid star branches then follow from the generic CSS phase diagram.
- Every observation, e.g. observing a $2M_\odot$ neutron star, $\Rightarrow$ constraint on CSS parameters.
  E.g., for soft NM we need $c^2_{\rm QM} \gtrsim 1/3$
  (But note that $c^2_{\rm QM} = 1/3 - \mathcal{O}(\alpha_s)$ in pert QCD).
- More measurements of M and R would strengthen the constraints.
- Models of quark matter tend to have $c^2_{\rm QM} \sim 1/3$ and high transition pressure $\Rightarrow$ very short hybrid branch.

Could we identify hybrid stars via $M(R)$?

We could identify a phase transition to a high-density phase

(A) Nuclear branch ends with $dM/dR \neq 0$
occurs if $\Delta\varepsilon/\varepsilon_{\rm trans}$ is large enough

(B,D) Disconnected branch
can occur with $M_{\rm max} \gtrsim 2M_{\odot}$ if nuclear *and* quark matter are both stiff ($c^2_{\rm QM} \sim 1$)



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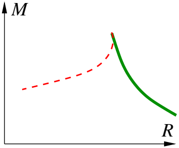

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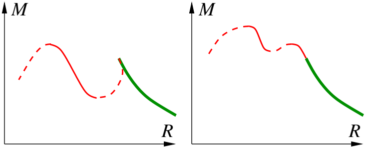

We need:

- better measurements of M and R
- knowledge of nuclear matter EoS

We could benefit from:

- theoretical constraints on parameters of QM EoS
  ($p_{\rm trans}/\varepsilon_{\rm trans}$, $\Delta\varepsilon/\varepsilon_{\rm trans}$, $c^2_{\rm QM}$)