Chiral Transition & Deconfinement in Magnetic QCD

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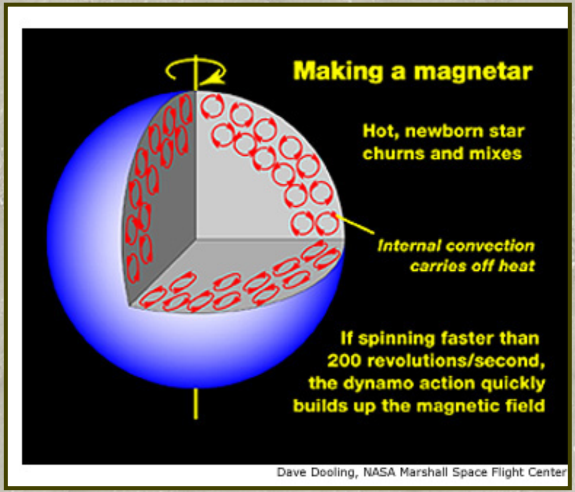
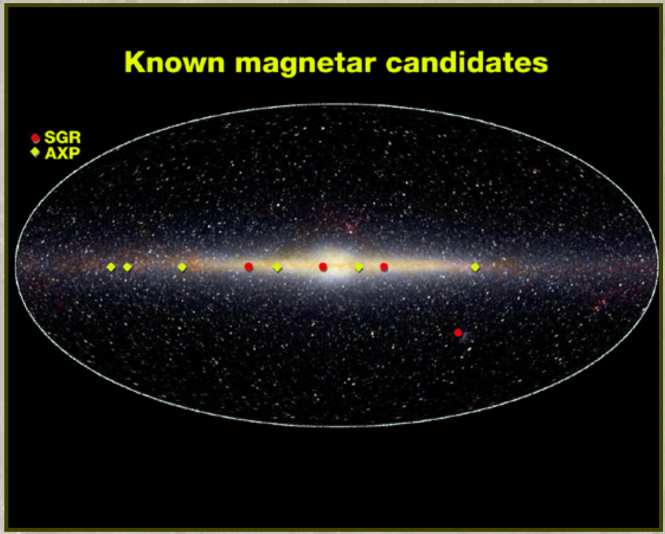
Initial motivation

Strong interactions under intense magnetic fields can be found, in principle, in a variety of systems:

❖ **High density and low temperature**

- "Magnetars": B ~ 10^{14}-10^{15} G at the surface, much higher in the core
  [Duncan & Thompson (1992/1993)]



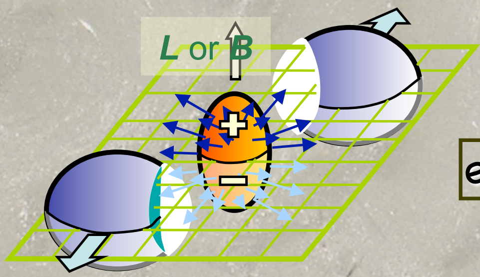


- Stable stacks of π^0 domain walls or axial scalars (η,η') domain walls in nuclear matter: B ~ 10^{17}-10^{19} G [Son & Stephanov (2008)]

❖ High temperature and low density

high magnetic fields in non-central RHIC collisions

[Kharzeev, McLerran & Warringa (2008)]



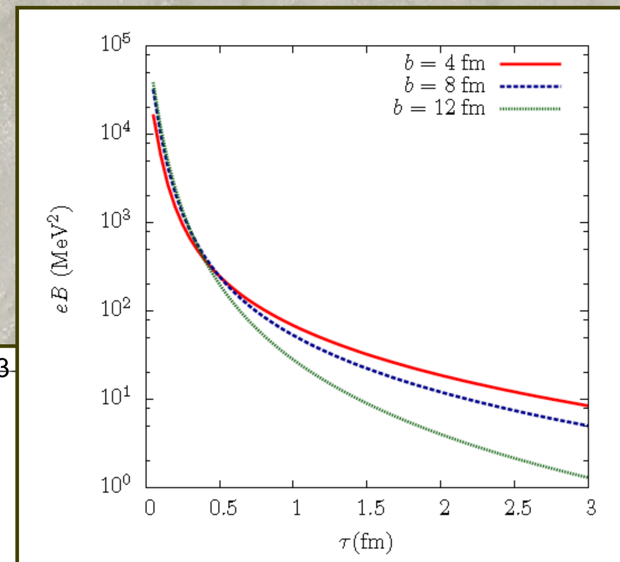
[Voloshin, QM2009]

$eB \sim 10^4\text{–}10^5\ \mathrm{MeV}^2 \sim 10^{19}\ \mathrm{G}$

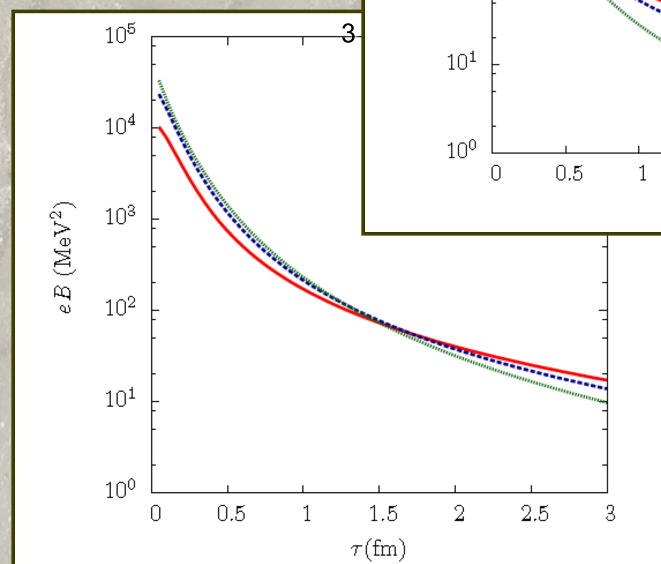
For comparison:

- "Magnetars": B ~ 10^{14}–10^{15} G at the surface, higher in the core [Duncan & Thompson (1992/1993)]
- Early universe (relevant for nucleosynthesis): B~ 10^{24} G for the EWPT epoch [Grasso & Rubinstein (2001)]

Plus: mechanism based on separation of charge for the detection of the Chiral Magnetic Effect and P-odd effects
[Voloshin (2000,2004), Kharzeev (2006); Kharzeev & Zhitnitsky (2007); Kharzeev, McLerran & Warringa (2008); Fukushima, Kharzeev & Warringa (2008)]



[Au-Au, 200 GeV]



[Au-Au, 62 GeV]

Magnetic QCD

Several theoretical/phenomenological questions arise:

How does the QCD phase diagram look like including a nonzero uniform B ? (another interesting "control parameter" ?)

Where are the possible metastable CP-odd states and how "stable" they are? What are their lifetimes ?

Are there modifications in the nature of the phase transitions ?

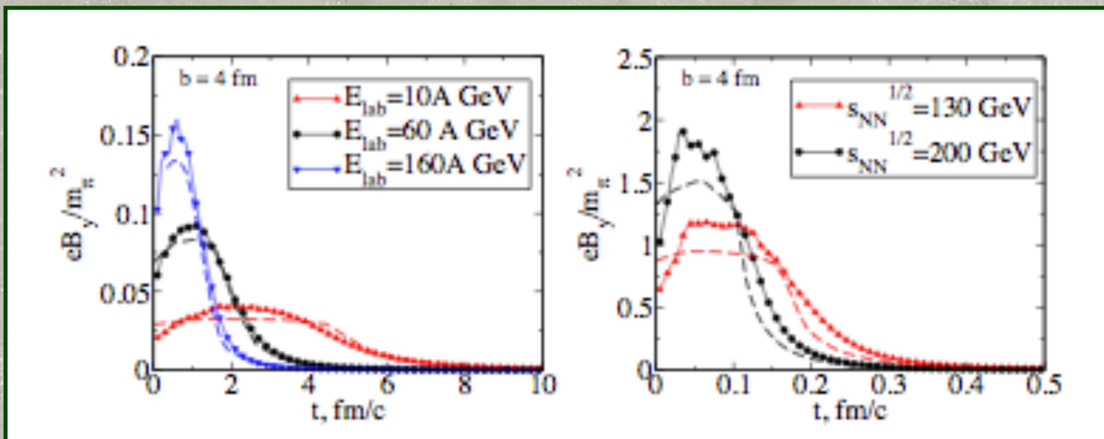
Are the relevant time scales for phase conversion affected ?

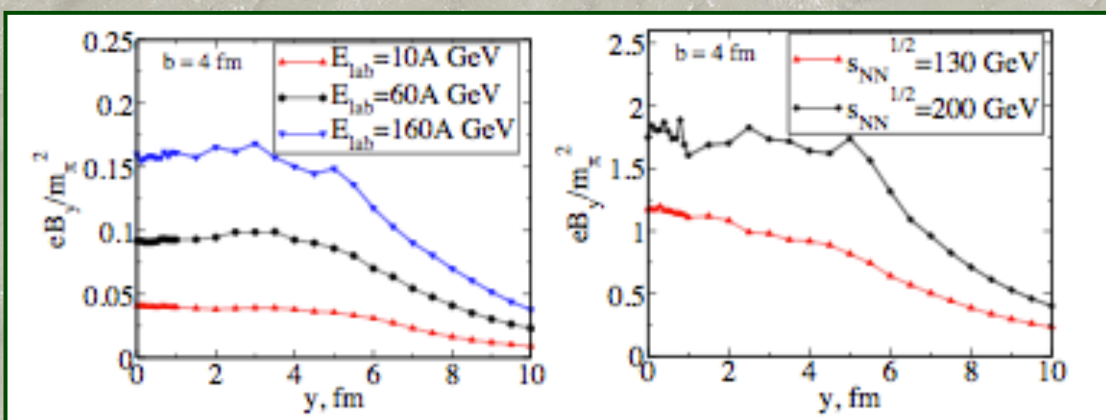
Are there other new phenomena (besides the chiral magnetic effect) ?

What is affected in the plasma formed in heavy ion collisions ?

Which are the good observables to look at ? Can we investigate it experimentally ? Can we simulate it on the lattice ?

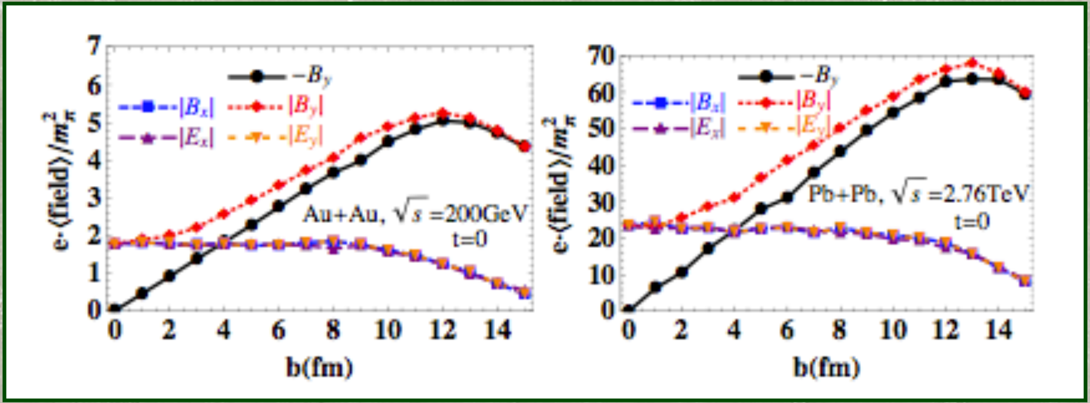
High magnetic fields in heavy-ion collisions have been computed...





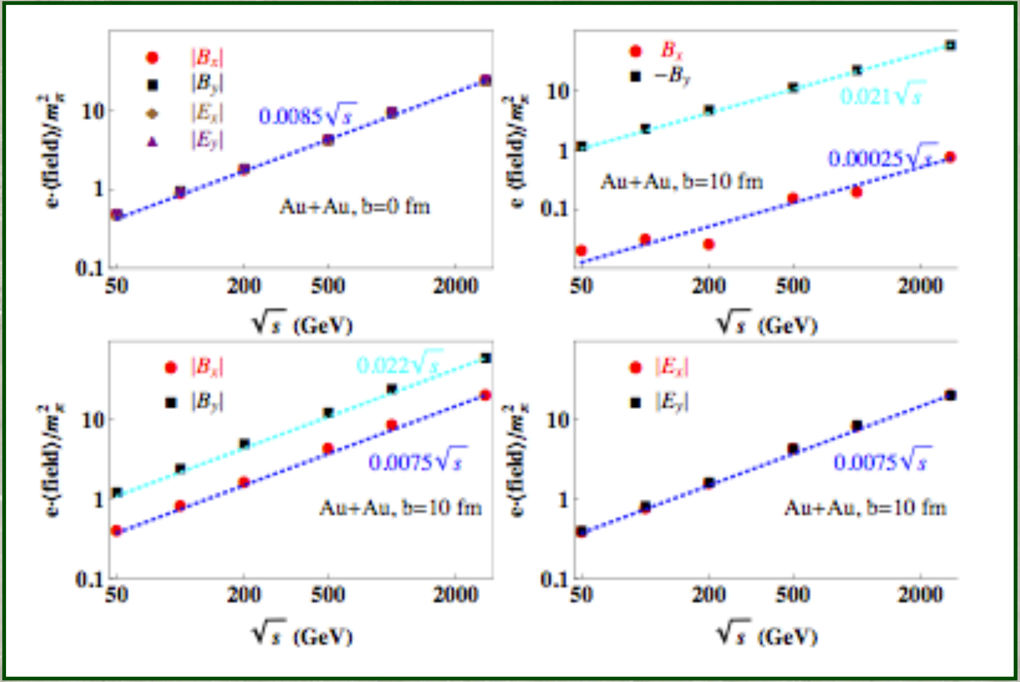
- RHIC energies, higher fields at the LHC
- Fields very flat in the central region (system may be deconfined/chiral)
- semi-analytic estimates & UrQMD agree well

[Skokov, Illariunov & Toneev (2009)]



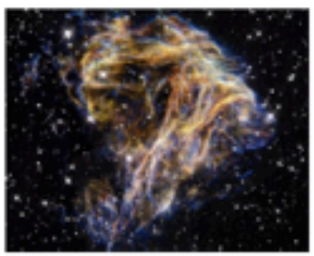
- HIJING computation
- RHIC vs LHC energies

- Huge fields for ultra-peripheral collisions due to event-by-event fluctuations
- Possible vacuum SUC via ρ meson condensation [Chernodub (2010)]
- Possible building of spin-charge correlation for quarks



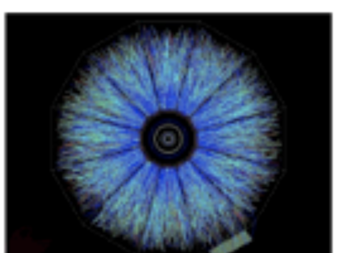
[Deng & Huang (2012)]

Comparison of magnetic fields



The Earths magnetic field	0.6 Gauss
A common, hand-held magnet	100 Gauss
The strongest steady magnetic fields achieved so far in the laboratory	4.5×10^5 Gauss
The strongest man-made fields ever achieved, if only briefly	10^7 Gauss
Typical surface, polar magnetic fields of radio pulsars	10^{13} Gauss
Surface field of Magnetars	10^{15} Gauss

http://solomon.as.utexas.edu/~duncan/magnetar.html



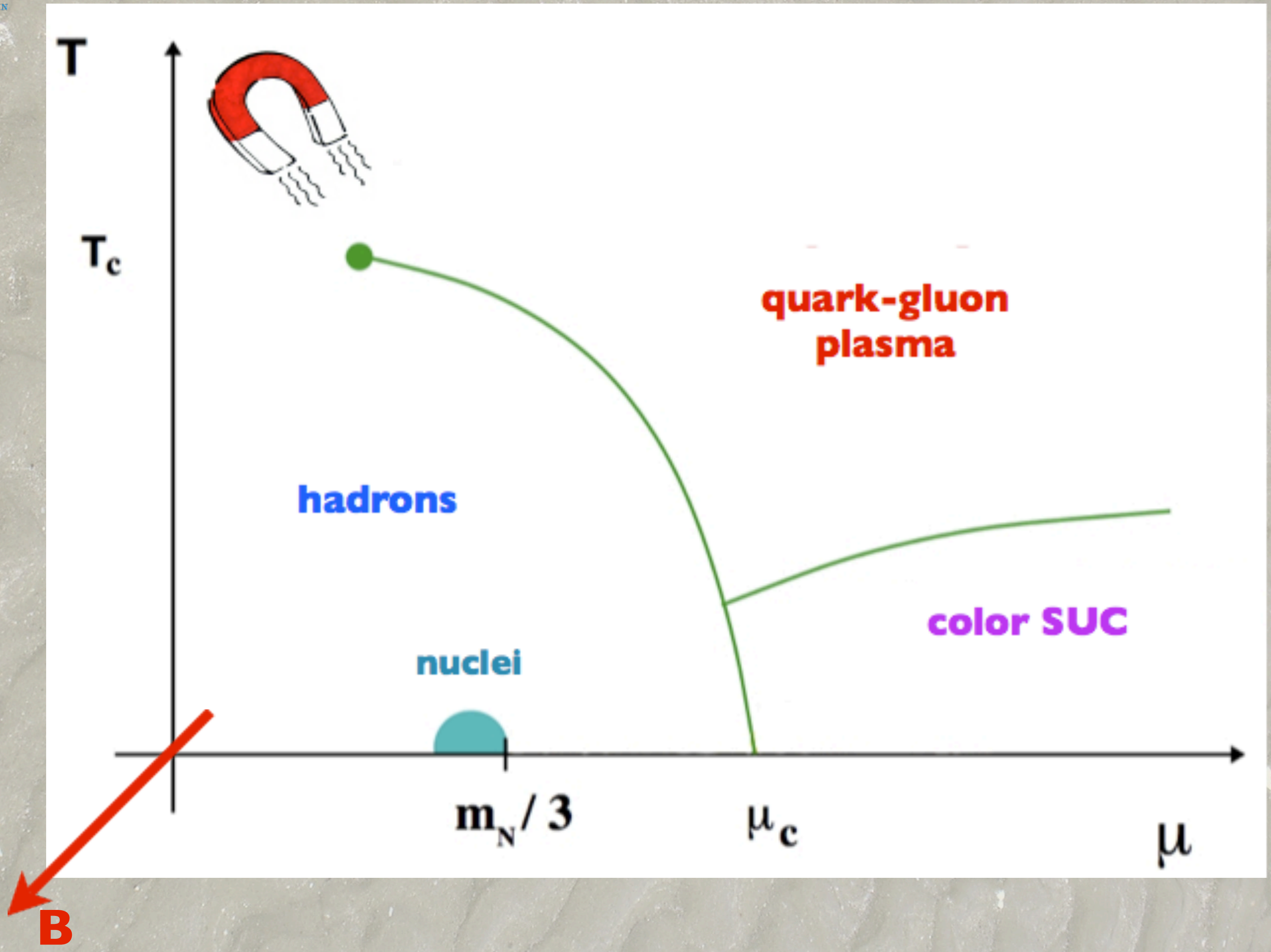
Heavy ion collisions: the strongest magnetic field ever achieved in the laboratory

Off central Gold-Gold Collisions at 100 GeV per nucleon

$$e\,B(\tau=0.2\,\mathrm{fm}) \;=\; 10^3 \sim 10^4\ \mathrm{MeV}^2 \sim 10^{17}\ \mathrm{Gauss}$$

12

Pictorially:



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Strong motivation: in-medium strong interactions under extreme magnetic fields are:

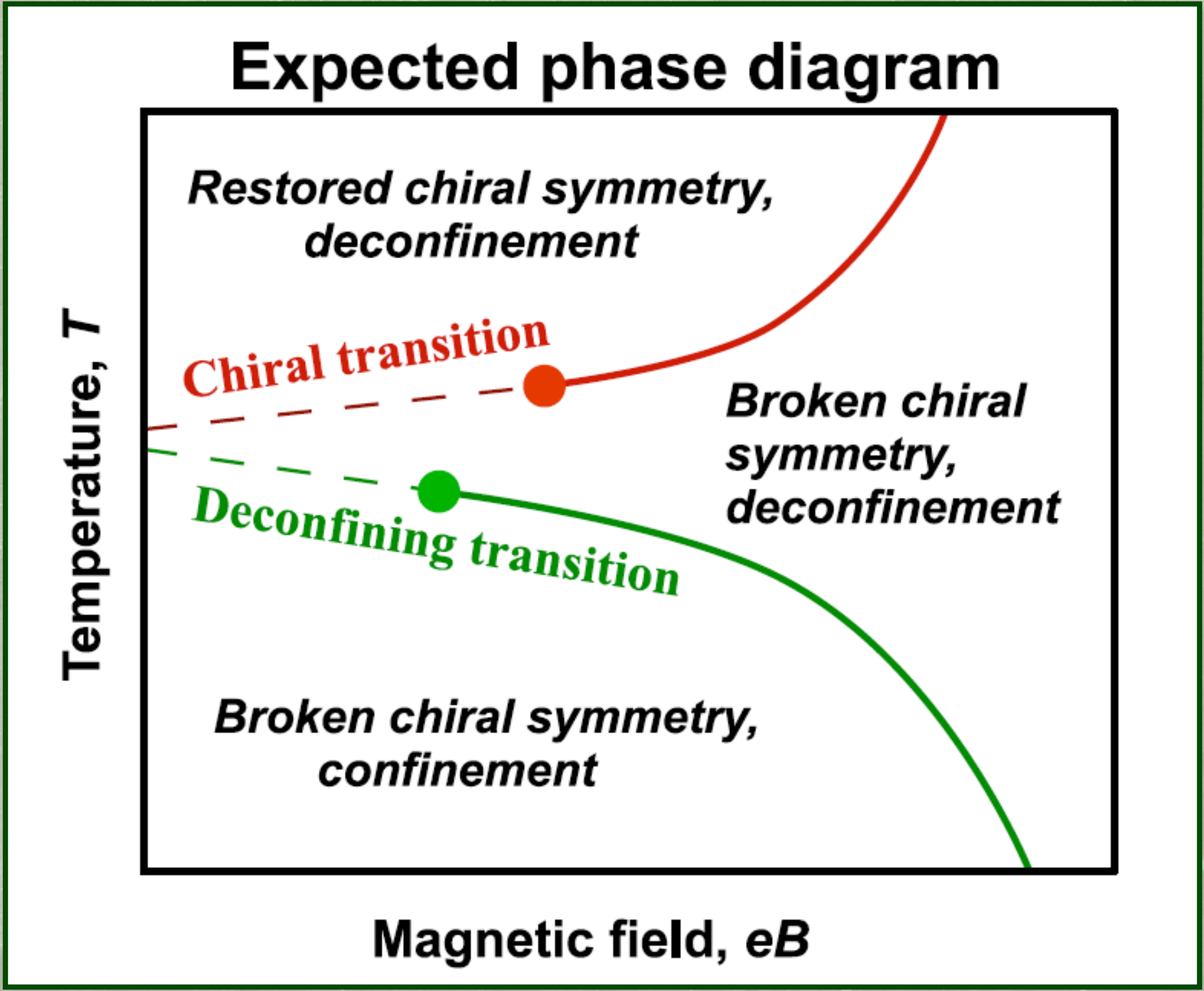
- of experimental relevance
  - HICs, early universe, magnetars
- rich in new phenomenology
  - Chiral magnetic effect, new QCD phase diagram, vacuum SUC
- amenable to lattice simulations: new open channel for comparison!
  - model constraining, tests for pQCD and nonpert. methods, ...

Outline

- Expected phase diagram and some questions
- Incorporating a magnetic background in loop integrals
- LSM coupled to the Polyakov loop & picture in 2011
- Magnetic MIT Bag model: a simple (but interesting) exercise
- Magnetic QCD in the `t Hooft limit
- pQCD in a nonperturbative magnetic background
- Final remarks

From first results, one could expect:

- Deconfining:
  Agasian & Fedorov (2008)
- Chiral:
  ESF & Mizher (2008)



[Mizher, Chernodub & ESF (2010)]

Several approaches (oldest usually concerned with vacuum properties)

NJL/LSM:

- Klevansky & Lemmer (1989)
- Gusynin, Miransky & Shovkovy (1994/1995)
- Klimenko et al. (1998-2008)
- Hiller, Osipov, ... (2007-2008)
- Rojas, Ayala, Bashir & Raya (2008)
- Boer & Boomsma (2009)
- Menezes et al (2009)
- Fukushima, Ruggieri & Gatto (2010-2011) – PNJL
- Andersen & Khan (2011)
- ...

χPT:

- Shushpanov & Smilga (1997)
- Agasian & Shushpanov (2000)
- Cohen, McGady & Werbos (2007)
- Agasian & Fedorov (2008)
- ...

Large-N QCD:

- Miransky & Shovkovy (2002)

Quark model:

- Kabat, Lee & Weinberg (2002)

FRG:

- Skokov (2011)
- Fukushima & Pawlowski (2012)
- Andersen & Tranberg (2012)

Lattice:

- D'Elia, Mukherjee & Sanfilippo (2010)
- D'Elia & Negro (2011)
- Bali et al (2011/2012)

Holographic:

- Johnson & Kundu (2008)
- Preis, Rebhan & Schmitt (2010)
- Callebaut, Dudal & Verschelde (2011)

Incorporating a magnetic background in loop integrals

Let us assume the system is in the presence of a strong magnetic field background that is constant and homogeneous:

$$\vec{B} = B\hat{z} \quad \xrightarrow{\text{choice of gauge}} \quad A^{\mu} = (A^0, \vec{A}) = (0, -By, 0, 0)$$

- charged mesons (new dispersion relation):

$$(\partial^2 + m^2)\phi = 0$$
$$\partial_{\mu} \to \partial_{\mu} + iqA_{\mu}$$

$$\longrightarrow \quad \varphi''(y) + 2m\left[\left(\frac{p_0^2 - p_z^2 - m^2}{2m}\right) - \frac{q^2B^2}{2m}\left(y + \frac{p_x}{qB}\right)^2\right]\varphi(y) = 0$$

Landau levels:

$$\varepsilon_n \equiv \left(\frac{p_{0n}^2 - p_z^2 - m^2}{2m}\right) = \left(n + \frac{1}{2}\right)\omega_B \qquad \omega_B \equiv \frac{|q|B}{m}$$

$$\Rightarrow \quad p_{0n}^2 = p_z^2 + m^2 + (2n+1)|q|B$$

- quarks (new dispersion relation):

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$
$$\partial_\mu \to \partial_\mu + iqA_\mu$$

→

$$u''(y) + 2m\left[\left(\frac{p_0^2 - p_z^2 - m^2 + qB\sigma}{2m}\right) - \frac{q^2B^2}{2m}\left(y + \frac{p_x}{qB}\right)^2\right]u(y) = 0$$

$$p_{0n}^2 = p_z^2 + m^2 + (2n + 1 - \sigma)|q|B$$

$$\sigma = \pm 1$$

- integration measure:

T = 0:

$$\int \frac{d^4k}{(2\pi)^4} \mapsto \frac{|q|B}{2\pi} \sum_{n=0}^{\infty} \int \frac{dk_0}{2\pi} \frac{dk_z}{2\pi}$$

T > 0:

$$T\sum_{\ell} \int \frac{d^3k}{(2\pi)^3} \mapsto \frac{|q|BT}{2\pi} \sum_{\ell} \sum_{n=0}^{\infty} \int \frac{dk_z}{2\pi}$$

l: Matsubara index

n: Landau level index

Linear Sigma Model coupled to Polyakov Loops

[Mizher, Chernodub & ESF (2010)]

A. Degrees of freedom and approximate order parameters

O(4) chiral field: $\phi = (\sigma, \vec{\pi})\,, \qquad \vec{\pi} = (\pi^+, \pi^0, \pi^-)$

quark spinors: $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$

Polyakov loop: $L(x) = \frac{1}{3} \mathrm{Tr}\, \Phi(x)\,, \quad \Phi = \mathcal{P} \exp\left[i \int_0^{1/T} \mathrm{d}\tau A_4(\vec{x}, \tau) \right]$

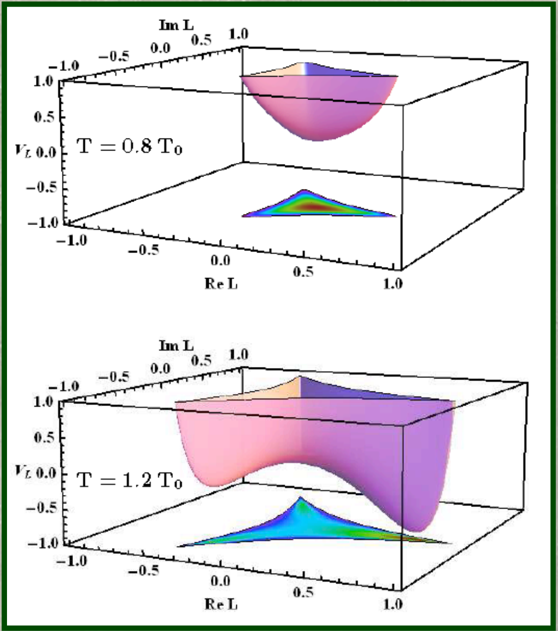
$$\text{Chiral symmetry}: \begin{cases} \langle \sigma \rangle \neq 0 \,, & \text{low } T \\ \langle \sigma \rangle = 0 \,, & \text{high } T \end{cases}$$

$$\text{Confinement}: \begin{cases} \langle L \rangle = 0 \,, & \text{low } T \\ \langle L \rangle \neq 0 \,, & \text{high } T \end{cases}$$

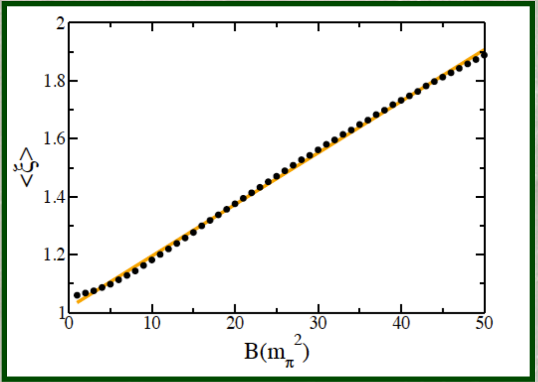
B. Summary of results

Phase diagram

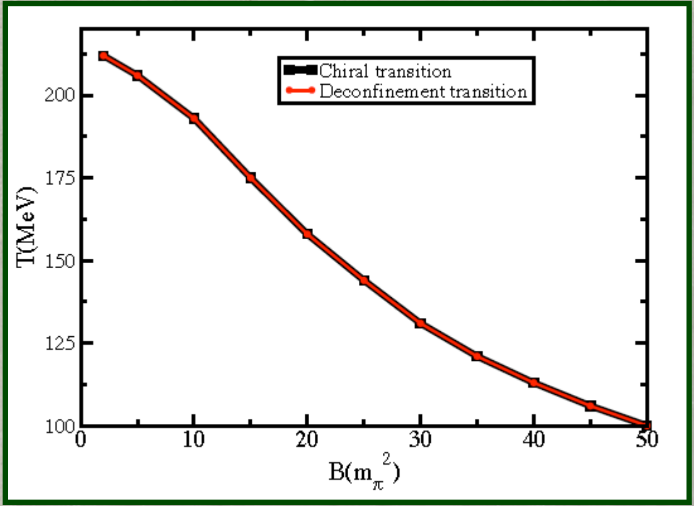
Paramagnetically-increased breaking of Z(3)



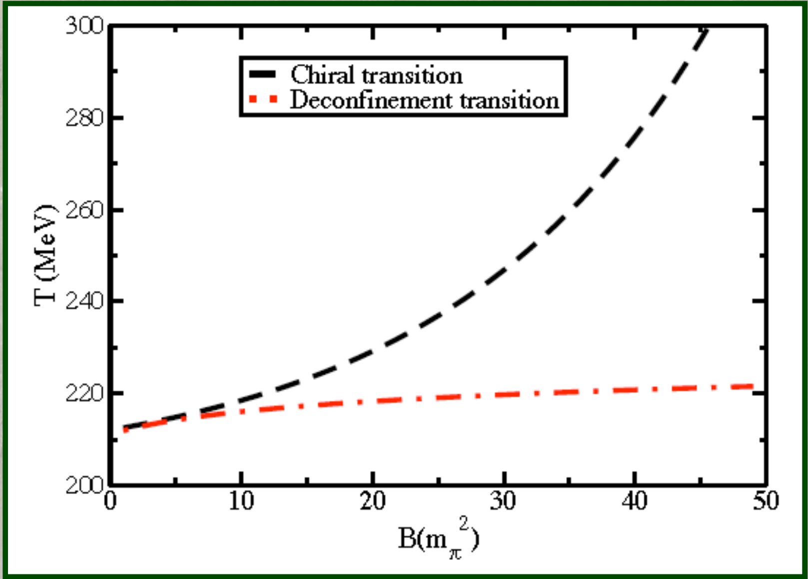
Linear chiral condensate



(without vacuum corrections)



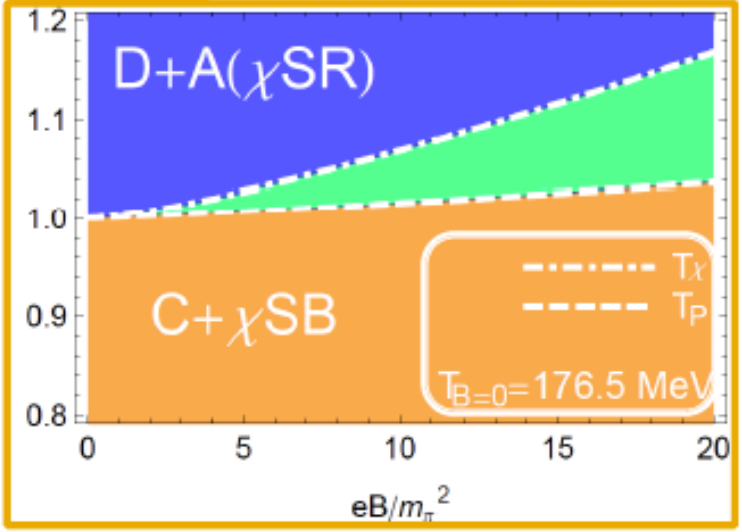
(with vacuum corrections)



Comparison with PNJL results

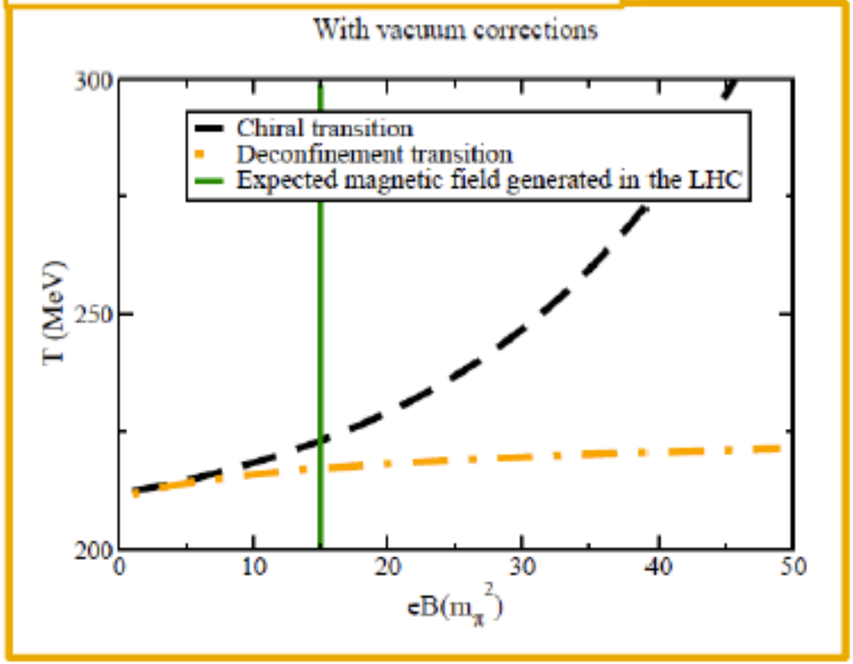
[Ruggeri & Gatto (2010)]

P-NJL



A. J. Mizher *et al*, **arXiv:1004.2712**

P-QM



Excellent agreement

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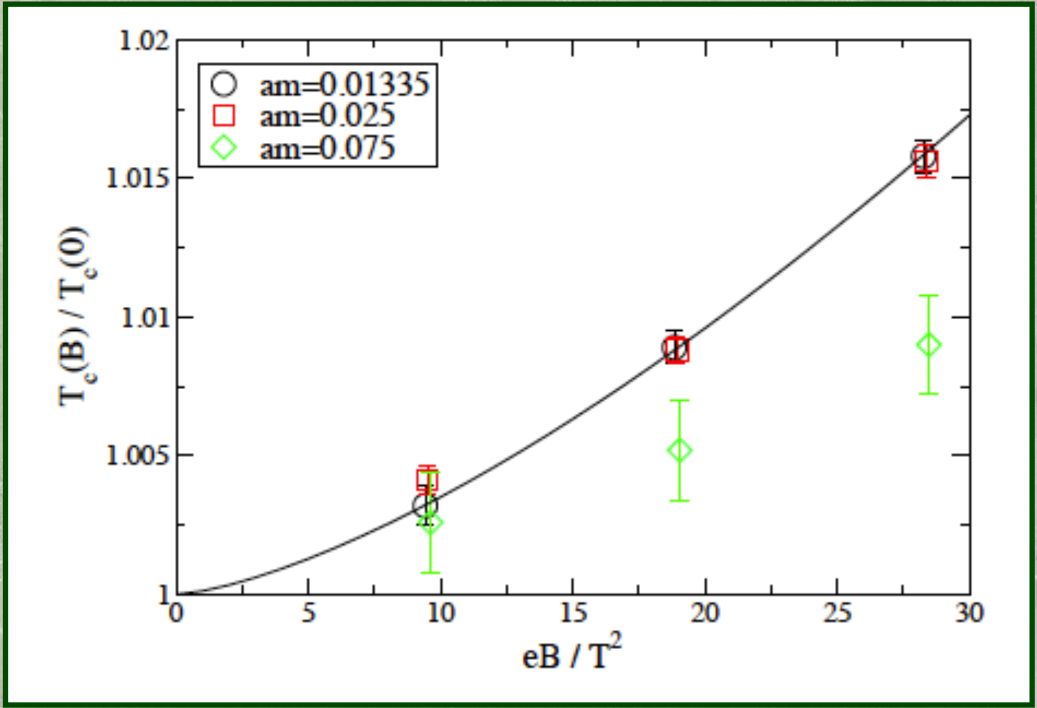
Comparison with first lattice results [N_f=2]

[D'Elia, Mukherjee & Sanfilippo (2010)]

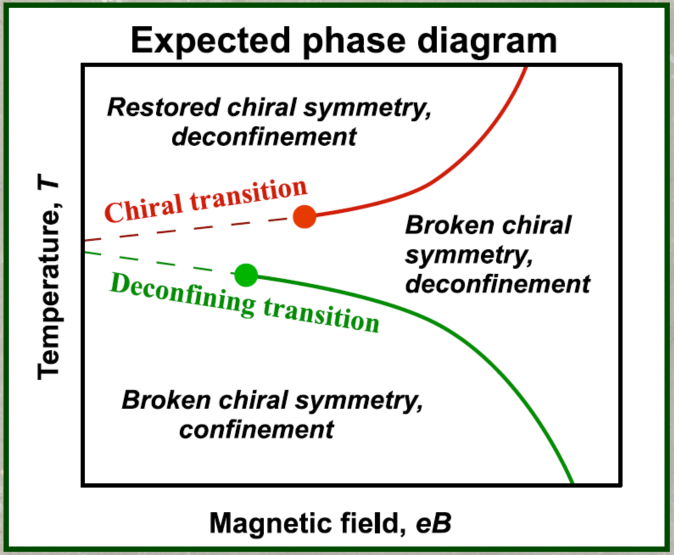


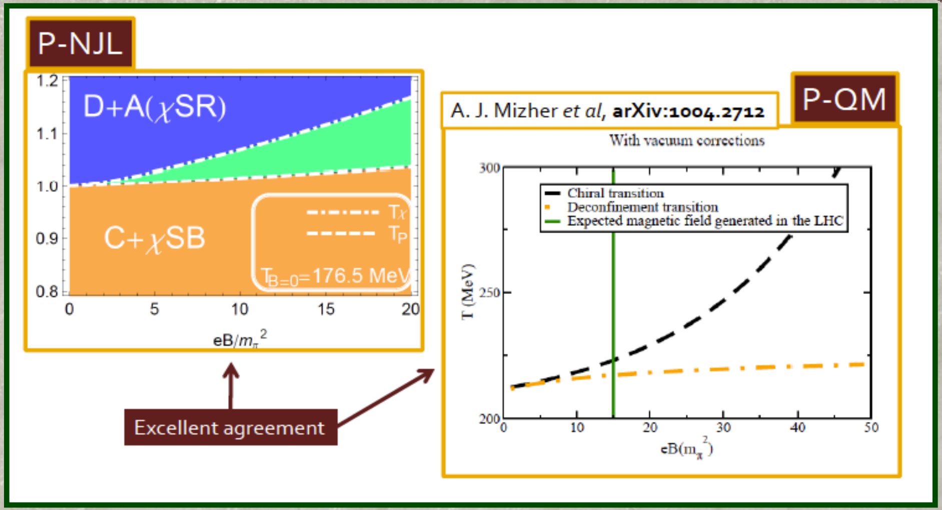
- Very small effect

« the deconfinement and chiral restoring temperatures both increase, even if we do not see any sign for a faster grow and splitting of the chiral transition till |e| B ~ 20 m_π^2 ».

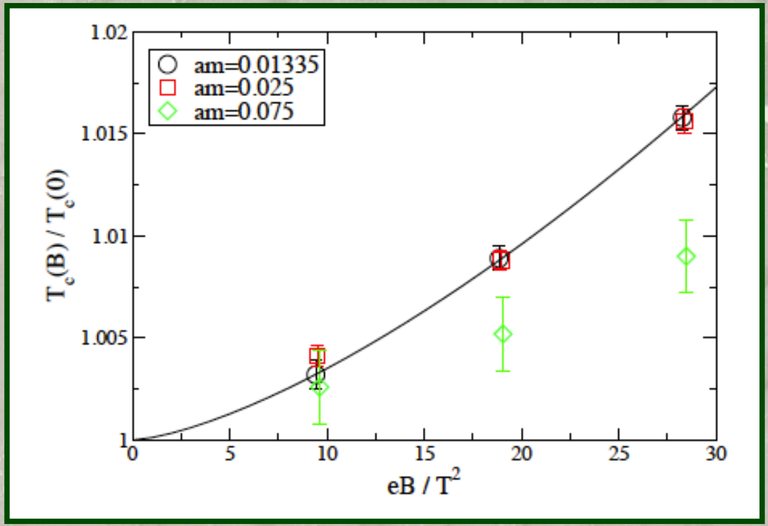
However, large pion masses: may have to go to higher B to see a splitting, if there is one.

An incomplete set of results

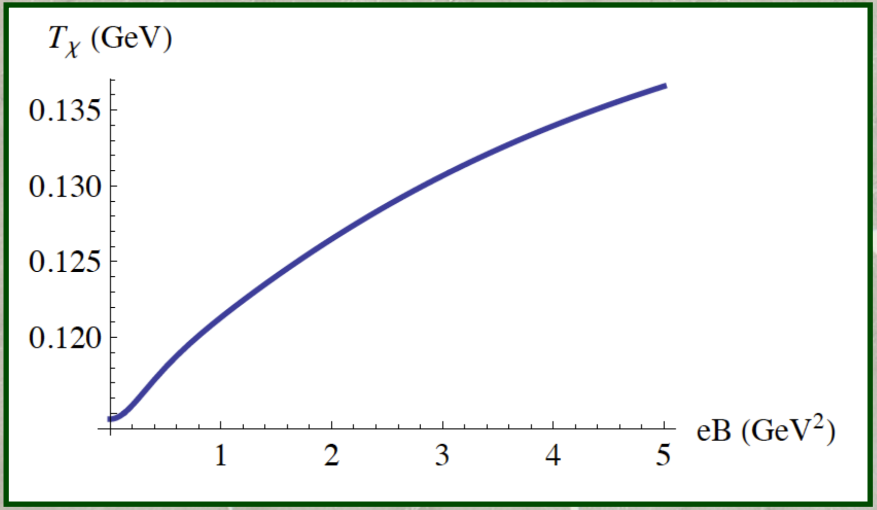




Lattice (1st results)



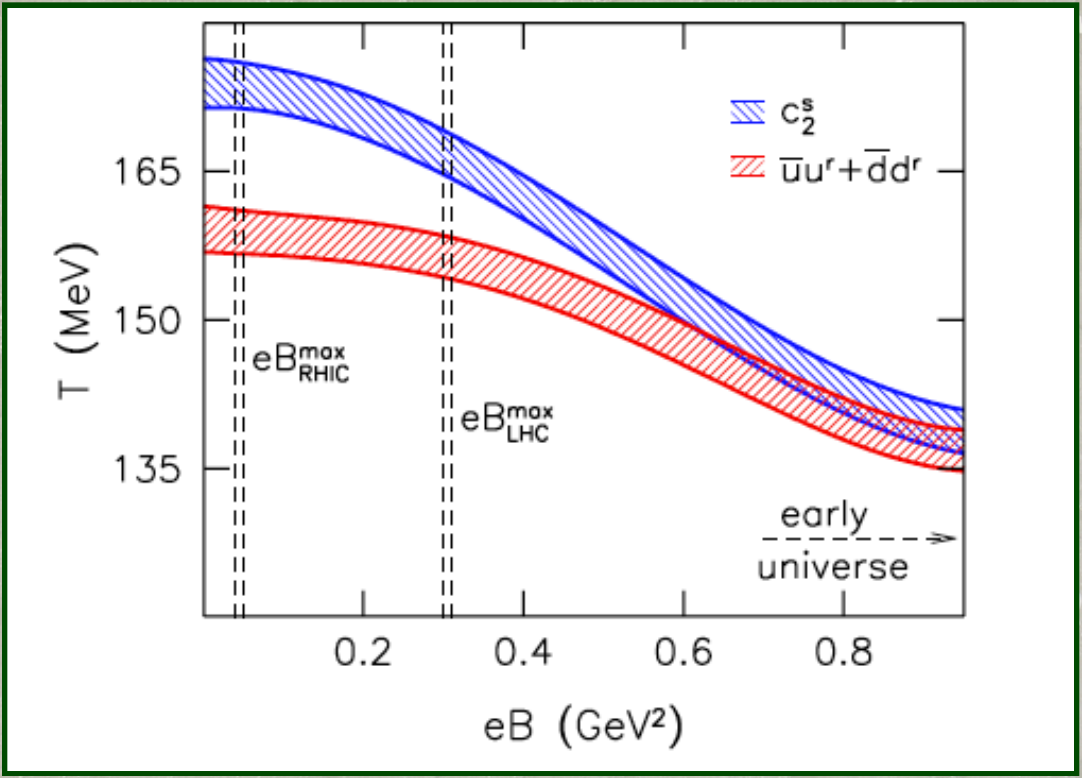
Holographic



Everything looking good and consistent in 2011, but...

... then came the newest lattice results [N_f=2+1, physical masses]:

T_c decreases and saturates!



[Bali et al (2012)]

???

Magnetic MIT Bag Model

[ESF & Palhares (2012)]

- Simplest phenomenological approach: MIT bag pressure for the QGP + pion gas for the hadronic sector

- Simple setup to discuss subtleties of vacuum & thermal contributions in each phase (no extra model complications)

- Presumably reasonable qualitative description of the behavior of $T_c(B)$

  - QGP sector:

$$P_{\rm QGP}^{B} = 2(N_c^2-1)\frac{\pi^2 T^4}{90} + P_q - \mathcal{B}$$

$$\frac{P_q}{N_c} \overset{\text{large } B}{=} \sum_f \frac{(q_f B)^2}{2\pi^2}\left[x_f \ln\sqrt{x_f}\right] + T\sum_f \frac{q_f B}{2\pi^2}\int dk_z \ln\left[1+e^{-\sqrt{k_z^2+m_f^2}/T}\right]$$

  - pion sector:

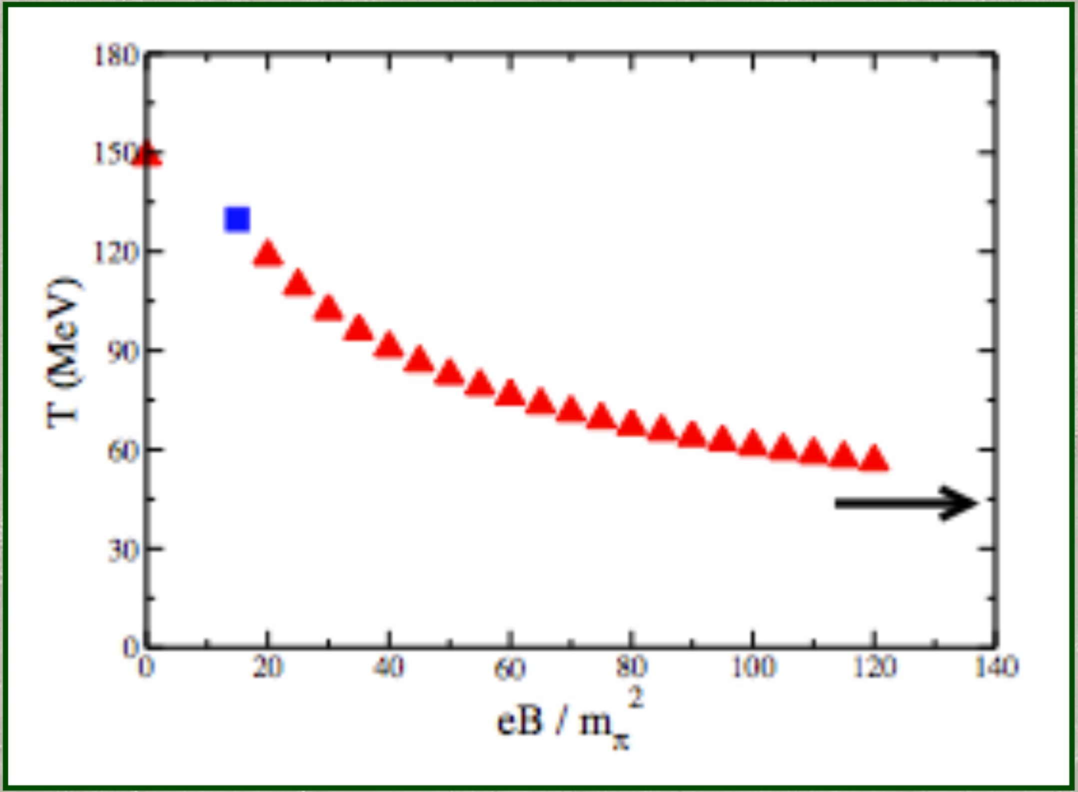
$$P_{\pi^0} = -\frac{T}{2\pi^2}\int dk k^2 \ln\left[1-e^{-\sqrt{k^2+m_\pi^2}/T}\right]$$

$$x_i \equiv \frac{m_i^2}{2q_i B}$$

$$P_{\pi^+} + P_{\pi^-} \overset{\text{large } B}{=} -\frac{(eB)^2}{4\pi^2}\zeta^{(1,1)}(-1,1/2)\; x_\pi$$

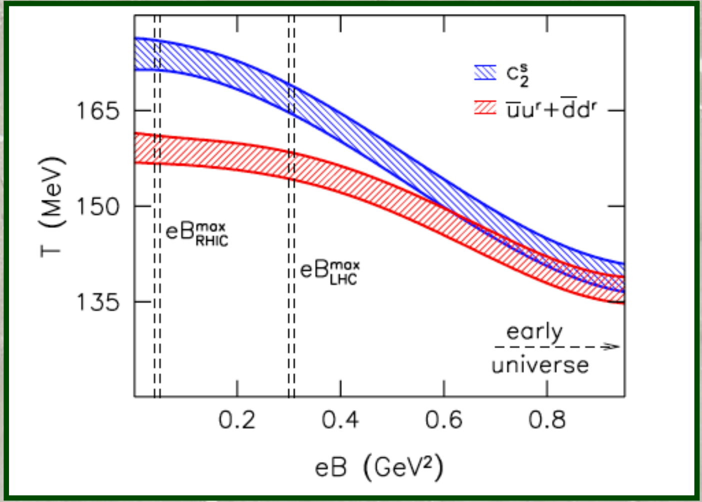
Phase diagram

[ESF & Palhares (2012)]



- Excellent qualitative agreement with recent lattice results

- Caveats (as in usual finite T): misses nature of the phase transition, quantitatively off



[Bali et al (2012)]

Q: So, what's the catch?

A: (i) Subtleties in renormalization

(ii) Is T_c a confinement-driven observable?

Effect also present in a large N_c analysis!

Subtleties of renormalization

Quark pressure after subtraction of P(T=0):

$$P_q^V = \frac{N_c}{2\pi^2}\sum_f (q_f B)^2\left[\zeta'(-1,x_f) + \frac{1}{2}(x_f - x_f^2)\ln x_f + \frac{x_f^2}{4} - \frac{1}{12}\left(2/\epsilon + \ln(\Lambda^2/2q_f B) + 1\right)\right]$$

• ***Reminiscent divergence: pure magnetic pressure spread throughout the (infinite) space.***

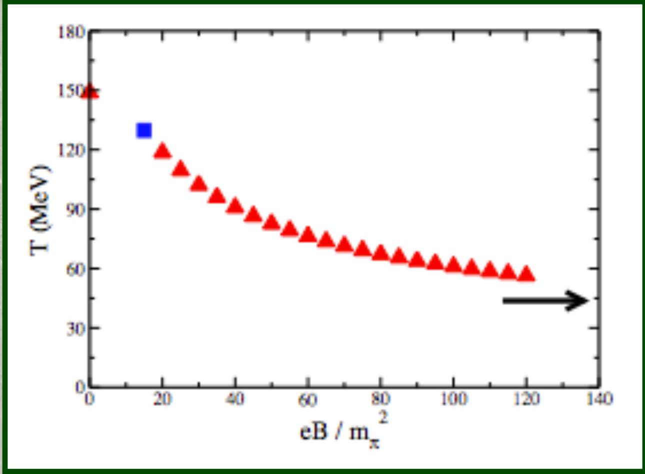
- related to wavefunction renormalization of the (classical) F_{em}^2 term.
- Modification of the permeability of vacuum, μ_0 (=1 in QFT). **['t Hooft; Nielsen (1980)]**
- T=0 terms essentially render $\mathcal{B} \mapsto \mathcal{B}_{\rm eff}(qB)$.

How to fix the finite part? Subtract all matter-independent T=0 pressure.

• For the pion case: guarantees consistency with magnetic catalysis in the vacuum.

• These terms are independent of order parameters and do not affect the effective potential.

• Not seen on the lattice: only 'measures' derivatives with respect to T (or m_q).



Is T_c a confinement-driven observable?

Magnetic QCD in the 't Hooft limit ($N_f/N_c \ll 1$)

[ESF, Noronha & Palhares (2012)]

★ Well-defined limit of QCD in which confinement properties dominate.

★ Different lattice QCD results indicate that N_c=3 is large.

$$\frac{T_c}{\sqrt{\sigma}} = 0.5949(17) + \frac{0.458(18)}{N_c^2} + O(1/N_c^4)$$

[Lucini, Rago & Rinaldi (2012)]

		$N_c \to \infty$	$O\left(\frac{N_f}{N_c}\right)$; $m_q = 0$; $B \neq 0$
Low energy:	• glueballs • (nearly free) mesons • heavy baryons	$P \sim c_0^4\, N_c^2\, \sigma^2$	Particle contributions are: $O\left(N_c^0\right)$
High energy:	• quarks and gluons • quark loops suppressed	$c_{SB} N_c^2 T^4 f_g\left(\frac{T_c}{\sqrt{\sigma}}\right)$	$+\, c_{qSB} N_f\, N_c\, f_q\left(\frac{T}{\sqrt{\sigma}}, \frac{eB}{T^2}\right)$

Obs.: all orders in $\lambda \equiv g^2 N_c$ (all planar diagrams)

★ Qualitative description of lattice results:

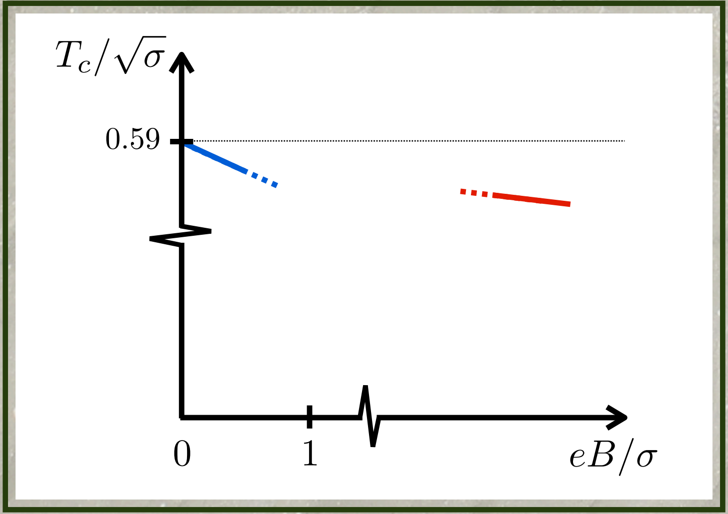
✓ $T_c/\sqrt{\sigma} = O(N_c^0)$

✓ Inclusion of $N_f \ll N_c$ light quark flavors reduces T_c.

✓ Magnetic background decreases T_c if quarks are paramagnetic.

✓ Similar reasoning with nonzero quark masses: competition m_q x B!

If one uses the result for the **free pressure of magnetically-dressed quarks**:
(probably not a good approximation around T_c, maybe for high B not so bad...)



[ESF, Noronha & Palhares (2012)]

✓ Qualitative description of lattice results

✓ Saturation of T_c for high magnetic fields

pQCD to O(g²) in a nonperturbative magnetic background

[Blaizot, ESF & Palhares (2013)]

Aims:

- Predictions within the fundamental gauge theory.
- Testing perturbation theory in the presence of a magnetic background.
- Providing results to compare with lattice data.

Framework: perturbative QCD in a nonperturbative magnetic background.

Quark-gluon interaction up to $O(g^2)$

Exact quark propagator in a constant and uniform magnetic field:

$$S_0 = \left[i\partial\!\!\!/ - q_f A\!\!\!/_{\rm cl}(x) - m_f \right]^{-1}$$

$$A_{\rm cl}(x) = (0, \vec{A}(x)) \ | \ \nabla \times \vec{A} = B\hat{z}$$

Basic ingredients

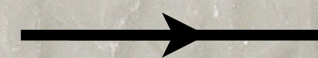
- Exact fermion propagator [Schwinger (1951); Chodos et al (1990)]

$$S_0(x,y) \;=\; \Phi(x,y)\,\overline{S}_0(x-y)$$

$$\Phi(x,y) \;\equiv\; \exp\left[iq\int_x^y dx'_\mu A^\mu_{\rm cl}(x')\right]$$

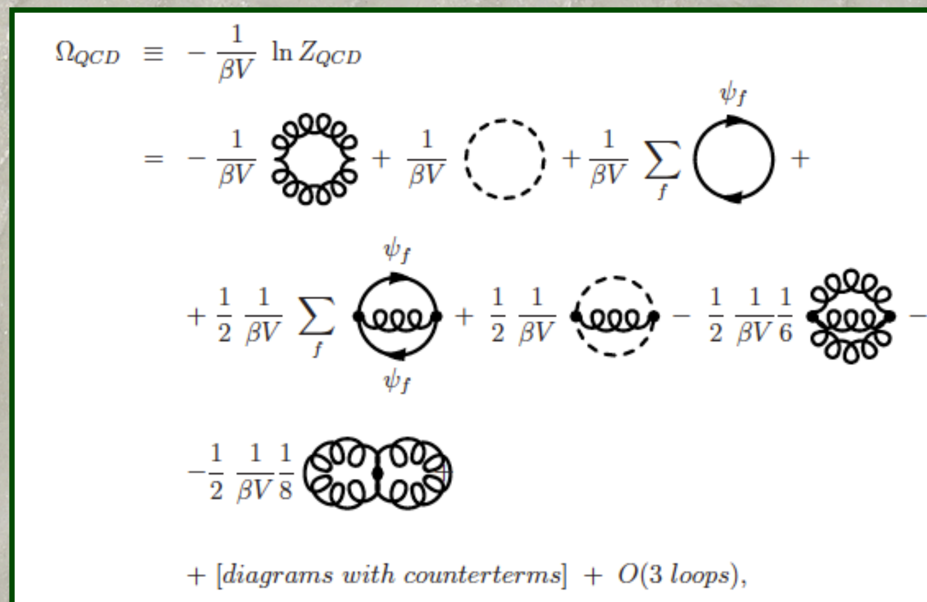
$$\overline{S}_0(P) \;=\; i\exp\left[-\frac{\mathbf{p}_T^2}{qB}\right]\sum_{n=0}^{\infty}(-1)^n\frac{D_n(qB,P)}{\mathbf{p}_L^2-m_f^2-2nqB}$$

=



- Thermodynamic potential:

(gluonic part from usual hot pQCD + magnetically-dressed quarks)



Exchange diagram in a magnetic background (in the LLL approx.):

[Palhares (2012); Blaizot, ESF & Palhares (2013)]

$$\Big[\;\Big]^{\rm LLL} = \left(\frac{q_f B}{2\pi}\right) \int \frac{dk_1 dk_2}{(2\pi)^2}\, \mathrm{e}^{-\frac{k_1^2+k_2^2}{2 q_f B}} \Big[\;\Big]^{\bar{d}=2;\; m_k^2 = k_1^2+k_2^2} .$$

Average over gluon "transverse mass"

exchange contribution in dim. 2 hot QCD with a "massive gluon"

Results:

- Clear dimensional reduction in the quark dynamics.
- There are no UV divergences.
- In D=1+1, the Dirac trace is proportional to the quark mass: trivial chiral limit!

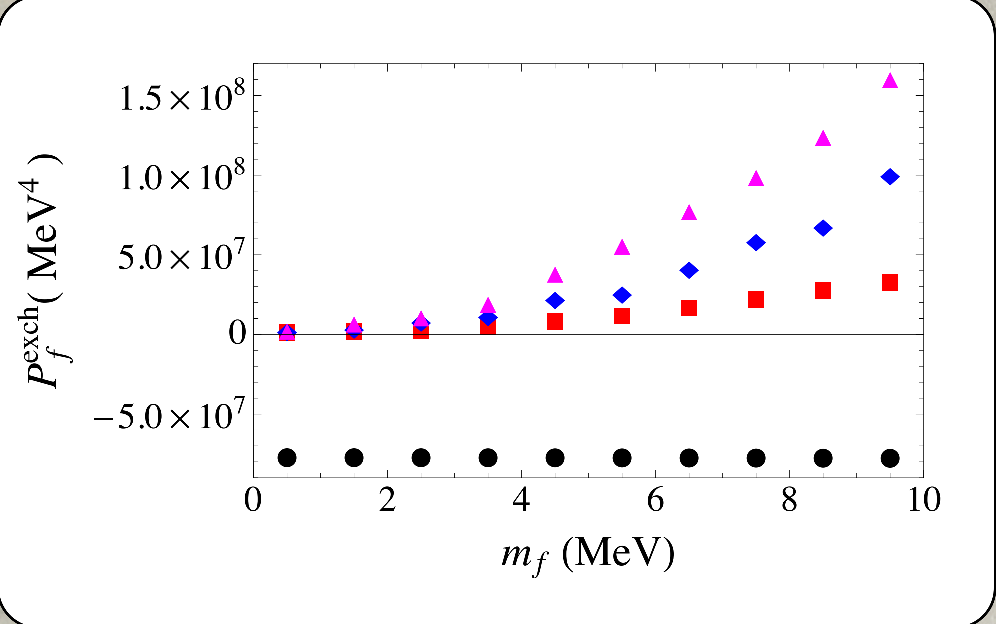
$$\gamma^\mu \gamma^\nu \gamma_\mu \overset{!}{=} -(\bar{d}-2)\gamma^\nu$$

- In D=1+1, spin and momenta are locked with the direction of the magnetic field

exchange couples different helicity/chirality states (forbidden for massless)

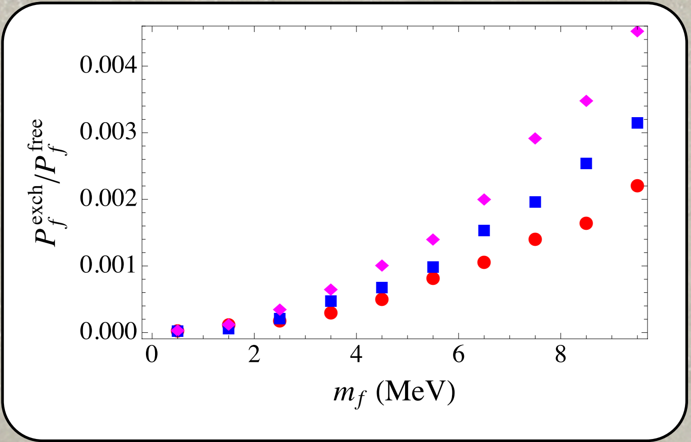
Exchange contribution to the pressure and the chiral limit

[Blaizot, ESF & Palhares (2013)]

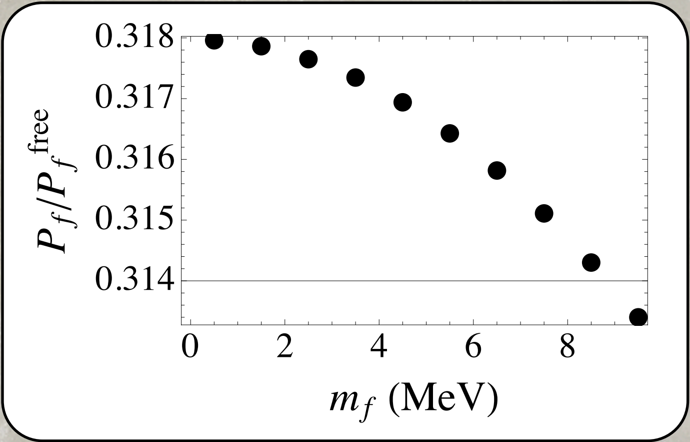


- eB = 100, 200 & 300 m_π^2
- T = 100 MeV ; α_s = 0.3 (fixed)
- Results valid for large B

- No IR divergence -> trivial chiral limit! [strong suppression for small masses]
- Qualitatively different from the B=0 case
- First QCD result: looking forward to comparison with lattice!



compare with ~70% correction at zero B



- eB = 50, 100 & 200 m_π^2
- T = 150 MeV ; α_s = 0.3 (fixed)
- Results valid for large B

- Always 1-2 orders of magnitude below the leading contribution.
  ➡ Improved convergence of the perturbative series at high T and extremely large B?

- Notice that the IR (Linde) problem in the gluon sector remains the same! (however: subdominant in B)

- One needs to go to higher orders...

A simple semiclassical picture:

• Large B shrinks the Landau orbits (since their radii go as $r_c^2 \sim 1/eB$).

• So, the orbital motion of quarks in the plane transverse to B becomes more and more constrained -> original helicoidal (tubular-like) paths become essentially straight lines parallel to the field direction.

• Of course, the entire motion is perturbed and partially randomized by the heat bath: tubes become "blurred" (noisy), as well as the straight lines. However, for $eB \gg T^2$, this effect will be minor -> steady flow with almost no scattering and no contribution to the pressure.

Final remarks

• Magnetic fields open new possibilities in the study of the phase diagram of strong interactions & in-medium pQCD.

• Lattice results show that T_c goes down.

- Not captured by PQM, PNJL, etc
- The qualitative success of the magbag description relies on [ESF & Palhares (2012)]

  ✳ full subtraction of purely B-dependent (matter-independent) contributions to the pressure.

  ✳ focus on confinement (T_c as a confinement-driven observable).

• Large N_c goes in the same direction, and seems to reinforce the role played by confinement. T_c goes down and B competes with m_q.

• Magnetic pQCD [Blaizot, ESF & Palhares (2013)]:

- first-principle pressure in pQCD to $O(g^2)$ shows trivial chiral limit.
- Maybe classical behavior induced by large B?
- Higher-order computations & direct comparison with lattice called for !!

• Quark mass effects should be studied in more detail in models & on the lattice !