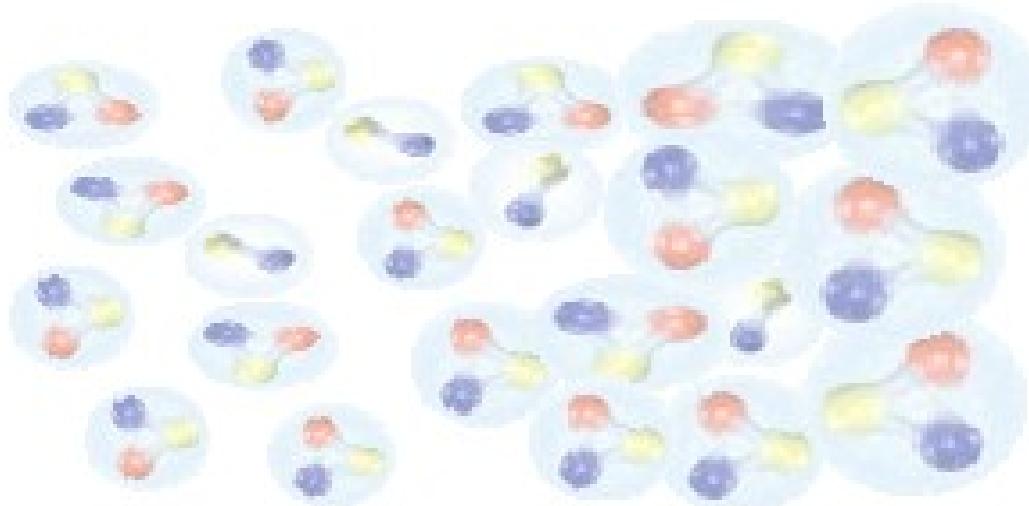


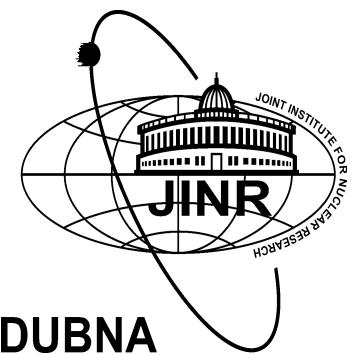
# From hadron resonance gas to quark matter

David Blaschke

University of Wroclaw, Poland & JINR Dubna, Russia



Nuclear Physics Colloquium, Goethe-Universitaet Frankfurt, May 21, 2015

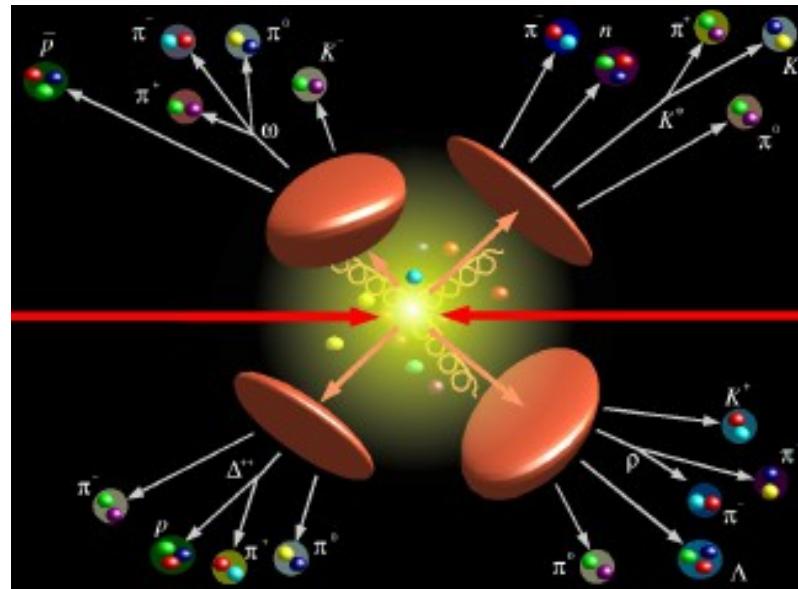


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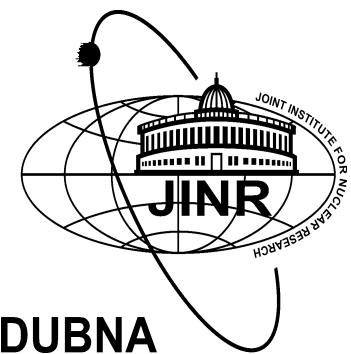
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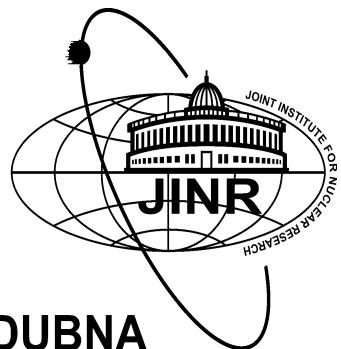
## 1. Introduction:

Mott-Anderson localization model for chemical freeze-out

## 2. Mott dissociation of pions in a PNJL model

## 3. Thermodynamics of Mott-HRG and lattice QCD data

Nuclear Physics Colloquium, Goethe-Universitaet Frankfurt, May 21, 2015



# Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Phys. Part. Nucl. Lett. 8 (2011) 811

**The basic idea:** Localization of (certain) multiquark states (“cluster”) = hadronization;  
Reverse process = delocalization by quark exchange between hadrons

Freeze-out criterion:  $\tau_{\text{exp}}(T, \mu) = \tau_{\text{coll}}(T, \mu)$

$$\tau_{\text{coll}}^{-1}(T, \mu) = \sum_{i,j} \sigma_{ij} n_j$$

Povh-Huefner law,  
PRC 46 (1992) 990

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle$$

$$r_\pi^2(T, \mu) = \frac{3}{4\pi^2} f_\pi^{-2}(T, \mu)$$

$$f_\pi^2(T, \mu) = -m_0 \langle \bar{q}q \rangle_{T, \mu} / M_\pi^2$$

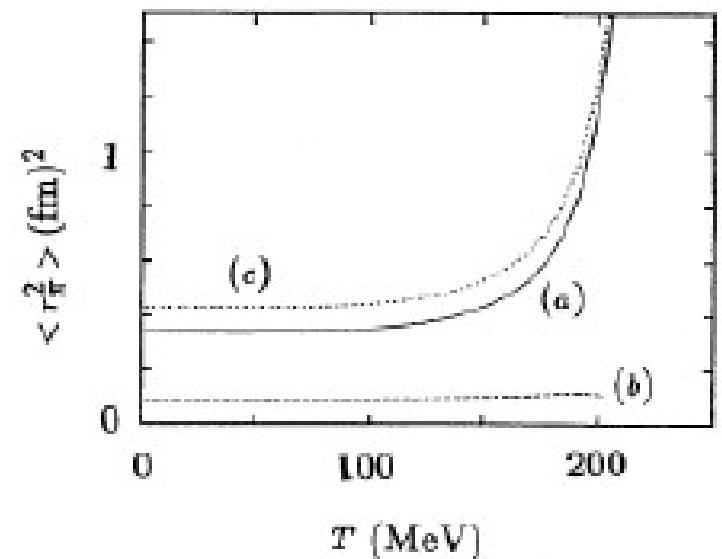


$$r_\pi^2(T, \mu) = \frac{3 M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T, \mu}|^{-1}$$

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_{\text{MF}} \left[ 1 - \frac{T^2}{8f_\pi^2(T, \mu)} - \frac{\sigma_N n_{s, N}(T, \mu)}{M_\pi^2 f_\pi^2(T, \mu)} \right]$$



Hippe & Klevansky, PRC 52 (1995) 2172



# Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Phys. Part. Nucl. Lett. 8 (2011) 811

Povh-Huefner law behaviour for quark exchange between hadrons

PHYSICAL REVIEW C

VOLUME 51, NUMBER 5

MAY 1995

## Quark exchange model for charmonium dissociation in hot hadronic matter

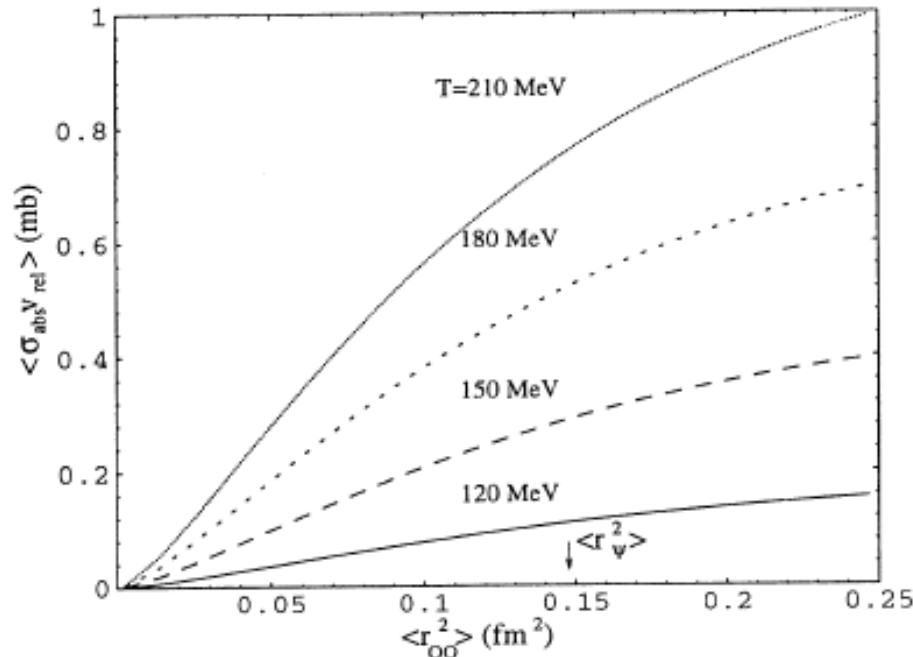
K. Martins\* and D. Blaschke†

Max-Planck-Gesellschaft AG "Theoretische Vielteilchenphysik," Universität Rostock, D-18051 Rostock, Germany

E. Quack‡

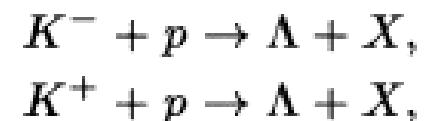
Gesellschaft für Schwerionenforschung mbH, Postfach 11 05 52, D-64220 Darmstadt, Germany

(Received 15 November 1994)



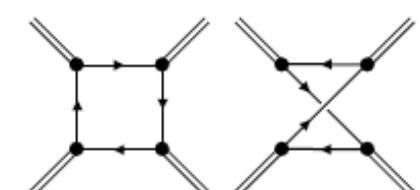
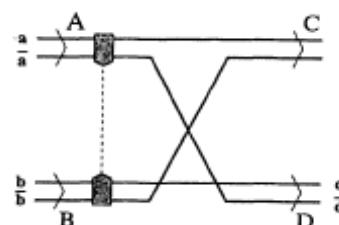
$$\langle \sigma_{\text{abs}} v_{\text{rel}} \rangle \propto \langle r^2 \rangle_{Q\bar{Q}} \langle r^2 \rangle_{q\bar{q}}$$

## Flavor exchange processes



## Nonrelativistic $\rightarrow$ rel. quark loop integrals

$$M_{fi} =$$



# Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Phys. Part. Nucl. Lett. 8 (2011) 811

Model results:

$$\tau_{\text{exp}}(T, \mu) = \tau_{\text{coll}}(T, \mu)$$

Collision time strongly T, mu dependent !

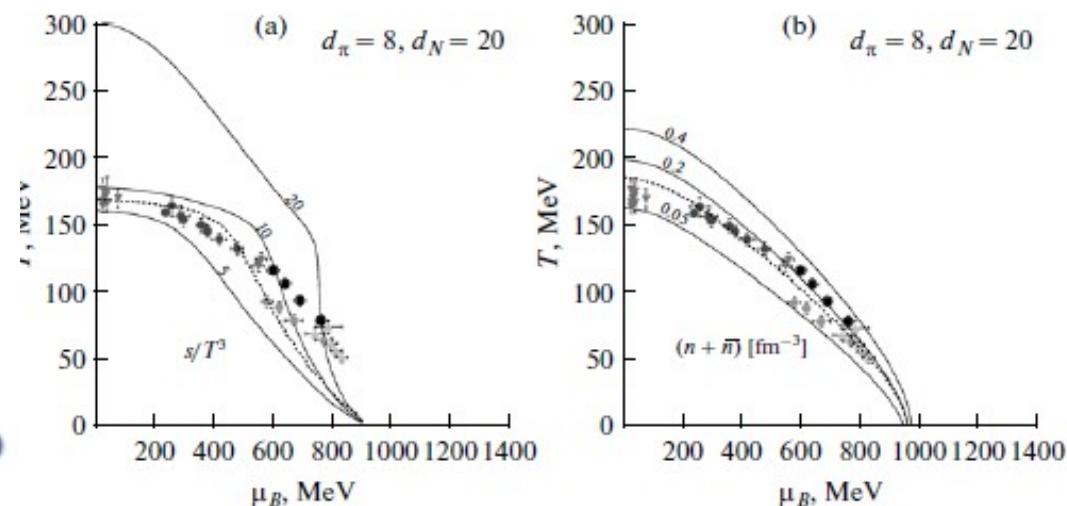
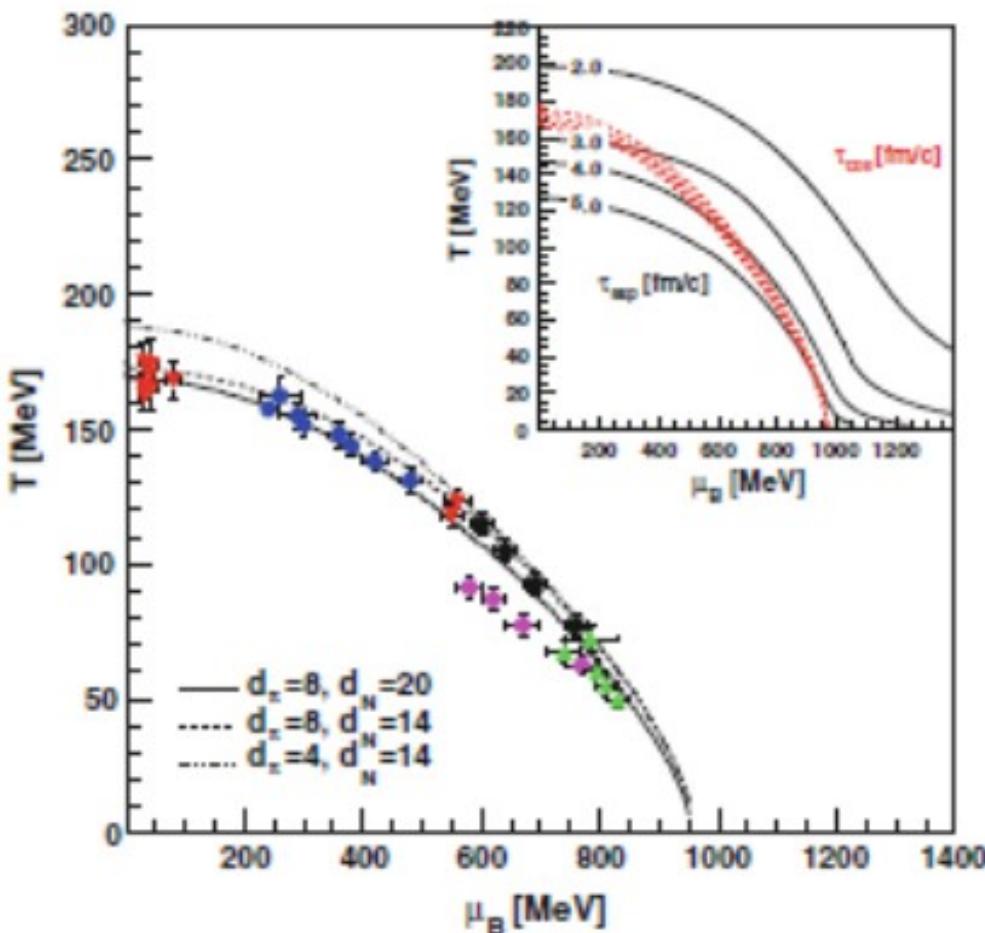
Schematic resonance gas: dp pions, dN nucleons

Expansion time scale from entropy conservation:

$$s(T, \mu) V(\tau_{\text{exp}}) = \text{const}$$

$$\tau_{\text{exp}}(T, \mu) = a s^{-1/3}(T, \mu),$$

Thermodynamics consistent with phenomenological Freeze-out rules:



# Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Few Body Syst. 53 (2012) 99

## Model results:

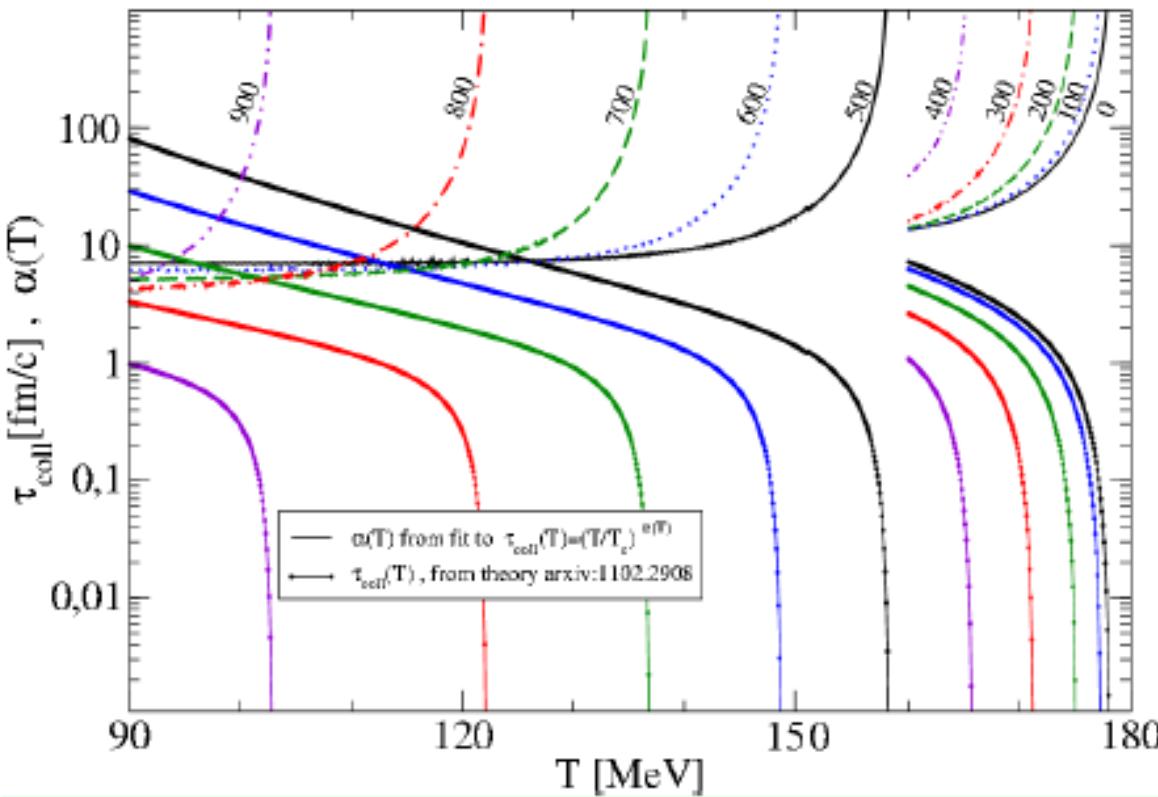
### Full hadron resonance gas model

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle ;$$

$$r_\pi^2(T, \mu) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T, \mu}|^{-1}$$

$$r_N^2(T, \mu) = r_0^2 + r_\pi^2(T, \mu)$$

$$\begin{aligned} \langle \bar{q}q \rangle &= 1 - \frac{m_0}{F_\pi^2 m_\pi^2} \left[ 4N_c \int \frac{dp}{2\pi^2} \frac{p^2}{s_p} [f_\Phi^+ + f_\Phi^-] \right. \\ &+ \sum_{M=f_0, \omega, \dots} d_M (2 - N_s) \int \frac{dp}{2\pi^2} \frac{p^2}{E_M(p)} f_M(E_M(p)) \\ &+ \sum_{B=N, \Lambda, \dots} d_B (3 - N_s) \int \frac{dp}{2\pi^2} \frac{m_B}{E_B(p)} [f_B^+(E_B(p)) + f_B^-(E_B(p))] \Big] \\ &- \sum_{G=\pi, K, \eta, \eta'} \frac{d_G r_G}{4\pi^2 F_G^2} \int dp \frac{p^2}{E_G(p)} f_G(E_G(p)). \end{aligned}$$



Collision time follows a power law  
 $t_{\text{coll}} \sim (T/T_c)^a$   
 with a large exponent  $a \sim 20$

See also: P. Braun-Munzinger, J. Stachel,  
 C. Wetterich, PLB (2004)

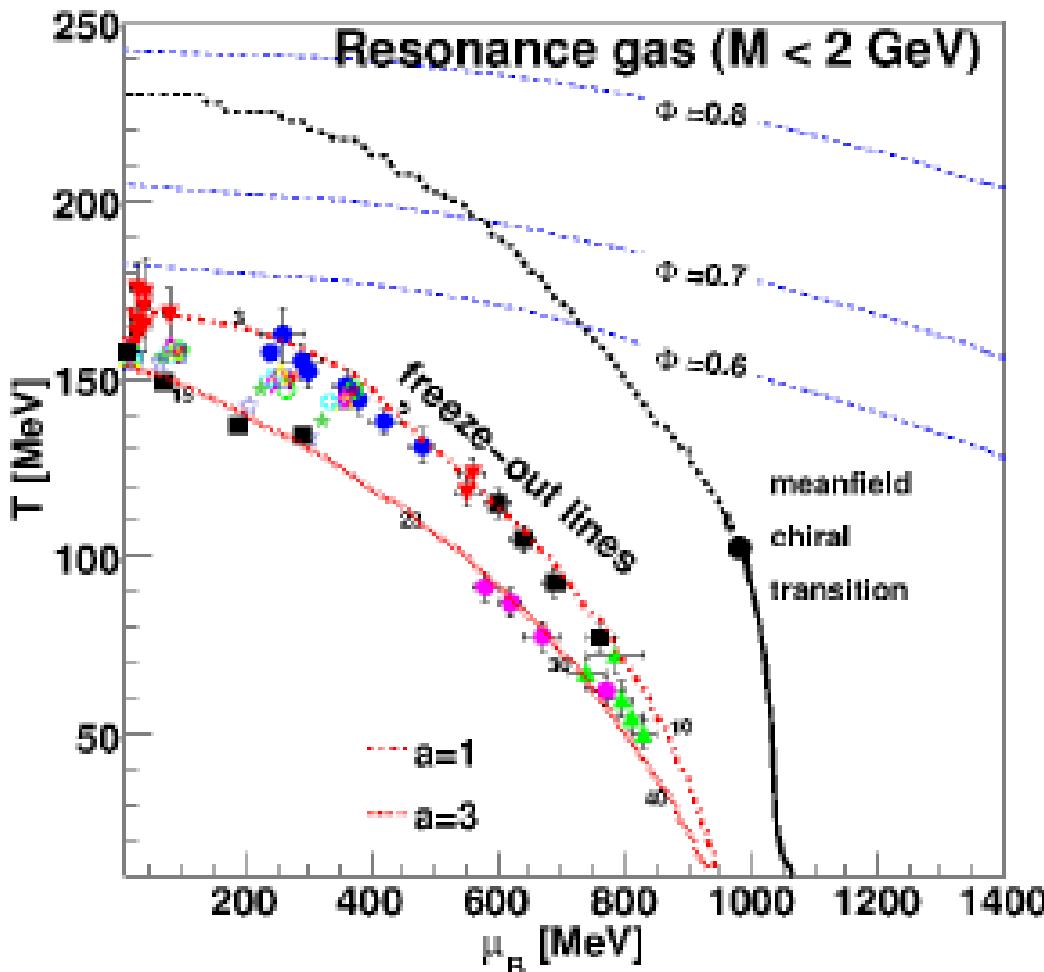
# Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Few Body Syst. 53 (2012) 99

## Model results:

Full hadron resonance gas model

See also: S. Leupold, J. Phys. G (2006)



$$\begin{aligned} \frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_{vac}} = 1 - \frac{m_0}{F_\pi^2 m_\pi^2} & \left[ 4N_c \int \frac{dp}{2\pi^2} \frac{p^2}{s_p} \left[ f_\Phi^+ + f_\Phi^- \right] \right. \\ + \sum_{M=f_0, \omega, \dots} d_M (2 - N_s) \int \frac{dp}{2\pi^2} \frac{m_M}{E_M(p)} f_M(E_M(p)) \\ + \sum_{B=N, \Lambda, \dots} d_B (3 - N_s) \int \frac{dp}{2\pi^2} \frac{m_B}{E_B(p)} \left[ f_B^+(E_B(p)) + f_B^-(E_B(p)) \right] \Big] \\ - \sum_{G=\pi, K, \eta, \eta'} \frac{d_G r_G}{4\pi^2 F_G^2} \int dp \frac{p^2}{E_G(p)} f_G(E_G(p)). \end{aligned}$$

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle ; \quad r_N^2(T, \mu) = r_0^2 + r_\pi^2(T, \mu)$$

$$r_\pi^2(T, \mu) = \frac{3 M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T, \mu}|^{-1}$$

The coefficient **a** stands for the inverse system size in the formula

$$\tau_{exp}(T, \mu) = \tau_{coll}(T, \mu)$$

for the 3D expansion time scale  
assuming entropy conservation

# Mott Dissociation of Hadrons in Hadron Matter

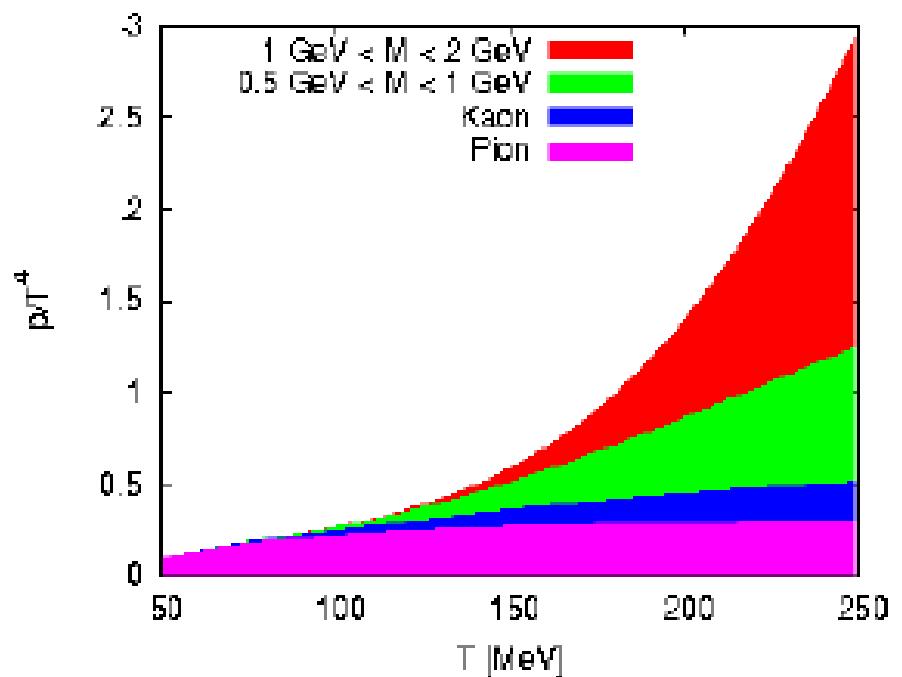
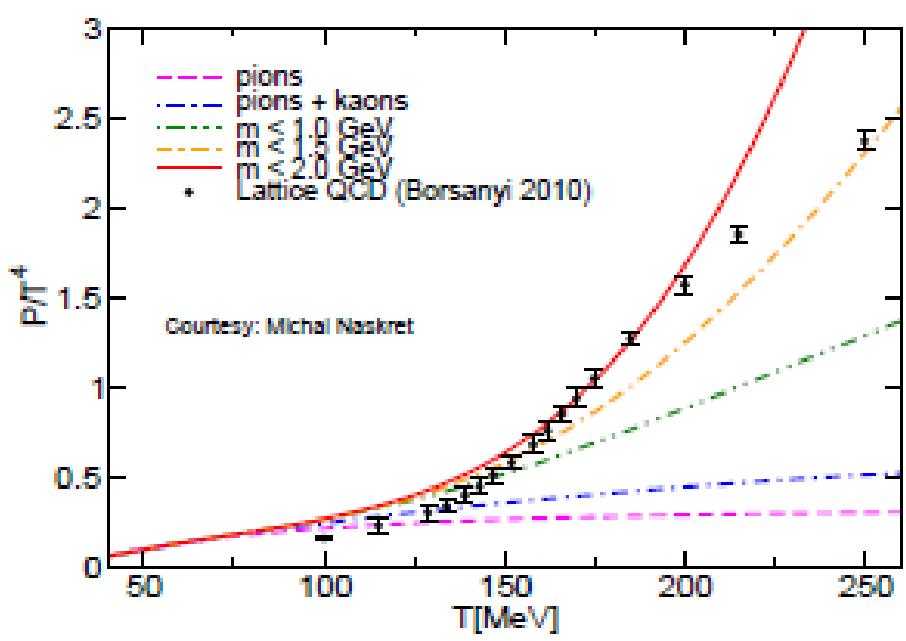
- Partition function as a Path Integral (imaginary time  $\tau = i t$ ,  $0 \leq \tau \leq \beta = 1/T$ )

$$Z[T, V, \mu] = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A \exp \left\{ - \int_0^\beta d\tau \int_V d^3x \mathcal{L}_{QCD}(\psi, \bar{\psi}, A) \right\}$$

- QCD Lagrangian, non-Abelian gluon field strength:  $F_{\mu\nu}^a(A) = \partial_\mu A^a \nu - \partial_\nu A^a \mu + g f^{abc}[A_\mu^b, A_\nu^c]$

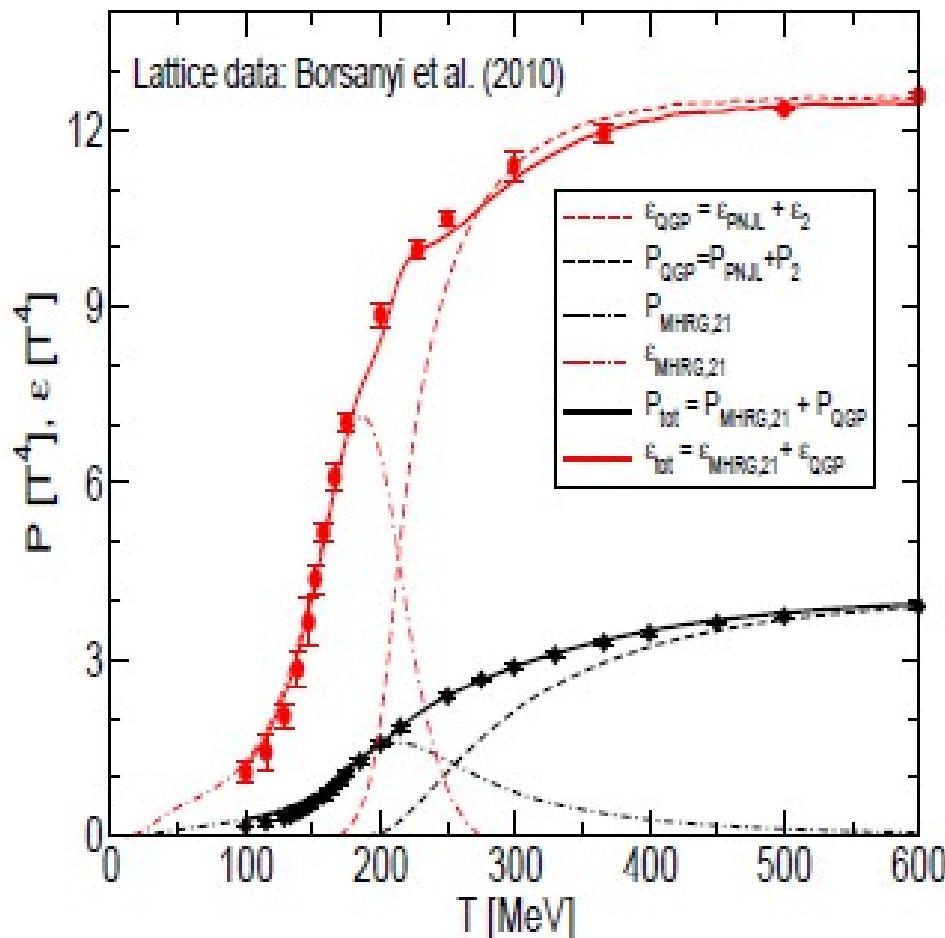
$$\mathcal{L}_{QCD}(\psi, \bar{\psi}, A) = \bar{\psi} [i\gamma^\mu (\partial_\mu - igA_\mu) - m - \gamma^0 \mu] \psi - \frac{1}{4} F_{\mu\nu}^a(A) F^{a,\mu\nu}(A)$$

- Numerical evaluation: Lattice gauge theory simulations ([hotQCD](#), Wuppertal-Budapest)



# Mott Dissociation of Hadrons in Hadron Matter

$$P_{\text{tot}}(T, \{\mu_j\}) = P_{\text{PNJL}}(T, \{\mu_i\}) + \sum_{r=M,B} \delta_r g_r \int ds A_r(s, m_r; T) \int \frac{d^3 p}{(2\pi)^3} T \ln \left\{ 1 + \delta_r \exp \left( \frac{\sqrt{p^2 + s} - \mu_r}{T} \right) \right\}$$



Spectral function for hadronic resonances:

$$A_r(s, m; T) = N_s \frac{m \Gamma_r(T)}{(s - m^2)^2 + m^2 \Gamma_r^2(T)}$$

Ansatz motivated by chemical freeze-out model:

$$\Gamma_r(T) = \tau_r^{-1}(T) = \sum_h \lambda < r_r^2 >_T < r_h^2 >_T n_h(T)$$

Apparent phase transition at  $T_c \sim 165$  MeV

Hadron resonances present up to  $T_{\text{max}} \sim 250$  MeV

Blaschke & Bugaev, Fizika B13, 491 (2004)

Prog. Part. Nucl. Phys. 53, 197 (2004)

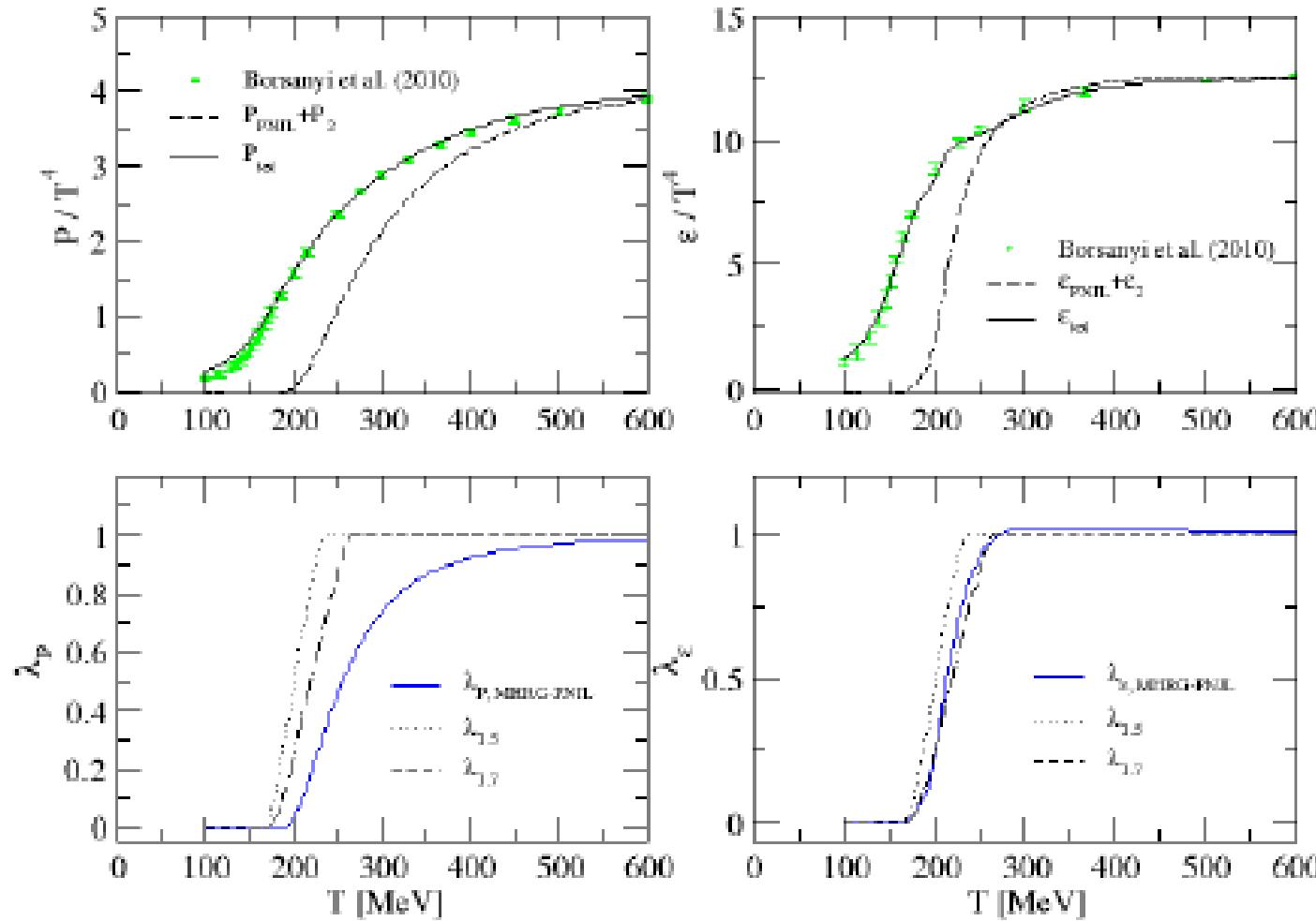
Turko, Blaschke, Prorok & Berdermann,

APPS 5, 485 (2012); J. Phys. Conf. Ser. 455, 012056 (2013)

Hadronic states above  $T_c$  ! See also: Ratti, Bellwied et al., arXiv:1109.6243 [hep-ph]

# Mott Dissociation of Hadrons in Hadron Matter

Possible application: parton fraction in the EoS at the hadronization transition



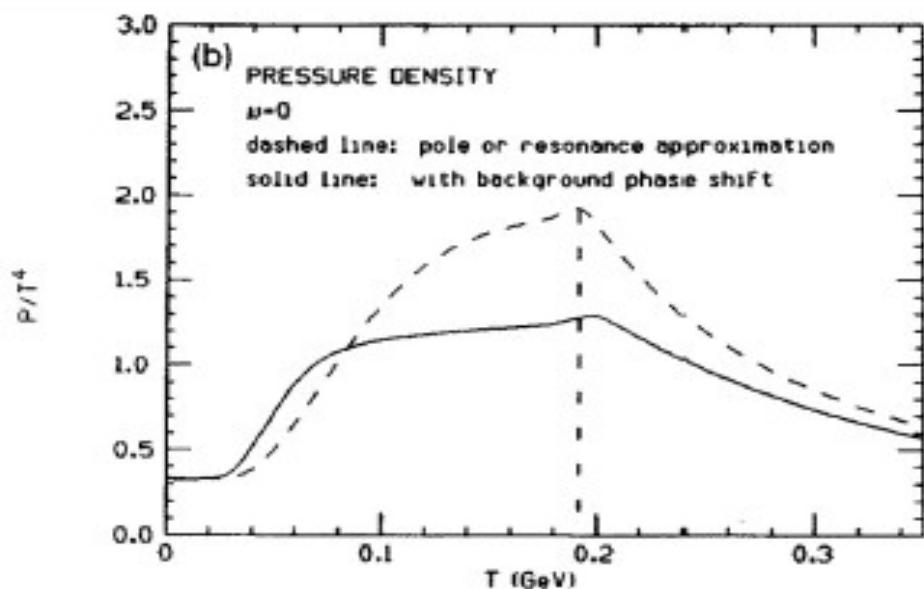
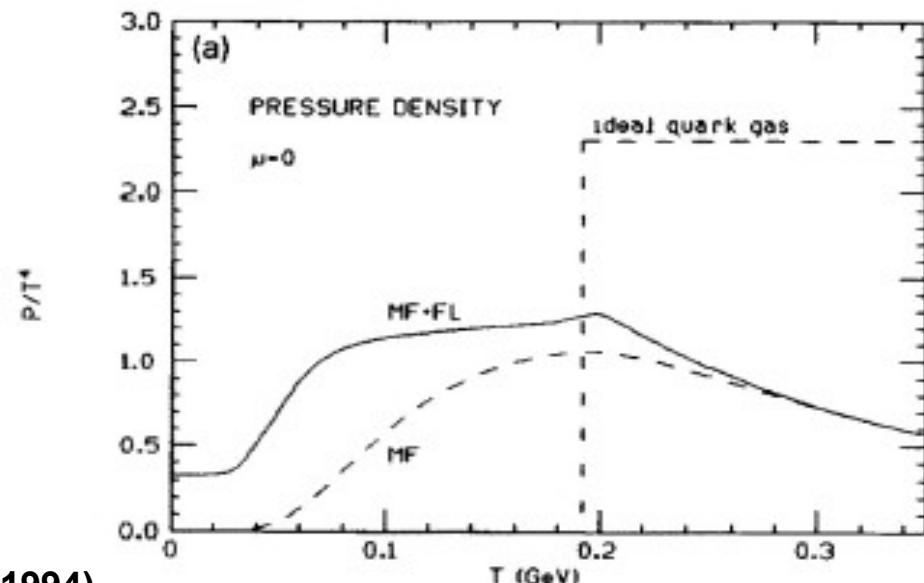
L. Turko et al. "Effective degrees of freedom in QCD ...", EPJ Web Conf. 71 (2014) 00134  
Compare:  
M. Nahrgang et al. "Influence of hadronic bound states above  $T_c$  ...", PRC 89 (2014) 014004

# Mott Dissociation of Mesons in Quark Matter

J. Huefner, S.P. Klevansky, P. Zhuang, H. Voss, Ann. Phys. 234, 225 (1994)



P. Zhuang, J. Huefner, S.P. Klevansky, NPA 576, 525 (1994)



# Mott Dissociation of Mesons in Quark Matter

D. Blaschke, M. Buballa, A. Dubinin, G. Roepke, D. Zablocki, Ann. Phys. 348, 228 (2014)

- Partition function as a Path Integral (imaginary time  $\tau = i t$ )

$$Z[T, V, \mu] = \int \mathcal{D}\bar{q} \mathcal{D}q \exp \left\{ - \int^{\beta} d\tau \int_V d^3x [\bar{q}(i\gamma^\mu \partial_\mu - m_0 - \gamma^0 \mu) q + \sum_{M=\pi,\sigma} G_M (\bar{q} \Gamma_M q)^2] \right\}$$

- Couplings:  $G_\pi = G_\sigma = G_S$  (chiral symmetry)
- Vertices:  $\Gamma_\sigma = \mathbf{1}_D \otimes \mathbf{1}_f \otimes \mathbf{1}_c$ ;  $\Gamma_\pi = i\gamma_5 \otimes \vec{\tau} \otimes \mathbf{1}_c$
- Bosonization (Hubbard-Stratonovich Transformation)

$$\exp [G_S (\bar{q} \Gamma_\sigma q)^2] = \text{const.} \int \mathcal{D}\sigma \exp \left[ \frac{\sigma^2}{4G_S} + \bar{q} \Gamma_\sigma q \sigma \right]$$

- Integrate out quark fields  $\rightarrow$  bosonized partition function

$$Z[T, V, \mu] = \int \mathcal{D}\sigma \mathcal{D}\pi \exp \left\{ -\frac{\sigma^2 + \pi^2}{4G_S} + \frac{1}{2} \text{Tr} \ln S^{-1}[\sigma, \pi] \right\}$$

- Systematic evaluation: Mean fields + Fluctuations
  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)

# Mott Dissociation of Mesons in Quark Matter

- Separate the mean-field part of the quark determinant

$$\text{Tr} \ln S^{-1}[\sigma, \pi] = \text{Tr} \ln S_{\text{MF}}^{-1}[m] + \text{Tr} \ln [1 + (\sigma + i\gamma_5 \vec{\tau} \vec{\pi}) S_{\text{MF}}[m]]$$

- Mean-field quark propagator

$$S_{\text{MF}}(\vec{p}, i\omega_n; m) = \frac{\gamma_0(i\omega_n + \mu) - \vec{\gamma} \cdot \vec{p} + m}{(i\omega_n + \mu)^2 - E_p^2}$$

- Expand the logarithm:  $\ln(1 + x) = -\sum_{n=1}^{\infty} (-1)^n x^n / n = x - x^2/2 + \dots$
- Thermodynamic potential in Gaussian approximation

$$\begin{aligned} \Omega(T, \mu) &= -T \ln Z(T, \mu) = \Omega_{\text{MF}}(T, \mu) + \sum_M \Omega_M^{(2)}(T, \mu) + \mathcal{O}[\phi_M^3] \\ \Omega_M^{(2)}(T, \mu) &= \frac{N_M}{2} \int \frac{d^2 p}{(2\pi)^3 \beta} \sum_n e^{i\nu_n \eta} \ln S_M^{-1}(\vec{p}, i\nu_n), \quad N_\sigma = 1, \quad N_\pi = 3 \end{aligned}$$

- Meson propagator  $S_M(\vec{p}, i\nu_n) = 1 / [1/(2G_S) - \Pi_M(\vec{p}, i\nu_n)]$
- Mesonic polarization loop

$$\Pi_M(\vec{p}, i\nu_n) = -\frac{1}{\beta} \sum_{n'} e^{i\nu_{n'} \eta} \int \frac{d^2 k}{(2\pi)^3} \text{Tr} \left[ \Gamma_M S_{\text{MF}}(-\vec{k}, -i\omega_{n'}) \Gamma_M S_{\text{MF}}(\vec{k} + \vec{p}, i\omega_{n'} + i\nu_n) \right]$$

# Mott Dissociation of Mesons in Quark Matter

- Polar representation of the analytically continued quark propagator

$$S_M = |S_M| e^{i\delta_M} = S_R + iS_I ,$$

- Phase shift  $\delta_M(\omega, \mathbf{q}) = -\text{Im} \ln S_M^{-1}(\omega - \mu_M + i\eta, \mathbf{q})$
- Thermodynamic potential for mesonic modes

$$\begin{aligned}\Omega_M(T, \mu) &= \text{Tr} \ln S_M^{-1}(iz_n, \mathbf{q}) = d_M T \sum_n \int \frac{d^3 q}{(2\pi)^3} \ln S_M^{-1}(iz_n, \mathbf{q}) , \\ &= -d_M T \sum_n \int \frac{d^3 q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{iz_n - \omega} \text{Im} \ln S_M^{-1}(\omega + i\eta, \mathbf{q})\end{aligned}$$

- Perform Matsubara summation  $\Omega_M(T, \mu) = d_M \int \frac{d^3 q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} n_M^-(\omega) \delta_M(\omega, \mathbf{q})$
- Using symmetries of Bose function  $n_M^-(-\omega) = -[1 + n_M^+(\omega)]$  and polarization loop

$$\Omega_M(T, \mu) = d_M \int \frac{d^3 q}{(2\pi)^3} \int_0^{\infty} \frac{d\omega}{2\pi} [1 + n_M^-(\omega) + n_M^+(\omega)] \delta_M(\omega, \mathbf{q})$$

- Partial integration gives field theoretic Beth-Uhlenbeck formula

$$\Omega_M = -d_M \int \frac{d^3 q}{(2\pi)^3} \int_0^{\infty} \frac{d\omega}{2\pi} \left[ \omega + T \ln \left( 1 - e^{-(\omega - \mu_M)/T} \right) + T \ln \left( 1 - e^{-(\omega + \mu_M)/T} \right) \right] \frac{d\delta_M(\omega, \mathbf{q})}{d\omega}$$

# Mott Dissociation of Mesons in Quark Matter

- When polarization loop integral can be expressed in the form

$$\Pi_M(z, \mathbf{q}) = \Pi_{M,0} + \Pi_{M,2}(z, \mathbf{q})$$

- Factorization of two-particle propagator possible with  $R_M(z, \mathbf{q}) = \frac{1 - G_M \Pi_{M,0}}{G_M \Pi_{M,2}(z, \mathbf{q})}$

$$S_M(z, \mathbf{q}) = \frac{1}{G_M^{-1} - \Pi_{M,0} - \Pi_{M,2}(z, \mathbf{q})} = \frac{1}{\Pi_{M,2}(z, \mathbf{q})} \frac{1}{R_M(z, \mathbf{q}) - 1}$$

- This entails  $\ln S_M(z, \mathbf{q})^{-1} = \ln \Pi_{M,2}(z, \mathbf{q}) + \ln [R_M(z, \mathbf{q}) - 1]$   
and thus a separation of the phase shift in two contributions

$$\delta_M(\omega, \mathbf{q}) = \delta_{M,c}(\omega, \mathbf{q}) + \delta_{M,R}(\omega, \mathbf{q})$$

- They correspond to continuum (state independent) and resonant phases

$$\delta_{M,c}(\omega, \mathbf{q}) = -\arctan \left( \frac{\text{Im} \Pi_{M,2}(\omega - \mu_M + i\eta, \mathbf{q})}{\text{Re} \Pi_{M,2}(\omega - \mu_M + i\eta, \mathbf{q})} \right)$$

$$\delta_{M,R}(\omega, \mathbf{q}) = \arctan \left( \frac{\text{Im} R_M(\omega - \mu_M + i\eta, \mathbf{q})}{1 - \text{Re} R_M(\omega - \mu_M + i\eta, \mathbf{q})} \right)$$

# Mott Dissociation of Mesons in Quark Matter

- Suppose  $\delta_{M,R}(\omega, \mathbf{q})$  corresponds to a resonance at  $\omega = \omega_M = \sqrt{\mathbf{q}^2 + M_M^2}$ , then the propagator shall have the representation with a complex pole at  $z = z_M = \omega_M + i\Gamma_M/2$ , where  $\Gamma_M$  is the width of the resonance.
- The position of the pole is found from the condition  $\text{Re}R_M(z_M, \mathbf{q}) = 1$ , where  $\delta_{M,R}(\omega \rightarrow \omega_M) \rightarrow \pi/2$  since  $\tan \delta_{M,R}(\omega \rightarrow \omega_M) \rightarrow \infty$
- Expanding  $R_M(z, \mathbf{q})$  at the complex pole  $z_M$  for small width, one obtains

$$1 - \text{Re}R_M(z_M, \mathbf{q}) = -(\omega^2 - \omega_M^2) \frac{dR_M(z, \mathbf{q})}{d\omega^2} \Big|_{z=z_M}, \quad \text{Im}R_M(z_M, \mathbf{q}) = \omega_M \Gamma_M \frac{dR_M(z, \mathbf{q})}{d\omega^2} \Big|_{z=z_M} \quad (1)$$

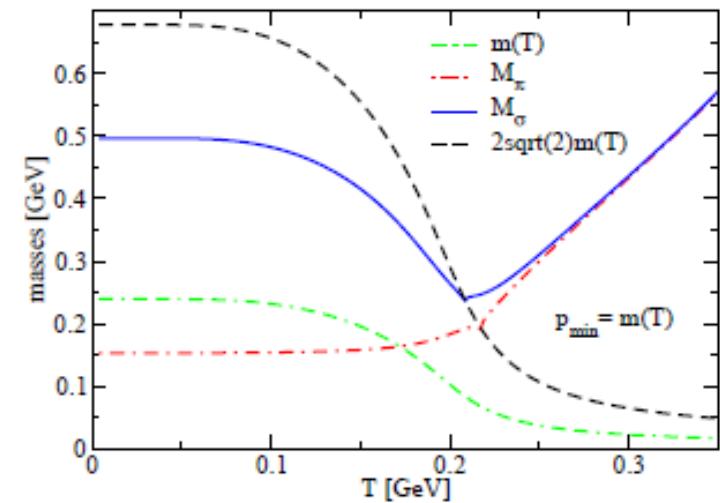
- The resonant shift becomes  $\delta_{M,R}(\omega, \mathbf{q}) = -\arctan\left(\frac{\omega_M \Gamma_M}{\omega^2 - \omega_M^2}\right)$  corresponding to a Breit-Wigner form of the spectral density in the Beth-Uhlenbeck EoS

$$\frac{d\delta_{M,R}}{d\omega} = \frac{2\omega \omega_M \Gamma_M}{(\omega^2 - \omega_M^2)^2 + \omega_M^2 \Gamma_M^2}.$$

- This takes the form of a bound state spectral density for  $\Gamma_M \rightarrow 0$

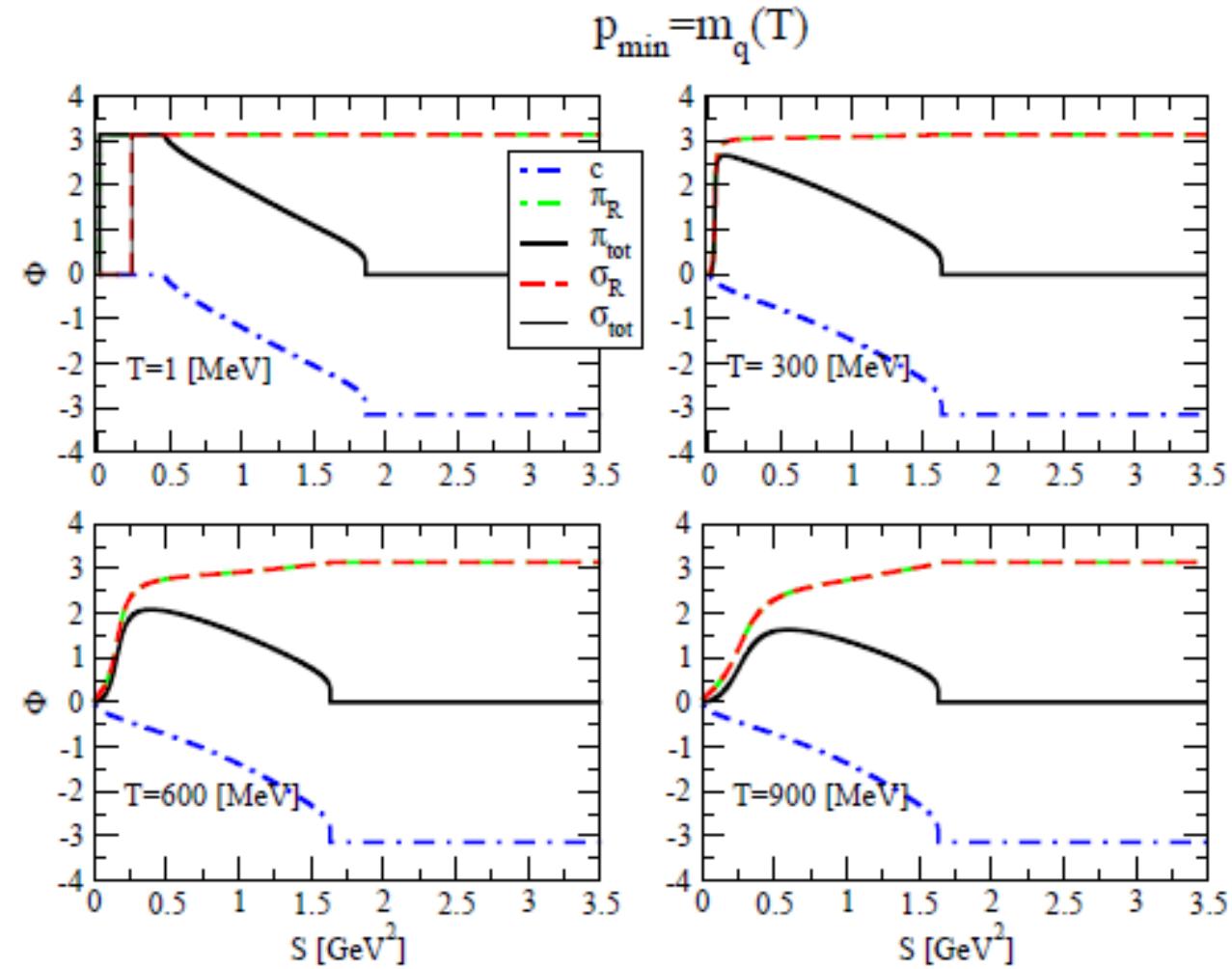
$$\lim_{\Gamma_M \rightarrow 0} \delta'_{M,R}(\omega) = \pi [\delta(\omega - \omega_M) + \delta(\omega + \omega_M)]$$

# Mott Dissociation of Mesons in Quark Matter



XXXI. Max Born Symposium,  
Wroclaw (2013)

D. Blaschke, A. Dubinin, Yu. Kalinovsky,  
Acta Phys. Pol. Suppl. 7 (2014)



# Hadron Resonance Gas with Mott Dissociation

$$P(T) = \sum_{i=M,B} P_i(T) + P_{\text{PNJL}}(T) + P_{\text{pert}}(T) ,$$

$$\begin{aligned} P_i(T) &= d_i \int \frac{d^3 p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{\pi} f_i(\omega) \delta_i(\omega; T) \\ &= d_i \int_0^\infty \frac{dp p^2}{2\pi^2} \int_0^\infty \frac{ds}{2\pi} \frac{1}{\sqrt{p^2 + s}} f_i(\sqrt{p^2 + s}) \delta_i(s; T), \end{aligned}$$

$$P_i(T) = d_i \int_0^\infty \frac{dp p^2}{2\pi^2} \int_0^\infty \frac{dM}{\pi} \frac{M}{\sqrt{p^2 + M^2}} f_i(\sqrt{p^2 + M^2}) \delta_i(M^2; T).$$

By partial integration over  $M$  we obtain

$$P_i(T) = \mp d_i \int_0^\infty \frac{dp p^2}{2\pi^2} \int_0^\infty dM T \ln(1 \mp e^{-\sqrt{p^2+M^2}/T}) \frac{1}{\pi} \frac{d\delta_i(M^2; T)}{dM} ,$$

when  $m(T) \rightarrow \infty$ ,  $\Gamma_i(T) \rightarrow 0$ , so that  $\delta_i(M^2; T) = \pi\theta(M - M_i)$ ,

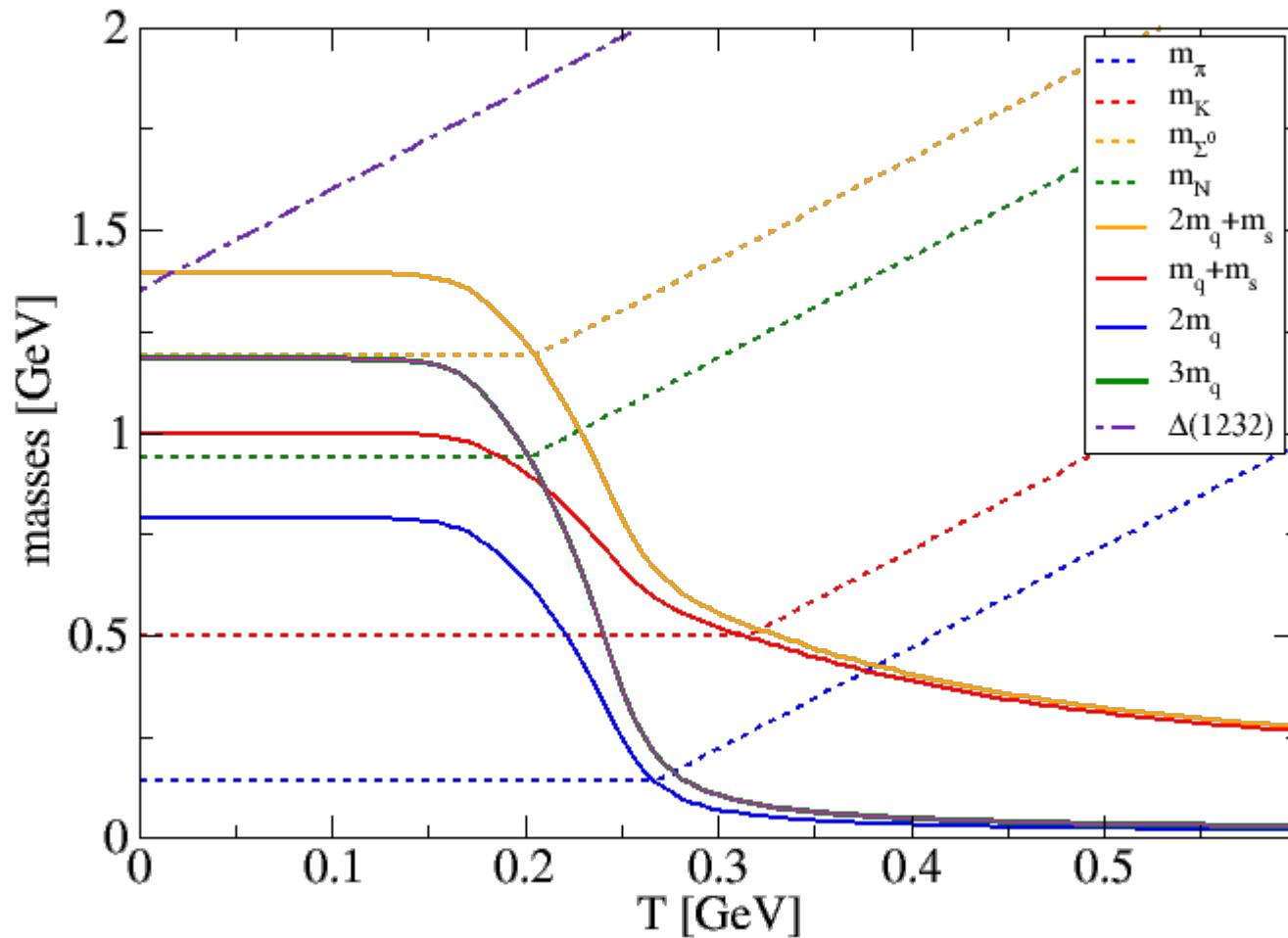
$$\frac{1}{\pi} \frac{d\delta_i(M^2; T)}{dM} = \delta(M - M_i),$$

so that the  $M$ -integration becomes trivial and gives

$$P_i(T) = \mp d_i \int_0^\infty \frac{dp p^2}{2\pi^2} T \ln \left( 1 \mp e^{-\sqrt{p^2+M_i^2}/T} \right),$$

# Hadron Resonance Gas with Mott Dissociation

D. Blaschke, A. Dubinin, L. Turko, Phys. Part. Nucl. 46 (2015); arxiv:1501.00485



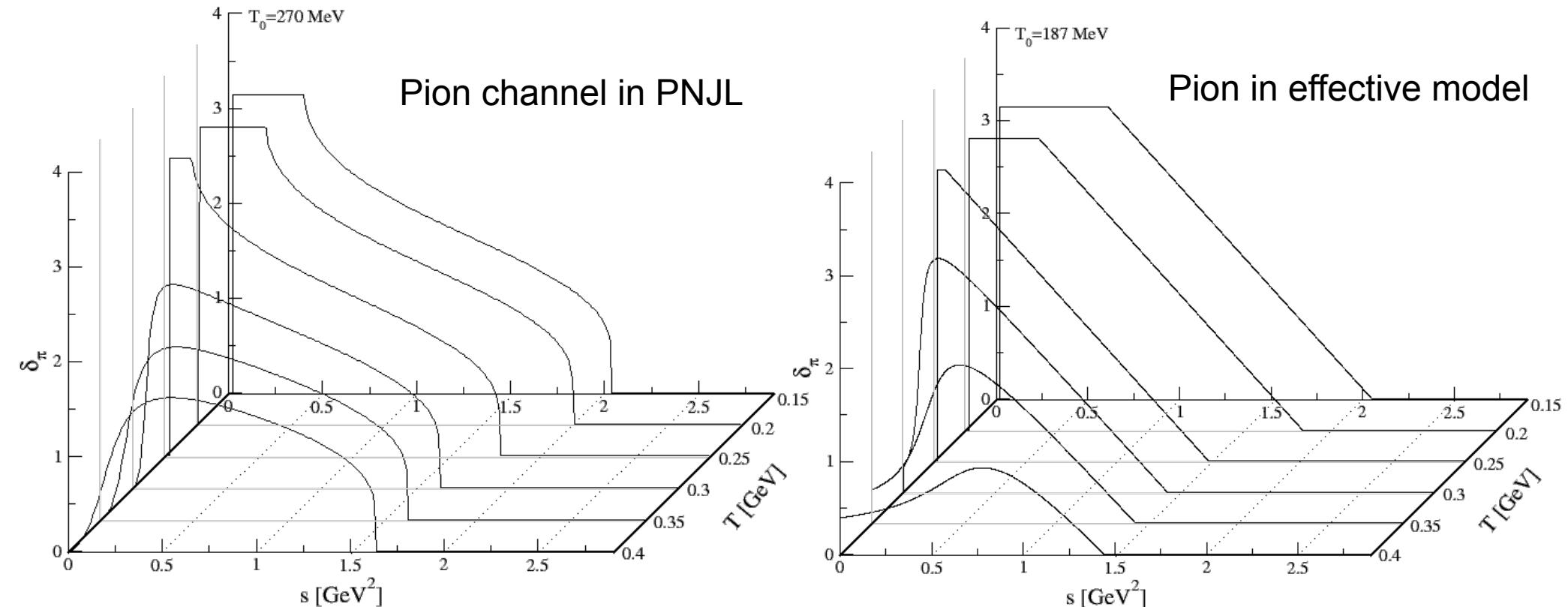
$$M_i(T) = M_i(0) + \Gamma_i(T) ,$$

$$\Gamma_i(T) = a (T - T_{\text{Mott},i}) \Theta(T - T_{\text{Mott},i})$$

$$\begin{aligned} M_i(T_{\text{Mott},i}) &= m_{\text{thr},i}(T_{\text{Mott},i}) , \\ m_{\text{thr},M}(T) &= (2 - N_s)m(T) + N_s m_s(T) \\ m_{\text{thr},B}(T) &= (3 - N_s)m(T) + N_s m_s(T) \end{aligned}$$

# Hadron Resonance Gas with Mott Dissociation

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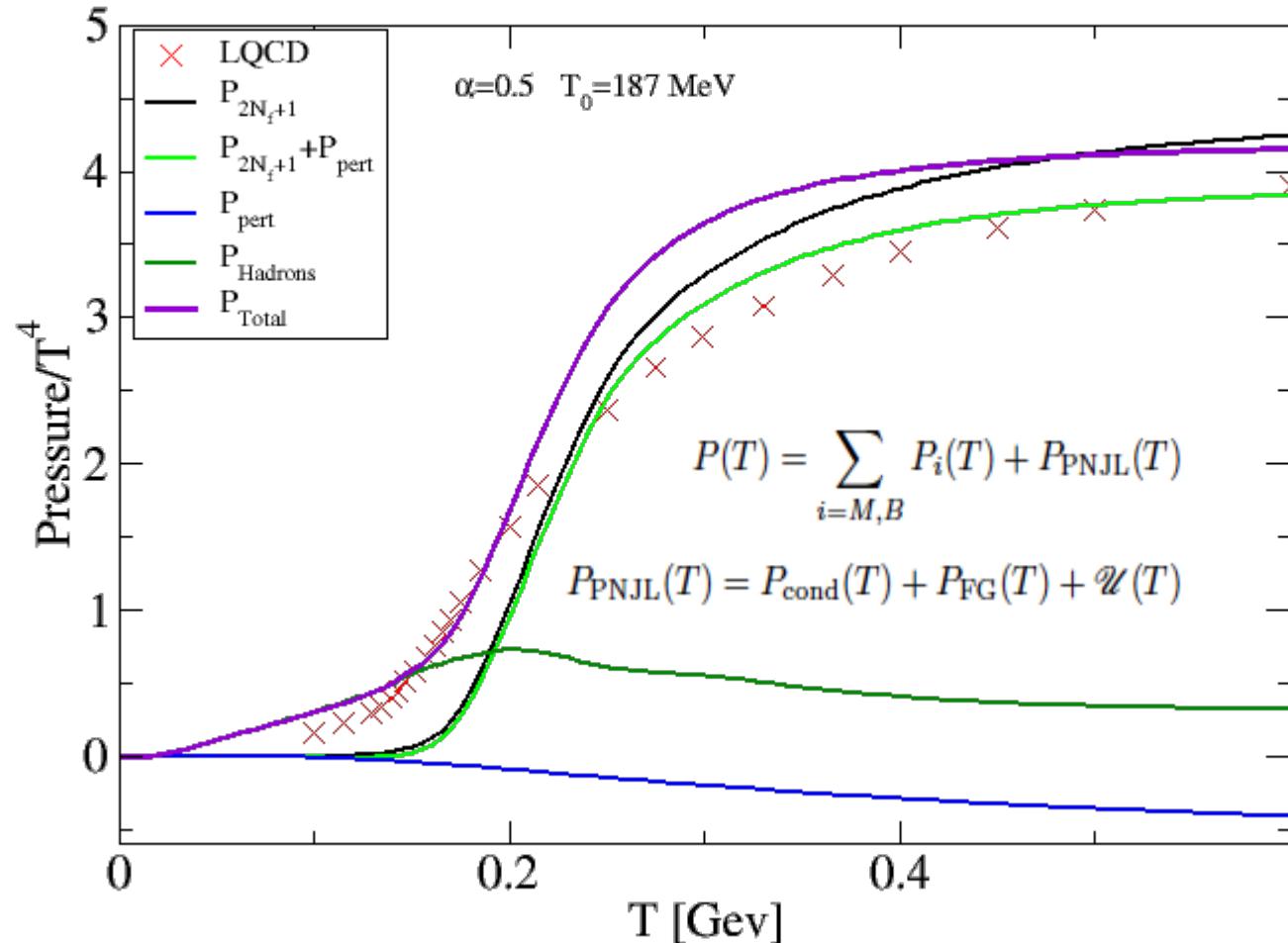


Effective model for in-medium hadron phase shifts

$$\delta_i(s; T) = \left[ \frac{\pi}{2} + \arctan \left( \frac{s - M_i^2(T)}{M_i(T)\Gamma_i(T)} \right) \right] \left\{ \Theta[m_{\text{thr},i}^2 - s] + \Theta[s - m_{\text{thr},i}^2] \Theta[m_{\text{thr},i}^2 + N_i^2 \Lambda^2 - s] \left[ \frac{[m_{\text{thr},i}^2 + N_i^2 \Lambda^2 - s]}{N_i^2 \Lambda^2} \right] \right\}$$

# Hadron Resonance Gas with Mott Dissociation

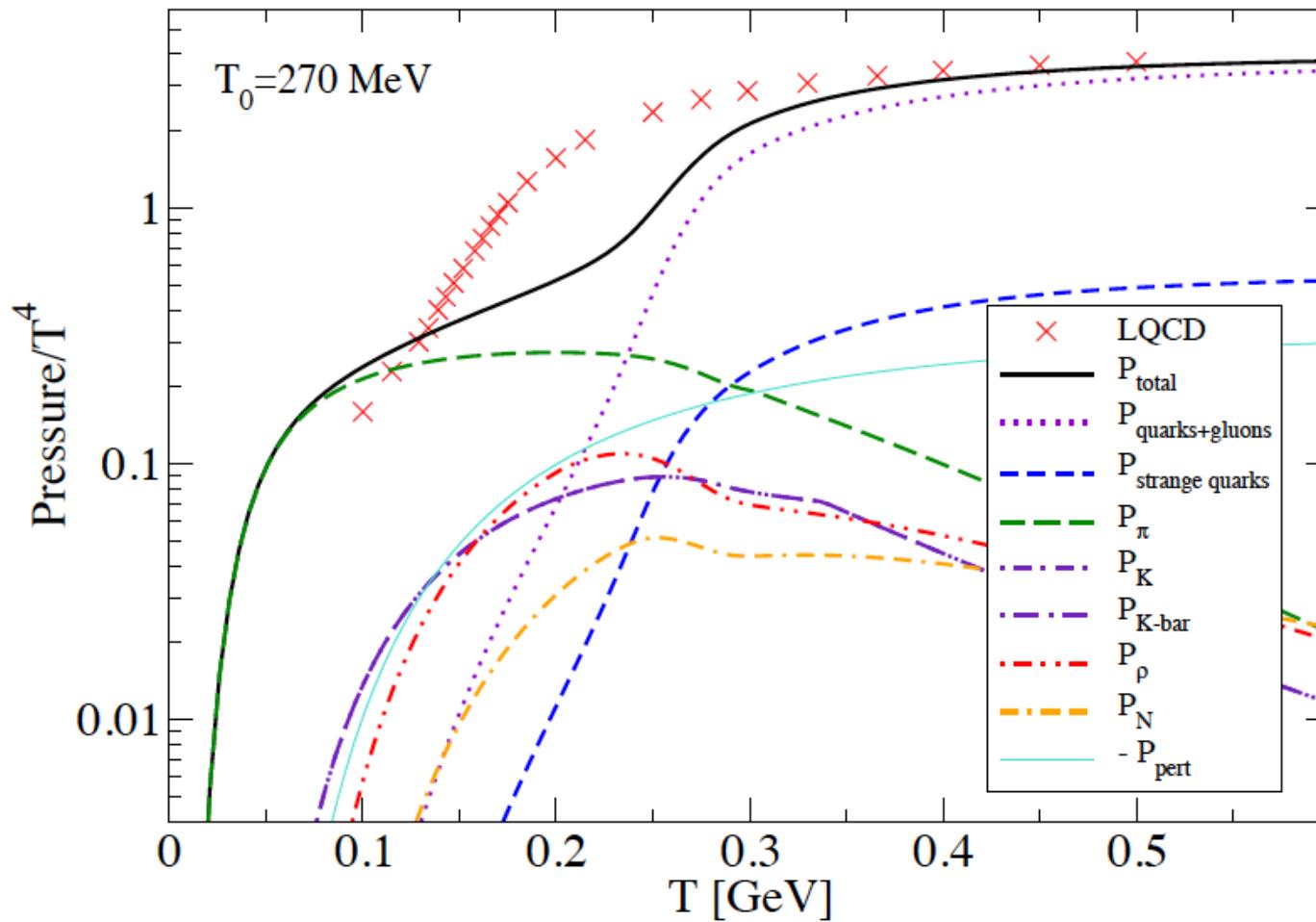
D. Blaschke, A. Dubinin, L. Turko, Phys. Part. Nucl. 46 (2015); arxiv:1501.00485



$$P_i(T) = d_i \int \frac{d^3 p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{\pi} f_i(\omega) \delta_i(\omega; T) = d_i \int_0^\infty \frac{dp}{2\pi^2} \int_0^\infty \frac{ds}{2\pi} \frac{1}{\sqrt{p^2 + s}} f_i(\sqrt{p^2 + s}) \delta_i(s; T)$$

# Hadron Resonance Gas with Mott Dissociation

D. Blaschke, A. Dubinin, L. Turko, Phys. Part. Nucl. 46 (2015); arxiv:1501.00485



$$P_2(T) = P_2^{\text{quark}}(T) + P_2^{\text{gluon}}(T) =$$



$$-1/2 \langle \cdots \rangle + 1/12 \langle \cdots \rangle$$



$$+ 1/8 \langle \cdots \rangle$$



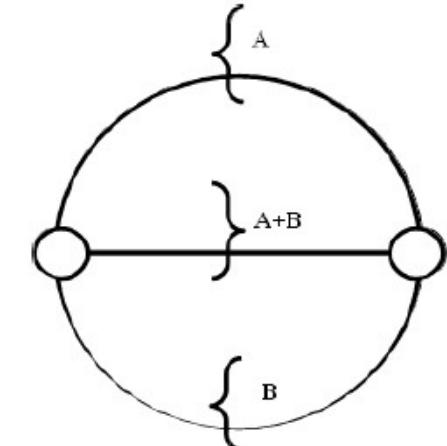
# Towards Selfconsistence: Cluster Virial Expansion

D. Blaschke, PoS Baldin XXII (2014); arxiv: 1502.06279

Clusters in nuclear matter: Roepke et al., Nucl. Phys. A (2013)

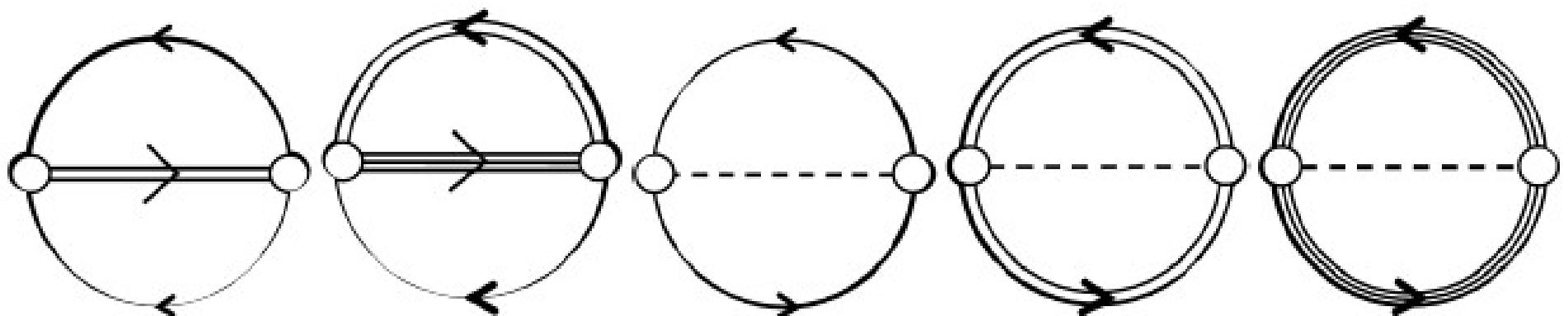
$$\Omega = \sum_A (-1)^A [\text{Tr} \ln (-G_A^{-1}) + \text{Tr} (\Sigma_A G_A)] + \sum_{A,B} \Phi [G_A, G_B, G_{A+B}] ,$$

$$G_A^{-1} = G_A^{(0)-1} - \Sigma_A , \quad \Sigma_A (1 \dots A, 1' \dots A', z_A) = \frac{\delta \Phi}{\delta G_A (1 \dots A, 1' \dots A', z_A)} .$$



Conserving approximation (Phi-derivable approach) for  
**Hadrons in Quark Matter**

$$\Omega = \sum_{i=Q,M,D,B} (-1)^{c_i} [\text{Tr} \ln (-G_i^{-1}) + \text{Tr} (\Sigma_i G_i)] + \Phi [G_Q, G_M, G_D, G_B] , \quad \Sigma_i = \frac{\delta \Phi [G_Q, G_M, G_D, G_B]}{\delta G_i} .$$

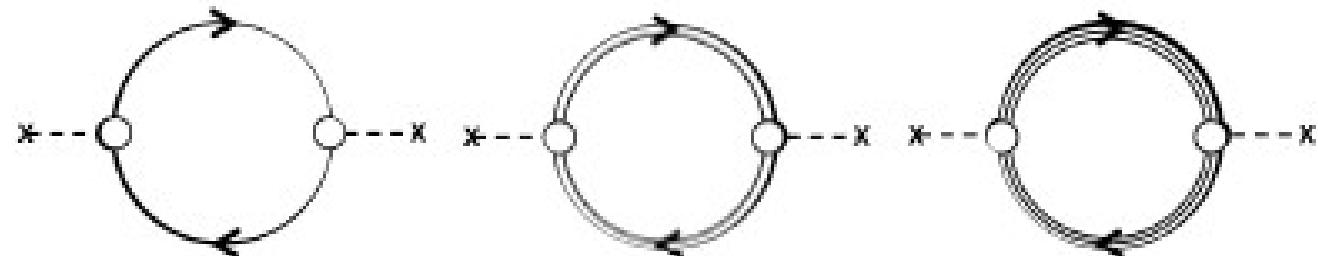


# Towards Selfconsistence: Cluster Virial Expansion

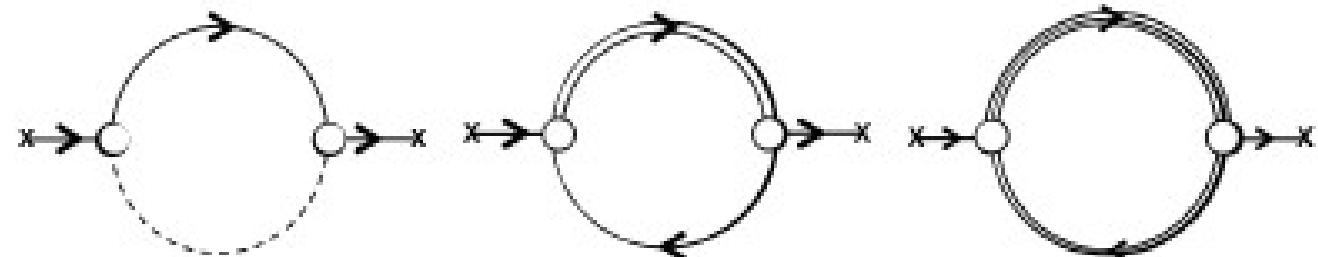
D. Blaschke, PoS Baldin XXII (2014); arxiv: 1502.06279

$$\Sigma_i = \frac{\delta \Phi[G_Q, G_M, G_D, G_B]}{\delta G_i}$$

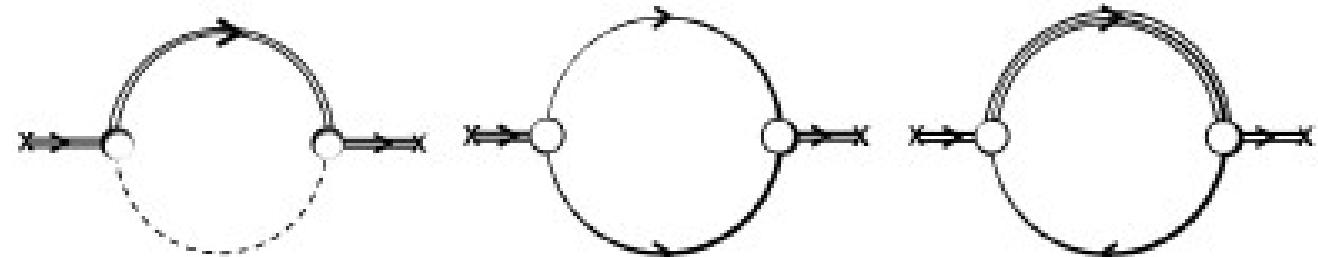
Mesons =



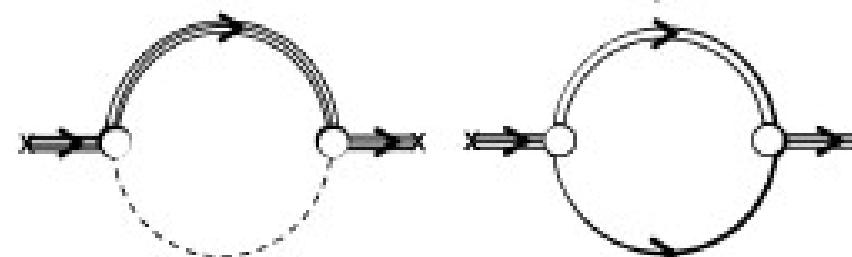
Quarks =



Diquarks =



Nucleons =

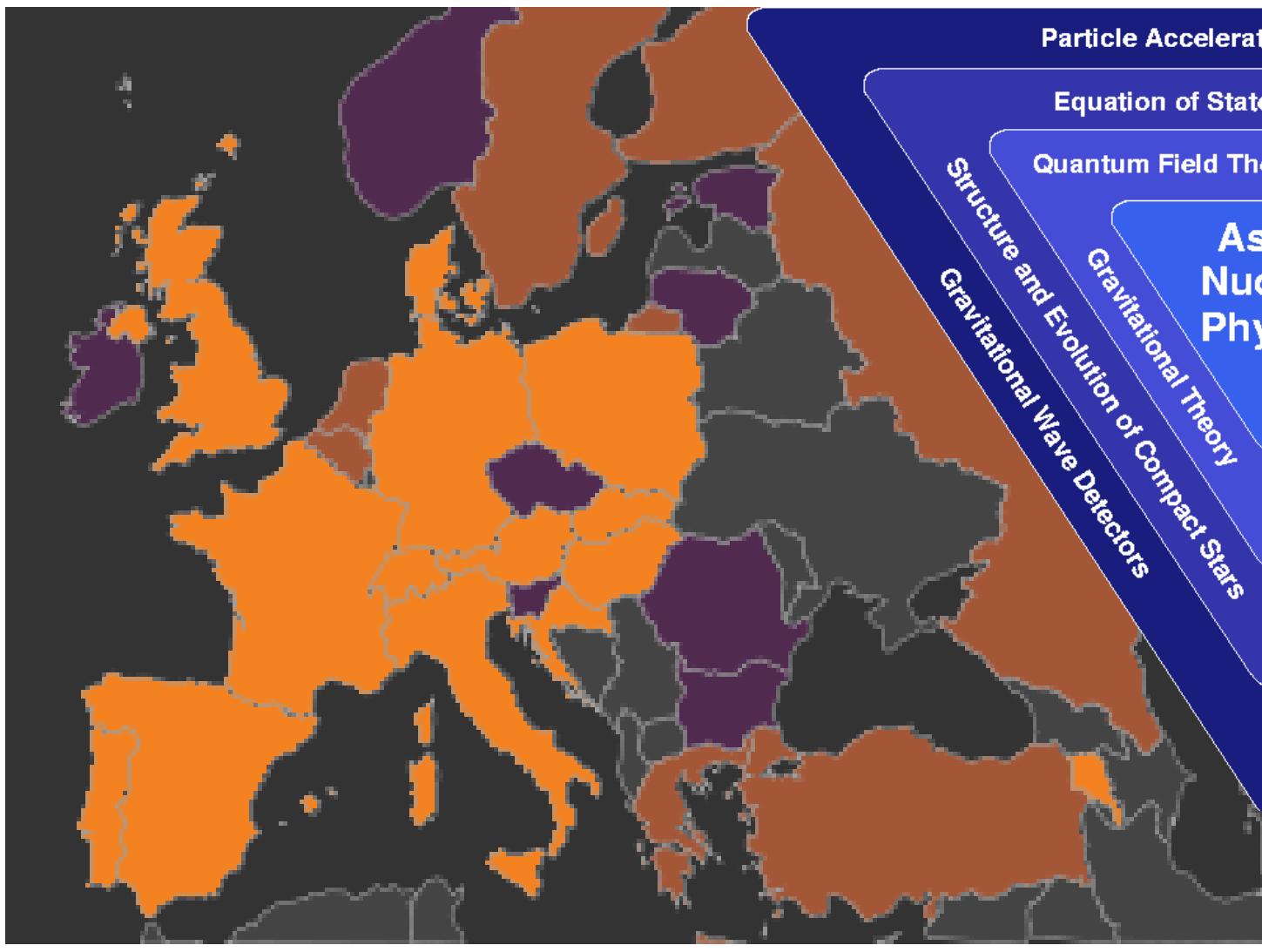


# Summary

- Generalized Beth-Uhlenbeck approach as microphysical basis to account for hadron dissociation (Mott effect) at extreme temperatures and densities
- Benchmark: pion and sigma Mott effect within NJL model, revised within nonlocal PNJL model
- Nonlocal PNJL model calibrated with lattice quark propagator data, EoS at finite  $T$  and  $\mu$ , Phase diagram with critical point
- Application of GBU to interpret chemical freeze-out as Mott-Anderson localization
- Effective GBU model description: Mott-Hagedorn resonance gas + PNJL model describes Lattice QCD thermodynamics

# Outlook

- RMF (Walecka) model as limit of the PNJL model: chiral transition effects in nuclear EoS
- Prospects for HIC (CBM & NICA) and Supernovae: color superconducting (quarkyonic) phases accessible!



Particle Accelerators and Detectors

Equation of State – Phase Diagram

Quantum Field Theory of Dense Matter

## Astro– Nuclear– Physics

Structure and Evolution  
Gravitational Wave Detectors  
Gravitational Theory  
Gravitational Evolution of Compact Stars

Astrophysics

Particle Production under Extreme Conditions  
Radio- and optical Telescopes; X-ray-, Gamma- Satellites

**28 member  
countries !!  
(MP1304)**

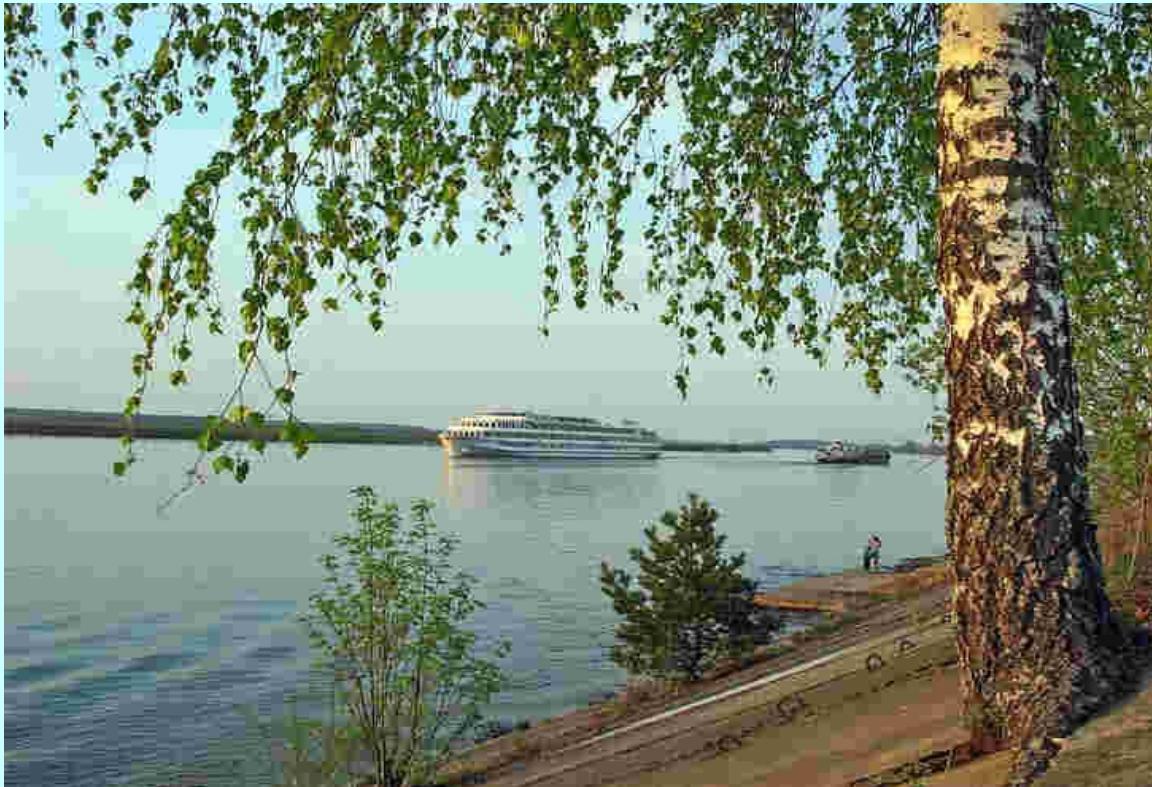
New  
comp  
star !



**Kick-off: Brussels, November 25, 2013**

# Strangeness in Quark Matter 2015

Dubna, 6.-11. July 2015



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Satellite Meetings:

Summer School “Dense Matter”, Dubna, June 29 – July 11, 2015

Roundtable “Physics at NICA”, Dubna, 5. July 2015