Flavor dependence of hadron melting temperatures

in collaboration with J. Aichelin and B. Sintes

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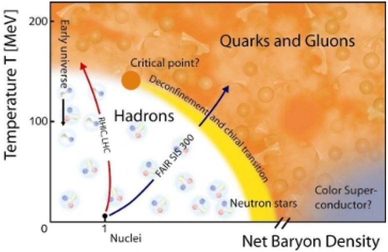
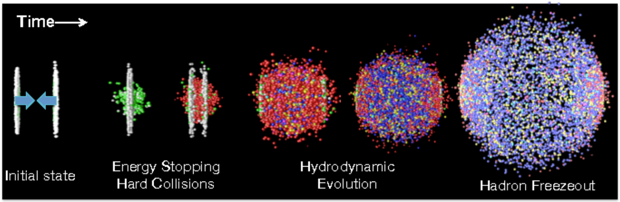
Frankfurt. June 18, 2015

Introduction: Relativistic Heavy-Ion Collisions



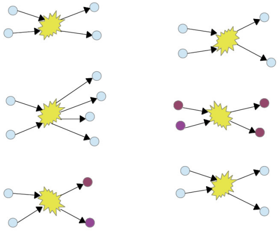
QCD phase diagram: Hadron and quark-gluon plasma phases



Heavy-ion collision to explore them (figure taken from T. Nayak, 2012).

Chemical freeze-out

Stage at which hadrons cease to have inelastic collisions in the plasma.



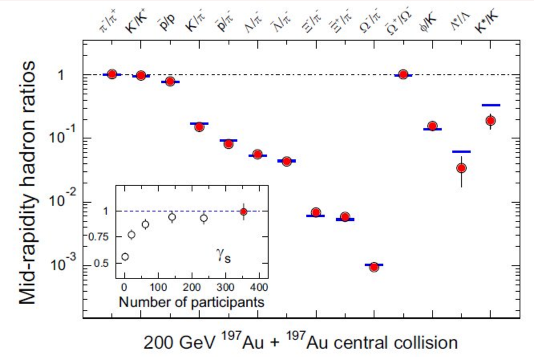
Collisions BEFORE chemical freeze-out — Collisions AFTER chemical freeze-out

What is "frozen"?

After the chemical freeze-out the total yield for each species is (approximately) fixed.

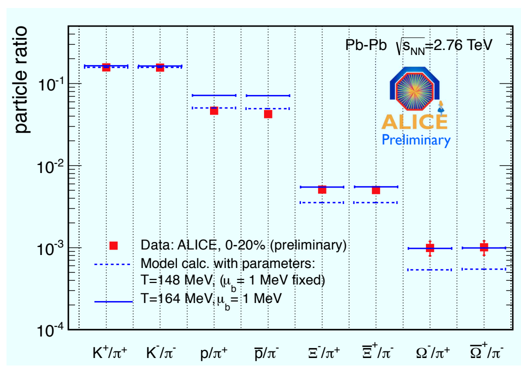
Chemical freeze-out determination

Global fit of total yields according to some statistical-thermal model



STAR Collaboration ($T_{ch} = 163$ MeV)

Same effect in LHC data:



ALICE Collaboration

The p^+/π^+ ratio seems to be inconsistent with other baryonic ratios. Strange baryons seem to have a larger chemical freeze-out temperature.

Possible solutions

Several proposals to explain this effect:

1. Missing hadronic resonances in thermal models?
2. Fit using non-equilibrium distributions?
3. Proton annihilation ($p + \bar{p} \to 5\pi$) beyond the chemical freeze out?
4. Flavor-dependent freeze-out temperature?

Several proposals to explain this effect:

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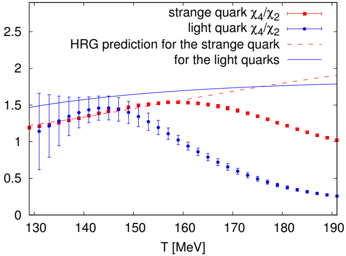
4. Flavor-dependent freeze-out temperature?

"Deconfinement" temperature seems to be flavor dependent in lattice-QCD computation
(Bellwied, Borsanyi, Fodor, Katz and Ratti, 2013)

$$\chi_{lmn}^{uds} = \frac{\partial^{l+m+n}(p/T^4)}{\partial(\mu_u/T)^l \partial(\mu_d/T)^m \partial(\mu_s/T)^n}$$



$$T_{light} \simeq 145 \text{ MeV}, \quad T_{strange} \simeq 160 \text{ MeV}$$

Contents

Goal

Study baryons in the context of an effective model and extract their melting temperature to compare non-strange with strange baryons.

(Polyakov)-Nambu-Jona–Lasinio Model

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Goal

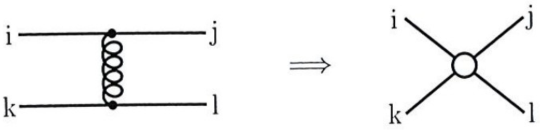
Study baryons in the context of an effective model and extract their melting temperature to compare non-strange with strange baryons.

(Polyakov)-Nambu-Jona–Lasinio Model

More ambitious goal

Adapt a simple model of QCD –able to describe partonic and hadronic phases– to simulate the whole evolution of the plasma after a heavy-ion collision.

NJL Lagrangian



From QCD, the NJL model is inspired on the limit $t \to 0$ of the gluon exchange

Interaction Lagrangian: color current interaction

$$\mathcal{L}_{int} = -g \quad [\bar{q}_i \gamma^\mu T^a \delta_{ij} q_j] \quad [\bar{q}_k \gamma_\mu T^a \delta_{kl} q_l]$$

Flavor: $i, j = 1...N_f = 3$; T^a : color representations $a = 1...N_c^2 - 1 = 8$.

And now we make a Fierz transformation to account to the relevant vertices that account for mesons $(\bar{q}q)_{1_c}$ and diquarks $(qq)_{\bar{3}c}$.

For reviews see Vogl and Weise (1991), Klevansky (1992), Ebert, Reinhardt and Volkov (1994), Hatsuda and Kunihiro (1994), Buballa (2004)...

NJL for mesons

Exchange Lagrangian: accounts for mesons

Pseudoscalar mesons

$$\mathcal{L}_{ex} = G \; (\bar{q}_i \; \tau^a_{ij} \; \mathbb{I}_c i\gamma_5 \; q_j) \; (\bar{q}_k \; \tau^a_{kl} \; \mathbb{I}_c i\gamma_5 \; q_l) \; ; \qquad G = (N_c^2 - 1)/N_c^2 g$$

$\mathbb{I}_c$: color singlet interaction
τ^a : flavor generators $a = 1...8$ ($N_f = 3$).

(Other terms not shown, e.g. the one accounting for scalar mesons $i\gamma_5 \to \mathbb{I}$)

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The effective Lagrangian should share the global symmetries of (massless) QCD:

Symmetries of massless NJL model

$$SU_V(3) \otimes SU_A(3) \otimes U_V(1) \otimes U_A(1)$$

In our scheme, chiral symmetry is explicitly broken to $SU_A(3)$ by the bare quark masses. We keep an isospin $SU_V(2)$ symmetry. The $U_A(1)$ is broken by quantum effects...

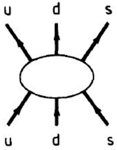
$U_A(1)$ breaking

$U_A(1)$ symmetry is broken by the axial anomaly.
It accounts for the mass difference between the η and η' (mixing between flavor octet and singlet).

't Hooft Lagrangian

$$\mathcal{L}_{'t\ Hooft} = H \det_{ij} [\bar{q}_i(1-\gamma_5)q_j] - H \det_{ij} [\bar{q}_i(1+\gamma_5)q_j]$$

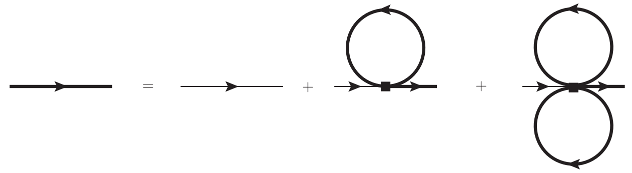
For $N_f = 3$ it represents a six-quark contact interaction.



H is fixed by the $\eta - \eta'$ mass difference.

Note: We will not directly work with this operator.
We will use the mean-field approximation to simplify it...

The dressed quark masses are calculated through the **gap equation** in the mean-field approximation



$$m_q \quad = \quad m_{q0} \quad - \quad 4G\langle \bar{q}q \rangle \quad + \quad 2H\langle \bar{q}'q' \rangle \langle \bar{q}''q'' \rangle$$

Quark condensate

$$\langle \bar{q}q \rangle = -iN_c \mathrm{Tr}\ S_q \ , \quad S_q : \text{ quark propagator}$$

Temperature and chemical potential

We use imaginary time formalism ($0 \leq it \leq \beta = 1/T$) with the prescription

Imaginary time formalism

$$k_0 \to i\omega_n, \qquad \int \frac{d^4k}{(2\pi)^4} \to iT \sum_{n\in\mathcal{Z}} \int \frac{d^3k}{(2\pi)^3}$$

with $i\omega_n = i\pi T(2n+1)$ the fermionic Matsubara frequencies.

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The quark chemical potential can be introduced by the Lagrangian

Quark chemical potential

$$\mathcal{L}_\mu = \sum_{ij} \bar{q}_i \mu_{ij} \gamma_0 q_j$$

where

$$\mu_{ij} = \mathrm{diag}\,(\mu_u, \mu_d, \mu_s)$$

Gluon (static) properties are implemented in the model through an effective potential for the Polyakov loop

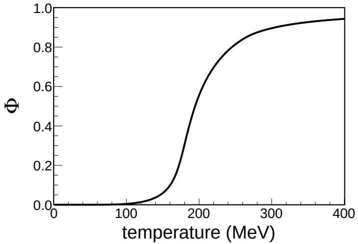
$$\frac{U(T,\Phi,\bar{\Phi})}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3 + \bar{\Phi}^3\right) + \frac{b_4}{4}(\bar{\Phi}\Phi)^3$$

$$b_2(T) = a_0 + a_1 \frac{T_0}{T} + a_2 \left(\frac{T_0}{T}\right)^2 + a_3 \left(\frac{T_0}{T}\right)^3$$

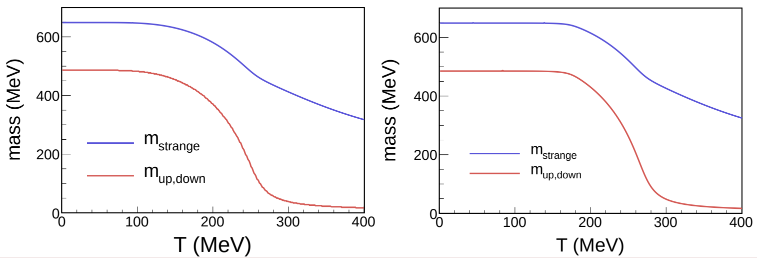
$a_0 = 6.75$, $a_1 = -1.95$, $a_2 = 2.625$, $a_3 = -7.44$, $b_3 = 0.75$, $b_4 = 7.5$ from fit to lattice-QCD results of Yang-Mills and $T_0 = 190$ MeV.

Φ is the order parameter
of the deconfinement transition

$$\Phi = \frac{1}{N_c} \mathrm{Tr}_c \left\langle \mathcal{P} \exp\left(-\int_0^\beta d\tau A_0(\mathbf{x},\tau)\right)\right\rangle$$



Quark masses as a function of the temperature (zero chemical potential)

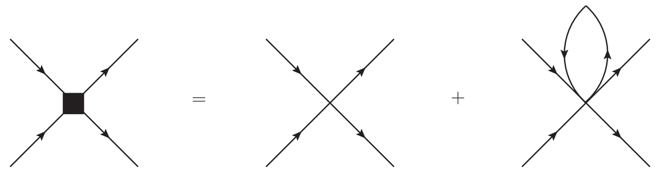


The masses from the PNJL model are systematically more stable at low T.

$\bar{q}q$ rule for mesons

Combine G and K at mean-field level into an effective coupling, e.g. for pion:

$$G_{eff} = G + \frac{1}{2} H \langle \bar{s} s \rangle$$



The complete Feynmann rule for the 4-point function

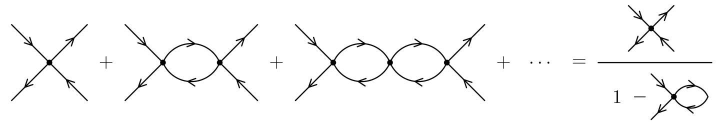
$$\mathcal{K} = \Omega \; 2G_{eff} \; \bar{\Omega}$$

where Ω encodes color, flavor and spin factors

$$\Omega = \mathbb{I}_c \otimes \tau^a \otimes \{1, i\gamma_5, \gamma_\mu, \gamma_5\gamma_\mu\}$$

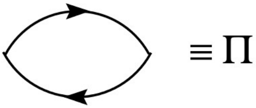
This $\mathcal{K}$ is used as the kernel for the Bethe-Salpeter equation

$$T(p) = \mathcal{K} + i \int \frac{d^4k}{(2\pi)^4} \, \mathcal{K} \, S\left(k+\frac{p}{2}\right) S\left(k-\frac{p}{2}\right) T(p)$$

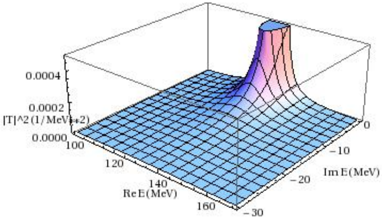


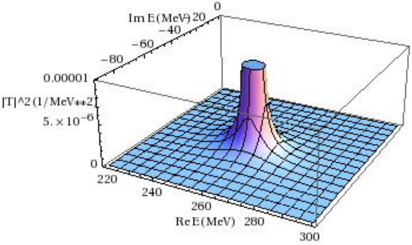
The solution of the $T-$amplitude at finite temperature

$$T(p) = \frac{\mathcal{K}}{1-\mathcal{K}\Pi(p)} \;, \qquad \Pi(p_0,\mathbf{p}) = -T\sum_n \int \frac{d^3k}{(2\pi)^3} \, S\left(k+\frac{p}{2}\right) S\left(k-\frac{p}{2}\right)$$



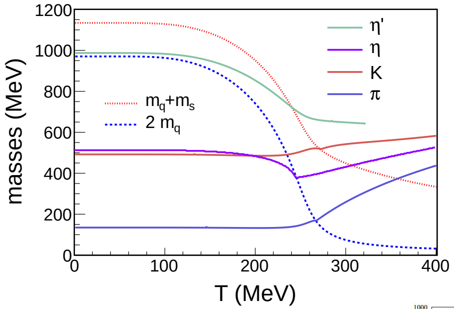
Mesons appear as bound states/resonances of $\bar{q}q$ scattering, i.e. poles of the $T-$matrix.





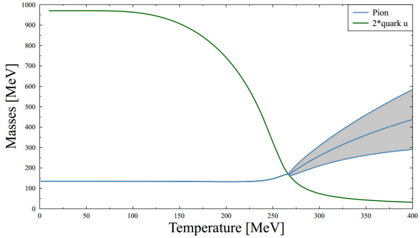
The position of the pole gives the mass and width of the meson as a function of the temperature.

$$1 - \mathcal{K}\ \Pi(p_0 = M_{meson} - i\Gamma_{meson}/2, \mathbf{p} = 0) = 0$$



Pseudoscalar mesons
in the NJL model at $\mu = 0$

Pion mass (blue line) and
decay width (grey band)



We can also combine two quarks to form a DIQUARK.

However, it cannot be color singlet $\not{\mathbb{1}}_c$
$\rightarrow$ they are not interesting as observable states

Allowed combinations

Color	Flavor	J^P	Denomination
$\mathbf{6}_S$	Not considered here		
$\bar{\mathbf{3}}_A$	$\mathbf{6}_S$	1^+_S	Axial
$\bar{\mathbf{3}}_A$	$\bar{\mathbf{3}}_A$	0^+_A	Scalar
$\bar{\mathbf{3}}_A$	$\bar{\mathbf{3}}_A$	0^-_A	Pseudoscalar
$\bar{\mathbf{3}}_A$	$\bar{\mathbf{3}}_A$	1^-_A	Vector

We focus on SCALAR and AXIAL diquarks to build baryons.
(the other diquarks have masses much higher and not suitable to form baryons)

A Fierz transformation provides que correct combination (Buballa, 2005) e.g. the SCALAR sector

Scalar diquark sector

$$\mathcal{L}_{qq} = G_{DIQ} \, (\bar{q}\tau_A \lambda_{A'} i\gamma_5 C \bar{q}^T)(q^T \tau_A \lambda_{A'} C i\gamma_5 q) \; , \quad G_{DIQ} = (N_c+1)g/(2N_c)$$

($C = i\gamma_0\gamma_2$; A, A' : antisymmetric members of the Gell-Mann matrices)

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($C = i\gamma_0\gamma_2$; A, A' : antisymmetric members of the Gell-Mann matrices)

The procedure is analogous to mesons taking care of the different reps, spin structure and charge conjugations.

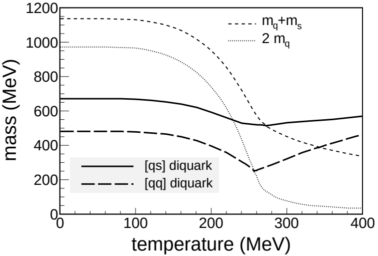
Solution of Bethe-Salpeter equation

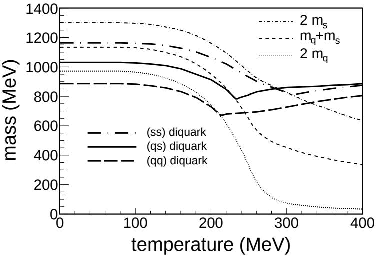
$$T(p) = \frac{2G_{DIQ}}{1-2G_{DIQ}\Pi(p)}$$

$$\Pi(p) = i\int \frac{d^4k}{(2\pi)^4} \, \mathrm{Tr} \, \left[\bar{\Omega}\, S\left(k+\frac{p}{2}\right)\Omega S^T\left(\frac{p}{2}-k\right)\right]$$

There is no flavor singlets $\rightarrow$ 't Hooft terms are not included

Result for [Scalar] and (Axial) diquarks (q denotes either u or d quark)



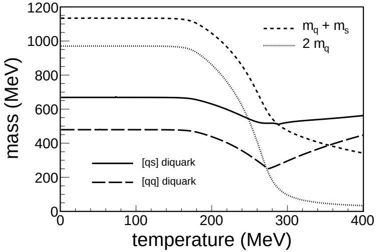


Mott temperatures

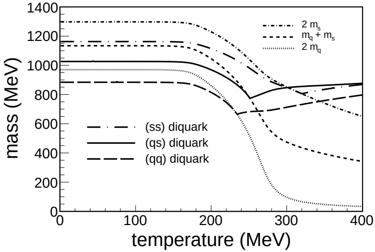
Diquark	$[qq]$	$[qs]$	(qq)	(qs)	(ss)
T_c **(MeV)**	256	273	212	233	307

Diquarks with strangeness have larger melting temperature!

More stable masses in the PNJL model:



SCALAR



AXIAL

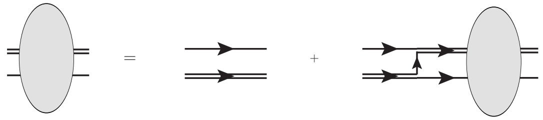
They will be used to contruct the members of the baryon octet and decuplet, respectively.

$$\mathbf{3} \otimes (\bar{\mathbf{3}} \oplus \mathbf{6}) = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$$

A genuine (3-body) Fadeev equation is effectively written as a two-body equation for diquark+quark interaction (Reinhardt 1990; Buck et al. 1992) Schematically:

Dyson equation

$$G(P) \quad = \quad G_0 \quad + \quad G_0 Z G(P)$$



$$G_0 = S_q \text{ (quark propagator)} \times t_D \text{ (diquark propagator)}$$

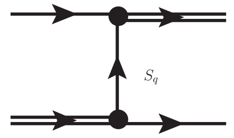
with

Diquark propagator

$$t_D(q) = -\frac{g_{eff}^2}{q_0^2 - \mathbf{q}^2 - m_{DIQ}^2}$$

The elementary interaction reads (note that effective coupling have been included in diquark propagator)

$$Z = \Omega \ S_q \ \Omega$$



The solution for the baryon propagator

$$G(P) = \frac{G_0}{1 - G_0 Z}$$

Baryon masses

$$1 - G_0 Z(P_0 = M_{baryon}, \mathbf{P} = 0) = 0$$

This equation can be reduced assuming the **static approximation**:
one neglects the transferred momentum of the exchanged quark with respect to its dressed mass.

Static Approximation

$$S_q(q) = \frac{1}{\not{q} - m_{quark}} \to -\frac{\mathbb{I}_{Dirac}}{m_{quark}}$$

This approximation seems to be quite drastic, but it has been tested providing a numerical error of 5 % for the nucleon (Buck et al. 1992).
We assume that this approximation is not too crude in our scheme.

Final equation for the members of the octet

The final equation looks like (omiting Dirac structure, flavor projections and coupled-channel indices)

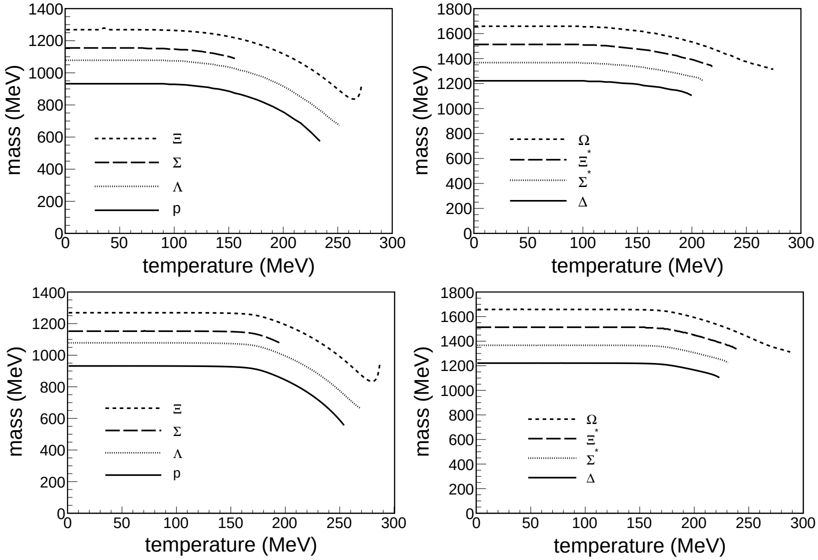
Baryon masses

$$\left[1 - \frac{2}{m_l} \; T \sum_n \int \frac{d^3q}{(2\pi)^3} S_q(i\omega_n, \mathbf{q}) \; t_D(i\nu_l - i\omega_n, -\mathbf{q}) \right] \Bigg|_{i\nu_l \to P_0 + i\epsilon = M_{Baryon}} = 0$$

where

- $-1/m_l$ comes for quark exchange in the static limit
- S_q and t_D are the quark and diquark propagators
- The intermediate momentum is integrated $T \sum_n \int_q$
- Baryon is taken at rest $\mathbf{P} = 0$
- The external frequency $i\nu_l$ is analitycally continued to real energies $P_0 + i\epsilon$.

Baryon masses in $\mathbf{8}_f$ and $\mathbf{10}_f$ representations, as a function of temperature



Summary of melting temperatures

Baryon	NJL T_c (MeV)	PNJL T_c (MeV)
p	234	254
Λ	252	269
Σ	156	195
Ξ	272	287
Δ	200	223
Σ^*	211	231
Ξ^*	219	239
Ω	275	288

We have defined the baryon melting temperature as

$$T_c \equiv \min\{T_{Mott}(\text{baryon}), T_{Mott}(\text{diquark})\}$$

Conclusions

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- T_c of Ξ and Ω baryons are very close as suggested by thermal fit (even with totally different diquark content) and the difference is of the order of the difference for the freeze-out temperature.
- However, we find not a clear hierarchy based on strangeness content. Look at the Σ!
- Relative temperatures are very robust within models. Although the absolute values provided by the model seem to be quite high with respect to the typical values of the lattice for the crossover temperature.

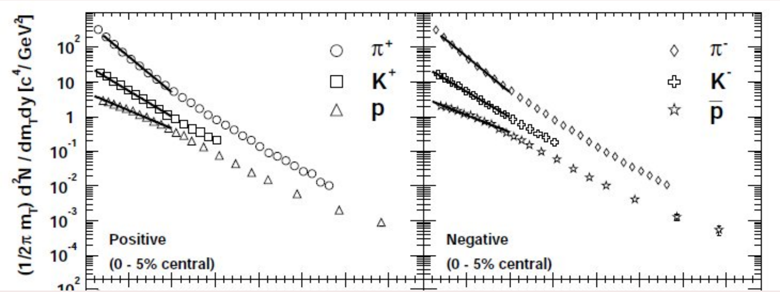
Summary

- We can study hadrons (mesons and baryons) in an effective model of QCD which does not present true confinement.
- We have a precise definition for the melting temperature, which can be understood as a hadronization temperature in our scheme.
- A robust conclusion is that the melting temperature depends on the flavor content (strangeness), albeit no clear hierarchy. Approximate consistency with experimental results.
- Many improvements can be done: break isospin limit, improve the static approximation, axial diquark contribution in octet baryons, results at finite chemical potential, stable baryons with unstable diquarks...
- Finally, we want to implement these hadron masses and their interaction into a dynamical evolution code for heavy-ion collisions, and predict results like multiplicities, elliptic flow...
  (see Marty and Aichelin, Phys.Rev. C87 (2013) 034912 for first results).

THANKS FOR YOUR ATTENTION!

Kinetic freeze-out

Fit of individual charged multiplicities to Boltzmann distribution

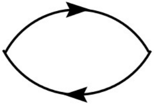


PHENIX Collaboration (Au+Au at $\sqrt{s_{NN}} = 200$ GeV)

$$m_T = \sqrt{p_T^2 + m_0^2}$$

$$\frac{d^2N}{2\pi m_T dm_T dy} = \frac{1}{2\pi T(T+m_0)} \cdot A \cdot \exp\left(-\frac{m_T - m_0}{T}\right)$$

$$\Pi_{12}^{P}(p_0,\mathbf{p}) = -\frac{N_c}{8\pi^2}\left\{A(m_1)+A(m_2)+\left[(m_1-m_2)^2-p_0^2+\mathbf{p}^2\right]B_0(\mathbf{p},m_1,m_2,p_0)\right\}$$



$$\equiv \Pi$$

with

$$A(m_1) = 16\pi^2 T\sum_n \int \frac{d^3k}{(2\pi)^3}\frac{1}{(i\omega_n)^2-\mathbf{k}^2-m_1^2}$$

and

$$B_0(\mathbf{p},m_1,m_2,i\nu_m\to p_0+i\epsilon) = 16\pi^2 T\sum_n \int \frac{d^3k}{(2\pi)^3}\frac{1}{(i\omega_n)^2-\mathbf{k}^2-m_1^2} \times \frac{1}{(i\omega_n-i\nu_m)^2-(\mathbf{p}-\mathbf{k})^2-m_2^2}$$

We take the inverse of the T-matrix

$$T^{-1}(p^2) = \frac{1 - \mathcal{K}\Pi(p^2)}{\mathcal{K}}$$

and make a Taylor expansion around the pole mass of the meson/diquark $p^2 = m^2$

$$T^{-1}(p^2) = \left.\frac{\partial T^{-1}(p^2)}{\partial p^2}\right|_{p^2=m^2} (p^2 - m^2) + \cdots$$

$$T^{-1}(p^2) = -\left.\frac{\partial \Pi(p^2)}{\partial p^2}\right|_{p^2=m^2} (p^2 - m^2) + \cdots$$

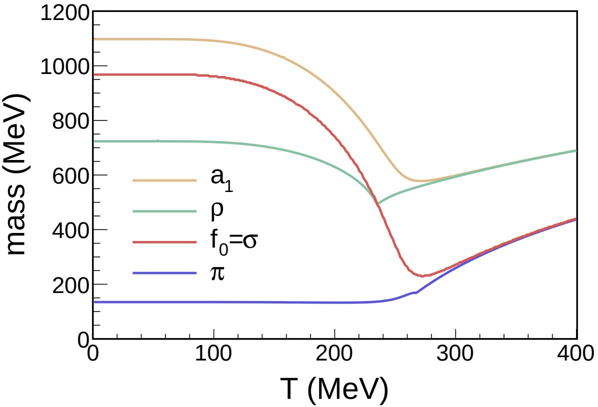
Finally, let us define and effective coupling constant

$$g_{eff}^2 \equiv \frac{1}{\left.\frac{\partial \Pi(p^2)}{\partial p^2}\right|_{p^2=m^2}} \tag{1}$$

to finally obtain the meson propagator from the $T-$matrix

$$t(p^2) \simeq \frac{-g_{eff}^2}{p^2 - m^2}$$

Restoration of chiral symmetry at large temperatures



We repeat the calculation with diquarks $\langle q^T \Omega q \rangle$.
There are many types of diquarks according to $\Omega = \Omega_c \otimes \Omega_f \otimes \Omega_{Dirac}$:

$$\Omega_c \in \mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$$

$$\Omega_f \in \mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$$

$$\Omega_{Dirac} \in 1/2 \otimes 1/2 = 0 \oplus 1$$

With the constraint of Pauli principle: total antisymmetry

$$\Omega^T = -\Omega$$

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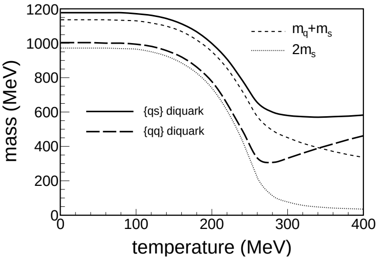
Allowed combinations

Color	Flavor	J^P	Denomination
$\mathbf{6}_S$	Not considered here		
$\bar{\mathbf{3}}_A$	$\mathbf{6}_S$	1^+_S	Axial
$\bar{\mathbf{3}}_A$	$\bar{\mathbf{3}}_A$	0^+_A	Scalar
$\bar{\mathbf{3}}_A$	$\bar{\mathbf{3}}_A$	0^-_A	Pseudoscalar
$\bar{\mathbf{3}}_A$	$\bar{\mathbf{3}}_A$	1^-_A	Vector

Pseudoscalar and vector diquarks have **too large masses** and they present decay widths even at $T = 0$



Therefore, we are not assume these states to take part of baryons.

The formal equation for the baryon wavefunction $\mathcal{X}$ (flavor octet) reads (Reinhardt 1990; Buck et al. 1992)

Two-body equation

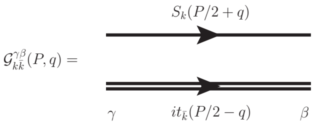
$$\mathcal{X}_j^{\bar{j}}(P,q) - \int \frac{d^4k}{(2\pi)^4} Z_{jk}^{\bar{k}\bar{j}}(q,k)\ G_{0,k\bar{k}}(P,q)\ \mathcal{X}_k^{\bar{k}}(P,k)\ |_{P^2=M^2_{Baryon}} = 0$$

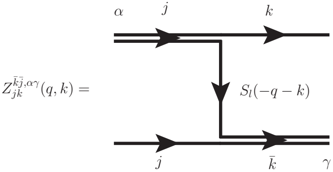
Quark index j, diquark index $\bar{j}$. The two-body quark-diquark propagator reads

$$G_{0,k\bar{k}}(P,q) = S_k(P/2+q)\ it_{\bar{k}}(P/2-q)$$

And the interaction subdiagram with two diquark-quark-quark vertices

$$Z_{jk}^{\bar{k}\bar{j}}(q,k) = \Omega_{jl}^{\bar{k}}\ S_l(-q-k)\ \Omega_{lk}^{\bar{j}}$$





It is convenient to use normalizated projectors on to the physical baryons.

$$(\mathcal{P}^{A}_{j\bar{i}})^{\dagger}\mathcal{P}^{A'}_{\bar{i}j} = \delta^{*,AA'}$$

In the octet representation of $SU(3)$ we can construct them from the Gell-Mann matrices

$$\mathcal{P}^{p}_{\bar{i}j} = \frac{1}{2}\left(\lambda^{4} - i\lambda^{5}\right)_{\bar{i}j} \; ; \qquad \mathcal{P}^{n}_{\bar{i}j} = \frac{1}{2}\left(\lambda^{6} - i\lambda^{7}\right)_{\bar{i}j}$$

$$\mathcal{P}^{\Lambda}_{\bar{i}j} = \mathcal{P}^{8}_{\bar{i}j} = \sqrt{\frac{1}{2}}\lambda^{8}_{\bar{i}j} \; ; \qquad \mathcal{P}^{\Sigma^0}_{\bar{i}j} = \mathcal{P}^{3}_{\bar{i}j} = \sqrt{\frac{1}{2}}\lambda^{3}_{\bar{i}j}$$

$$\mathcal{P}^{\Sigma^{\pm}}_{\bar{i}j} = \frac{1}{2}\left(\lambda^{1} \mp i\lambda^{2}\right)_{\bar{i}j} \; , \qquad \mathcal{P}^{\Xi^0}_{\bar{i}j} = \frac{1}{2}\left(\lambda^{6} + i\lambda^{7}\right)_{\bar{i}j}$$

$$\mathcal{P}^{\Xi^-}_{\bar{i}j} = \frac{1}{2}\left(\lambda^{4} + i\lambda^{5}\right)_{\bar{i}j} \; , \qquad \mathcal{P}^{0}_{\bar{i}j} = \sqrt{\frac{1}{3}}\mathbb{I}_{\bar{i}j}$$

Where the singlet flavor mixes with the $\Lambda - \Sigma^0$, producing a coupled channel problem (note that the Λ, Σ and singlet contains the same quark content). Because of this fact, the projectors are not totally orthogonal (*).
Similar for the decuplet representation of $SU(3)$ to obtain the $\Delta, \Sigma^*, \Xi^*, \Omega$.