Deconfinement and Equation of State in QCD

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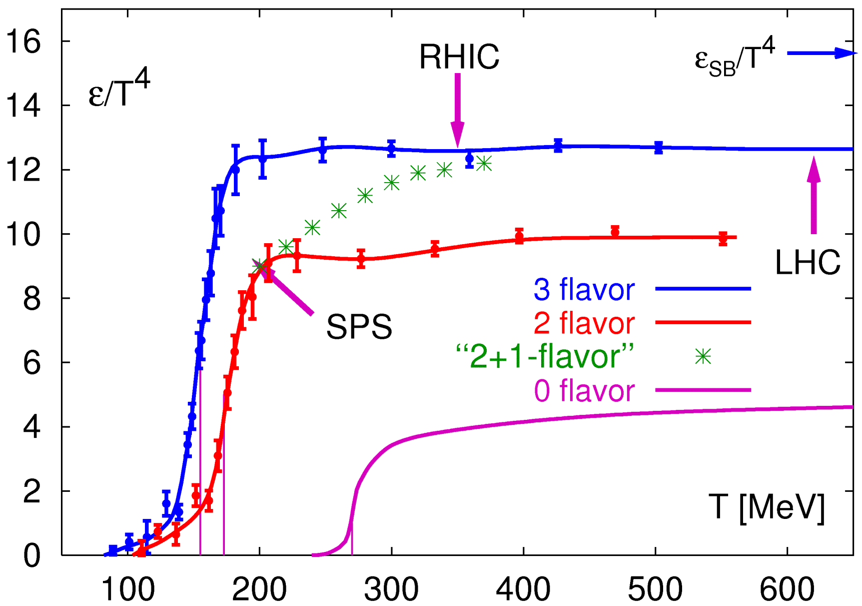
- Introduction (Symmetries of QCD, Computational challenges etc.)
- Deconfinement and Color Screening
- QCD Equation of State in the Continuum Limit at Zero Net Baryon Density
- Taylor expansion: Equation of State at Non-Zero Baryon Density
- Taylor expansion: Fluctuations and Correlations of Conserved Charges and Deconfinement
- Conclusions

Frankfurt, July 13, 2015

Lattice QCD at T>0 now and then

Lattice QCD calculations at T>0 around 2002:



$T_c \simeq 173\mathrm{MeV}$

for both chiral transition and deconfinement transition (in terms of Polyakov loop)

Problems:

$N_\tau = 4 : a \equiv 1/(N_\tau a) = 1/(4T)$

$m_\pi = (500 - 800)\mathrm{MeV}$

Continuum limit and physical masses are needed

$N_\tau \to \infty$

$m_\pi = 140\mathrm{MeV}$

$costs \sim N_\tau^{11}$

$\sim 1/m_\pi^3$

2014: Calculations using Highly Improved Staggered Quark (HISQ) formulations

⇒ Largely reduced discretization effects, continuum extrapolation possible

$m_\pi = 160\mathrm{MeV}$

Fluctuations of conserved charges: new look into deconfinement and QGP properties

Symmetries of QCD at T>0

- Chiral symmetry : $m_{u,d} \ll \Lambda$

  $SU_A(2)$ symmetry $\psi \to e^{i\phi T^a \gamma_5}\psi$ $\quad \psi_{L,R} \to e^{i\phi_{L,R} T^a}\psi_{L,R}$

  $$\langle\bar\psi\psi\rangle = \langle\bar\psi_L\psi_R\rangle + \langle\bar\psi_R\psi_L\rangle \neq 0$$

  $U_A(1)$ is borken by anomaly

- Center (Z3) symmetry : invariance under global gauge transformation

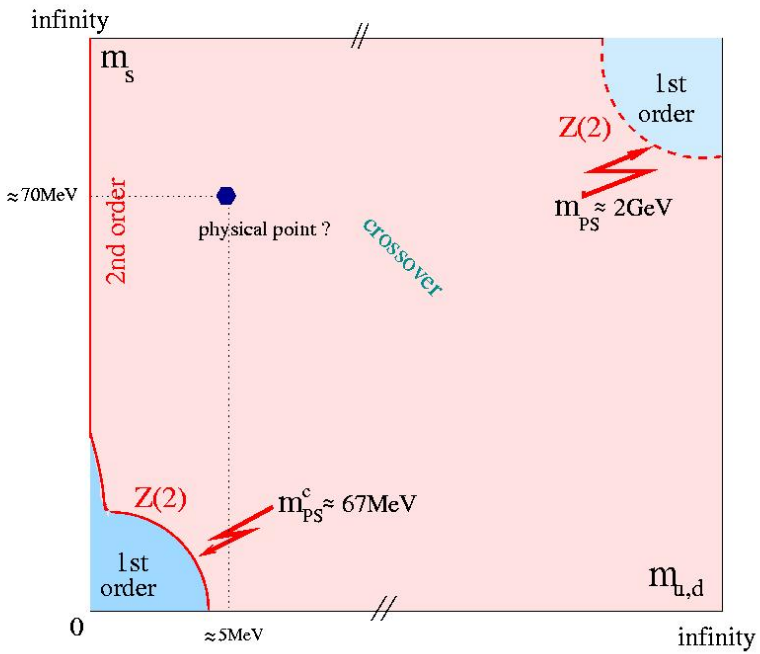
  $$A_\mu(0,\mathbf{x}) = e^{i2\pi N/3} A_\mu(1/T,\mathbf{x}), \;\; N = 1,2,3$$

  Exact symmetry for infinitely heavy quarks $\quad \langle L\rangle = 0$

  Polyakov loop : $\quad L = \mathrm{tr}\mathcal{P} e^{ig\int_0^{1/T} d\tau A_0(\tau,\vec{x})}$

$\langle\bar\psi\psi\rangle = 0$ restored

$\langle L\rangle \neq 0$ broken



LQCD calculations with staggered quarks suggest crossover, e.g. Aoki et al, Nature 443 (2006) 675

Evidence for 2nd order transition in the chiral limit
⇒ universal properties of QCD transition:

$SU_A(2) \sim O(4)$
relation to spin models

$U_A(1)$ restoration ?

Center symmetry does not seem to play any role in QCD

O(N) scaling and the transition temperature

The notion of the transition temperature is only useful if it can be related to the critical temperature in the chiral limit : fit the lattice data on the chiral condensate with scaling form + simple Ansatz for the regular part

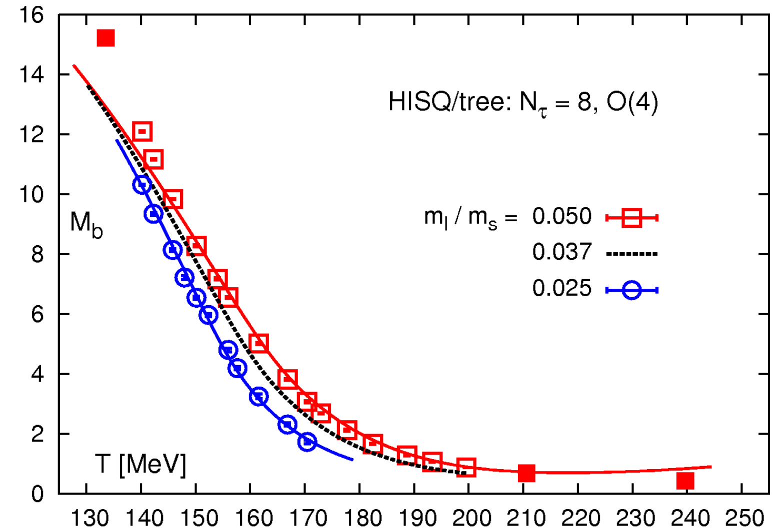
$$M_b = \frac{m_s \langle \bar{\psi}\psi \rangle_l}{T^4} = h^{1/\delta} f_G(t/h^{1/\beta\delta}) + f_{M,reg}(T,H)$$

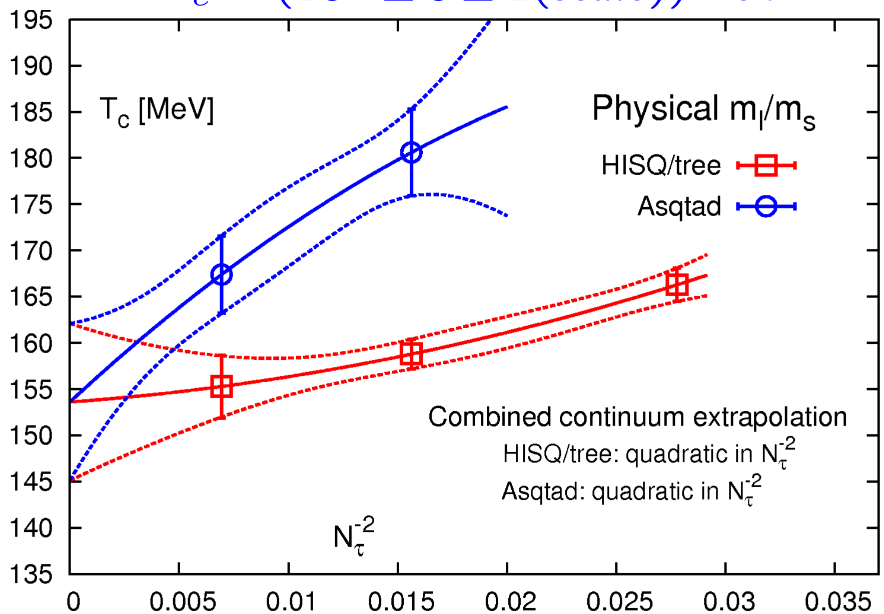
$$f_{reg}(T,H) = (a_1(T - T_c^0) + a_2(T - T_c^0)^2 + b_1)H$$

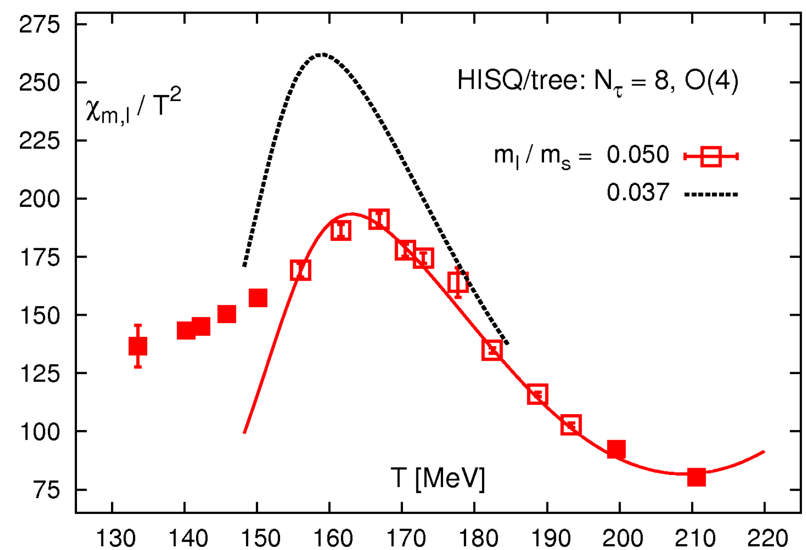
$$t = \frac{1}{t_0}\left(\frac{T - T_c^0}{T_c^0} + \kappa \frac{\mu_q^2}{T^2}\right),\ H = \frac{m_l}{m_s}, h = \frac{H}{h_0}$$

6 parameter fit : T_c^0, t_0, h_0, a_1, a_2, b_1

$$T_c = (154 \pm 8 \pm 1(scale))\text{MeV}$$







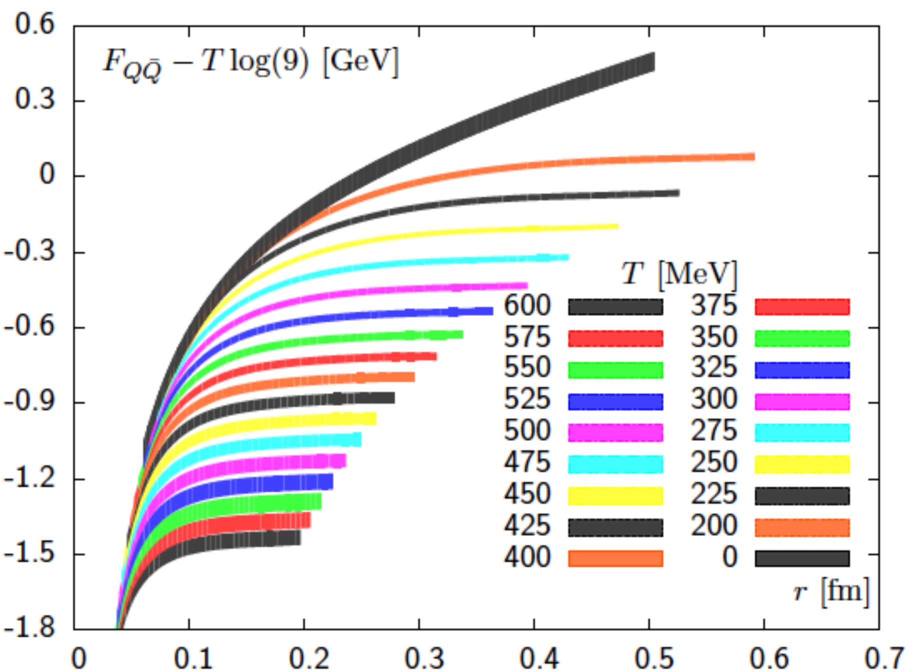
Deconfinement and color screening

Onset of color screening is described by Polyakov loop

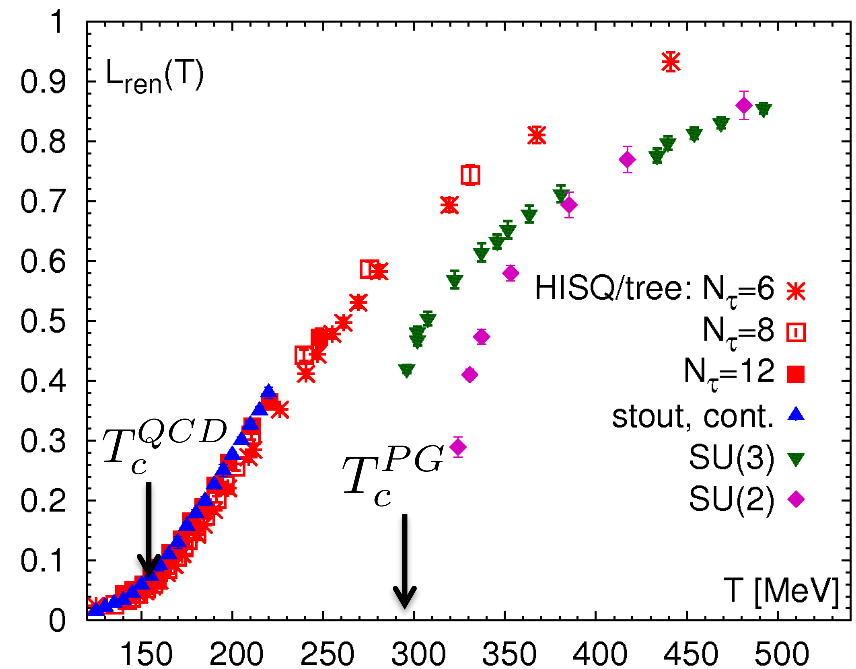
$$L = \mathrm{tr}\mathcal{P}e^{ig\int_0^{1/T} d\tau A_0(\tau,\vec{x})} \qquad \exp(-F_{Q\bar{Q}}(r,T)/T) = \frac{1}{9}\langle \mathrm{tr}L(r)\mathrm{tr}L^\dagger(0)\rangle$$

$$F_{Q\bar{Q}}(r\to\infty,T) = 2F_Q(T) \quad \Rightarrow \quad L_{ren} = \exp(-F_Q(T)/T)$$

2+1 flavor QCD, continuum extrapolated:



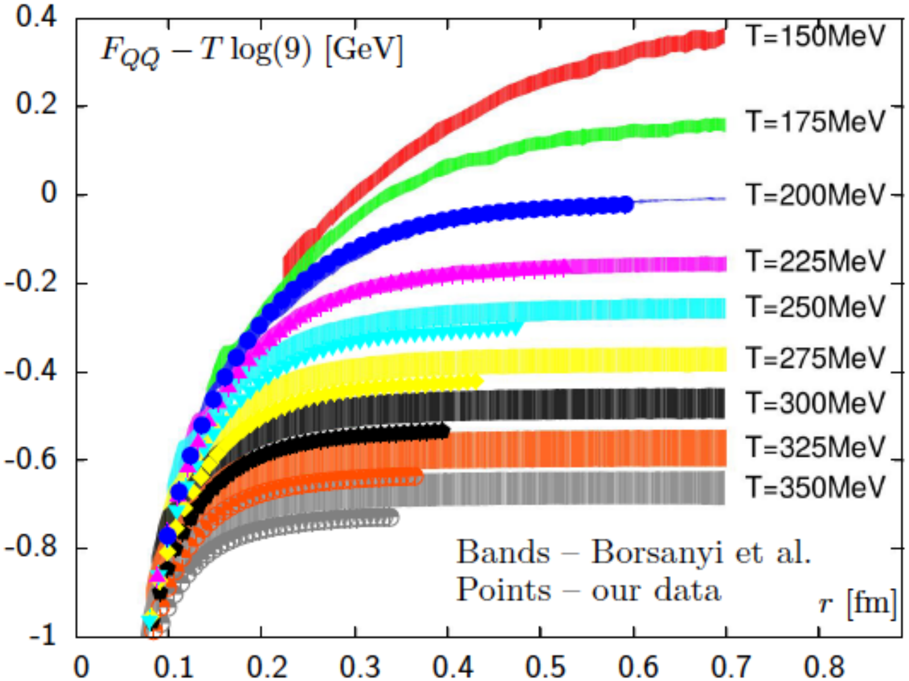
free energy of static quark anti-quark pair shows Debye screening at high temperatures

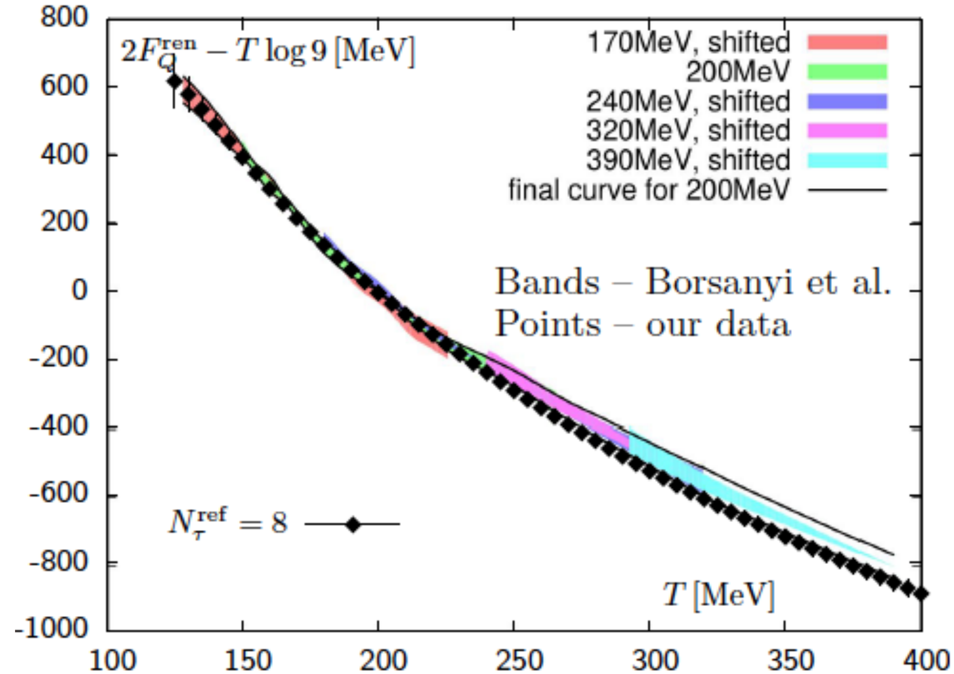


Pure glue ≠ QCD !

Polyakov loop correlators in the continuum limit

Based on work with Bazavov, Berweinm Brambilla, Vairo, Weber





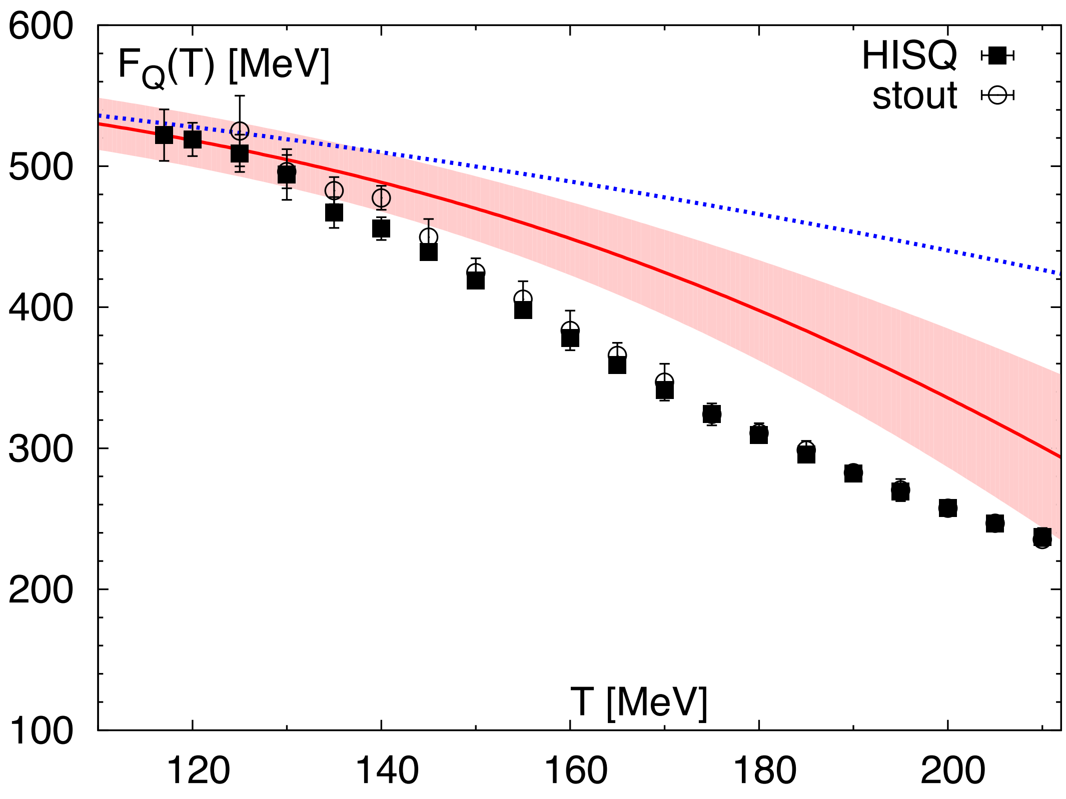
Calculations with HISQ action agree with the calculations performed with stout action (WB, Borsanyi et al, JHEP 1504 (2015) 138)

Polyakov and gas of static-light hadrons

$$Z_{Q\bar{Q}}(T)/Z(T) = \sum_n \exp(-E_n^{Q\bar{Q}}(r \to \infty)/T)$$

Energies of static-light mesons: $E_n^{Q\bar{Q}}(r \to \infty) = M_n - m_Q$

Free energy of an isolated static quark: $F_Q(T) = -\frac{1}{2}(T \ln Z_{Q\bar{Q}}(T) - T \ln Z(T))$



Megias, Arriola, Salcedo, PRL 109 (12) 151601

Bazavov, PP, PRD 87 (2013) 094505

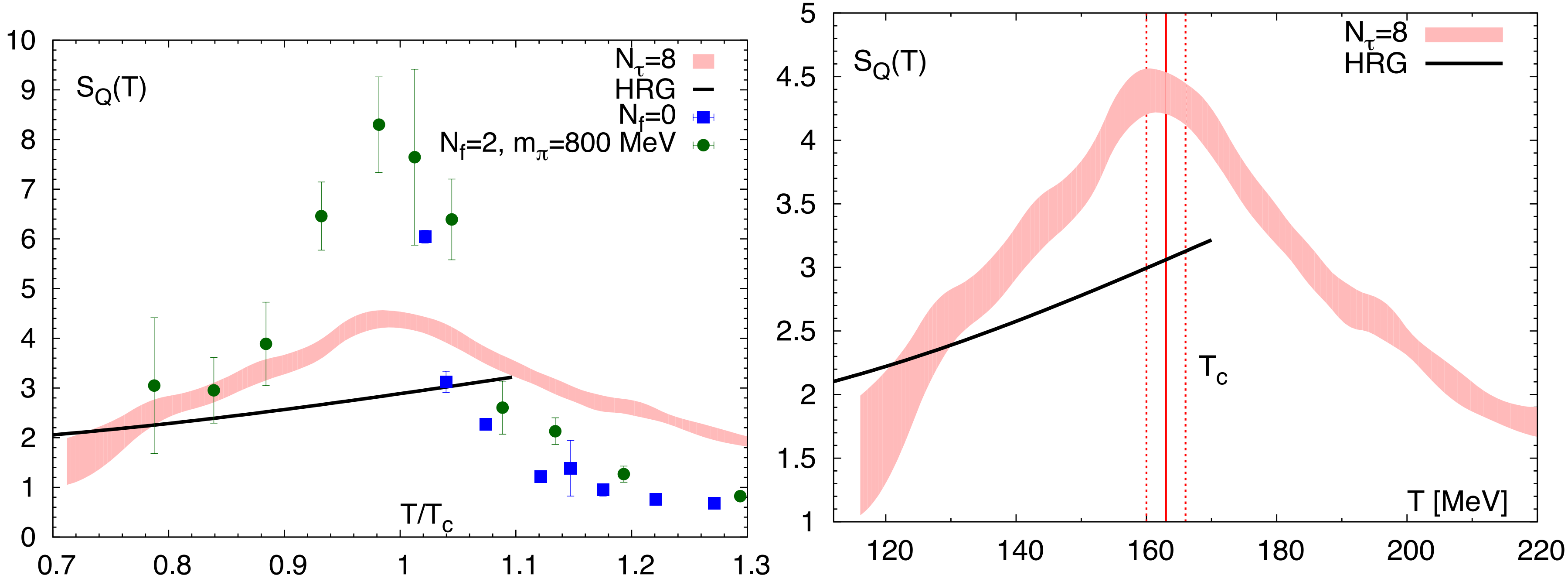
Ground state and first excited states are from lattice QCD
Michael, Shindler, Wagner, arXiv1004.4235
Wagner, Wiese, JHEP 1107 016,2011

Higher excited state energies are estimated from potential model

Gas of static-light mesons only works for *T < 145* MeV

The entropy of static quark

$$S_Q = -\frac{\partial F_Q}{\partial T}$$



At low T the entropy S_Q increases reflecting the increase of states the heavy quark can be coupled to

At high temperature the static quark only "sees" the medium within a Debye radius, as T increases the Debye radius decreases and S_Q also decreases

The onset of screening corresponds to peak is S_Q and its position coincides with T_c

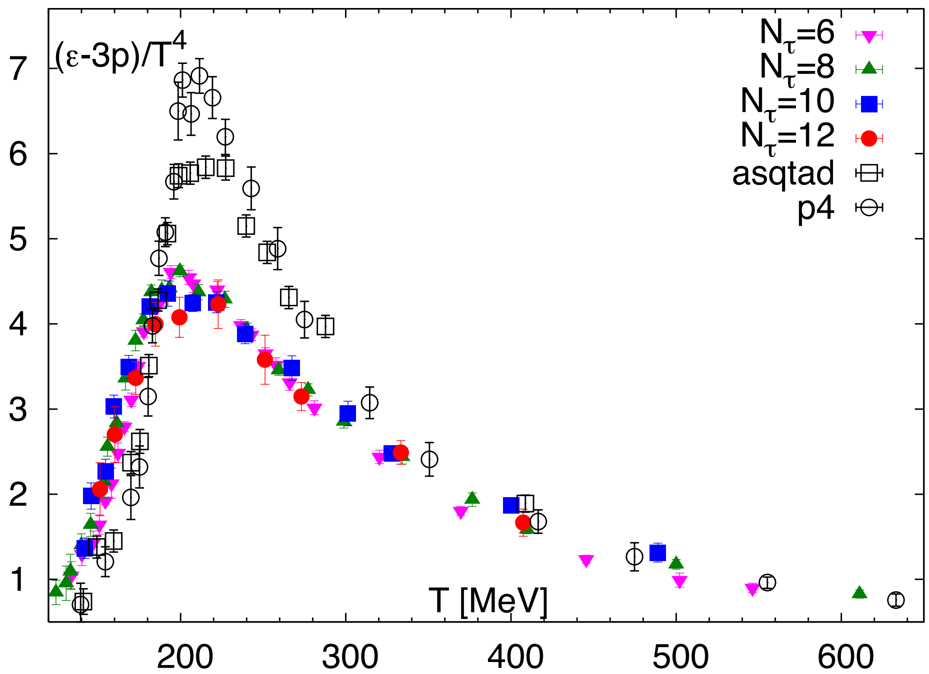
Trace anomaly and the integral method

$$\frac{\Theta^{\mu\mu}(T)}{T^4} = \frac{\varepsilon - 3p}{T^4} = T\frac{d}{dT}\left(\frac{p}{T^4}\right) \Rightarrow \frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \frac{\Theta^{\mu\mu}(T')}{T'^5},$$

$$\frac{\Theta^{\mu\mu}(T)}{T^4} = \frac{\epsilon - 3p}{T^4} = R_\beta \{\langle S_G\rangle_0 - \langle S_G\rangle_T\} - R_\beta R_m \{2m_l(\langle\bar{q}q\rangle_0 - \langle\bar{q}q\rangle) + m_s(\langle\bar{s}s\rangle_0 - \langle\bar{s}s\rangle_T)\}$$

$\Theta_G^{\mu\mu}$ $\Theta_F^{\mu\mu}$

$$R_\beta(\beta) = -a\frac{d\beta}{da}, \quad R_m = \frac{1}{m_q(\beta)}\frac{dm_q(\beta)}{d\beta}, \quad \beta = 10/g^2$$



Bazavov et al, arXiv:1407:6387

The peak height is much reduced compared to the asqtad and p4 N_τ=8 calculations

Agreement with p4 and asqtad calculations for T>350 MeV

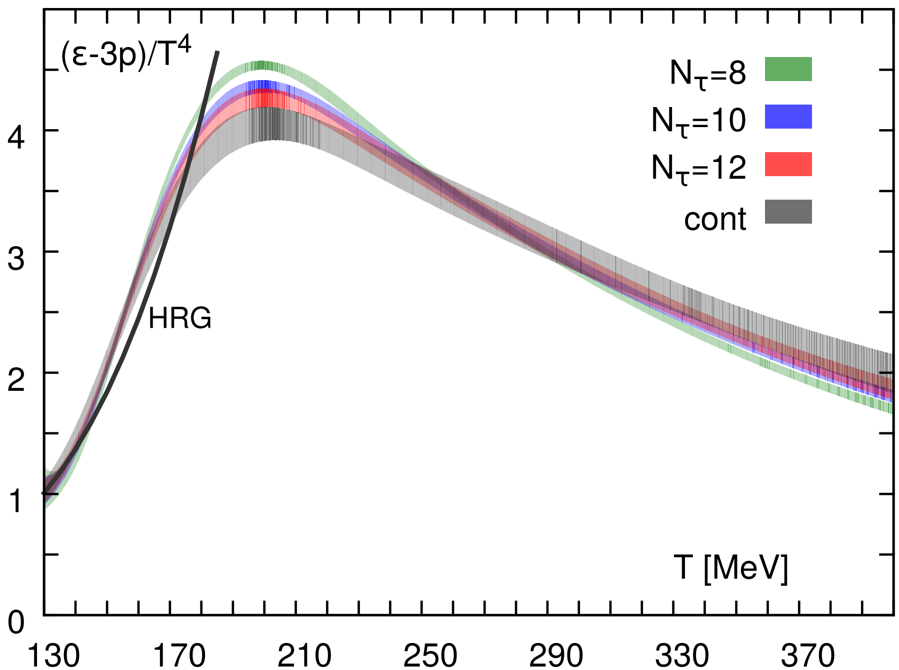
Small cutoff effects for HISQ except for N_τ=6

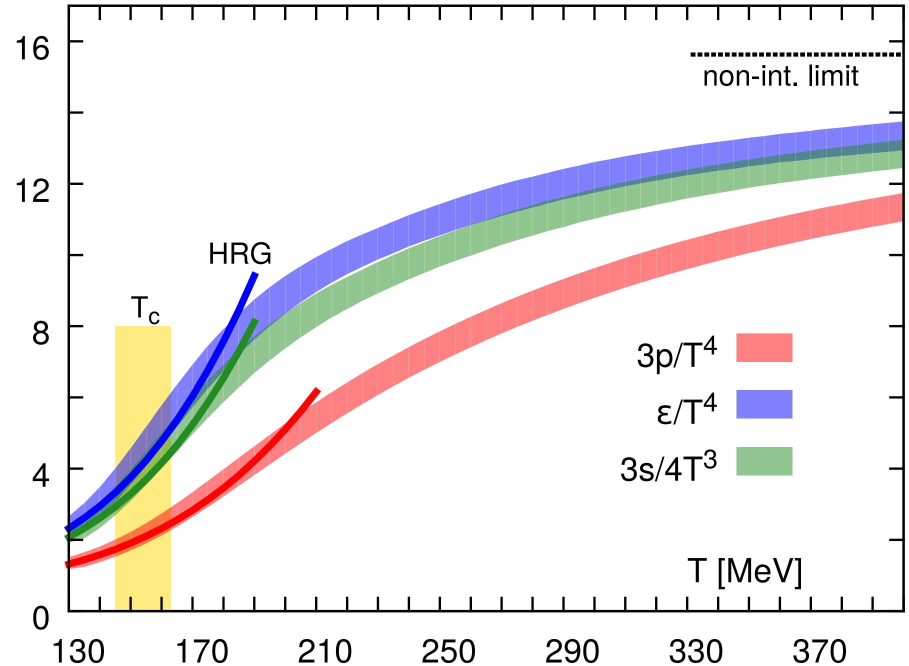
Equation of state in the continuum limit

Perform spline interpolation of all the $N_\tau > 6$ data with spline coefficients of the form $a + b/N_\tau^2$, stabilize the spline demanding that $\varepsilon - 3p$ is given by HRG at $T = 130$ MeV

Set the lower integration limit to $T_0 = 130$ MeV and take $p_0 = p^{HRG}(T = 130 \text{ MeV})$ ➡ $p(T)$

Bazavov et al, arXiv:1407:6387





Hadron resonance gas (HRG):
Interacting gas of hadrons = non-interacting gas of hadrons and hadron resonances
(virial expansion, Prakash & Venugopalan)

HRG agrees with the lattice for $T < 145$ MeV

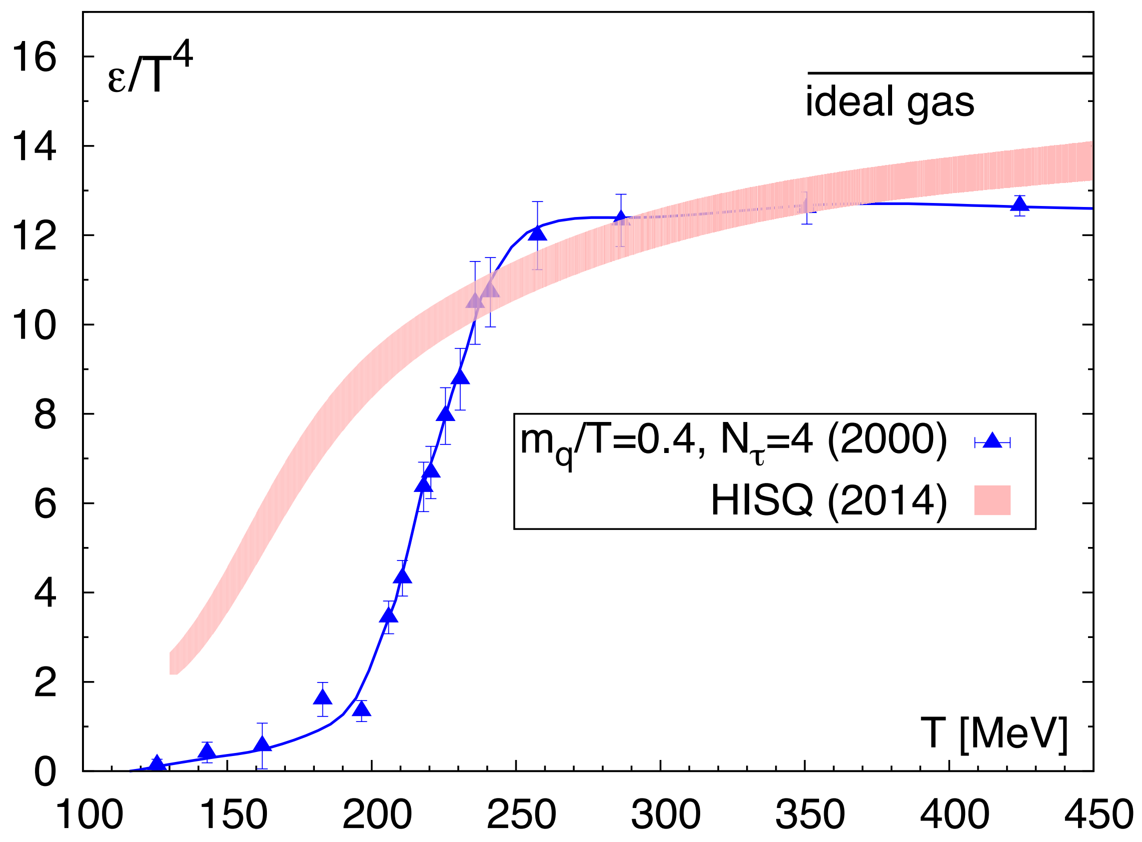
$$T_c = (154 \pm 9)\text{MeV}$$

$$\epsilon_c \simeq 300\text{MeV}/\text{fm}^3$$

$$\epsilon_{low} \simeq 180\text{MeV}/\text{fm}^3 \leftrightarrow \epsilon_{nucl} \simeq 150\text{MeV}/\text{fm}^3$$

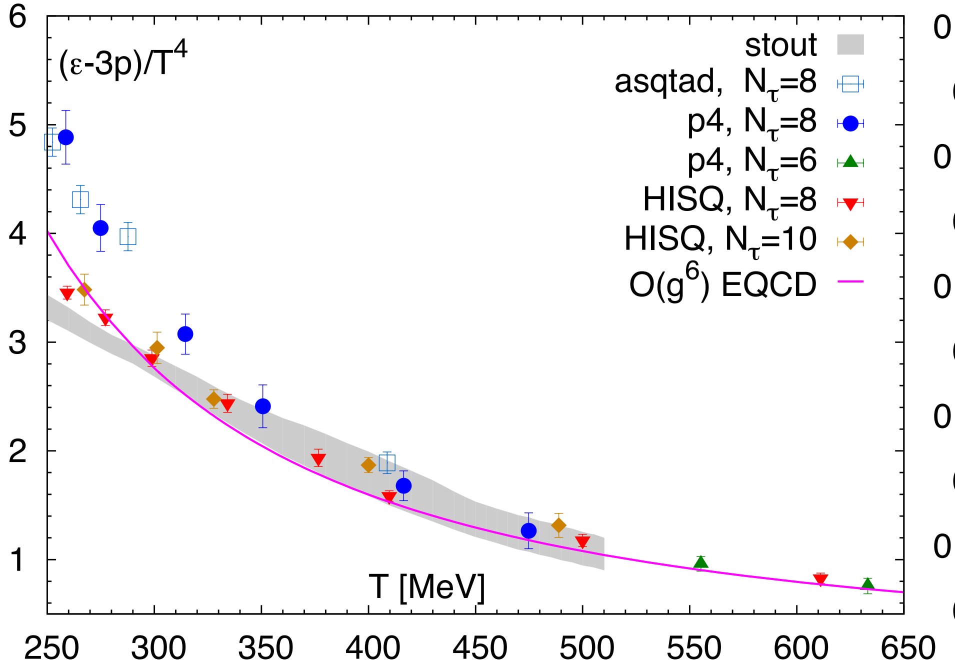
$$\epsilon_{high} \simeq 500\text{MeV}/\text{fm}^3 \leftrightarrow \epsilon_{proton} \simeq 450\text{MeV}/\text{fm}^3$$

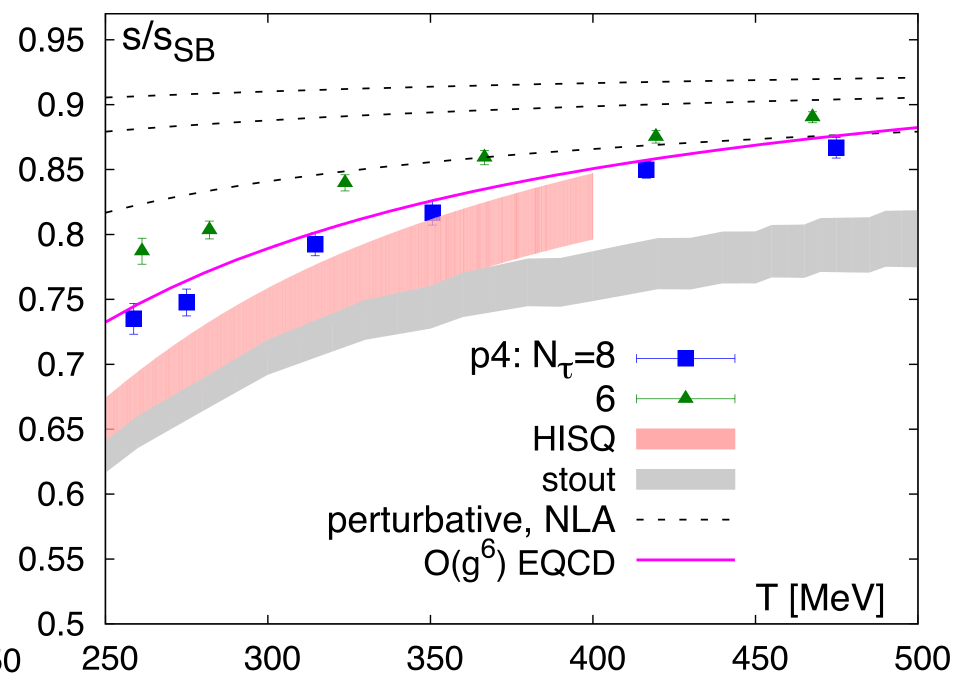
How Equation of state changed since 2002



- Much smoother transition to QGP
- The energy density keeps increasing up to 450 MeV instead of flattening

Equation of State on the lattice and in the weak coupling





The high temperature behavior of the trace anomaly is not inconsistent with weak coupling calculations (EQCD) for $T>300$ MeV

For the entropy density the continuum lattice results are below the weak coupling calculations For $T< 500$ MeV

At what temperature can one see good agreement between the lattice and the weak coupling results ?

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \cdot \left(\frac{\mu_B}{T}\right)^i \cdot \left(\frac{\mu_Q}{T}\right)^j \cdot \left(\frac{\mu_S}{T}\right)^k$$

hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds} \cdot \left(\frac{\mu_u}{T}\right)^i \cdot \left(\frac{\mu_d}{T}\right)^j \cdot \left(\frac{\mu_s}{T}\right)^k$$

quark

$$\chi_{ijk}^{abc} = T^{i+j+k} \frac{\partial^i}{\partial \mu_a^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{1}{VT^3} \ln Z(T,V,\mu_a,\mu_b,\mu_c)|_{\mu_a=\mu_b=\mu_c=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3}(\langle X^2 \rangle - \langle X \rangle^2) \qquad \chi_{11}^{XY} = \frac{1}{VT^3}(\langle XY \rangle - \langle X \rangle \langle Y \rangle)$$

⬇

information about carriers of the conserved charges (hadrons or quarks)

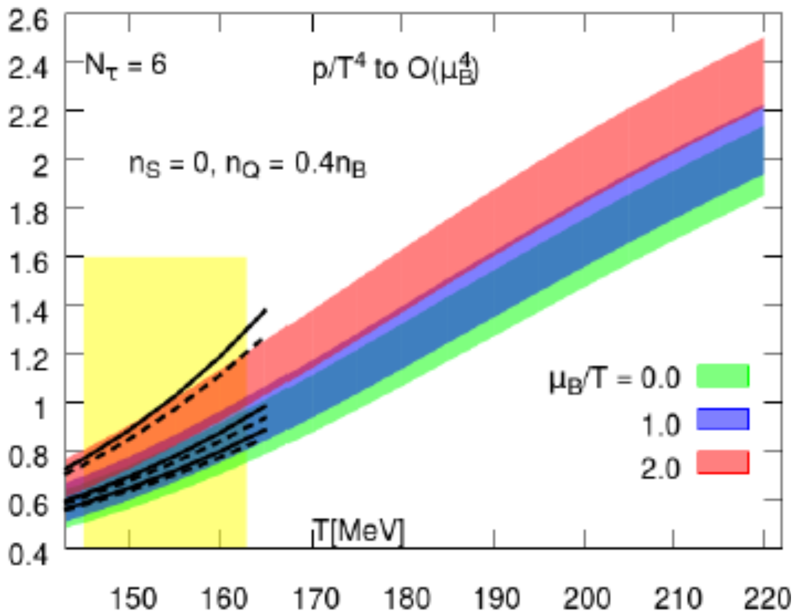
⬇

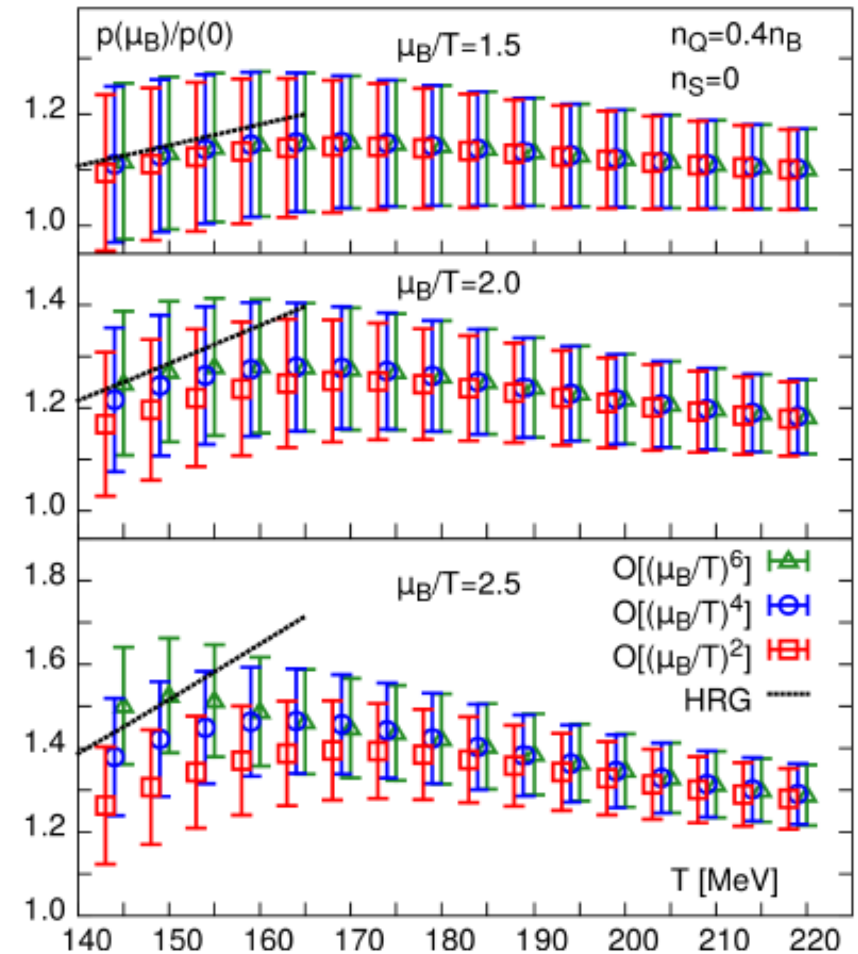
probes of deconfinement

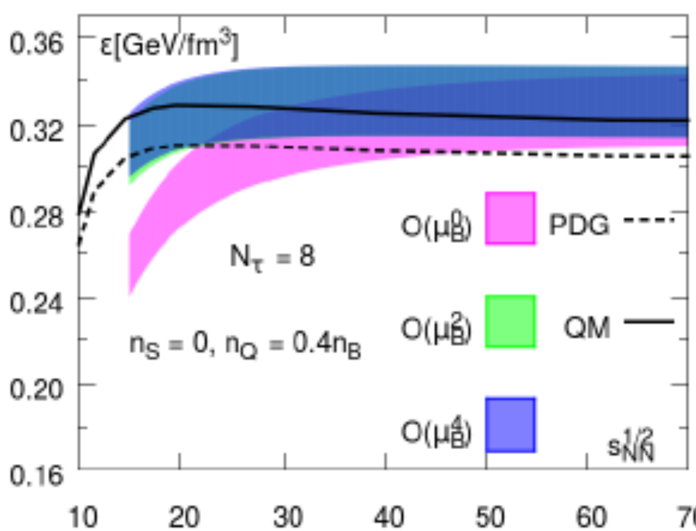
Equation of state at non-zero baryon density

Taylor expansion up to 4th order for net zero strangeness $n_S = 0$ and $r = n_Q/n_B = Z/A = 0.4$

BNL-Bielefeld-CCNU







Moderate effects due to non-zero baryon density up to $\mu_B/2 = 2 \leftrightarrow \sqrt{s} \sim 20\mathrm{GeV}$

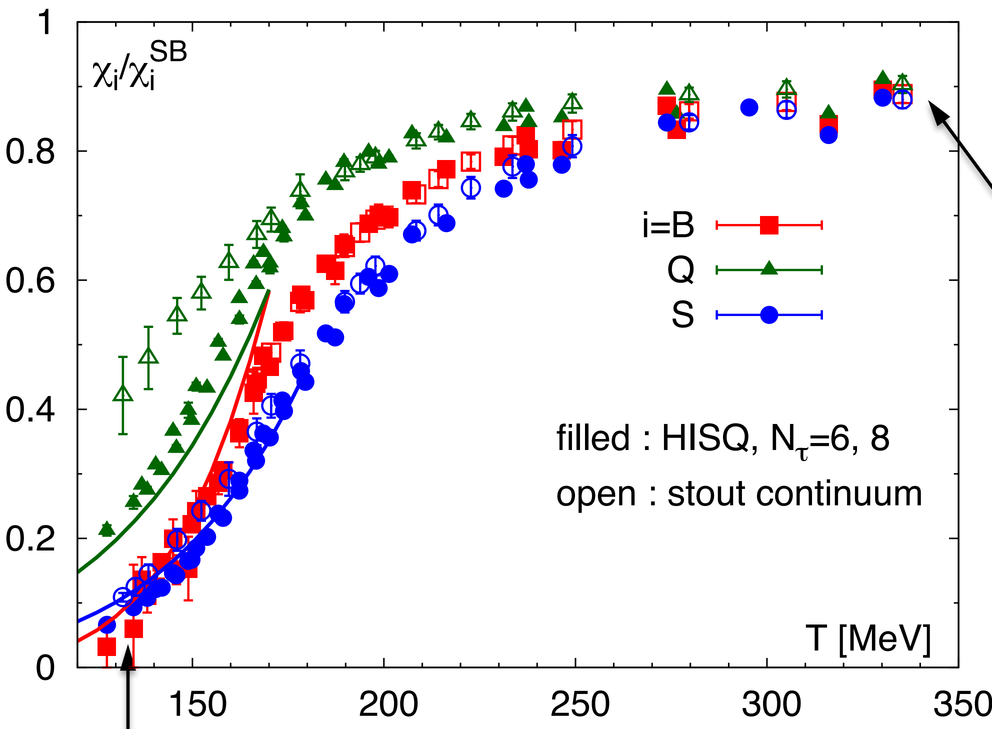
Energy density at freeze-out is independent of μ_B

Deconfinement : fluctuations of conserved charges

$$\chi_B = \frac{1}{VT^3}\left(\langle B^2\rangle - \langle B\rangle^2\right)$$ baryon number

$$\chi_Q = \frac{1}{VT^3}\left(\langle Q^2\rangle - \langle Q\rangle^2\right)$$ electric charge

$$\chi_S = \frac{1}{VT^3}\left(\langle S^2\rangle - \langle S\rangle^2\right)$$ strangeness



conserved charges are carried by massive hadrons

Ideal gas of massless quarks :

$$\chi_B^{SB} = \frac{1}{3} \qquad \chi_Q^{SB} = \frac{2}{3}$$

$$\chi_B^{SB} = 1$$

conserved charges carried by light quarks

HotQCD: PRD86 (2012) 034509

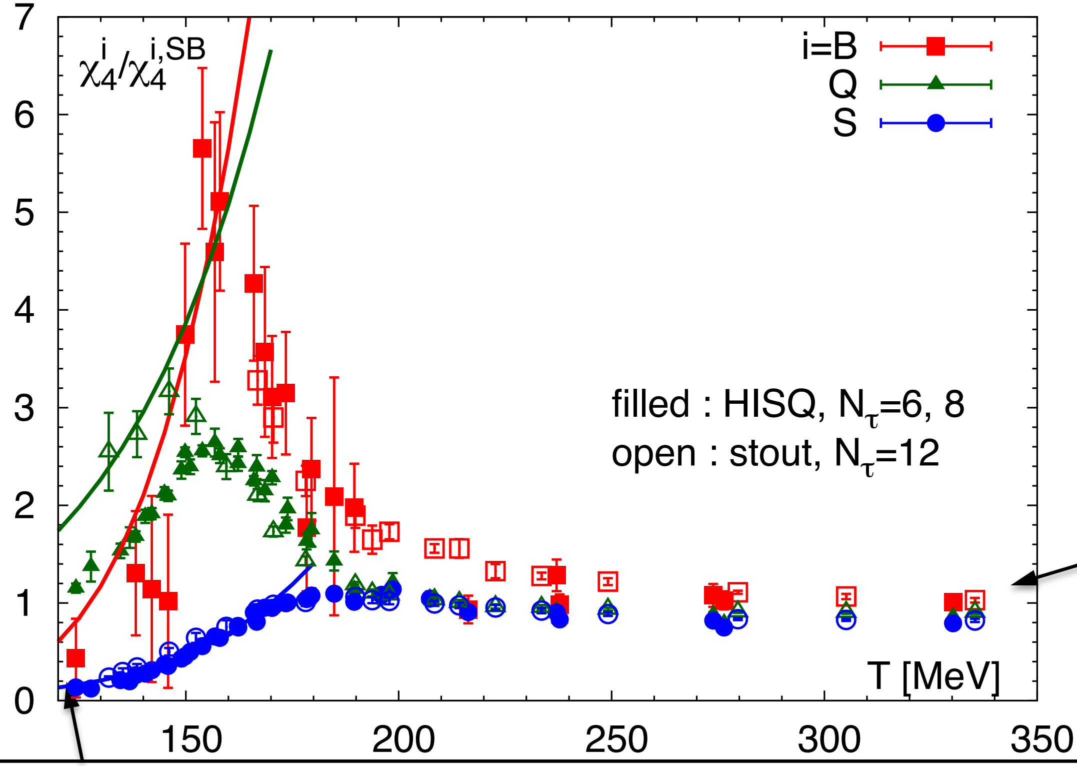
BW: JHEP 1201 (2012) 138,

Deconfinement : fluctuations of conserved charges

$$\chi_4^B = \frac{1}{VT^3}(\langle B^4\rangle - 3\langle B^2\rangle^2)$$ baryon number

$$\chi_4^Q = \frac{1}{VT^3}(\langle Q^4\rangle - 3\langle Q^2\rangle^2)$$ electric charge

$$\chi_4^S = \frac{1}{VT^3}(\langle S^4\rangle - 3\langle S^2\rangle^2)$$ strangeness



Ideal gas of massless quarks :

$$\chi_4^B{}_{SB} = \frac{2}{9\pi^2} \qquad \chi_4^Q{}_{SB} = \frac{4}{3\pi^2}$$

$$\chi_4^S{}_{SB} = \frac{6}{\pi^2}$$

conserved charges carried by light quarks

conserved charges are carried by massive hadrons

BNL-Bielefeld : talk by C. Schmidt
BW: talk by Borsanyi
@ Confinement X conference

Deconfinement of strangeness

Partial pressure of strange hadrons in uncorrelated hadron gas:

$$P_S = \frac{p(T) - p_{S=0}(T)}{T^4} = M(T)\cosh\left(\frac{\mu_S}{T}\right) + B_{S=1}(T)\cosh\left(\frac{\mu_B - \mu_S}{T}\right) + B_{S=2}(T)\cosh\left(\frac{\mu_B - 2\mu_S}{T}\right) + B_{S=3}(T)\cosh\left(\frac{\mu_B - 3\mu_S}{T}\right)$$

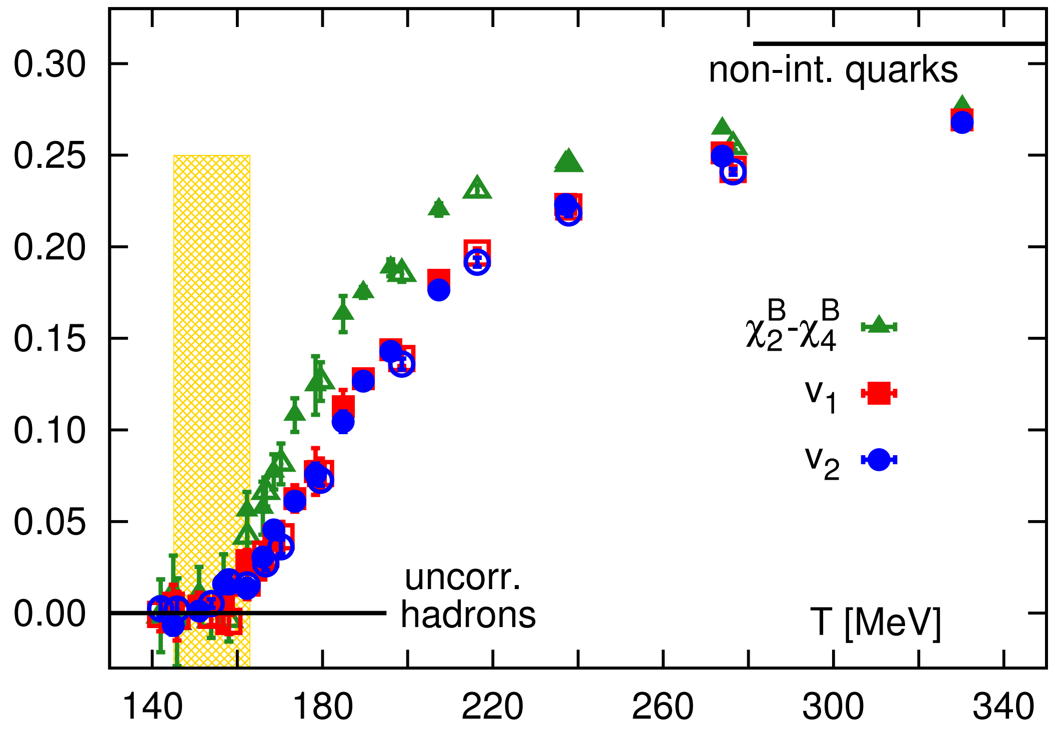
$$v_1 = \chi_{31}^{BS} - \chi_{11}^{BS}$$

$$v_2 = \frac{1}{3}\left(\chi_4^S - \chi_2^S\right) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}$$

should vanish !

- v_1 and v_2 do vanish within errors at low T
- v_1 and v_2 rapidly increase above the transition region, eventually reaching non-interacting quark gas values

Bazavov et al, PRL 111 (2013) 082301



Deconfinement of strangeness (cont'd)

Using the six Taylor expansion coefficients related to strangeness

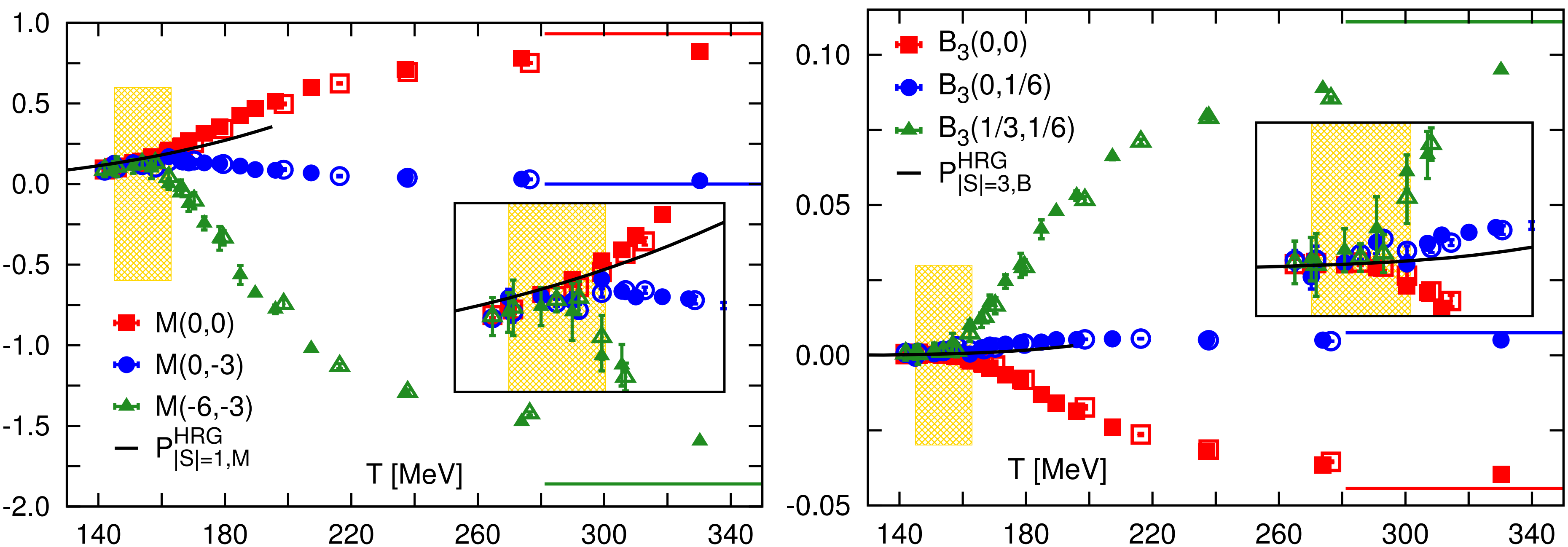
$$\chi_2^S,\ \chi_4^S,\ \chi_{13}^{BS},\ \chi_{22}^{BS},\ \chi_{31}^{BS}$$

it is possible to construct combinations that give

$$M(T),\ B_{S=1}(T),\ B_{S=2}(T),\ B_{S=3}(T)$$

up to terms $c_1 v_1 + c_2 v_2$

Bazavov et al, PRL 111 (2013) 082301



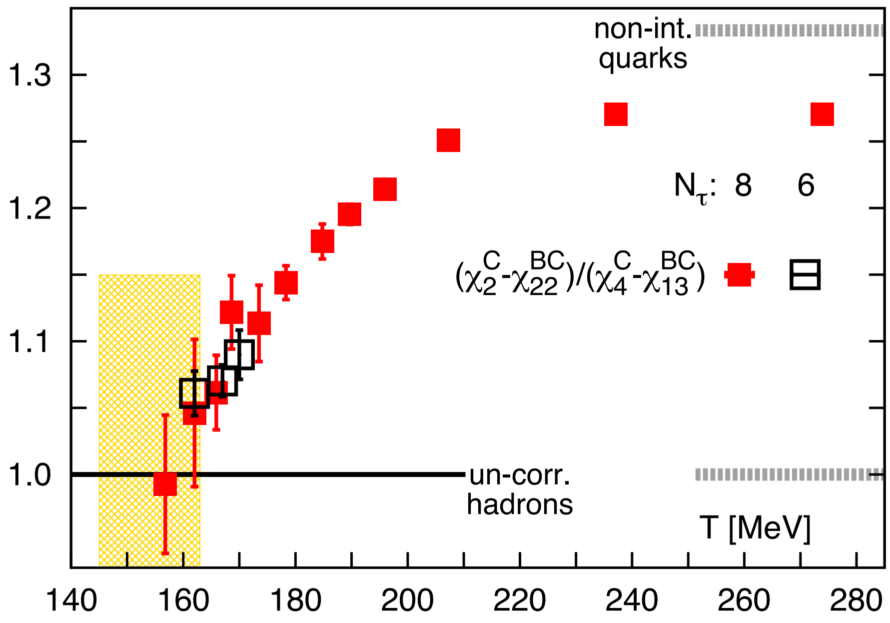
Hadron resonance gas descriptions breaks down for all strangeness sectors above T_c
⇒ Strangeness deconfines at T_c

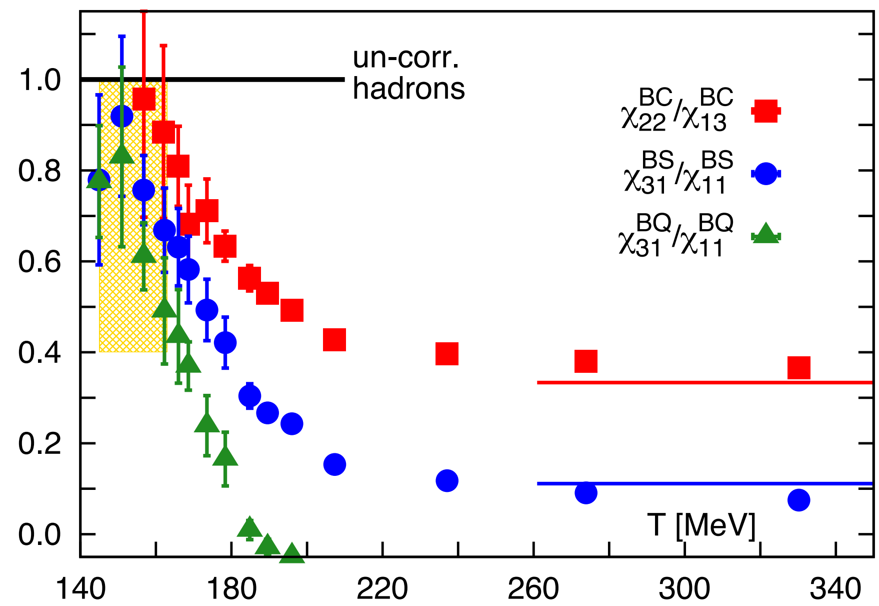
What about charm hadrons ?

We could introduce chemical potential for charm quarks and study the derivatives of the pressure with respect to the charm chemical potential Bazavov et al, PLB737 (2014) 210

$m_c \gg T$ ⇨ only |*C*|=*1* sector contributes

In the hadronic phase all *BC*-correlations are the same !



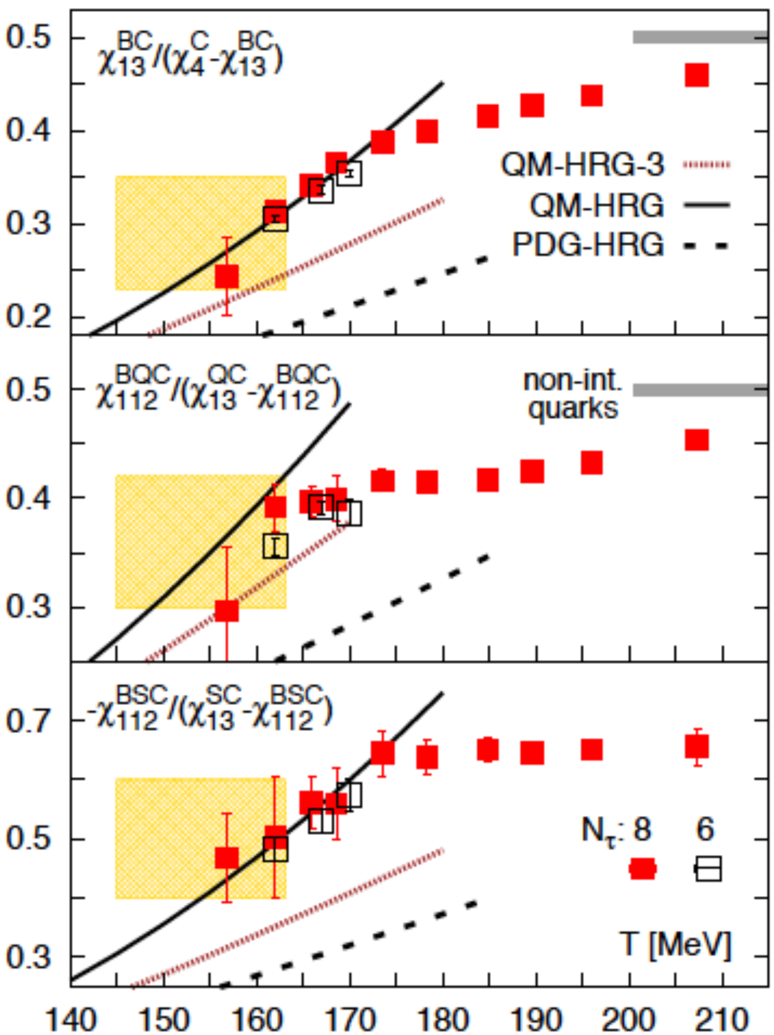


Hadronic description breaks down just above T_c
⇒ open charm deconfines above T_c

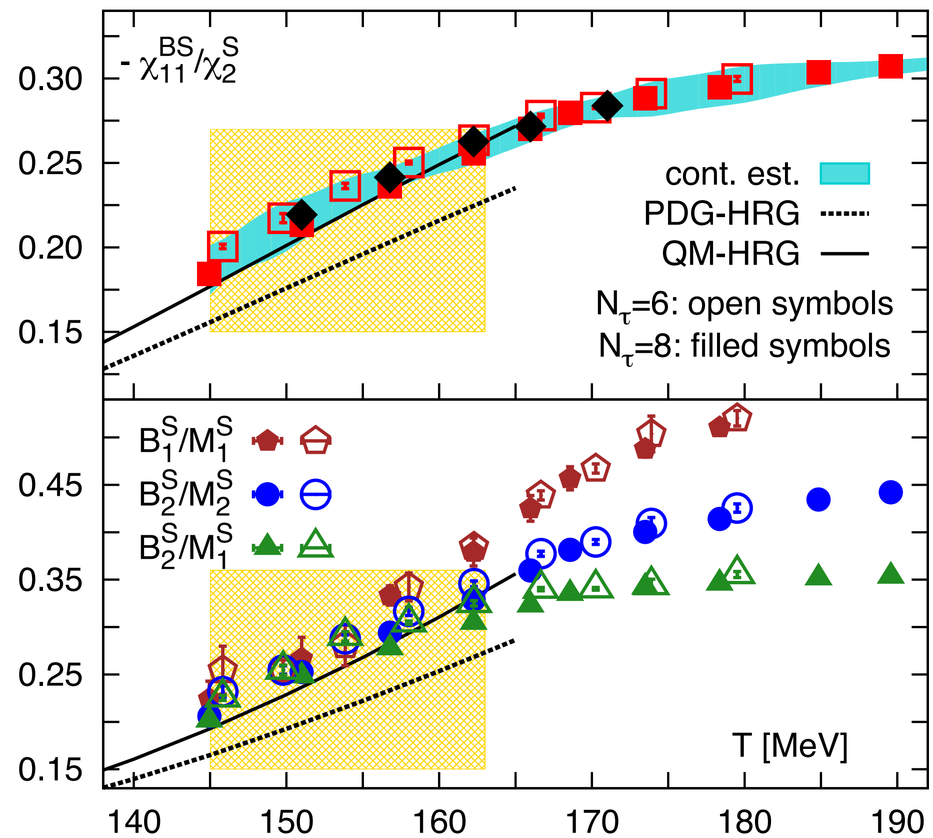
The description in terms of un-correlated gas of hadrons breaks down at T_c for all *BX*-correlations, *X=Q,S,C*

Thermodynamics and “missing” baryons

Bazavov et al, PLB737 (2014) 210 , PRL 113 (2014) 072001



Many baryons predicted by quark model (QM) and LQCD are missing from PDG

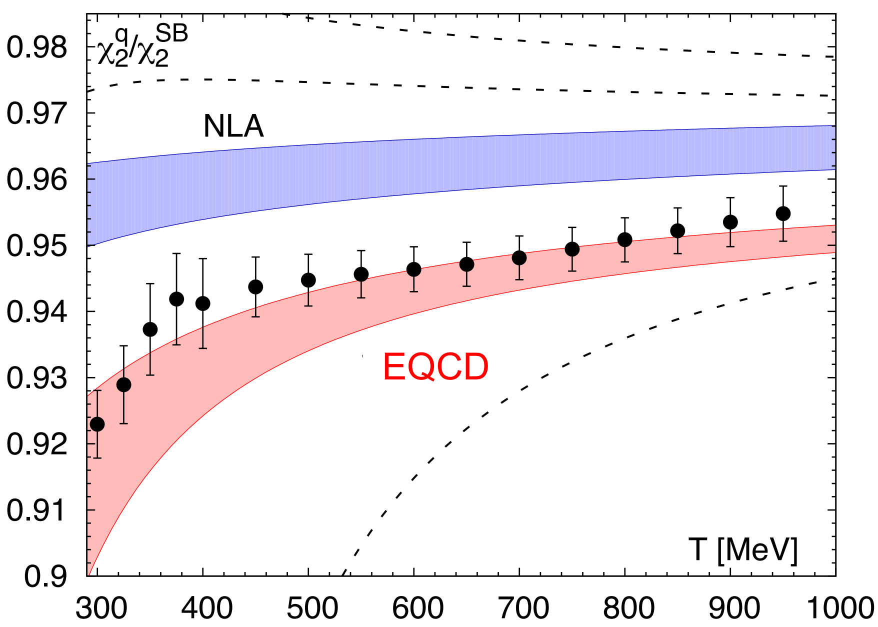


“Missing” strange baryons have to be included to obtain a good agreement between HRG and the lattice results

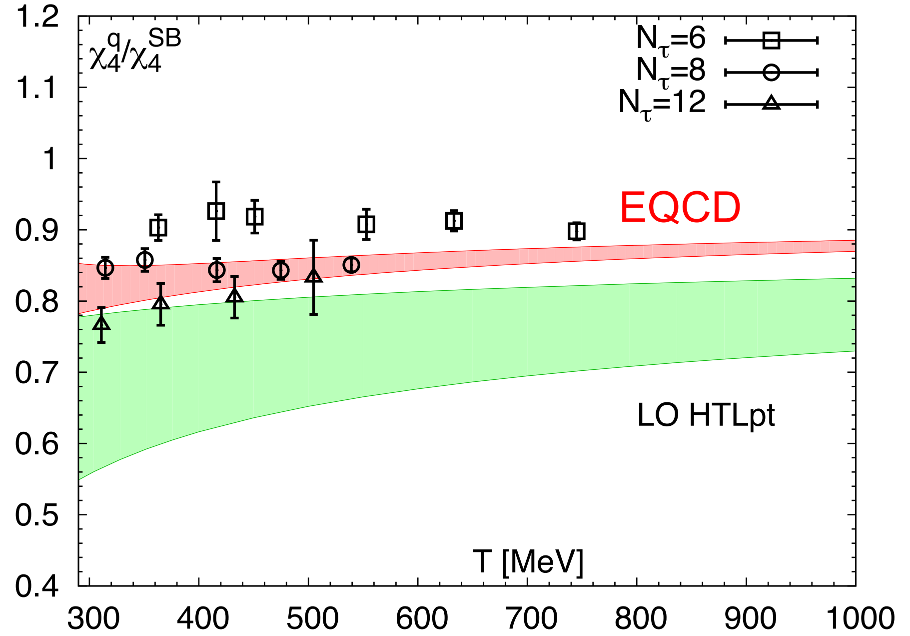
Quark number fluctuations at high T

At high temperatures quark number fluctuations can be described by weak coupling approach due to asymptotic freedom of QCD

2nd order quark number fluctuations



4th order quark number fluctuations



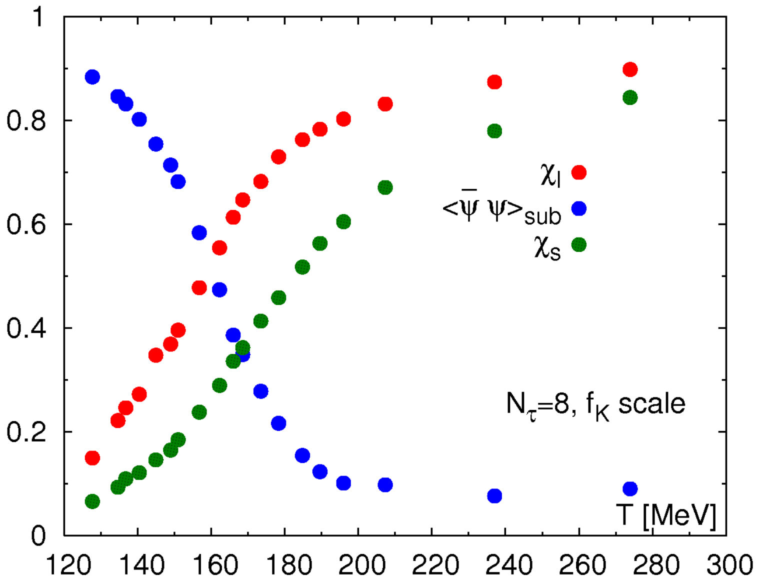
Bazavov et al, PRD88 (2013) 094021

- Lattice results converge as the continuum limit is approached
- Good agreement between lattice and the weak coupling approach for 2nd order quark number fluctuations
- For 4th order the weak coupling results are in reasonable agreement with lattice

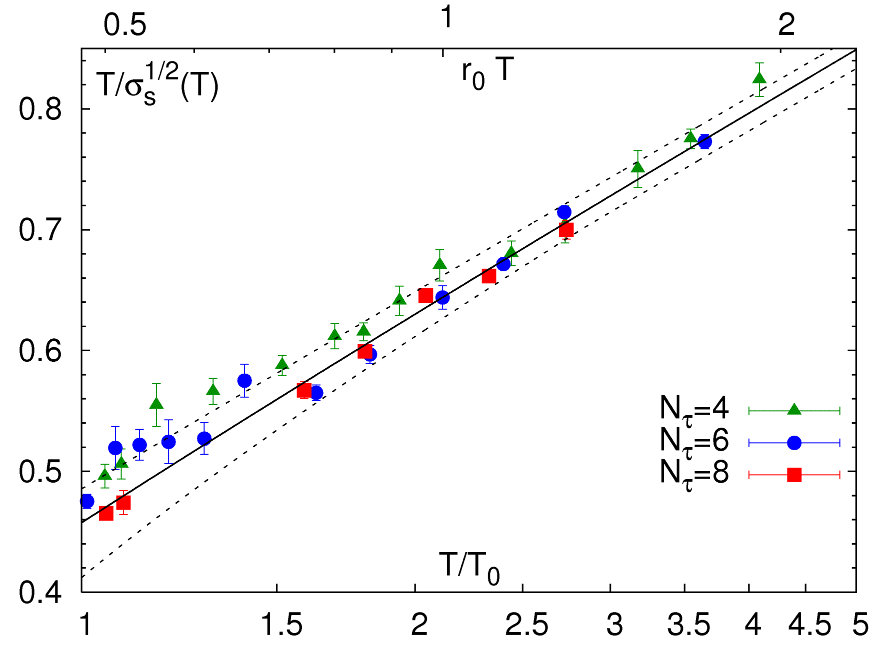
Summary

- The value chiral transition temperature is now well established in the continuum
  T_c=*154(9)* MeV
- Equation of state are known in the continuum limit up to T=*400* MeV
- Hadron resonance gas can describe various thermodynamic quantities at low temperatures
- Deconfinement transition can be studied in terms of fluctuations and correlations of conserved charges, it manifest itself as a abrupt breakdown of hadronic description that occurs around the chiral transition temperature
- The approach to the weakly interacting quark gluon gas for $T>T_c$ is rather slow and the matter is strongly interacting for $T<$ *300* MeV with no apparent quasi-particle composition
- For T > *(300-400)* MeV weak coupling expansion works well for certain quantities (e.g. quark number susceptibilities), more work is needed to connect lattice and weak coupling results
- Comparison of lattice and HRG results for certain strangeness and charm correlations hints for existence of yet undiscovered excited baryons

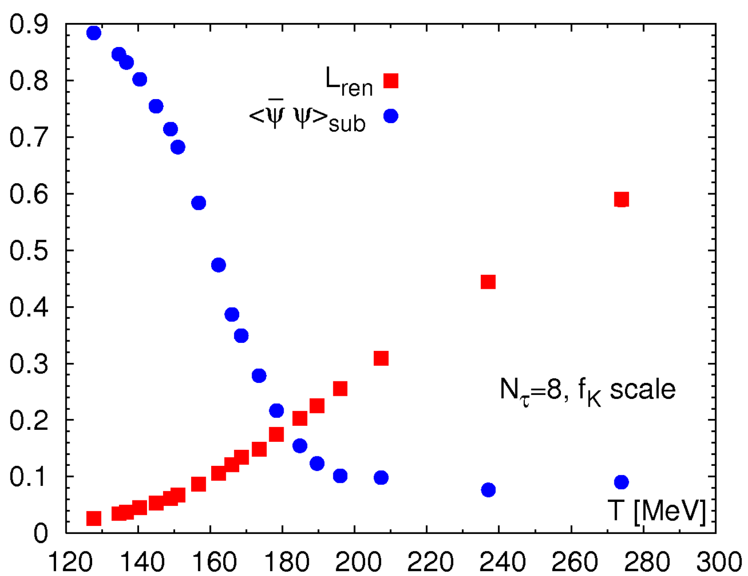
Back-up:



χ_l
$<\bar{\psi}\psi>_{sub}$
χ_s
N_τ=8, f_K scale
T [MeV]

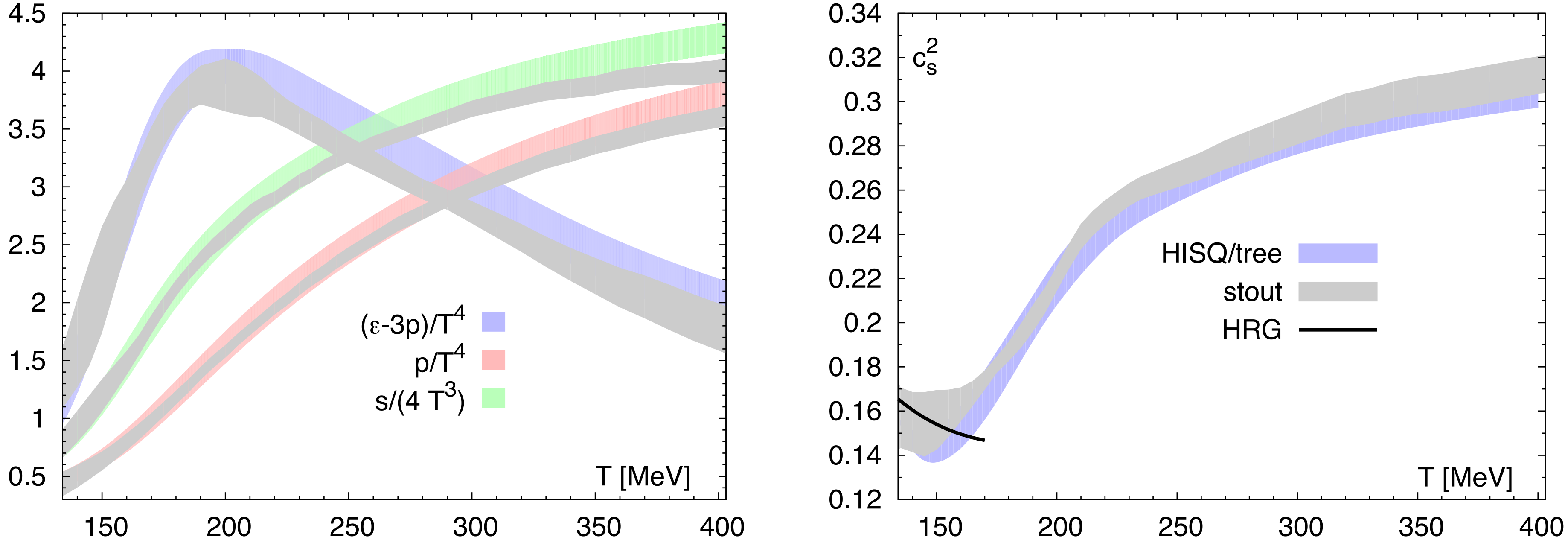


$T/\sigma_s^{1/2}(T)$
r_0 T
T/T_0
N_τ=4
N_τ=6
N_τ=8



L_{ren}
$<\bar{\psi}\psi>_{sub}$
N_τ=8, f_K scale
T [MeV]

HISQ action vs. stout action



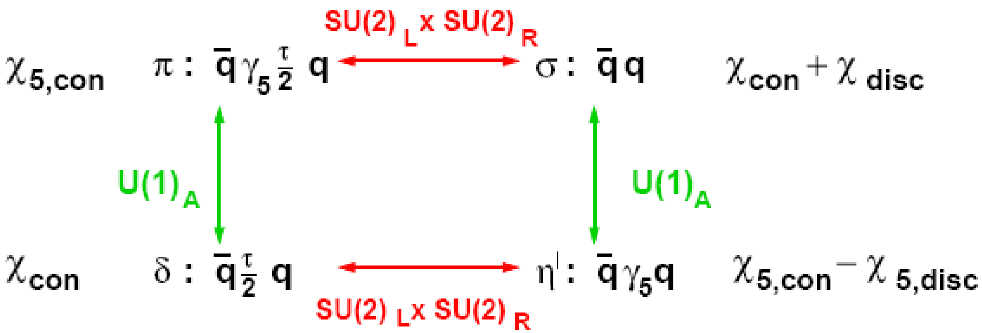
Continuum results obtained with stout and HISQ action agree reasonably well given their errors (some tension for the entropy density)

Even in the transition region the speed of sound is not much smaller than the HRG speed of sound (the EoS is never really soft)

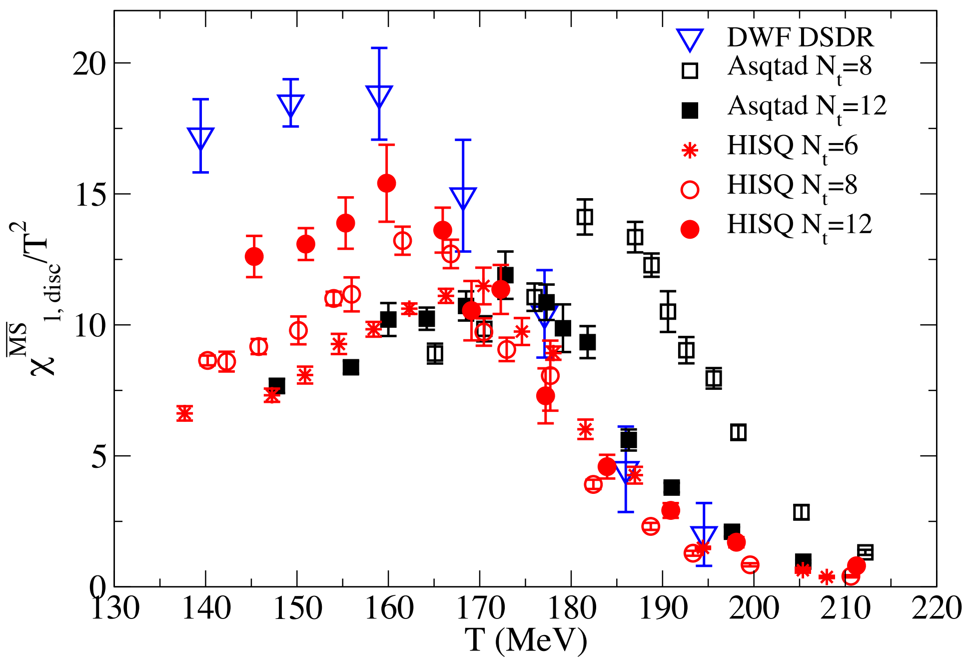
Domain wall Fermions and $U_A(1)$ symmetry restoration

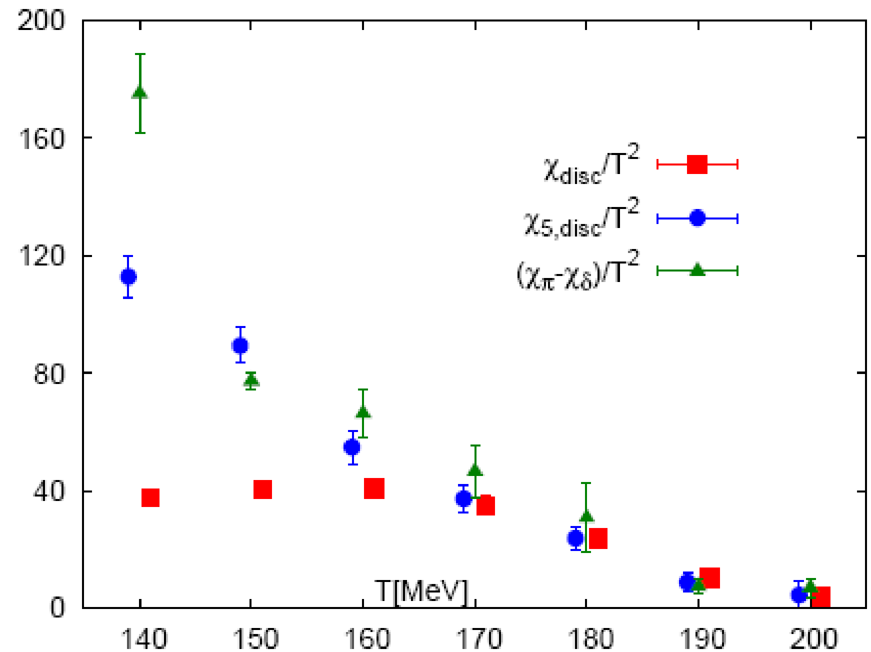
Domain Wall Fermions, Bazavov et al (HotQCD), PRD86 (2012) 094503

$$\chi_i = \int d^4x G_i(x)$$



chiral:	axial:
$\chi_\pi = \chi_\delta + \chi_{disc}$	$\chi_\pi = \chi_\delta$
$\chi_\delta = \chi_\pi - \chi_{5,disc}$	$\chi_\delta + \chi_{disc} = \chi_\pi - \chi_{5,disc}$
$\chi_{disc} = \chi_{5,disc}$	$\chi_{disc} = -\chi_{5,disc}$





Peak position roughly agrees with previous staggered results

axial symmetry is effectively restored *T>200* MeV !

Improved staggered calculations at finite temperature

low T region

$T<200$ MeV

$\mathcal{O}(\alpha_s^n(a\Lambda_{QCD})^2)$ errors

$a>0.125fm$

hadronic degrees of freedom

improvement of the flavor symmetry is important $\rightarrow$ fat links

cutoff effects are different in :

$a = 1/(TN_\tau)$

$N_\tau = 8$

for #flavors < 4 rooting trick

$$detD \rightarrow (detD)^{\frac{n_f}{4}}$$

high-T region

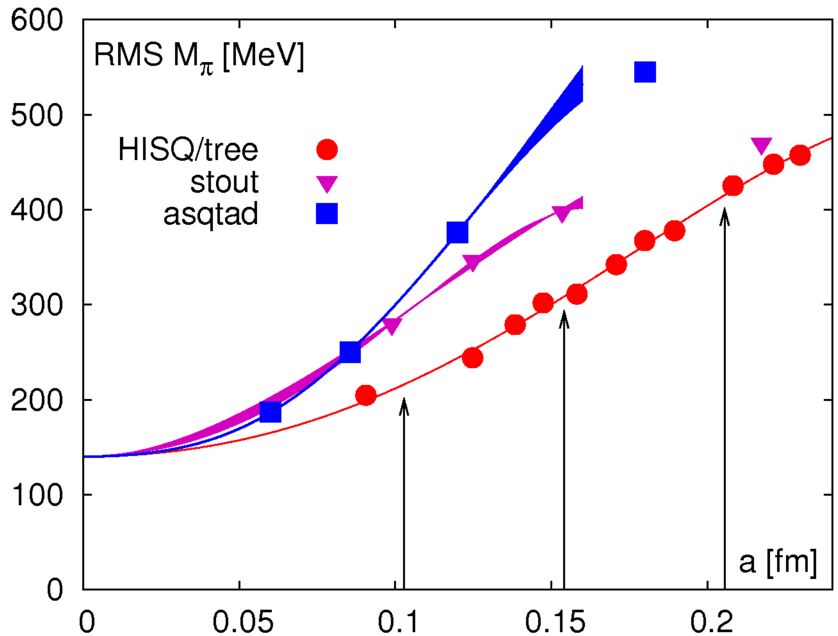
$T>200$MeV

$\mathcal{O}((aT)^2)$ errors

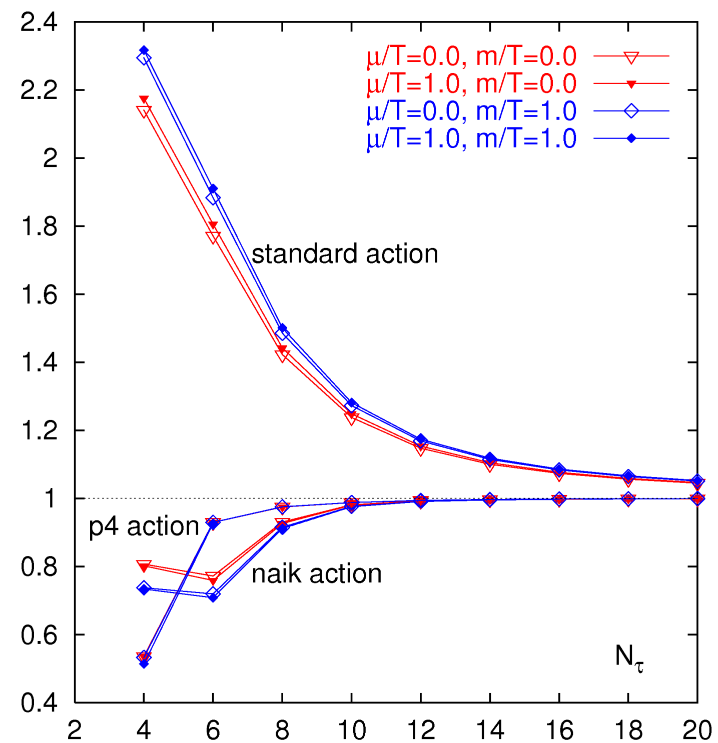
$a<0.125fm$

quark degrees of freedom

quark dispersion relation



p4, asqtad, HISQ, stout



The Highly Improved Staggered Quark (HISQ) Action

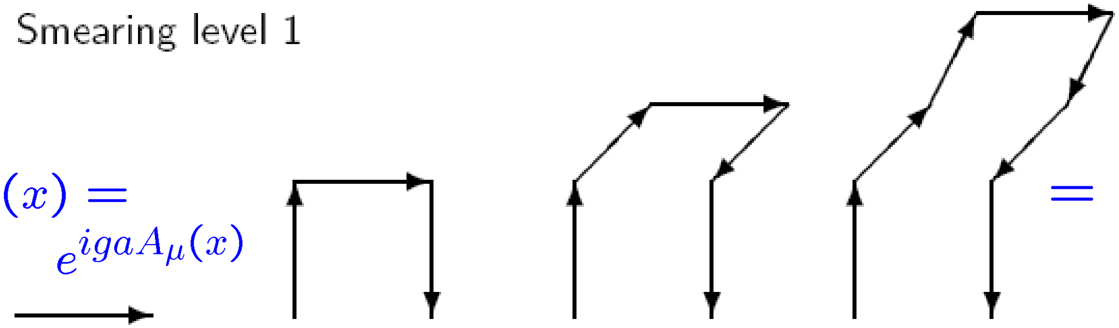
HISQ action

two levels of gauge field smearing with re-unitarization

Follana et al, PRD75 (07) 054502

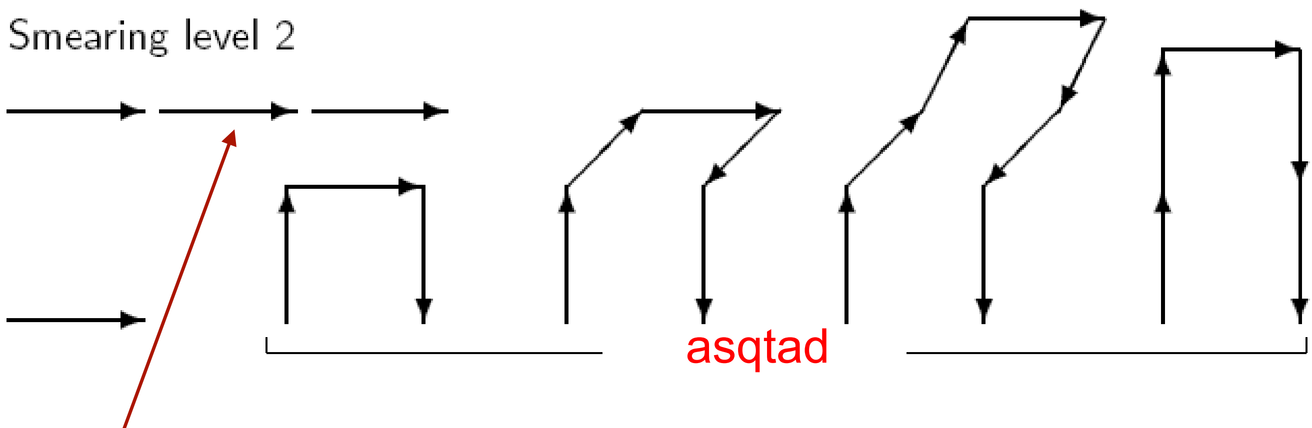
Smearing level 1

$U_\mu(x) = e^{igaA_\mu(x)}$



$= U_\mu^{fat7} \rightarrow \tilde{U}_\mu = \frac{U_\mu^{fat7}}{\sqrt{U_\mu^{fat7} U_\mu^{fat7\dagger}}}$

Smearing level 2



projection onto U(3) improves flavor symmetry
Hasenfratz, arXiv:hep-lat/0211007

3-link (Naik) term to improve the quark dispersion relation + asqtad smearing