Francesco Becattini,
University of Florence and FIAS Franfkurt

Relativistic distribution function of particles with spin at local thermodynamical equilibrium

(Cooper-Frye with spin)

F. B., V. Chandra, L. Del Zanna, E. Grossi, arXiv:1303.3431

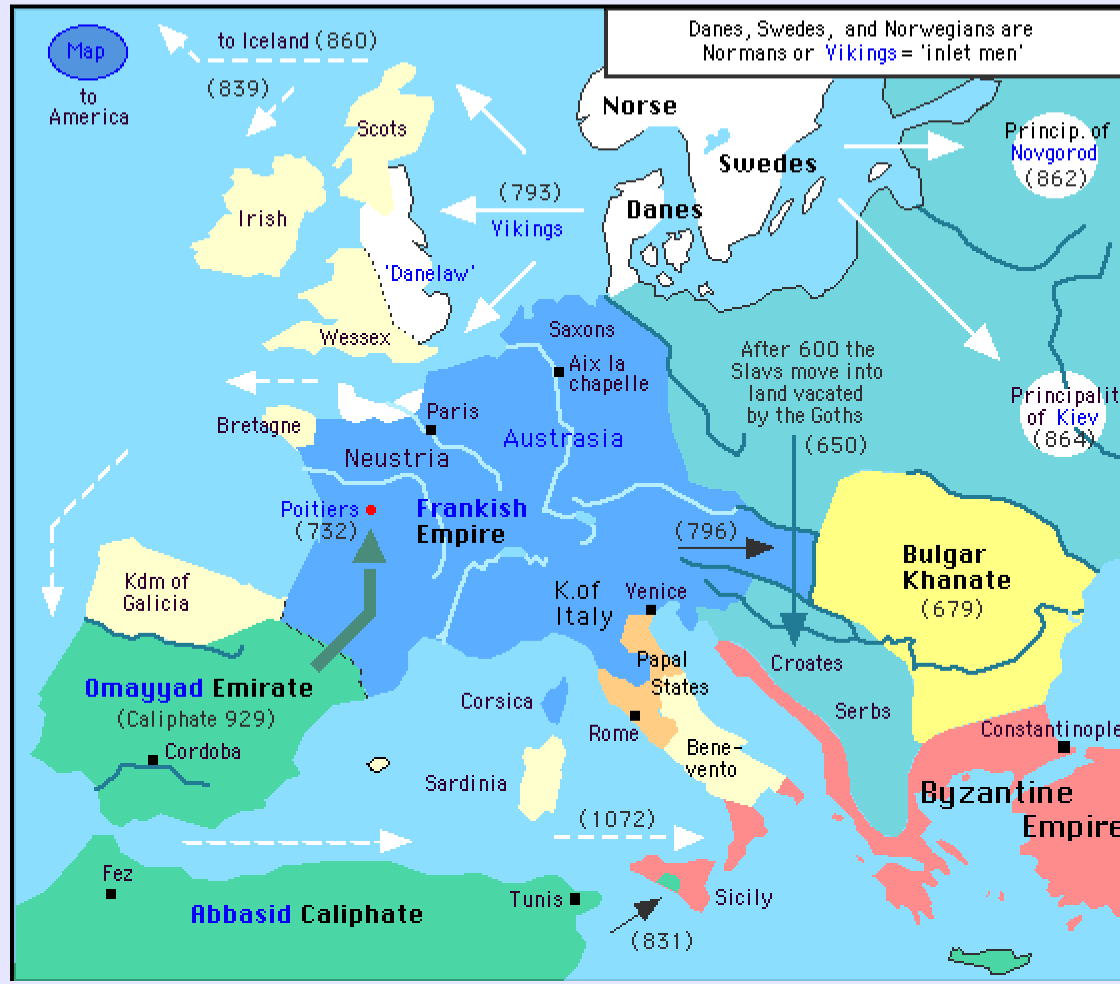
OUTLINE

- Intoduction and formalism
- Global thermodynamical equilibrium with rotation
- Local thermodynamical equilibrium
- Polarization of spin ½ fermions in a "thermally vorticous" flow
- Λ polarization in relativistic heavy ion collisions
- Conclusions

Personal digression (historical)

*Fran*cesco and *Fran*kfurt: common root?

Franks: the founders of the
Holy Roman Empire around 800 AD



France and *Frankfurt* are named after Franks as everybody knows...

Four centuries later...

Giovanni di Pietro di Bernardone
Also known as St. Francis (Assisi 1182 - La Verna 1226)

His father had a successful cloth trading business with southern France, where he used to go, and nicknamed his son *Francesco* (in old central italian = *frenchman*). Thereafter, he adopted it as his first name.

This has been the first occurrence of a person named Francesco (along with translations Francis, Franziskus, Francisco, Francois etc.)

Introduction

The single particle distribution function at local thermodynamical equilibrium (known as Juttner distribution) reads (spinless bosons):

$$f(x,p) = \frac{1}{e^{\beta(x)\cdot p - \xi(x)} - 1}$$

$$\beta^\mu = \frac{1}{T_0} u^\mu \qquad \xi = \mu_0 / T_0$$

In HIC often used, e.g., in the so-called Cooper-Frye formula:

$$\varepsilon \frac{dN}{d^3p} = \int_\Sigma d\Sigma_\mu p^\mu f(x,p)$$

QUESTION: *What happens if particles have a spin?*

Answering this question urges us to review several of the "familiar" concepts of statistical mechanics and hydrodynamics. Quantum features cannot be neglected.

What is the distribution function?

Cannot say "the density of particles in phase space" because it does not take into account polarization degrees of freedom.
The answer can be found in the book: S.R. De Groot et al. *Relativistic kinetic theory*

Covariant Wigner function: scalar field

$$\langle \ \rangle = \mathrm{tr}(\widehat{\rho} \ )$$

$$W(x,k) = \frac{1}{(2\pi)^4} \int \mathrm{d}^4 y \, 2\, \mathrm{e}^{-ik\cdot y} \langle : \widehat{\psi}^\dagger(x+y/2)\,\widehat{\psi}(x-y/2) : \rangle$$

For quasi-free theory, neglecting Compton-wavelength scale variations

$$W^+(x,k) \equiv \theta(k^0) W(x,k) = \int \frac{\mathrm{d}^3\mathrm{p}}{\varepsilon}\, \delta^4(k-p) f(x,p)$$

$$W^-(x,k) \equiv \theta(-k^0) W(x,k) = \int \frac{\mathrm{d}^3\mathrm{p}}{\varepsilon}\, \delta^4(k+p) \bar{f}(x,p)$$

which *define* the distribution functions of particles and antiparticles

For a free field, previous equations lead to:

$$f(x,p) = \frac{1}{2(2\pi)^3} \int \mathrm{d}^4u\ \delta(u\cdot p)\, \mathrm{e}^{-iu\cdot x} \langle a^\dagger_{p-u/2} a_{p+u/2} \rangle$$

$$\bar{f}(x,p) = \frac{1}{2(2\pi)^3} \int \mathrm{d}^4u\ \delta(u\cdot p)\, \mathrm{e}^{-iu\cdot x} \langle b^\dagger_{p-u/2} b_{p+u/2} \rangle$$

thus:

$$\int \mathrm{d}^3\mathrm{x}\ f(x,p) = \frac{1}{2\varepsilon} \langle a^\dagger_p a_p \rangle = \frac{\mathrm{d}N}{\mathrm{d}^3p} \qquad \int \mathrm{d}^3x\ \bar{f}(x,p) = \frac{1}{2\varepsilon} \langle b^\dagger_p b_p \rangle = \frac{\mathrm{d}\bar{N}}{\mathrm{d}^3p}$$

Wigner function of the free Dirac field

$$W(x,k)_{AB} = -\frac{1}{(2\pi)^4} \int \mathrm{d}^4 y \, \mathrm{e}^{-ik\cdot y} \langle : \Psi_A(x - y/2) \overline{\Psi}_B(x + y/2) : \rangle$$
$$= \frac{1}{(2\pi)^4} \int \mathrm{d}^4 y \, \mathrm{e}^{-ik\cdot y} \langle : \overline{\Psi}_B(x + y/2) \Psi_A(x - y/2) : \rangle$$

$$W^+(x,k) \equiv \theta(k^0) W(x,k) = \frac{1}{2} \int \frac{\mathrm{d}^3 \mathrm{p}}{\varepsilon} \, \delta^4(k-p) \sum_{r,s} u_r(p) f_{rs}(x,p) \bar{u}_s(p)$$

$$W^-(x,k) \equiv \theta(-k^0) W(x,k) = -\frac{1}{2} \int \frac{\mathrm{d}^3 \mathrm{p}}{\varepsilon} \, \delta^4(k+p) \sum_{r,s} v_s(p) \bar{f}_{rs}(x,p) \bar{v}_r(p)$$

The *u*, *v* spinors are the usual solution of the free Dirac equation, with all of their well known properties (orthogonality and completeness).

Thus, the distribution function for spin ½ particles is a 2x2 matrix

Densities of conserved quantities

Preliminary

$$\int \mathrm{d}^3\mathrm{x}\; f_{rr}(x,p) = \frac{1}{2\varepsilon}\langle a^\dagger_{p,r} a_{p,r}\rangle = \frac{\mathrm{d}N_r}{\mathrm{d}^3 p} \qquad \int \mathrm{d}^3 x\; \bar{f}_{rr}(x,p) = \frac{1}{2\varepsilon}\langle b^\dagger_{p,r} b_{p,r}\rangle = \frac{\mathrm{d}\bar{N}_r}{\mathrm{d}^3 p}$$

$$\int \mathrm{d}^3\mathrm{x}\; \mathrm{tr}_2 f(x,p) = \int \mathrm{d}^3\mathrm{x} \sum_{r=1}^{2} f_{rr}(x,p) = \frac{\mathrm{d}N}{\mathrm{d}^3 p}$$

Stress-energy tensor

$$T^{\mu\nu}(x) \equiv \frac{i}{2}\langle :\overline{\Psi}(x)\gamma^\mu \overleftrightarrow{\partial}^\nu \Psi(x): \rangle = \int \frac{\mathrm{d}^3\mathrm{p}}{\varepsilon} p^\mu p^\nu \left(\mathrm{tr}_2 f(x,p) + \mathrm{tr}_2 \bar{f}(x,p)\right)$$

Current

$$j^\mu(x) \equiv \langle :\overline{\Psi}(x)\gamma^\mu \Psi(x): \rangle = \int \frac{\mathrm{d}^3\mathrm{p}}{\varepsilon} p^\mu \left(\mathrm{tr}_2 f(x,p) - \mathrm{tr}_2 \bar{f}(x,p)\right)$$

Spin tensor

$$\mathcal{S}^{\lambda,\mu\nu}(x) \equiv \frac{1}{2}\langle : \overline{\Psi}(x)\{\gamma^\lambda, \Sigma^{\mu\nu}\}\Psi(x) : \rangle = \frac{1}{2}\int \frac{\mathrm{d}^3\mathrm{p}}{2\varepsilon}\mathrm{tr}_2\left(f(x,p)\bar{U}(p)\{\gamma^\lambda, \Sigma^{\mu\nu}\}U(p)\right) - \mathrm{tr}_2\left(\bar{f}^T(x,p)\bar{V}(p)\{\gamma^\lambda, \Sigma^{\mu\nu}\}V(p)\right)$$

We have introduced here a compact spinorial notation, with *U* and *V* being 4x2 matrices
In the Weyl representation:

$$U(p) = \sqrt{m}\begin{pmatrix} D^S([p]) \\ D^S([p]^{\dagger -1}) \end{pmatrix} \qquad V(p) = \sqrt{m}\begin{pmatrix} D^S([p]C^{-1}) \\ D^S([p]^{\dagger -1}C) \end{pmatrix}$$

D^S	Representation (2S+1)-dimensional of SL(2,C), of the kind (0,S)
$[p] \in SL(2,C)$	"Standard" transformation taking (1,***0***) into *p/m*
$C = i\sigma_2$	

What is the form of the distribution function matrix *f* at local thermodynamical equilibrium?

Global thermodynamical equilibrium with rotation

Density operator (see e.g. Landau, *Statistical physics*; A. Vilenkin, Phys. Rev. D 21 2260)

$$\widehat{\rho} = \frac{1}{Z_\omega} \exp\left[-\widehat{H}/T + \boldsymbol{\omega} \cdot \widehat{\mathbf{J}}/T + \mu/T \right]$$

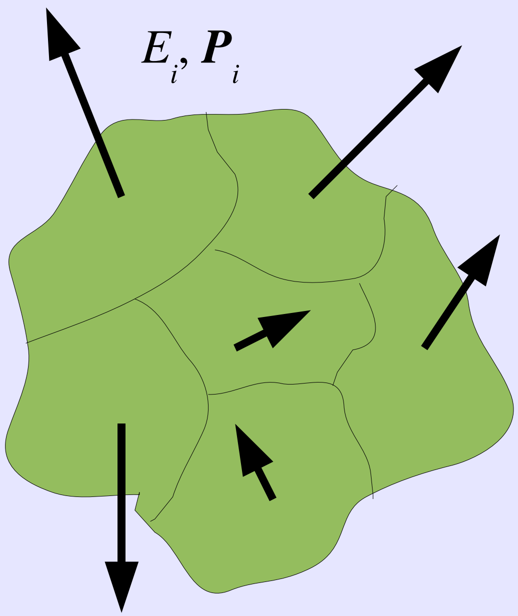
Grand-canonical rotational partition function

Obtained by maximizing the entropy $S = -\mathrm{tr}(\widehat{\rho} \log \widehat{\rho})$ with respect to $\widehat{\rho}$ with the constraints of total mean energy, mean momentum and mean angular momentum Fixed (equivalent to exact conservation for a *large* system)

ω/T is the Lagrange multiplier of the angular momentum conservation constraint and its physical meaning is that of an *angular velocity*

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{x}$$

Classical (=non-quantum) Landau's argument

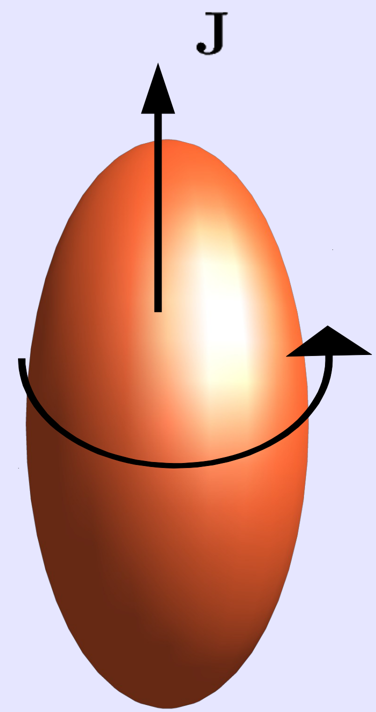


$$S = \sum_i S_i(\sqrt{E_i^2 - \mathbf{P}_i^2}) \qquad \frac{\partial S_i}{\partial E_i} = \frac{E_i}{M_i}\frac{\partial S_i}{\partial M_i} = \frac{\gamma_i}{T_i}$$

Maximize entropy with constraints

$$\sum_i S_i - \frac{\boldsymbol{\beta}}{T}\cdot\sum_i \mathbf{P}_i - \frac{1}{T}(\sum_i E_i - E_0) - \frac{\boldsymbol{\omega}}{T}\cdot(\sum_i \mathbf{x}_i \times \mathbf{P}_i - \mathbf{J})$$

$$\frac{\gamma_i}{T_i} = \frac{1}{T} \quad \forall i \qquad \mathbf{v}_i = \boldsymbol{\omega} \times \mathbf{x}_i \quad \forall i$$



Local temperature

$$T_i = \frac{T}{\sqrt{1 - (\boldsymbol{\omega} \times \mathbf{x}_i)^2}}$$

Single particle distribution function at global thermodynamical equilibrium

In the Boltzmann limit, for an ideal relativistic gas, this is a calculation which can be done without the explicit use of quantum field theory, just with quantum statistical mechanics and group theory (F. B., L. Tinti, Ann. Phys. 325, 1566 (2010)).

More explicitely: maximal entropy (equipartition), angular momentum conservation and Lorentz group representation theory.

$$f(x,p)_{rs} = \mathrm{e}^{\xi}\, \mathrm{e}^{-\beta\cdot p}\ \frac{1}{2}\left(D^S([p]^{-1}\mathsf{R}_{\hat\omega}(i\omega/T)[p]) + D^S([p]^{\dagger}\mathsf{R}_{\hat\omega}(i\omega/T)[p]^{\dagger-1})\right)_{rs}$$

$\mathsf{R}_{\hat\omega}(i\omega/T) = \exp[D^S(J_3)\omega/T]$ = SL(2,C) matrix representing a rotation around $\boldsymbol{\omega}$ axis (z or 3) by an imaginary angle $i\omega/T$.

$$\mathrm{tr}_{2S+1} f = \mathrm{e}^{\xi}\, \mathrm{e}^{-\beta\cdot p}\mathrm{tr}_{2S+1}\mathsf{R}_{\hat\omega}(i\omega/T) = \mathrm{e}^{\xi}\, \mathrm{e}^{-\beta\cdot p}\sum_{\sigma=-S}^{S}\mathrm{e}^{-\sigma\omega/T} \equiv \mathrm{e}^{\xi}\, \mathrm{e}^{-\beta\cdot p}\chi\left(\frac{\omega}{T}\right)$$

As a consequence, particles with spin get polarized in a rotating gas

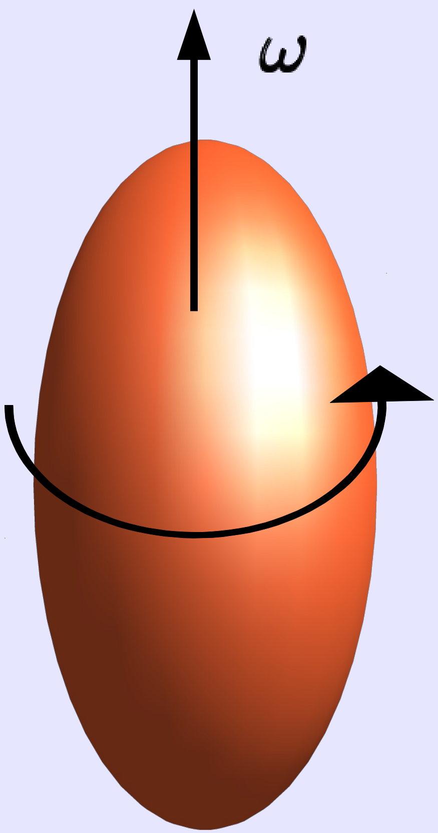
$$\mathbf{\Pi}_0 = \frac{\sum_{n=-S}^{S} n \mathrm{e}^{n\omega/T}}{\sum_{n=-S}^{S}\mathrm{e}^{n\omega/T}}\left[\frac{\varepsilon}{m}\hat{\boldsymbol{\omega}} - \frac{\hat{\boldsymbol{\omega}}\cdot\mathbf{p}\mathbf{p}}{m(\varepsilon+m)}\right]$$

F.B., F. Piccinini, Ann. Phys. 323, 2452 (2008)

Barnett effect

S. J. Barnett, Magnetization by Rotation, Phys. Rev.. 6, 239–270 (1915).

Spontaneous magnetization of an uncharged body when spun around its axis, in quantitative agreement with the previous polarization formula



$$M = \frac{\chi}{g}\omega$$

It is a dissipative transformation of the orbital angular momentum into spin of the constituents. The angular velocity decreases and a small magnetic field appears; this phenomenon is accompanied by a heating of the sample.

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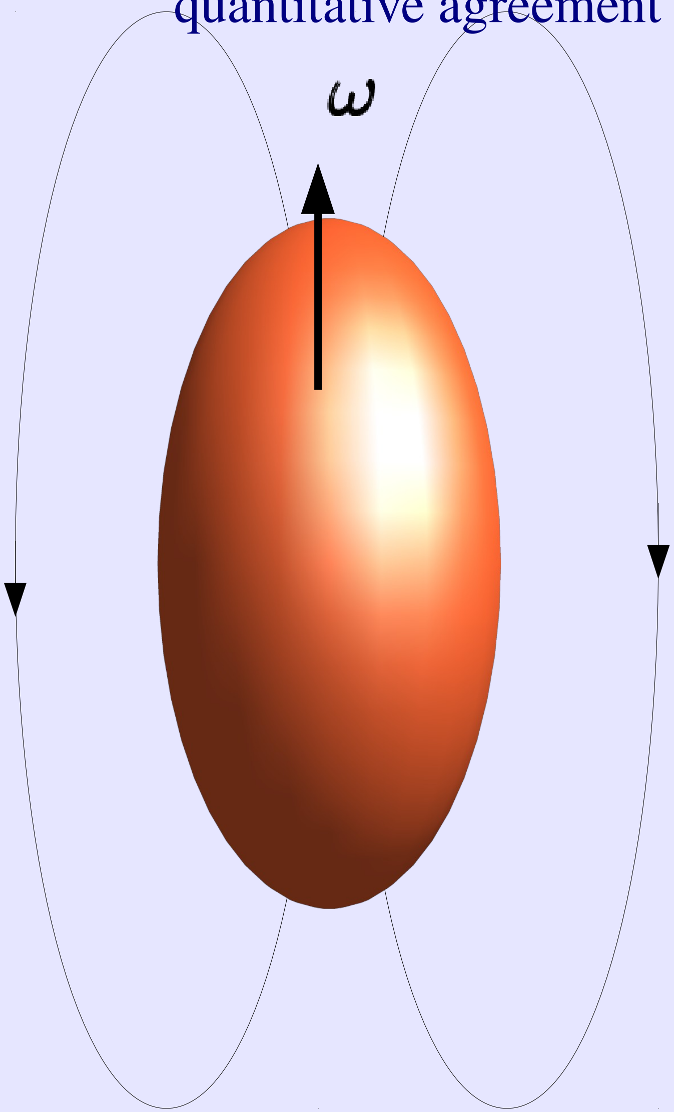


$$M = \frac{\chi}{g}\omega$$

It is a dissipative transformation of the orbital angular momentum into spin of the constituents. The angular velocity decreases and a small magnetic field appears; this phenomenon is accompanied by a heating of the sample. Requires a spin-orbit coupling.

Converse: Einstein-De Haas effect

the only experiment by Einstein

A. Einstein, W. J. de Haas, Koninklijke Akademie van Wetenschappen te Amsterdam, Proceedings, 18 I, 696-711 (1915)

Rotation of a ferromagnet originally at rest when put into an external H field

An effect of angular momentum conservation:
spins get aligned with H (irreversibly) and this must be compensated by a on overall orbital angular momentum

Dirac-ization of f

For the case S=1/2 the formulae can be rewritten using Dirac spinors

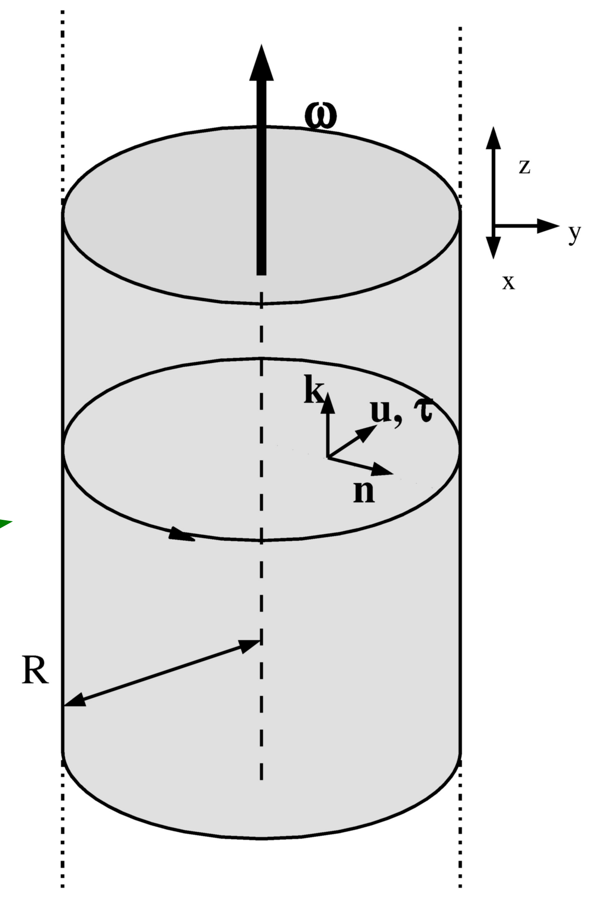
$$f(x,p) = \mathrm{e}^{\xi}\,\mathrm{e}^{-\beta\cdot p}\ \frac{1}{2m}\bar{U}(p)\exp[(\omega/T)\,\Sigma_z]U(p) \qquad \Sigma_z = \frac{1}{2}\begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$$

$$\bar{f}(x,p) = -\mathrm{e}^{-\xi}\,\mathrm{e}^{-\beta\cdot p}\ \frac{1}{2m}\left[\bar{V}(p)\exp[-(\omega/T)\,\Sigma_z]V(p)\right]^T$$

They can be also rewritten in a fully covariant form taking into account that

$$\varpi_{\mu\nu} = (\omega/T)(\delta^1_\mu\delta^2_\nu - \delta^1_\nu\delta^2_\mu) = \sqrt{\beta^2}\,\Omega_{\mu\nu}$$

Ω being the acceleration tensor of the Frenet-Serret tetrad of the velocity field lines



and the generators of the Lorentz group representation

$$\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu,\gamma^\nu]$$

$$f(x,p) = \mathrm{e}^{\xi}\,\mathrm{e}^{-\beta\cdot p}\ \frac{1}{2m}\bar{U}(p)\exp\left[\frac{1}{2}\varpi^{\mu\nu}\Sigma_{\mu\nu}\right]U(p) \qquad \bar{f}(x,p) = -\mathrm{e}^{-\xi}\,\mathrm{e}^{-\beta\cdot p}\ \frac{1}{2m}\left[\bar{V}(p)\exp\left[-\frac{1}{2}\varpi^{\mu\nu}\Sigma_{\mu\nu}\right]V(p)\right]^T$$

Single particle distribution function
at local thermodynamical equilibrium

In principle, it should be calculated from the covariant Wigner function with the local thermodynamical equilibrium quantum density operator

$$\widehat{\rho}_{\rm LE}(t) = \frac{\exp[-\int {\rm d}^3{\rm x} \left(\widehat{T}^{0\nu}\beta_\nu(x) - \widehat{j}^0\xi(x) - \frac{1}{2}\widehat{\mathcal{S}}^{0,\mu\nu}\omega_{\mu\nu}(x)\right)]}{{\rm tr}(\exp[-\int {\rm d}^3{\rm x} \left(\widehat{T}^{0\nu}\beta_\nu(x) - \widehat{j}^0\xi(x) - \frac{1}{2}\widehat{\mathcal{S}}^{0,\mu\nu}\omega_{\mu\nu}(x)\right)])}$$

Obtained by maximizing the entropy $S = -{\rm tr}(\widehat{\rho}\log\widehat{\rho})$ with respect to $\widehat{\rho}$ with the constraints of fixed mean energy-momentum density and fixed mean angular momentum density.

$$W(x,k) = {\rm tr}(\widehat{\rho}_{LE}(t)\text{Combination of quantum fields})$$

A complicated calculation (PhD student E. Grossi at work).

One can make a reasonable ansatz which

- reduces to the global equilibrium solution in the Boltzmann limit
- reduces to the known Fermi-Juttner or Bose-Juttner formulae at the LTE in the non-rotating case

Ansatz

$$f(x,p) = \frac{1}{2m}\bar{U}(p)\left(\exp[\beta(x)\cdot p - \xi(x)]\exp[-\frac{1}{2}\varpi(x):\Sigma] + I\right)^{-1} U(p)$$

$$\bar{f}(x,p) = -\frac{1}{2m}(\bar{V}(p)\left(\exp[\beta(x)\cdot p + \xi(x)]\exp[\frac{1}{2}\varpi(x):\Sigma] + I\right)^{-1} V(p))^T$$

Example:

Recalling:

$$U(p)\bar{U}(p) = (\not{p} + m)I \qquad V(p)\bar{V}(p) = (\not{p} - m)I$$

$$\mathrm{tr}_2 f = \frac{1}{2m}\mathrm{tr}_2(\bar{U}(p)XU(p)) = \frac{1}{2m}\mathrm{tr}(XU(p)\bar{U}(p)) = \frac{1}{2m}\mathrm{tr}(X(\not{p}+m)) = \frac{1}{2}\mathrm{tr}X$$

$$\mathrm{tr}_2 \bar{f} = -\frac{1}{2m}\mathrm{tr}_2(\bar{V}(p)\bar{X}V(p)) = -\frac{1}{2m}\mathrm{tr}(\bar{X}V(p)\bar{V}(p)) = -\frac{1}{2m}\mathrm{tr}(\bar{X}(\not{p}-m)) = \frac{1}{2}\mathrm{tr}\bar{X}$$

with

$$X = \left(\exp[\beta(x)\cdot p - \xi(x)]\exp\left[-\frac{1}{2}\varpi(x):\Sigma\right] + I\right)^{-1}$$

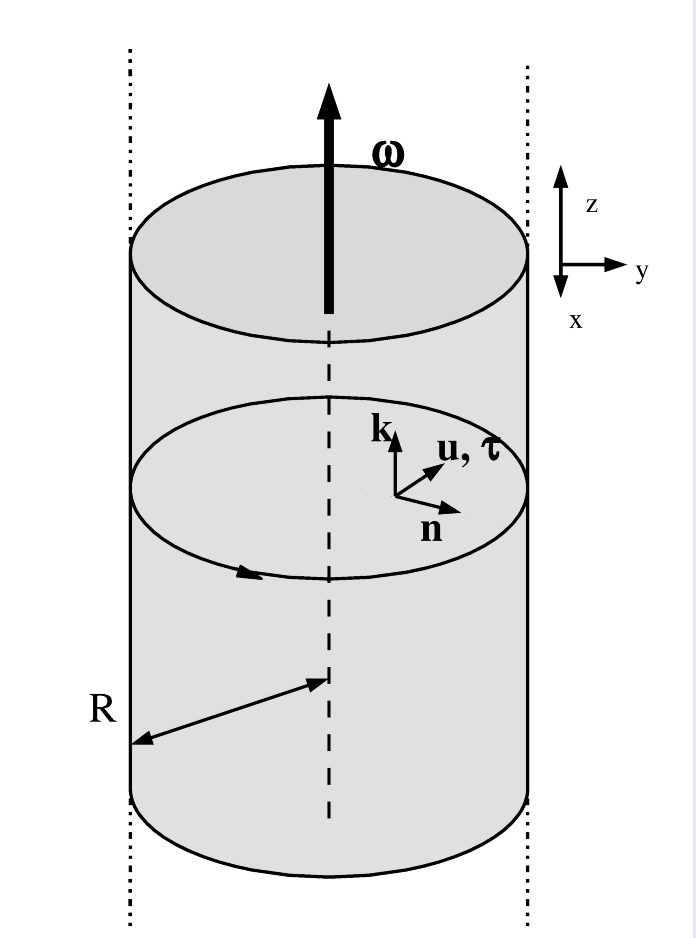
What is $\varpi(x)$?

This is a crucial issue to calculate polarization

At global equilibrium:

$$\varpi_{\mu\nu} = (\omega/T)(\delta^1_\mu \delta^2_\nu - \delta^1_\nu \delta^2_\mu) = \sqrt{\beta^2}\Omega_{\mu\nu}$$

$$\sqrt{\beta^2} = \frac{1}{T_0} \qquad \Omega^{\mu\nu} = \sum_{i=1}^{4} \frac{De_i^\mu}{\mathrm{d}\tau} e^{i\nu}$$



$$e_0 = u \quad e_1 = (0, \hat{\mathbf{n}}) \quad e_2 = (0, \hat{\mathbf{k}}) \quad e_3 = \tau$$

At the same time

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$$

being, for global equilibrium

$$\beta = \frac{1}{T}(1, \boldsymbol{\omega} \times \mathbf{x}) = \frac{1}{T_0}(\gamma, \gamma \mathbf{v})$$

The latter equation can be checked explicitely, but it is form is indeed a deeper consequence of relativity coupled with thermodynamics

Equilibrium in relativity can be achieved only if the inverse four-temperature field is a Killing vector

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0$$

$$\beta_\mu = b_\mu + \varpi_{\mu\nu} x^\nu \qquad b \text{ and } \varpi \text{ constants}$$

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$$

"Thermal vorticity"

If deviations from equilibrium are *small*, we know that the tensor $\varpi(x)$ should differ from the above expression only by terms which vanish at equilibrium, i.e. second-order terms in the gradients of the β field

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu) + \mathcal{O}(\partial^2 \beta)$$

This is what we need for leading-order hydrodynamics!

Polarization in a relativistic fluid

Definition:

$$\Pi_\mu = -\frac{1}{2}\epsilon_{\mu\rho\sigma\tau}S^{\rho\sigma}\frac{p^\tau}{m}$$

also known as Pauli-Lubanski vector

should be the total angular momentum vector of the particle

For a kinetic system

$$\langle \Pi_\mu(x,p)\rangle = -\frac{1}{2}\frac{1}{\mathrm{tr}_2 f}\epsilon_{\mu\rho\sigma\tau}\frac{\mathrm{d}\mathcal{J}^{0,\rho\sigma}(x,p)}{\mathrm{d}^3 p}\frac{p^\tau}{m}$$

Total angular momentum tensor

$$\mathcal{J}^{\lambda,\rho\sigma}(x) = x^\rho T^{\lambda\sigma}(x) - x^\sigma T^{\lambda\tau}(x) + \mathcal{S}^{\lambda,\rho\sigma}(x)$$

$$\frac{\mathrm{d}\mathcal{J}^{0,\rho\sigma}(x)}{\mathrm{d}^3 p} = (x^\rho p^\sigma - x^\sigma p^\rho)\mathrm{tr}_2 f(x,p) + \frac{\mathrm{d}\mathcal{S}^{\lambda,\rho\sigma}(x)}{\mathrm{d}^3 p}$$

vanished by the Levi-Civita symbol

Canonical spin tensor

$$\mathcal{S}^{\lambda,\mu\nu}(x) \equiv \frac{1}{2}\langle : \overline{\Psi}(x)\{\gamma^\lambda, \Sigma^{\mu\nu}\}\Psi(x) : \rangle = \frac{1}{2}\int \frac{\mathrm{d}^3\mathrm{p}}{2\varepsilon} \mathrm{tr}_2 \left(f(x,p)\bar{U}(p)\{\gamma^\lambda, \Sigma^{\mu\nu}\}U(p)\right) - \mathrm{tr}_2 \left(\bar{f}^T(x,p)\bar{V}(p)\{\gamma^\lambda, \Sigma^{\mu\nu}\}V(p)\right)$$

...tracing the γ's, expanding in $\varpi(x)$ which is usually a small number (at global equilibrium $\hbar\omega/KT \ll 1$)...

$$\frac{\mathrm{d}\mathcal{S}^{\lambda,\rho\sigma}(x)}{\mathrm{d}^3 p} \simeq \frac{1}{2\varepsilon}\left(p^\lambda n_F(1-n_F)\varpi^{\rho\sigma} + \text{rotation of indices}\right)$$

$$n_F = \frac{1}{\mathrm{e}^{\beta(x)\cdot p - \xi(x)} + 1}$$

Polarization four-vector in the LAB frame

Final formulae:

$$\langle \Pi_\mu(x,p) \rangle \simeq \frac{1}{16} \epsilon_{\mu\rho\sigma\tau} \, (1-n_F) \left(\partial^\rho \beta^\sigma - \partial^\sigma \beta^\rho \right) \frac{p^\tau}{m} = \frac{1}{8} \epsilon_{\mu\rho\sigma\tau} \, (1-n_F) \partial^\rho \beta^\sigma \frac{p^\tau}{m}$$

$$\Pi = (\Pi^0, \mathbf{\Pi}) = \frac{1-n_F}{8m} ((\nabla \times \boldsymbol{\beta}) \cdot \mathbf{p}, \varepsilon (\nabla \times \boldsymbol{\beta}) - \frac{\partial \boldsymbol{\beta}}{\partial t} \times \mathbf{p} - \nabla \beta^0 \times \mathbf{p})$$

As a by-product, a new effect is predicted: particles in a steady temperature gradient (here with $\boldsymbol{v} = 0$) should be transversely polarized:

$$\Pi = (\Pi^0, \mathbf{\Pi}) = (1-n_F) \frac{\hbar p}{8mKT^2} (0, \nabla T \times \hat{\mathbf{p}})$$

Cooper-Frye for polarization

$$\langle \Pi_\mu(p) \rangle \equiv \frac{\displaystyle\int \mathrm{d}\Sigma_\lambda \, \frac{p^\lambda}{\varepsilon} (-1/2) \epsilon_{\mu\rho\sigma\tau} \frac{\mathrm{d}\mathcal{S}^{0,\rho\sigma}}{\mathrm{d}^3\mathrm{p}} \frac{p^\tau}{m}}{\displaystyle\int \mathrm{d}\Sigma_\lambda \, \frac{p^\lambda}{\varepsilon} \mathrm{tr}_2 f(x,p)} = -\frac{1}{4} \epsilon_{\mu\rho\sigma\tau} \frac{p^\tau}{m} \frac{\displaystyle\int \mathrm{d}\Sigma_\lambda \, p^\lambda \, \Theta^{\rho\sigma}}{\varepsilon \dfrac{\mathrm{d}N}{\mathrm{d}^3\mathrm{p}}}$$

$$\langle \Pi_\mu(p) \rangle \simeq -\frac{1}{4} \epsilon_{\mu\rho\sigma\tau} \frac{p^\tau}{m} \frac{\displaystyle\int \mathrm{d}\Sigma_\lambda \, p^\lambda \, n_F (1 - n_F) \varpi^{\rho\sigma}}{\varepsilon \dfrac{\mathrm{d}N}{\mathrm{d}^3\mathrm{p}}} \simeq \frac{1}{8} \epsilon_{\mu\rho\sigma\tau} \frac{p^\tau}{m} \frac{\displaystyle\int \mathrm{d}\Sigma_\lambda \, p^\lambda \, n_F (1 - n_F) \partial^\rho \beta^\sigma}{\displaystyle\int \mathrm{d}\Sigma_\lambda \, p^\lambda n_F}$$

Polarization in relativistic heavy ion collisions

There have been several papers in the past years about this subject:

A. Ayala et al., Phys. Rev. C 65 024902 (2002)
Z. T. Liang, X. N. Wang, Phys. Rev. Lett. 94 102301 (2005) and others
B. Betz, M. Gyulassy and G. Torrieri, Phys. Rev. C 76 044901 (2007)
F. B., F. Piccinini and J. Rizzo, Phys. Rev. C 77 024906 (2008)

yet no definite formula connecting the polarization of hadrons to the hydrodynamical model.

Now we have it:

$$\Pi_\mu(p) = \epsilon_{\mu\rho\sigma\tau} \, \frac{p^\tau}{8m} \frac{\int \mathrm{d}\Sigma_\lambda p^\lambda \, n_F (1 - n_F) \partial^\rho \beta^\sigma}{\int \mathrm{d}\Sigma_\lambda p^\lambda \, n_F}$$

and we can use it to predict Λ polarization in peripheral heavy ion collisions
(F.B., L. Csernai, D.J. Wang in preparation)

Distribution of protons in the Λ rest frame

$$\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}\Omega^*} = \frac{1}{4\pi}\left(1 + \alpha\, \mathbf{\Pi}_0 \cdot \hat{\mathbf{p}}^*\right) \qquad \mathbf{\Pi}_0(p) = \mathbf{\Pi}(p) - \frac{\mathbf{p}}{\varepsilon(\varepsilon + m)} \mathbf{\Pi}(p) \cdot \mathbf{p}$$

Vorticity of the *u* field

L. Csernai, V. Magas,
D.J. Wang,
Phys. Rev. C 87 034906 (2013)

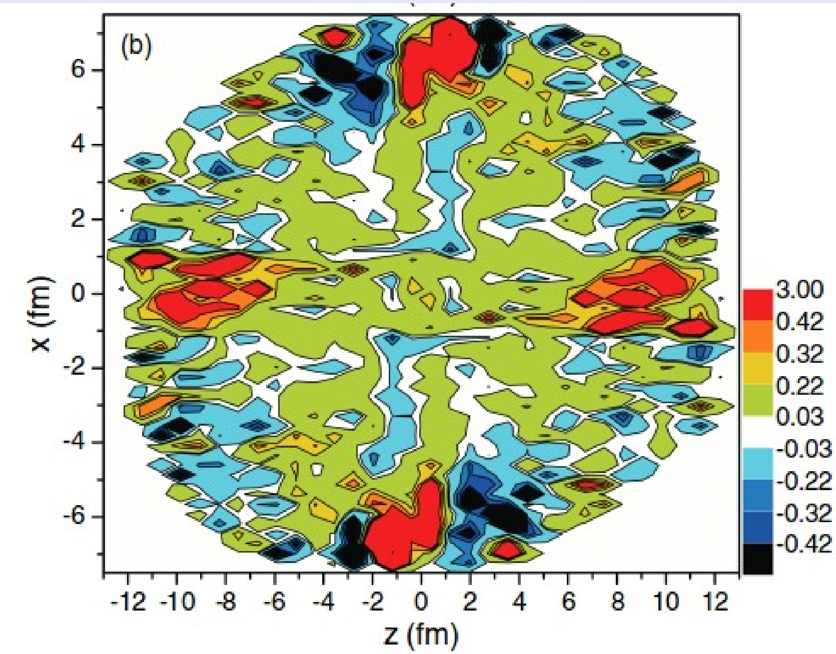


FIG. 6. (Color online) The classical (a) and relativistic (b) weighted vorticity Ω_{zx} (c/fm), calculated in the reaction xz plane at $t = 6.94$ fm/c. The collision energy is $\sqrt{s_{NN}} = 2.76$ TeV and $b = 0.7\, b_{\max}$; the cell size is $dx = dy = dz = 0.4375$ fm. The average vorticity in the reaction plane is 0.01555 (0.05881) c/fm for the classical (relativistic) weighted vorticity respectively.

Vorticity of the β field (thermal vorticity)

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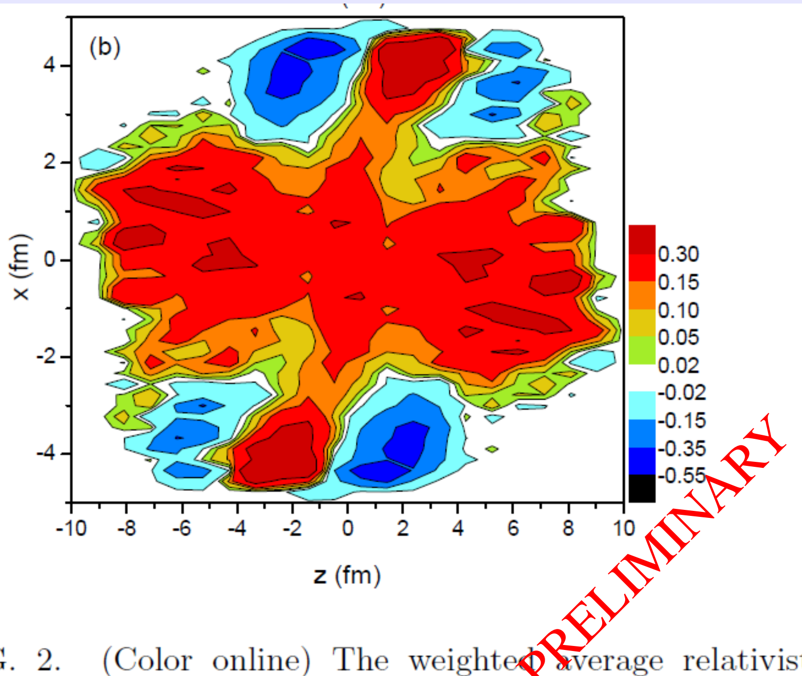
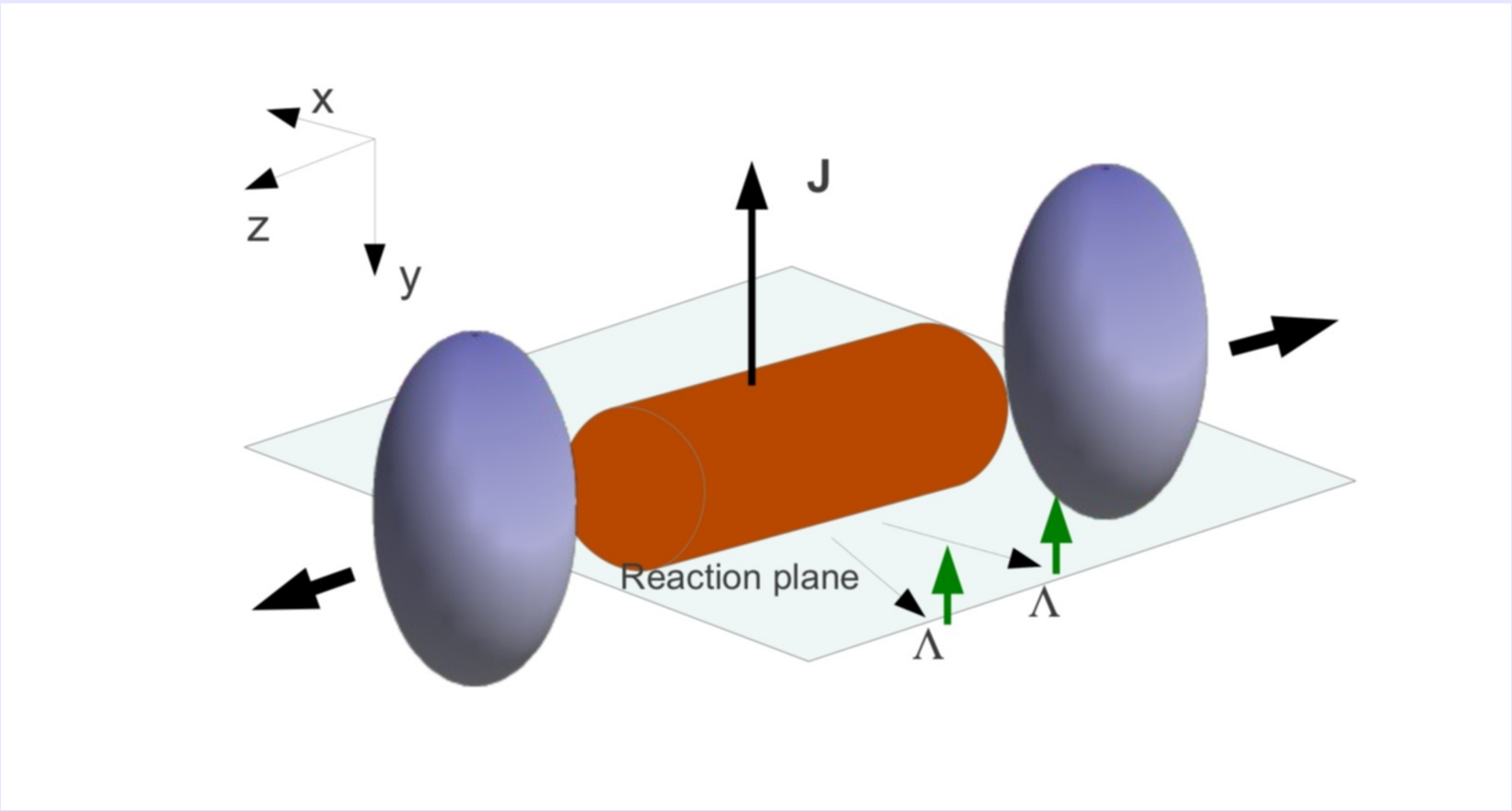


FIG. 2. (Color online) The weighted average relativistic vorticity, $\Omega_{zx}(x, z)$, of temperature 4-vector, $\hat{\beta}^{\mu}$, calculated for all [x-z] layers at t=3.56 fm/c. The collision energy is $\sqrt{s_{NN}} = 2.76$ TeV, $b = 0.5\, b_{max}$ for (a) and $b = 0.7\, b_{max}$ for (b). The configuration (a) is not favoring KHI while (b) is. The cell size is $dx = dy = dz = 0.585\,/\,0.4375$ fm, while the average weighted vorticity is $\langle\Omega_{zx}\rangle = 0.033\,/\,0.078$ for (a) / (b) respectively.

Because of the parity symmetry of the collision

$$\mathbf{\Pi}(p) = \frac{\varepsilon}{8m} \frac{\int \mathrm{d}V \, n_F (\nabla \times \boldsymbol{\beta})}{\int \mathrm{d}V \, n_F}$$

The most polarized Λ are those in the reaction plane (normal to angular momentum).



$$\mathbf{\Pi}_0(p) = \mathbf{\Pi}(p) - \frac{\mathbf{p}}{\varepsilon(\varepsilon+m)}\mathbf{\Pi}(p)\cdot\mathbf{p}$$

Average polarization consistent with the bound set by RHIC (<0.02).

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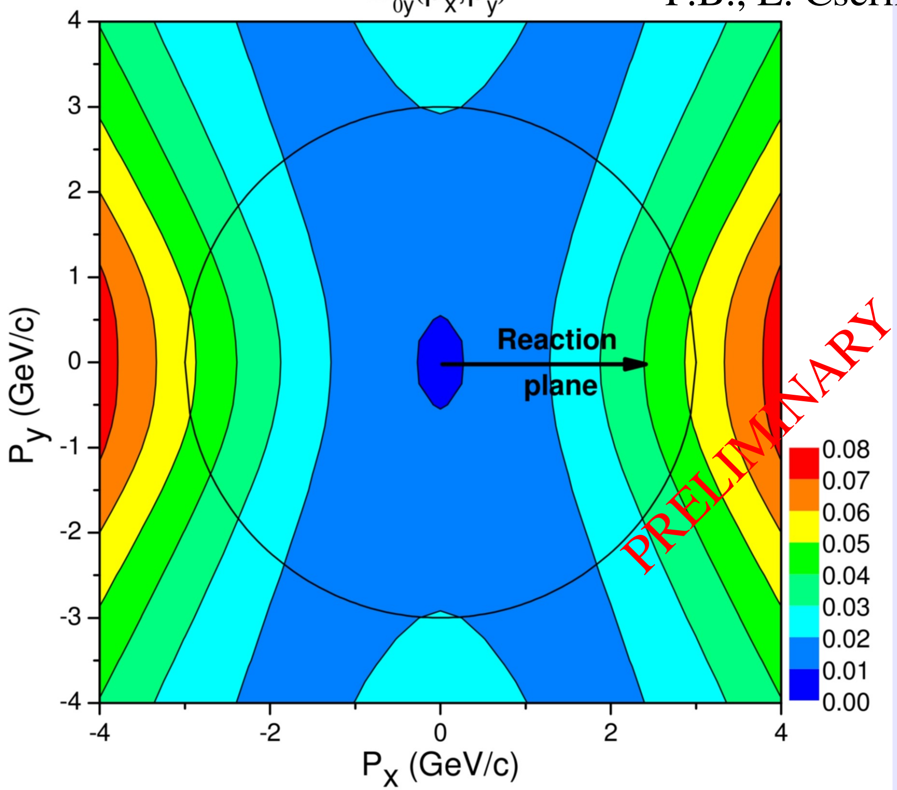

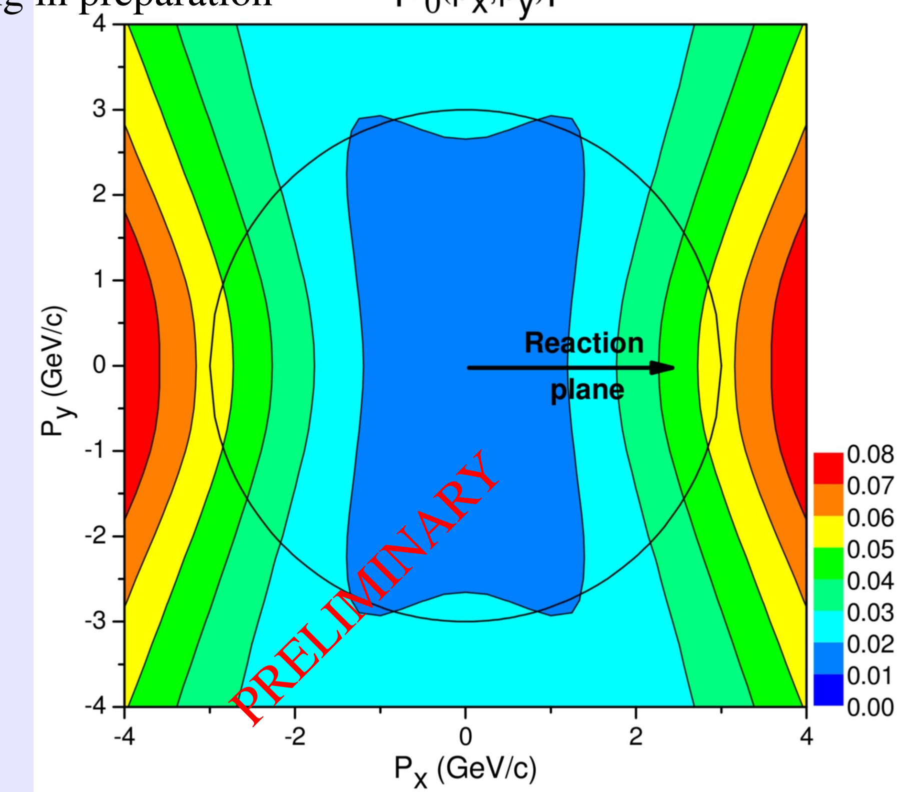

NOTE: the polarization owing to the spectator's magnetic field (E. Bratkovskaya et al.) is at least 4 orders of magnitude less than the one shown above

Conclusions and Outlook

- We have determined the relativistic distribution function of particles with spin ½ at local thermodynamical equilibrium.

- At the leading order hydro, particle polarization is proportional to the vorticity of the inverse temperature four-vector.

- A new (quantum statistical) effect is predicted: transverse polarization in a steady T gradient

- This formula allows to *quantitatively* determine polarization of baryons in peripheral relativistic heavy ion collisions at the freeze-out and its momentum dependence. It is likely to have applications in the so-called CME and CVE.

- The detection of a polarization (in agreement with the prediction of the hydro model) would be a striking confirmation of the local thermodynamical equilibrium picture and, to my knowledge, it would be the first direct observation of polarization induced by rotation for single particles (Barnett effect sees the induced B field)

- It would also have theoretical implications for the existence of the spin tensor.