Vorkurs Mathematik, Blatt 5

(1)(a) $\int_0^\pi dx \, \sin x \cos^2 x = -\int_1^{-1} du \, u^2 = \int_{-1}^1 du \, u^2 = 2\int_0^1 du \, u^2 = 2\frac{u^3}{3}\Big|_0^1$

$u = \cos x \, ; \; du = -\sin x \, dx$

$= \frac{2}{3}$

(b) $I = \int_{\sqrt{2}}^{b} dx \, \frac{3x}{\sqrt{x^2-2}} = \int_0^{b^2-2} du \, \frac{3}{2} \, u^{-1/2}$

$u = x^2 - 2 \, ; \; du = 2x$

$I = \frac{3}{2} \frac{1}{\frac{1}{2}} u^{1/2} \Big|_0^{b^2-2} = 3\sqrt{b^2-2}$

(2)(a) $I = \int_0^b dx \, \frac{x}{\sqrt{1-x^2}} \; ; \; u = 1-x^2 \, ; \; du = -2x \, dx$

$\Rightarrow I = -\int_1^{\sqrt{1-b^2}} \frac{du}{2} u^{-\frac{1}{2}} = \int_{\sqrt{1-b^2}}^{1} du \, \frac{1}{2} u^{-1/2}$

$= \sqrt{u} \Big|_{\sqrt{1-b^2}}^{1} = 1 - \sqrt{1-b^2}$

(b) $\int_{-\pi/2}^{0} dx \, \frac{\cos x}{2-\sin x} = I \, ; \; u = 2 - \sin x \, ; \; du = -\cos x \, dx$

$= -\int_3^2 \frac{du}{u} = \int_2^3 \frac{du}{u} = \ln u \Big|_2^3 = \ln \frac{3}{2}$

(c) $\int_0^b dx \, \frac{x^4}{1-ax^5} = I \; ; \; u = 1-ax^5 \; ; \; du = -5ax^4$ ②

$$I = -\int_1^{1-ab^5} \frac{du}{5a} \frac{1}{u} = \int_{1-ab^5}^{1} du \, \frac{1}{5au} = \frac{1}{5a} \ln u \Big|_{1-ab^5}^{1}$$

$$= -\frac{1}{5a} \ln(1-ab^5)$$

(d) $\int_0^b dx \, \frac{\sin x \cos x}{1+\cos^2 x} = I$

$u = \cos x \; ; \; du = -\sin x \, dx$

$$\Rightarrow I = -\int_1^{\cos b} du \, \frac{u}{1+u^2} = \int_{\cos b}^{1} du \, \frac{u}{1+u^2}$$

$v = 1+u^2 \; ; \; dv = 2u \, du$

$$\Rightarrow I = \int_{1+\cos^2 b}^{2} dv \, \frac{1}{2v} = \frac{1}{2} \ln u \Big|_{1+\cos^2 b}^{2} = \frac{1}{2} \ln\left(\frac{2}{1+\cos^2 b}\right)$$

(e) $\int_0^{\pi/(2a)} dx \, a \sin(ax) \cos^n(ax) = I$

$u = \cos(ax) \; ; \; du = -dx \, a \sin(ax)$

$$I = \int_0^1 du \, u^n = \frac{1}{n+1} u^{n+1} = \frac{1}{n+1}$$

(f) $I = \int_0^b dx\, x^2 \exp(-x^3)$

$u = x^3$; $du = dx\, 3x^2$

$\Rightarrow I = \frac{1}{3}\int_0^{b^3} du\, \exp(-u) = -\frac{1}{3}\exp(-u)\Big|_0^{b^3} = \frac{1}{3}\left[1-\exp(-b^3)\right]$

(g) $\int_0^{(\pi/4+1)^{1/3}} dx\, \frac{x^2}{\cos^2(x^3-1)} = I$

$u = x^3-1$; $du = dx 3x^2$

$I = \frac{1}{3}\int_{-1}^{\pi/4} \frac{du}{\cos^2 u} = \frac{1}{3}\tan u\Big|_0^{\pi/4} = \frac{1}{3}$ (1+tan 1) //