# Heavy Probes in Heavy-Ion Collisions
## Theory Part IV

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# Outline

1. Heavy quarkonia in the vacuum
    - “Stable” charmonium and bottomonium states ($M < 2m_D$)
    - Non-relativistic potential models

2. Heavy quarkonia in the sQGP
    - $J/\psi$ suppression
    - Heavy-quarkonium dissociation
    - In-medium modification of bound-state potentials
    - Heavy-quarkonia dissociation and regeneration in the QGP
    - Dissociation Cross Sections

# Charmonium states

- no flavor-changing neutral currents in weak interactions ⇒ prediction of fourth quark
  - GIM mechanism (Glashow, Iliopolous, Maiani)
  - CKM-quark-mixing matrix (Cabibbo, Kobayashi, Maskawa)
  - discovered as $\bar{c}c$ bound state
  - simultaneously by Ting (BNL) and Richter (RHIC) ⇒ name $J/\psi$
- today many charmonia known (only “stable” states)
- higher excitations can decay strongly to $\bar{D} + D$

| Name | $\eta_c$ | $J/\psi$ | $\chi_{c0}$ | $\chi_{c1}$ | $\chi_{c2}$ | $\psi'$ |
|---|---|---|---|---|---|---|
| mass (GeV) | 2.98 | 3.10 | 3.42 | 3.51 | 3.56 | 3.69 |
| $E_B$ (GeV) | 0.75 | 0.64 | 0.32 | 0.22 | 0.18 | 0.05 |
| state | 1S | 1S | 1P | 1P | 1P | 2S |
| $J^{PC}$ | $0^{-+}$ | $1^{--}$ | $0^{++}$ | $1^{++}$ | $2^{++}$ | $1^{--}$ |

[L. Kluberg, H. Satz, Landolt-Börnstein **23/I**, 6-1 (2010)]

# Bottomonium states

- $\Upsilon$ as first $\bar{b}b$-bound state discovered by Ledermann (Fermi Lab) 1977
- even more "stable" bottomonium states (due to stronger binding)
- higher excitations $\Rightarrow$ can decay strongly to $\bar{B}+B$

| Name | $\Upsilon$ | $\chi_{b0}$ | $\chi_{b1}$ | $\chi_{b2}$ | $\Upsilon'$ | $\chi_{b0}'$ | $\chi_{b1}'$ | $\chi_{b2}'$ | $\Upsilon''$ |
|---|---|---|---|---|---|---|---|---|---|
| mass (GeV) | 9.46 | 9.86 | 9.89 | 9.91 | 10.02 | 10.23 | 10.26 | 10.27 | 10.36 |
| $E_B$ (GeV) | 1.10 | 0.70 | 0.67 | 0.64 | 0.53 | 0.34 | 0.3 | 0.29 | 0.20 |
| state | 1S | 1P | 1P | 1P | 2S | 1P | 1P | 1P | 3S |
| $J^{PC}$ | $1^{--}$ | $0^{++}$ | $1^{++}$ | $2^{++}$ | $1^{--}$ | $0^{++}$ | $1^{++}$ | $2^{++}$ | $1^{--}$ |

[L. Kluberg, H. Satz, Landolt-Börnstein **23/I**, 6-1 (2010)]

- light-quark mesons: mass from strong interaction (confinement)
- heavy quarkonia: mass due to quark masses
- can be treated as (quasi-)non-relativistic bound states

# Non-relativistic potential models

- use phenomenological static potentials, e.g., Cornell potential

$$V(r) = \sigma r - \frac{\alpha}{r}$$

- long-range scale: confining (non-perturbative QCD), string tension, $\sigma \simeq 0.2\ \mathrm{GeV}^2$
- short-range scale: Coulomb-like (pQCD), $\alpha \simeq \pi/12$
- heavy-quarkonium states from non-relativistic Schrödinger equation

$$\left[2m_Q - \frac{1}{m_Q}\Delta + V(r)\right]\Phi_i(\vec{r}) = M_i \phi_i(r)$$

- fit to spin-averaged heavy-quarkonium spectra
  $\Rightarrow m_c = 1.25\ \mathrm{GeV},\ m_b = 4.65\ \mathrm{GeV},\ \sqrt{\sigma} = 0.445\ \mathrm{GeV},\ \alpha = \pi/12$
- from wave function $\left\langle r_i^2 \right\rangle = \left\langle \Phi_i \left| \vec{\mathbf{r}} \right| \Phi_i \right\rangle$

| Name | $J/\psi$ | $\chi_c$ | $\psi'$ | $\Upsilon$ | $\chi_b$ | $\Upsilon'$ | $\chi_b'$ | $\Upsilon''$ |
|---|---|---|---|---|---|---|---|---|
| mass (GeV) | 3.10 | 3.53 | 3.68 | 9.46 | 9.99 | 10.02 | 10.26 | 10.36 |
| $E_B$ (GeV) | 0.64 | 0.20 | 0.05 | 1.10 | 0.67 | 0.54 | 0.31 | 0.20 |
| $\Delta M_i$ (GeV) | 0.02 | -0.03 | 0.03 | 0.036 | -0.06 | -0.06 | -0.08 | -0.07 |
| $r_0$ (fm) | 0.50 | 0.72 | 0.90 | 0.28 | 0.44 | 0.56 | 0.68 | 0.78 |

# $J/\psi$ suppression as probe for QGP formation

- heavy quarkonia break up in sQGP
  - dissociation through inelastic scattering with medium particles
  - gluon dissociation, quasi-free knock-out reactions
  - in-medium modification of strong interaction; color screening
- suppression of heavy quarkonia as signal for QGP formation
  [T. Matsui, H. Satz, $J/\psi$ PLB **178**, 416 (1986)]
  - caveat! already suppression in pA collisions compared to pp
  - absorption, shadowing, Cronin effect as cold-nuclear-matter effects
  - must be taken into account to determine "anomalous suppression"

# Heavy-quarkonium dissociation

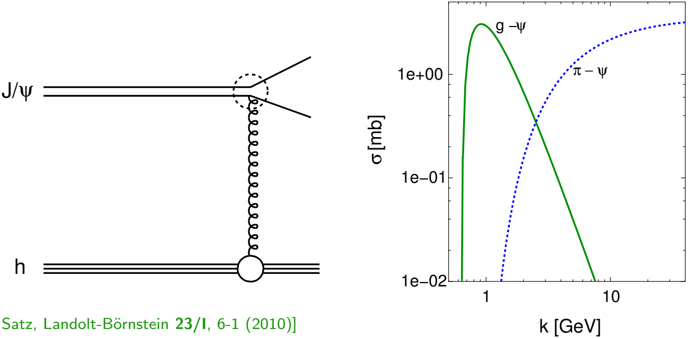
- from non-rel. potential models: tightly bound states of small size
- need hard parton to dissociate heavy quarkonia
- leading-order process via hard gluon
- hot hadron gas
  - hadron of high momentum $p_h \Rightarrow$ distribution of gluons with momentum $x p_h$:
    $$g(x) \propto (1-x)^2$$
  - average gluon-momentum fraction
    $$\langle x \rangle = \frac{\int_0^1 \mathrm{d}x x g(x)}{\int_0^1 \mathrm{d}x \; g(x)} = \frac{1}{5}$$
  - in hadronic medium with $T < T_c$ average momentum $\langle p_{\text{gluon}} \rangle = 3T/5 \leq 0.1 \text{ GeV} \ll E_B \simeq 0.6$ GeV
- deconfined matter (QGP)
  - $p_{\text{gluon}} \simeq 3T \Rightarrow$ for $T \gtrsim 1.2 T_c$ gluo dissociation of $J/\psi$ possible

# Heavy-quarkonium dissociation

- dissociation cross section (similar to photo effect in QED)
  $g + J/\psi \rightarrow \bar{c} + c$
  [M.E. Peskin, NPB **156**, 365 (1979), G. Bhanot, M.E. Peskin, NPB **156**, 391 (1979)]

$$\sigma_{g+J/\psi\rightarrow\bar{c}+c} \propto \frac{1}{m_c^2}\frac{(k/E_B-1)^{3/2}}{(k/E_B)^5}$$

- for hadron: convolution with parton-distribution function $g(x)$



[L. Kluberg, H. Satz, Landolt-Börnstein **23/I**, 6-1 (2010)]

# Potential models in the medium

- modify Cornell potential by Debye screening
- confining part: Laplace equation in 1D
- perturbative part: Laplace equation in 3D
- Debye-screened Cornell potential

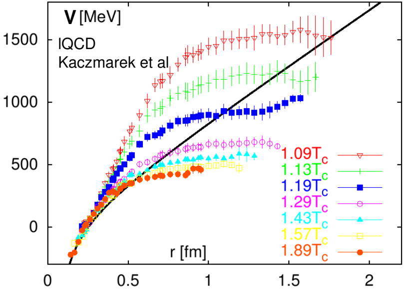
$$V(r,T) \simeq \sigma r \frac{1-\exp(-\mu_D r)}{\mu_D r} - \frac{\alpha}{r} \exp(-\mu_D r)$$

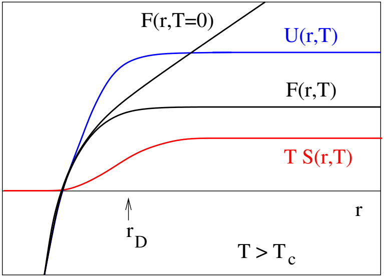
- shortcomings of this model
  - confining part treated as "1D-gauge theory" $\Rightarrow$ different in $3D$
  - $\mu_D$ taken in high-energy form $\mu \propto T$
  - lQCD $\mu_D$ different close to $T_c$ (strong interactions $\Rightarrow$ sQGP!)

[F. Karsch, M.-T. Mehr, H. Satz. ZPC **37**, 617 (1988); F. Karsch, H. Satz, ZPC **51**, 209 (1991)]

# Static heavy-quark potentials from lattice QCD

- lattice QCD at finite temperature: calculate free energy $F = U - TS$
- calculate difference between $F_{\bar{Q}Q}$ with $Q$ and $\bar{Q}$ at distance, $r$ and $F$
- average Hamiltonian for static $\bar{Q}Q$: $\langle H \rangle_T = U = -T^2 \partial(F/T)/\partial T$ $\Rightarrow M_{\text{quarkonium}}$
- long-distance limit: $2M_D(T) \simeq 2m_c + U(\infty, T)$
- can be reinterpreted as medium-modified heavy-quark mass
- short-distance: polarization zones overlap $\Rightarrow$ enhancement of $U$ over $T = 0$ Cornell potential
- "right" potential $V = xU + (1 - x)F$?





# Heavy-quarkonium spectral functions from lQCD

- lQCD: thermal expectation values of imaginary-time operators
- Monte Carlo evaluation of path integrals of discreticed QCD action
- current-correlation function for heavy quarkonia

$$G_\alpha(\tau,\vec{r}) = \left\langle \mathbf{j}_\alpha(\tau,\vec{r})\mathbf{j}_\alpha^\dagger(0,0)\right\rangle_T$$

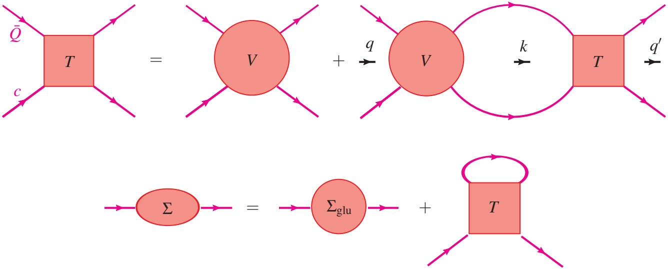
- connection with spectral function

$$G_\alpha(\tau,p;T) = \int_0^\infty \mathrm{d}E\sigma_\alpha(E,p;T)K(E,\tau;T) \quad \text{with}$$
$$K(E,\tau;T) = \frac{\cosh[E(\tau-\beta/2)]}{\sinh(\beta E/2)}$$

- inversion problematic on (discretized) imaginary time $0 \leq \tau \leq \beta$
- statistical maximum-entropy method (MEM)

# T-matrix approach for quarkonium-bound-state problem

- T-matrix Brückner approach for heavy quarkonia as for HQ diffusion
- consistency between HQ diffusion and $\bar{Q}Q$ suppression!



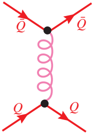
- 4D Bethe-Salpeter equation → 3D Lippmann-Schwinger equation
- relativistic interaction → static heavy-quark potential (lQCD)

$$T_\alpha(E;q',q) = V_\alpha(q',q) + \frac{2}{\pi}\int_0^\infty \mathrm{d}k\, k^2 V_\alpha(q',k) G_{Q\bar{Q}}(E;k) T_\alpha(E;k,q)$$
$$\times \{1 - n_F[\omega_1(\vec{k})] - n_F[\omega_2(k)]\}$$

- $q$, $q'$, $k$ relative 3-momentum of initial, final, intermediate $\bar{Q}Q$ state

[F. Riek, R. Rapp, arXiv:1005.0769 [hep-ph]]

# The potential



- non-perturbative static gluon propagator

$$D_{00}(\vec{k}) = 1/(\vec{k}^2 + \mu_D^2) + m_G^2/(\vec{k}^2 + \tilde{m}_D^2)^2$$

- finite-T HQ color-singlet-free energy from Polyakov loops

$$\begin{aligned}\exp[-F_1(r,T)/T] &= \left\langle \mathrm{Tr}[\Omega(x)\Omega^\dagger(y)]/N_c \right\rangle \\ &= \exp\left[\frac{g^2}{2N_c T^2}\left\langle A_{0,\alpha}(x)A_{0,\alpha}(y) - A_{0,\alpha}^2(x)\right\rangle\right] + \mathcal{O}(g^6)\end{aligned}$$

- identify $\langle A_{0,\alpha}(x)A_{0,\alpha}(y)\rangle = D_{00}(x-y)$
- color-singlet free energy

$$F_1(r,T) = -\frac{4}{3}\alpha_s\left\{\frac{\exp(-m_D r)}{r} + \frac{m_G^2}{2\tilde{m}_D}[\exp(-\tilde{m}_D r) - 1] + m_D\right\}$$

- in vacuo $m_D, \tilde{m}_D \to 0$

$$F_1(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r, \quad \sigma = \frac{2\alpha_s m_G^2}{3}$$

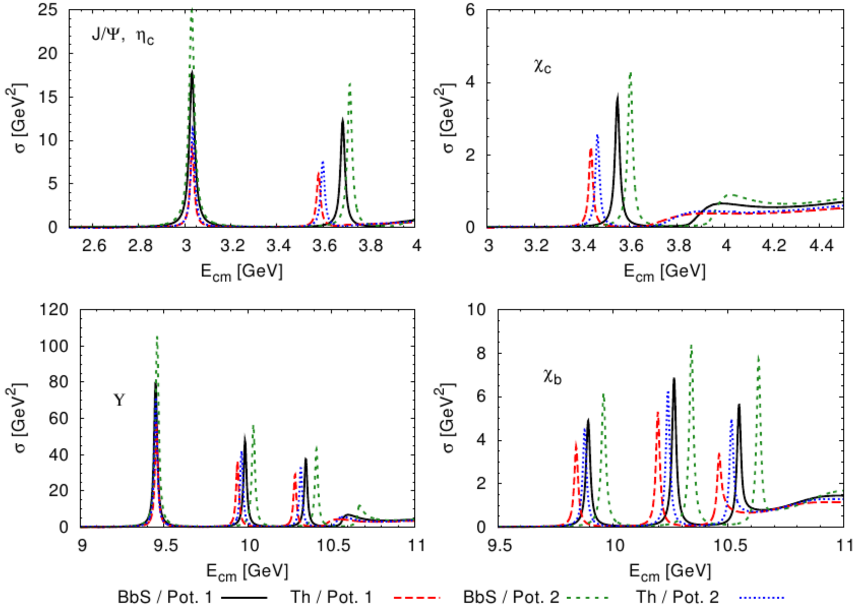
[F. Riek, R. Rapp, arXiv:1005.0769 [hep-ph]]

# Heavy quarkonia

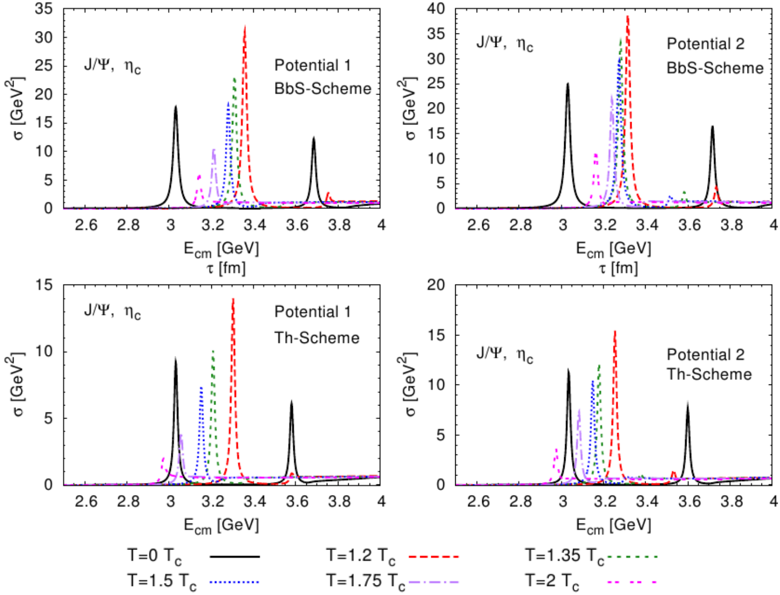
- fit parameters, $\alpha_s(T)$, $m_D(T)$, $\tilde{m}_D(T)$, $\tilde{m}_G(T)$ to lQCD
- calculate internal energy $U(r,T) = F(r,T) - T\frac{\partial}{\partial T}F(r,T)$
- solve Lippmann-Schwinger equation $\Rightarrow$ adjust $m_Q$ to get $s$-wave charmonia/bottomonia masses in vacuum
- in the following
  - potential 1: $N_f = 2+1$ [O. Kaczmarek]
  - potential 2: $N_f = 3$ [P. Petreczky]
  - BbS: Blancenblecler-Sugar reduction scheme
  - Th: Thompson reduction scheme
- vacuum-mass splittings
  - uncertainty for charmonia $50\text{-}100\ \mathrm{MeV}$
  - uncertainty for bottomonia $30\text{-}70\ \mathrm{MeV}$
  - overall uncertainty $\simeq 10\%$
- melting temperatures with $U$ and $F$
  - $s$-wave ($\eta_c$, $J/\psi$): $2\text{-}2.5T_c$, $\gtrsim 1.3T_c$,
    $\Upsilon$: $> 2T_c$, $\gtrsim 1.7T_c, 1T_c$, $\gtrsim 2T_c$, $1T_c$, $1T_c$
  - $p$-wave ($\chi_c$): $\gtrsim 1.2T_c$, $\gtrsim 1T_c$, $\chi_b$: $\gtrsim 1.7T_c, 1.2T_c$, all $\gtrsim 1T_c$

[F. Riek, R. Rapp, arXiv:1005.0769 [hep-ph]]

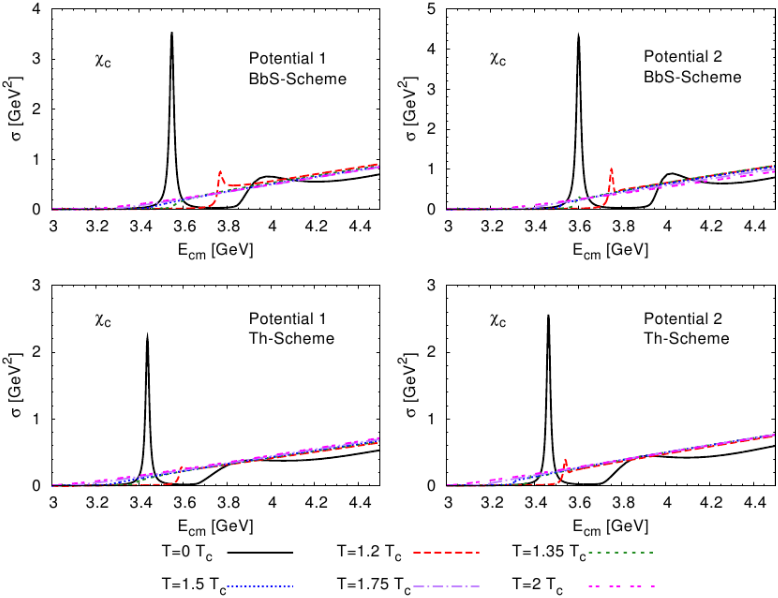
# Quarkonium-spectral functions in the vacuum



# In-medium charmonium-spectral functions (s states)



# In-medium charmonium-spectral functions (p states)



# $J/\psi$ suppression and regeneration

- dissociation rate

$$\Gamma_\Psi = \sum_i \int \frac{\mathrm{d}^3 k}{(2\pi)^3} f_i(\omega_k, T) v_{\mathsf{rel}} \sigma_{\Psi i}^{(\mathsf{diss})}(s)$$

- $g + \Psi \rightarrow \bar{Q} + Q$ ("gluon dissociation")

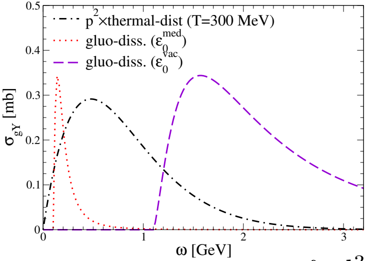
$$\sigma_{g\Psi}(k_0) = \frac{2\pi}{3} \left(\frac{32}{3}\right)^2 \left(\frac{m_Q}{E_b}\right)^{1/2} \frac{1}{m_Q^2} \frac{(k_0/E_B - 1)^{3/2}}{(k_0/E)^5}$$

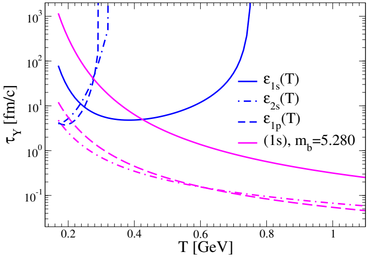
- for decreasing binding energy: cross section sharply peaked at low $k_0$
- gluon dissociation becomes inefficient for loosely bound states
- additional channel: quasi-free dissociation $g + \Psi \rightarrow g + \bar{Q} + Q$

[L. Granchamp, R. Rapp, PLB **523**, 60 (2001); R. Rapp EPC **43**, 91 (2005)]

# Dissociation Cross Sections

- need dissociation cross sections to evaluate $\Upsilon$ yield
- Usual mechanism: gluon dissociation (in dipole approximation)
- Problem: becomes inefficient for loosely bound states





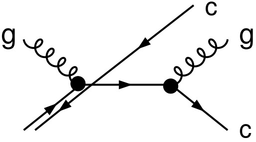
$$\Gamma_Y = \tau_Y^{-1} = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} f_{q,g}(\omega_k, T) v_{\text{rel}} \sigma_Y^{\text{diss}}(s)$$

$$m_Y = 2 m_b(T) - \epsilon_Y(T) = \text{const}$$

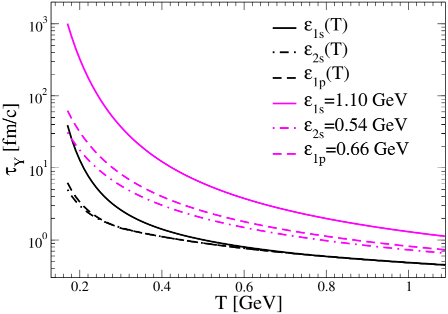
- $\epsilon_Y(T)$ from Schrödinger eq. with screened Cornell potential [Karsch, Mehr, Satz 88]

# Dissociation Cross Sections

- breakup mechanism for loosely bound states: quasifree dissociation

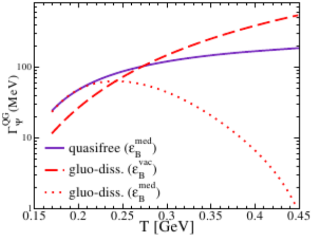


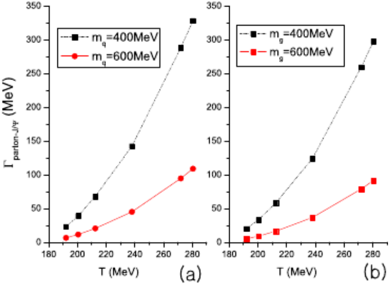
- use LO pQCD cross sections for elastic scattering [Combridge 79]



- Color screening reduces $\Upsilon$ lifetime by factor of 10!

# $J/\psi$ suppression and regeneration





- use of in-medium binding energies:
  need both gluon absorption + quasi-free scattering

[L. Granchamp, R. Rapp, PLB **523**, 60 (2001); R. Rapp EPC **43**, 91 (2005)]

# Quarkonium transport in heavy-ion collisions

- quarkonium transport in the sQGP

$$\frac{p^\mu}{p_0}\partial_\mu f_\Psi(x,\vec{p}) = -\Gamma_\Psi(x,\vec{p}) + \beta_\Psi(x,\vec{p})$$

- gain/regeneration term (e.g., for $Q+\bar{Q} \rightarrow g+\Psi$)

$$\begin{aligned}\beta_\Psi(x,\vec{p}) = & \frac{1}{2p_0}\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3 2\omega_k}\int \frac{\mathrm{d}^3\vec{p}_Q}{(2\pi)^3 2\omega_Q}\int \frac{\mathrm{d}^3\vec{p}_{\bar{Q}}}{(2\pi)^3 2\omega_{\bar{Q}}} \\ & \times f_Q(x,\vec{p}_Q) f_{\bar{Q}}(x,\vec{p}_{\bar{Q}}) W_{Q\bar{Q}}^{g\Psi}(s)\Theta[T_{\text{diss}} - T(x)] \\ & \times (2\pi)^4 \delta^{(4)}(p+q-p_Q-p_{\bar{Q}})\end{aligned}$$

- $W_{Q\bar{Q}}^{g\Psi} = \sigma_{Q\bar{Q}\rightarrow g\Psi} v_{\text{rel}} 4\omega_Q\omega_{\bar{Q}}$
- cross section must be the same as for dissociation (up to kinematics)
- detailed balance

# Rate equations for quarkonia

- integrate Boltzmann equation over $x, \vec{p}$
- assume thermalized $Q/\bar{Q}$ distributions in the sQGP
- rate equation

$$\frac{\mathrm{d}N_\Psi}{\mathrm{d}\tau} = -\Gamma_\Psi (N_\Psi - N_\Psi^{(\mathsf{eq})})$$

- detailed balence ensures correct equilibrium limit
- conservation of heavy-quark number $N_{Q\bar{Q}} = N_Q = N_{\bar{Q}}$ over whole evolution of the medium
- HQ fugacity factors

$$N_{Q\bar{Q}} = \frac{1}{2} N_{\mathsf{op}} \frac{\mathrm{I}_1(N_{\mathsf{op}})}{\mathrm{I}_0(N_{\mathsf{op}})} + V_{\mathsf{FB}} \gamma_Q^2 \sum_\Psi n_\Psi^{(\mathsf{eq})}(T)$$

$$N_{\mathsf{op}} = \begin{cases} V_{\mathsf{FB}} \gamma_Q 2 n_Q^{(\mathsf{eq})}(m_Q^*, T) & \text{for QGP} \\ V_{\mathsf{FB}} \gamma_Q \sum_\alpha n_\alpha^{(\mathsf{eq})}(T, \mu_B) & \text{for hadron gas} \end{cases}$$

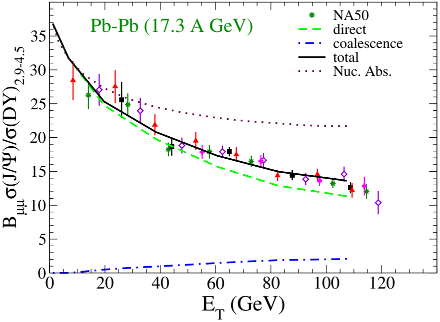
# Initial conditions

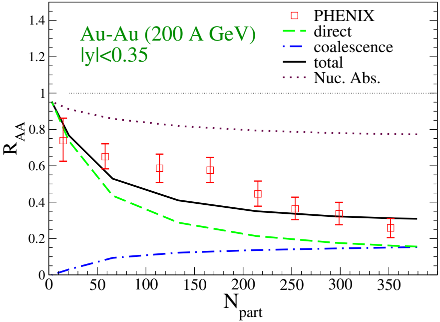
- $\bar{Q}Q$ pairs produced in primordial hard collisions only
- subject to cold-nuclear-matter effects
    - nuclear absorption: dissociation by interaction with surrounding nucleons
    - Cronin effect: broadening of $\Psi$-$p_T$ spectra due to rescattering of gluons before charmonium formation
    - (anti-)shadowing: modification of the parton-distribution functions in nuclei
- after formation time: assume equilibrium distributions
- $p_T$ distributions
    - direct part: from $pp$ + cold-nuclear-matter effects
    - regenerated part: boosted Boltzmann distribution (blast wave)

$$\frac{\mathrm{d}N_\Psi}{p_T \mathrm{d}p_T} \propto m_T \int_0^R \mathrm{d}r \; r \; K_1 \left( \frac{m_T \cosh y_T}{T} \right) \mathrm{I}_0 \left( \frac{p_T \sinh y_t}{T} \right)$$

# Centrality dependence of $J/\psi$ in AA collisions

- mid rapidity

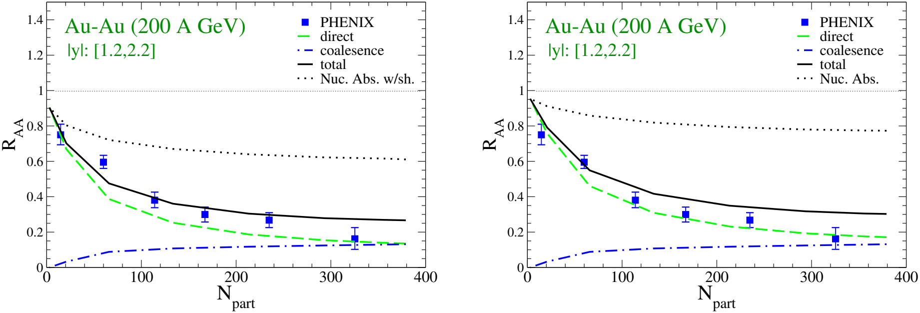




[X. Zhao, R. Rapp, EPC **62**, 109 (2009)]

# Centrality dependence of $J/\psi$ in AA collisions

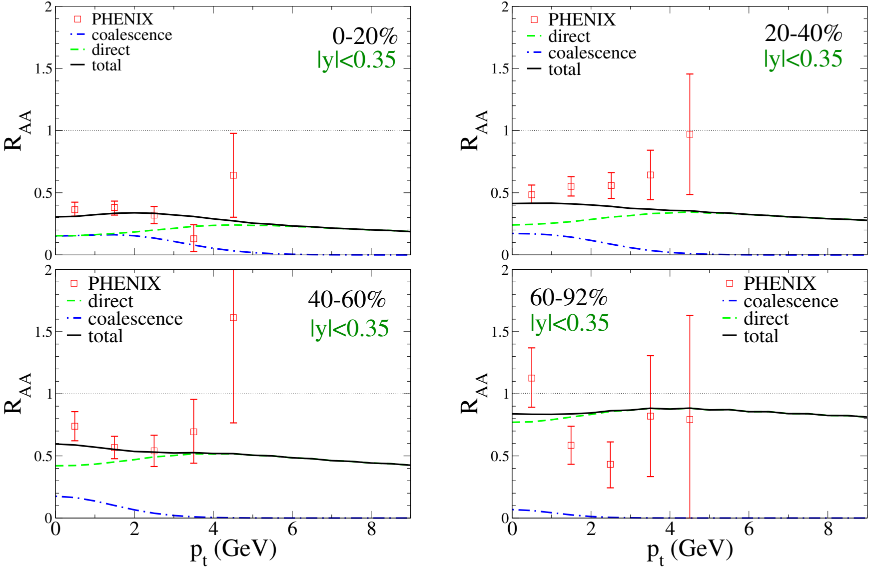
- forward rapidity
- with and without shadowing



[X. Zhao, R. Rapp, EPC **62**, 109 (2009)]

## $p_T$ dependence of $J/\psi$ $R_{AA}$

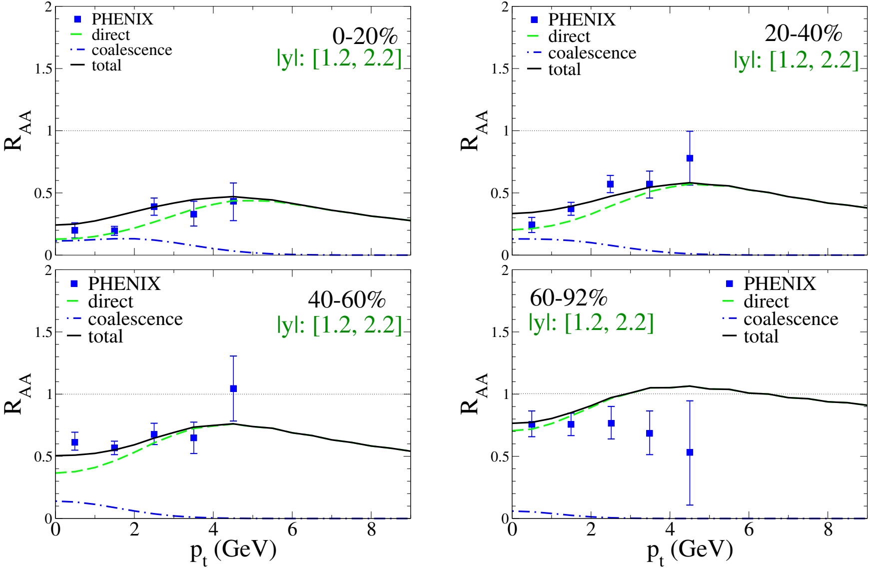
- mid rapidity



[X. Zhao, R. Rapp, EPC **62**, 109 (2009)]

# $p_T$ dependence of $J/\psi$ $R_{AA}$

- forward rapidity



[X. Zhao, R. Rapp, EPC **62**, 109 (2009)]

# Instead of a summary: questions

- How are heavy quarkonia in the vacuum theoretically described?
- What can we learn from that about fundamental properties of QCD?
- Which cold-nuclear matter (initial state) effects are important for heavy quarkonia?
- What are the main mechanisms behind "heavy-quarkonium suppression" in the sQGP?
- How are the bound-state properties of heavy quarkonia in the medium described?
- How are the heavy-quarkonium observables in heavy-ion collisions described?

# Summary

- Heavy quarkonium states
  - (non-relativistic) bound $\bar{Q}Q$ states
  - potential: "color-Coulomb" (pert.) + "confining" (non-pert.) part
  - good description of charmonia, $J/\psi$, $\chi_C$,... and bottomonia, $\Upsilon$, $\chi_b$,...
  - tightly bound states with small size
- Heavy quarkonia in the medium
  - dissociation via scattering with medium particles
  - main mechanism: gluon dissociation, quasi-free break-up reaction
  - $\Rightarrow$ $J/\psi$ suppression as signal for QGP formation in HICs
  - cold-nuclear-matter effects (absorption, shadowing Cronin effect)
  - $\Rightarrow$ "anomalous suppression" QGP signal

# Summary

- Potential models in the medium
    - heavy quarkonia with lQCD
    - difficult to extract spectral properties (MEM)
    - $\Rightarrow$ potential models in the medium
    - use in-medium potentials from the lattice
    - free energy or internal energy?
    - use screened color-Coulomb $+$ confining ansatz for potential
    - fit medium dependent parameters to lQCD
    - leads to survival of some quarkonia above $T_c$ $\Rightarrow$ regeneration important
- Dissociation/Regeneration of heavy quarkonia in the QGP
    - initial conditions: production cross sections, cold-nuclear matter effects
    - dissociation cross sections for gluon absorption $+$ quasi-free scattering
    - Transport approach to dissociation and regeneration of heavy quarkonia
    - in-medium bound-state properties