

# Hadrons in hot and dense matter II

Hendrik van Hees

Goethe University Frankfurt and FIAS

July 17-21, 2017



**FIAS** Frankfurt Institute  
for Advanced Studies



- 1 Theory of electromagnetic probes
  - The McLerran-Toimela formula
- 2 In-medium current-current correlator
  - Relation to chiral symmetry
  - QCD sum rules
- 3 Hadronic models for vector mesons
  - chiral symmetry constraints
  - Vector-meson dominance model (hadronic part)
  - Realistic hadronic models for light vector mesons
  - Hadronic many-body theory (HMBT)
- 4 References
- 5 Quiz

# Theory of electromagnetic probes

# The McLerran-Toimela formula

- derivation of dilepton-production rate [MT85, GK91]

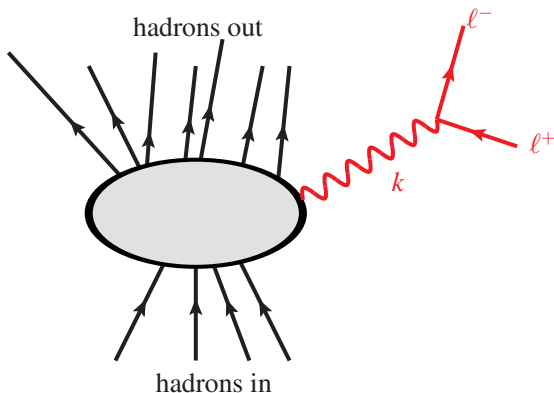
$$\frac{dR_{\ell^+\ell^-}}{d^4k} = \frac{dN_{\ell^+\ell^-}}{d^4x d^4k}$$

- radiation of **dileptons** from **thermalized strongly interacting particles** with total pair four-momentum  $k$
- dileptons** escape fireball without any final-state interactions
- calculation exact concerning **strong interactions**
- leading-order  $\mathcal{O}(\alpha^2)$  in **QED**
- implies assumption that leptons don't suffer final-state interactions

$$\mathbf{H}_{\text{em}}^{(\text{int})} = e \int d^3\vec{x} \mathbf{J}_\mu(t, \vec{x}) A^\mu(t, \vec{x}), \quad A^\mu(t, \vec{x}) = \frac{\epsilon^\mu}{2\omega V} \exp(ik \cdot x)$$

- $\mathbf{J}_\mu$ : exact (wrt. strong interaction!) em. current operator of quarks or hadrons in the Heisenberg picture wrt. strong interactions
- $e = \sqrt{4\pi\alpha}$ ,  $\alpha \simeq 1/137$

# The McLerran-Toimela formula



- Fermi's golden rule  $\Rightarrow$  transition-matrix element for process  $|i\rangle \rightarrow |f'\rangle = |f\rangle + |e^+e^-(k)\rangle$
- QED Feynman rules

$$S_{f'i} = \left\langle f \left| \int d^4x \mathbf{J}_\mu(x) \right| i \right\rangle D_\gamma^{\mu\nu}(x, x') e \bar{u}_\ell(x') \gamma_\nu v_\ell(x')$$

# The McLerran-Toimela formula

- Fourier transformation: energy-momentum conservation  $|f'\rangle = |f, \ell^+ \ell^-(k)\rangle$

$$S_{f'i} = T_{f'i} (2\pi)^4 \delta^{(4)}(P_f + k - P_i)$$

- Fermi's trick: Rate

$$R_{f'i} = \frac{|S_{f'i}|^2}{\tau V} = (2\pi)^4 \delta^{(4)}(P_f + k - P_i) |T_{f'i}|^2$$

- summing over  $|f\rangle$  and polarizations of **dilepton states**
- averaging over initial hadron states: heat bath (grand canonical)

$$\rho = \frac{1}{Z} \exp[-\beta(\mathbf{H}_{\text{QCD}} - \mu_B \mathbf{Q}_{\text{baryon}})]$$

# The McLerran-Toimela formula

- result (derivation see [GK91], Appendices)

$$\frac{dR_{\ell^+\ell^-}}{d^4k} = -\frac{\alpha^2}{3\pi^3} \frac{k^2 + 2m_\ell^2}{(k^2)^2} \sqrt{1 - \frac{4m_\ell^2}{k^2}} g_{\mu\nu} n_B(k^0) \text{Im}\Pi_{\text{ret}}^{\mu\nu}(k)$$

- **em. current-current correlator**

$$i\Pi_{\text{ret}}^{\mu\nu}(k) := \int d^4x \exp(ik \cdot x) \langle [J^\mu(x), J^\nu(0)] \rangle_{T, \mu_B} \Theta(x^0)$$


- written in (local) **restframe of the medium**
- in principle measurable: in **linear response approximation** Green's function for lepton current running through medium
- $k^2 = M^2 > 0$  **invariant mass of dilepton**
- probing medium with photons: **same correlator** for  $k^2 = M^2 = 0$
- then correlator  $\Leftrightarrow$  dielectric function  $\epsilon(\omega)$  in electrodynamics!

# The McLerran-Toimela formula

- for **real photons**

$$\omega \frac{dR}{d^3\vec{k}} = -\frac{\alpha g_{\mu\nu}}{2\pi^2} \text{Im} \Pi_{\text{ret}}^{\mu\nu}(k) n_B(k^0), \quad k^0 \omega = |\vec{k}|$$

- written in (local) **restframe of the medium**
- Phenomenological **effective hadronic model**: **vector-meson dominance model**
- em. current  $\propto V^\mu$  (with  $V \in \{\rho, \omega, \phi\}$ )

$$\Sigma_{\mu\nu}^\gamma = \text{---} G_\rho \text{---}$$


- Dilepton/photon rates:  $\propto A_V = -2 \text{Im} D_V^{(\text{ret})}$   
(**vector-meson spectral function!**)
- measuring **in-medium vector-meson** spectral function!?!)



# Em. current-current correlator

# Vector Mesons and electromagnetic Probes

- **photon** and **dilepton** thermal emission rates given by **same** electromagnetic-current-correlation function ( $J_\mu = \sum_f Q_f \bar{\psi}_f \gamma_\mu \psi_f$ )
- **McLerran-Toimela formula**

$$\Pi_{\mu\nu}^<(q) = \int d^4x \exp(iq \cdot x) \langle J_\mu(0) J_\nu(x) \rangle_T = -2n_B(q_0) \text{Im} \Pi_{\mu\nu}^{(\text{ret})}(q)$$

$$q_0 \frac{dN_\gamma}{d^4x d^3\vec{q}} = -\frac{\alpha_{\text{em}}}{2\pi^2} g^{\mu\nu} \text{Im} \Pi_{\mu\nu}^{(\text{ret})}(q, u) \Big|_{q_0=|\vec{q}|} f_B(p \cdot u)$$

$$\frac{dN_{e^+e^-}}{d^4x d^4k} = -g^{\mu\nu} \frac{\alpha^2}{3q^2 \pi^3} \text{Im} \Pi_{\mu\nu}^{(\text{ret})}(q, u) \Big|_{q^2=M_{e^+e^-}^2} f_B(p \cdot u)$$

- manifestly Lorentz covariant (**dependent on four-velocity of fluid cell,  $u$** )
- to lowest order in  $\alpha$ :  $4\pi\alpha\Pi_{\mu\nu} \simeq \Sigma_{\mu\nu}^{(\gamma)}$
- derivable from underlying thermodynamic potential,  $\Omega$ !

- **vector** and **axial-vector** mesons  $\leftrightarrow$  respective current correlators

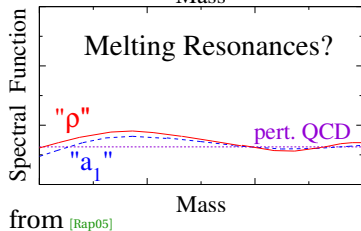
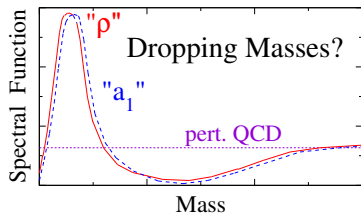
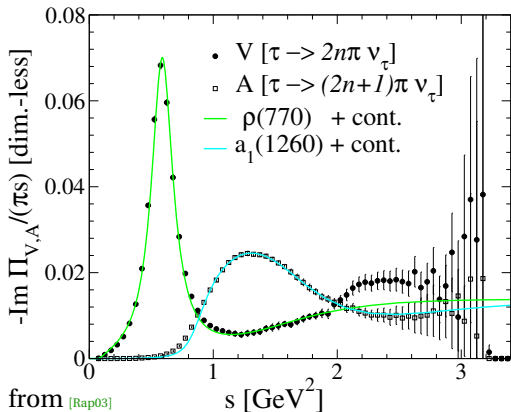
$$\Pi_{V/A}^{\mu\nu}(p) := \int d^4x \exp(ipx) \langle J_{V/A}^\nu(0) J_{V/A}^\mu(x) \rangle_{\text{ret}}$$

- Ward-Takahashi Identities of  $\chi$  symmetry  $\Rightarrow$  **Weinberg-sum rules**

$$f_\pi^2 = - \int_0^\infty \frac{dp_0^2}{\pi p_0^2} [\text{Im} \Pi_V(p_0, 0) - \text{Im} \Pi_A(p_0, 0)]$$

- spectral functions of vector (e.g.  $\rho$ ) and axial vector (e.g.  $a_1$ ) directly related to **order parameter of chiral symmetry!**

# Vector Mesons and chiral symmetry



- at high enough **temperatures and or densities**: melting of  $\langle \bar{q}q \rangle$
- $\Rightarrow$  spontaneous breaking of **chiral symmetry** suspended
- $\Rightarrow$  **chiral phase transition**; chiral-symmetry restoration ( $\chi$ SR)
- which scenario is right? microscopic mechanisms behind  $\chi$ SR?

# QCD Sum Rules

- based on [LPM98]
- calculate current correlator, e.g., the vector part of the **em. current**

$$j_\mu = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)$$

- corresponds to the  $\rho$  meson!
- use **pQCD** to determine correlator

$$\Pi_{\mu\nu}(k) = \left( g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \Pi(k^2)$$

in deep spacelike region,  $Q^2 = -k^2 \gg \Lambda_{\text{QCD}}$

- related to **time-like** region  $\Rightarrow$  **sum rule**

$$\Pi(k^2) = \Pi(0) + cQ^2 + \frac{Q^4}{\pi} \int_0^\infty ds \frac{\text{Im} \Pi(s)}{s^2(s + Q^2 - i\epsilon)}$$

- dispersion relation: **spectral function**  $\text{Im} \Pi!$

- left-hand side of **sum rule**
- pQCD + chiral models for baryon-pion interactions [see, e.g., [DGH92]]

$$R(Q^2) := \frac{\Pi(k^2 = -Q^2)}{Q^2} = -\frac{1}{8\pi^2} \left(1 + \frac{\alpha_s}{\pi}\right) \ln\left(\frac{Q^2}{\mu^2}\right) + \frac{1}{Q^4} m_q \langle \bar{q}q \rangle + \frac{1}{24Q^4} \left\langle \frac{\alpha_s}{\pi} F_{\mu\nu}^a F^{a\mu\nu} \right\rangle - \frac{112}{81Q^6} \kappa \langle \bar{q}q \rangle^2$$

- additional cold-nuclear matter contributions

$$\Delta R(Q^2) = \frac{m_N}{4Q^4} A_2 \rho_N - \frac{5m_N^3}{12Q^6} A_4 \rho_N$$

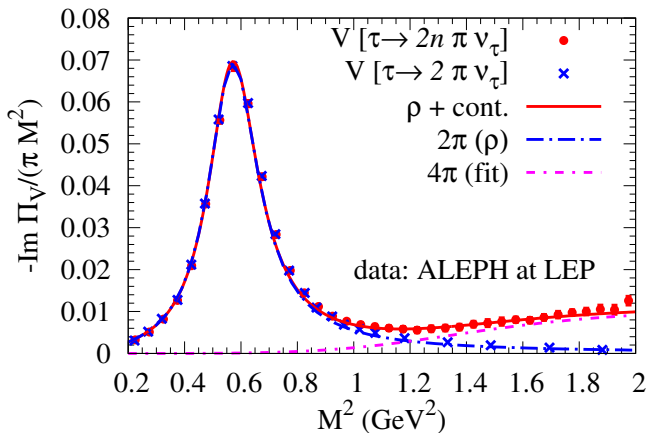
- $A_{2,4}$  from parton-distribution functions
- also condensates medium-modified (in low-density approximation)

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_{\text{vac}} + \frac{\sigma_N}{2m_q} \rho_N,$$

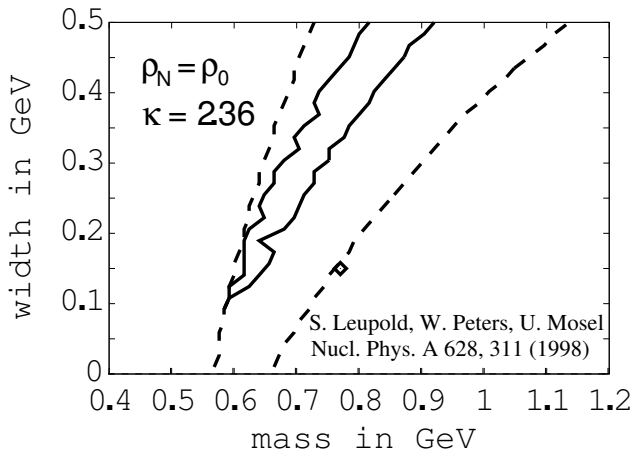
$$\left\langle \frac{\alpha_s}{\pi} F_{\mu\nu}^a F^{a\mu\nu} \right\rangle = \left\langle \frac{\alpha_s}{\pi} F_{\mu\nu}^a F^{a\mu\nu} \right\rangle_{\text{vac}} - \frac{8}{9} m_N^{(0)} \rho_N$$

# QCD Sum Rules

- right-hand side of **sum rule**
- use hadronic models to fit measured **vector-current correlator**
- e.g., ALEPH/OPAL data of  $\tau \rightarrow \nu + 2n\pi$



- typical result from [LPM98]

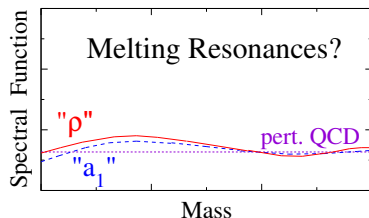
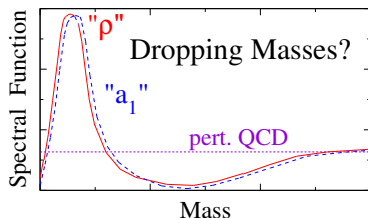


- possible **medium effects** on  **$\rho$  meson**
  - dropping mass, unchanged/small width
  - unchanged mass, broadened spectrum (large width)



# Scenarios for chiral symmetry restoration

- hadron spectrum must become **degenerate** between chiral partners



- models alone of little help (realization of  $\chi$ S not unique!)
  - “vector manifestation”  $\rho_{\text{long}} = \chi$  partner of  $\pi \Rightarrow$  dropping mass
  - “standard realization”  $\rho = \chi$  partner of  $a_1$ , extreme broadening + little mass shifts
- theory “shopping list”
  - effective hadronic models (well constrained in vacuum!)
  - and concise evaluation in the medium!**
  - models for **fireball evolution** (blast-wave parametrizations, hydro, transport, and transport-hydro hybrids)
  - must include partonic  $\rightarrow$  phase transition  $\rightarrow$  hadronic evolution
- precise  $l^+l^-$  ( $\gamma$ ) data from HICs mandatory!**

# Hadronic models

# Effective hadronic models: chiral-symmetry constraints

- different realizations of **chiral symmetry**
- equivalent only on shell (“**low-energy theorems**”)
- model-independent conclusions only in **low-temperature/density limit** (chiral perturbation theory) or from **lattice-QCD calculations**
- QCD sum rules: allow dropping-mass or melting-resonance scenario
- use **phenomenological hadronic many-body theory** (HMBT) to assess medium modifications of vector mesons
  - build models with **hadrons** as effective degrees of freedom
  - based on **(chiral) symmetries**
  - constrained by data on cross sections, branching ratios,... in the vacuum
  - in-medium properties assessed by **many-body (thermal) field theory**

# Example: vector meson dominance model

- early model for **electromagnetic interaction** of charged pions

[Sak60, KLZ67, GS68, Her92, Hee00]

- QED like U(1)-gauge model with massive vector meson for  $\rho_0$  and  $\pi^\pm$
- Stückelberg: introduce auxiliary scalar field for free vector mesons:

$$\mathcal{L}_\rho = -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} m^2 V_\mu V^\mu + \frac{1}{2} (\partial_\mu \varphi)(\partial^\mu \varphi) + m \varphi \partial_\mu V^\mu$$

- gauge invariant under local transformation

$$\delta V_\mu(x) = \partial_\mu \chi(x), \quad \delta \varphi = m \chi(x)$$

- Coupling to pions: **obey gauge invariance!** (like scalar QED)

$$\mathcal{L}_\pi = (D_\mu \pi)^*(D^\mu \pi) - m_\pi^2 |\pi|^2 - \frac{\lambda}{8} |\pi|^4$$

- $D_\mu = \partial_\mu + ig V_\mu$ ;  $g$ :  $\rho\pi\pi$  coupling

# VMD model (photon part)

- add photons:  $D_\mu = \partial_\mu + ig V_\mu + ie A_\mu$
- Lagrangian for photons: usual (gauge fixed) QED
- additional direct  $\rho\gamma$  mixing [KLZ67]


$$\mathcal{L}_{\rho\gamma} = -\frac{e}{2g_{\rho\gamma}} V_{\mu\nu} A^{\mu\nu}$$

- classical field equations:  $\Rightarrow$  **electromagnetic current**


$$j_{\text{em}}^\nu = \partial_\mu A^{\mu\nu} = ie \left( 1 - \frac{g}{g_{\rho\gamma}} \right) \pi \overleftrightarrow{D}^\nu \pi^* + \frac{e}{g_{\rho\gamma}} m^2 V^\mu + \frac{e^2}{g_{\rho\gamma}^2} \partial_\mu A^{\mu\nu}$$

- for  $g_{\rho\gamma} = g$ :  $j_{\text{em}}^\nu = \frac{e}{g} m^2 V^\nu + \mathcal{O}(e^2)$ :  $\Rightarrow$  **“vector-meson dominance”**

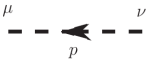
# VMD model (Feynman rules in Feynman gauge)



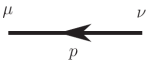
$$= -i \frac{\eta^{\mu\nu}}{p^2 - M^2 + i0^+}$$



$$= -i \frac{\eta^{\mu\nu}}{k^2 + i0^+},$$



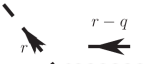
$$= \frac{i}{p^2 - m_\pi^2 + i0^+},$$



$$= i \frac{\not{p} + m_c}{p^2 - m_c^2 + i0^+},$$

- $\eta^{\mu\nu} = \text{diag}(1, -1, -1, -1)$

- $\Theta^{\mu\nu}(p) = \eta^{\mu\nu} - p^\mu p^\nu / p \cdot p$




$$= -ig(q^\mu + r^\mu),$$





$$= ig^2,$$





$$= -i \frac{\lambda}{8}$$



$$= -ie(q^\mu + r^\mu),$$


$$= ie\gamma^\mu,$$


$$= ie^2\eta^{\mu\nu},$$


$$= 2ie g\eta^{\mu\nu},$$


$$= -i \frac{e}{g} p^2 \Theta^{\mu\nu}(p).$$

# VMD model ( $\rho$ -self-energy and dressed $\gamma\pi\pi$ vertex)

- calculate  $\rho$ -self-energy (transversality from gauge invariance)

$$i\Pi_{\rho\pi\pi}^{\mu\nu}(p) = \text{Diagram 1} = is\Pi_{\rho\pi\pi}(s)\Theta^{\mu\nu}(p), \quad s = p^2$$

$$i\Pi_{\rho ee}^{\mu\nu}(p) = \text{Diagram 2} = is\Pi_{\rho ee}(s)\Theta^{\mu\nu}(p), \quad s = p^2$$

- Dressed Green's function

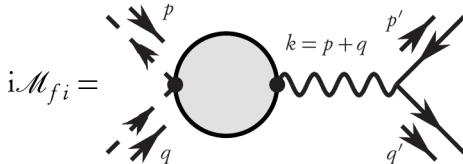
$$G_{\rho}^{\mu\nu}(p) = -\frac{\Theta^{\mu\nu}(p)}{p^2 - M^2 - p^2\Pi_{\rho\pi\pi}(p^2)} - \frac{\Lambda^{\mu\nu}(p)}{p^2 - M^2 + i0^+}$$

- dressed  $\gamma\pi\pi$  vertex to  $\mathcal{O}(e)$

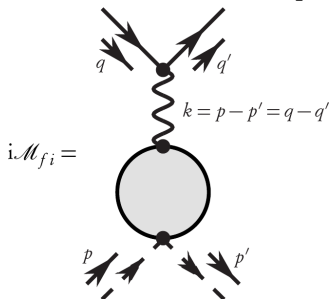
$$i\Gamma_{\gamma\pi\pi}^{\mu} = \text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5}$$

# VMD model (em. form factor of the $\pi$ )

- $\pi^+ + \pi^- \rightarrow e^+ + e^-$  (“time-like form factor”)



- $\Rightarrow |F(s)|^2$  with Mandelstam  $s = (p + q)^2$
- physical region  $s > 4m_\pi^2$
- $\pi^+ + e^- \rightarrow \pi^+ + e^-$  (“space-like form factor”)

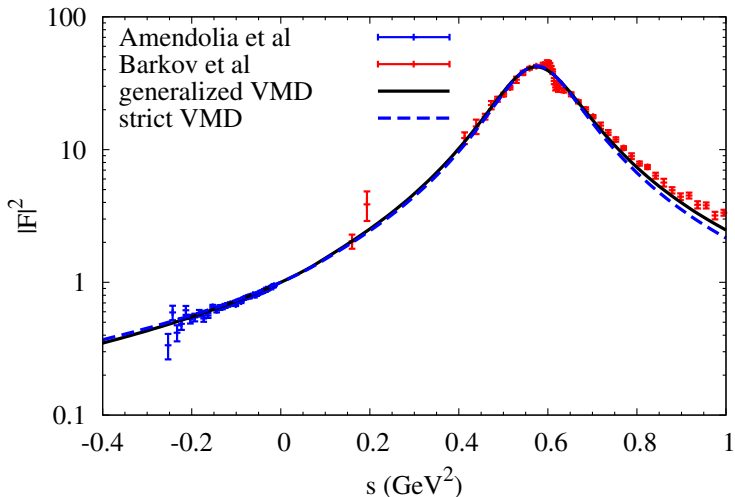


- $\Rightarrow |F(t)|^2$  with Mandelstam  $t = (p - p')^2$
- physical region  $t < 0$



# VMD model: (fit of parameters)

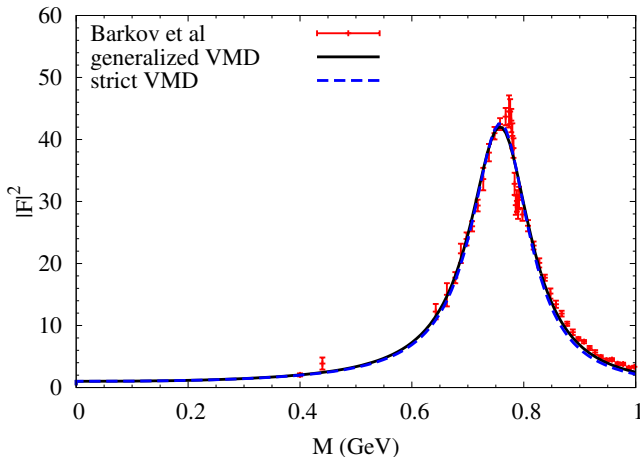
- best fit to form-factor data:  $g = 5.461$ ,  $g' = 5.233$ ,  $m_\rho = 763.1 \text{ MeV}/c^2$   
strict VMD:  $g \stackrel{!}{=} g' = 5.328$ ,  $m_\rho = 763.1 \text{ MeV}/c^2$



data from [A+86, BCE+85]

# VMD model: (fit of parameters)

- best fit to form-factor data:  $g = 5.461$ ,  $g' = 5.233$ ,  $m_\rho = 763.1 \text{ MeV}/c^2$   
strict VMD:  $g \stackrel{!}{=} g' = 5.328$ ,  $m_\rho = 763.1 \text{ MeV}/c^2$



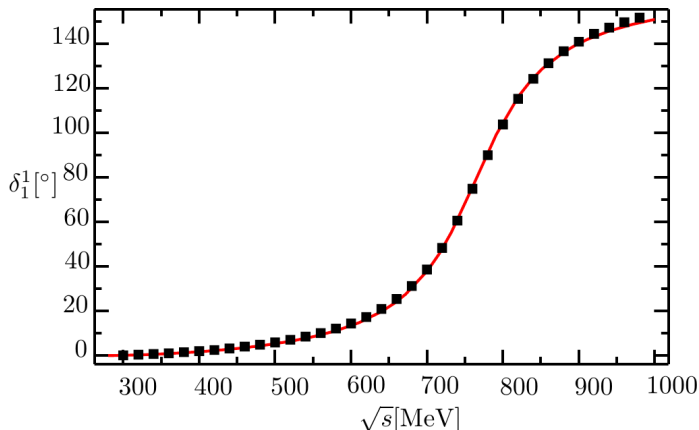
data from [BCE<sup>+</sup>85]

- small discrepancies around  $\rho$  peak: **contribution from  $\omega(782)$  meson!**

# VMD (elastic $\pi\pi$ phase shift)

- $\pi\pi \rightarrow \pi\pi$  phase shift in  $I = 1$  channel

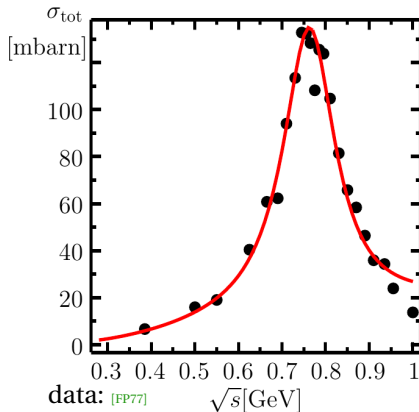
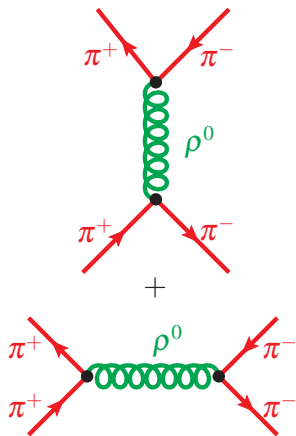
$$\delta_1^1 = \arccos \frac{\text{Re } G_\rho}{|G_\rho|}$$



data: [FP77]

# VMD: (total $\pi\pi$ elastic scattering cross section)

- $\pi\pi \rightarrow \pi\pi$  total cross section

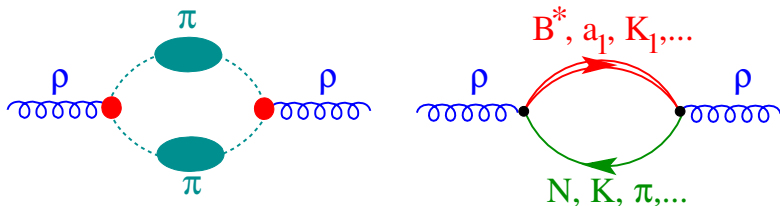


# Realistic hadronic models for light vector mesons

- CERES data: pion- $\rho$  model too simplistic
- many approaches to more realistic models
  - gauged linear  $\sigma$ -model + vector-meson dominance [Pis95, UBW02]  
gauge-symmetry breaking  $\Rightarrow$  pions still in physical spectrum!
  - massive Yang-Mills model; gauged non-linear chiral model with explicitly broken gauge symmetry [Mei88, LSY95]
  - hidden local symmetry: Higgs-like chiral model [BKU<sup>+</sup>85, HY03]  
allows for vector manifestation or usual manifestation (with  $a_1$ )
- here we concentrate on the phenomenological model by Rapp, Wambach, et al [RW99]

# Hadronic many-body theory

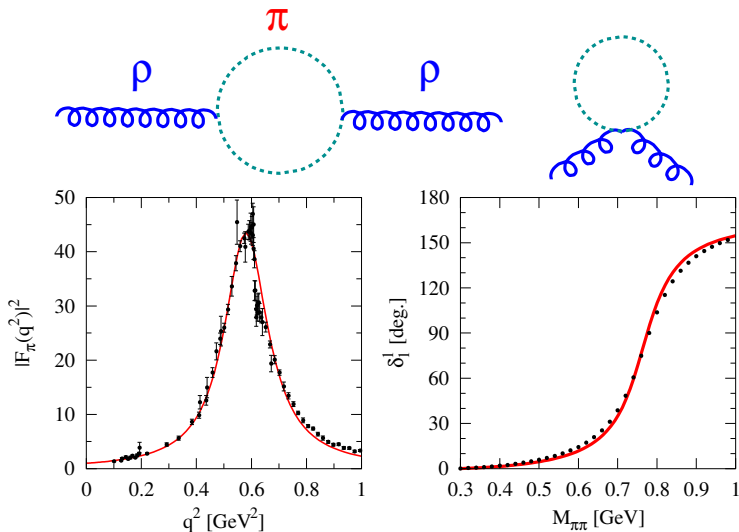
- Phenomenological HMBT [RW99] for vector mesons
- $\pi\pi$  interactions and **baryonic excitations**



- **Baryon (resonances)** important, even at RHIC with low **net** baryon density  $n_B - n_{\bar{B}}$
- reason:  $n_B + n_{\bar{B}}$  relevant (CP inv. of strong interactions)

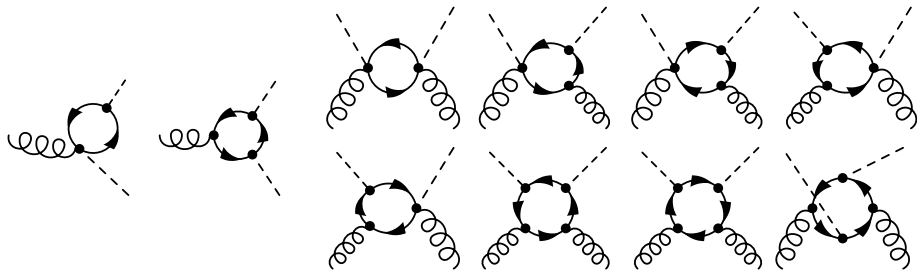
# The meson sector (vacuum)

- most important for  $\rho$ -meson: pions



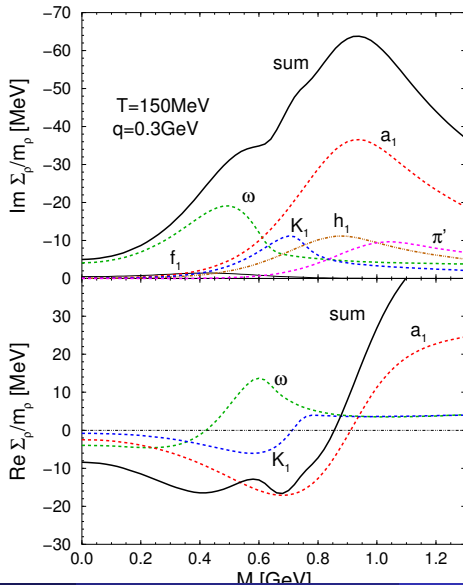
# The meson sector (matter)

- Pions dressed with **N-hole-**,  **$\Delta$ -hole** bubbles
- Ward-Takahashi  $\Rightarrow$  **vertex corrections** mandatory!

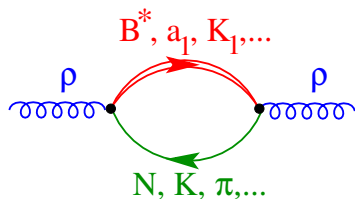




# The meson sector (contributions from higher resonances)

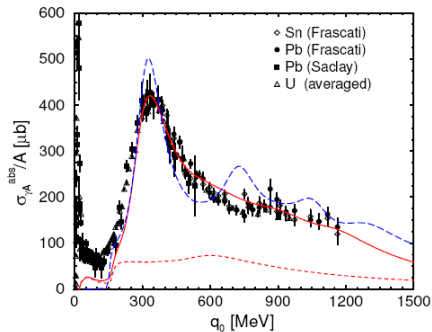
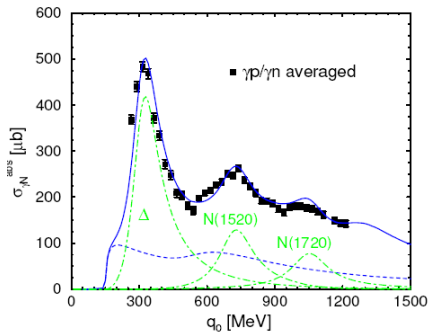


# The baryon sector (vacuum)

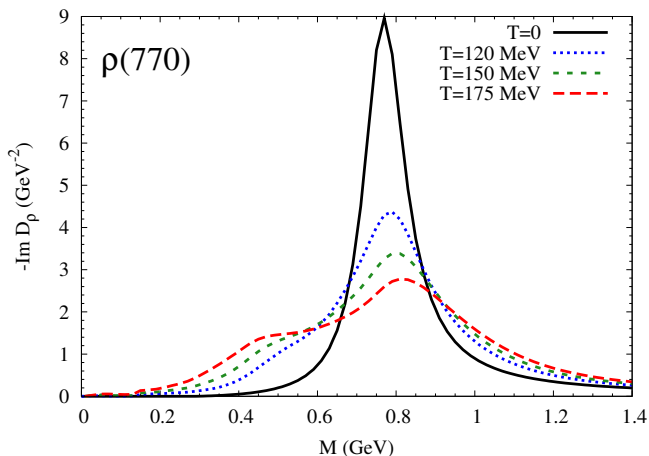


- $P = 1$ -baryons:  $p$ -wave coupling to  $\rho$ :  
 $N(939), \Delta(1232), N(1720), \Delta(1905)$
- $P = -1$ -baryons:  $s$ -wave coupling to  $\rho$ :  
 $N(1520), \Delta(1620), \Delta(1700)$

# Photoabsorption on nucleons and nuclei



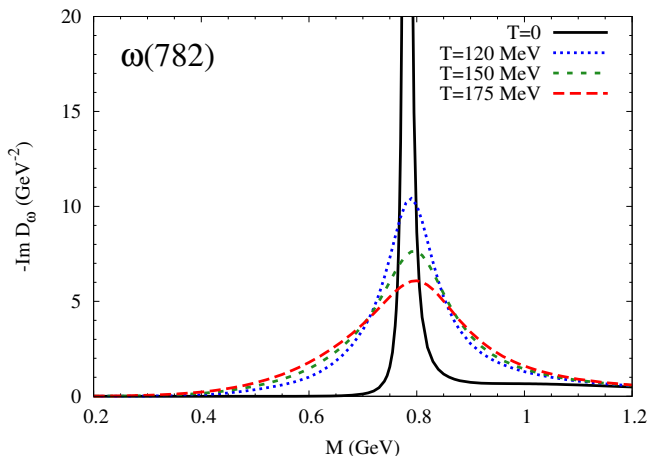
# In-medium spectral functions and baryon effects



[R. Rapp, J. Wambach 99]

- **baryon effects** important
  - large contribution to broadening of the peak
  - responsible for most of the strength at small  $M$

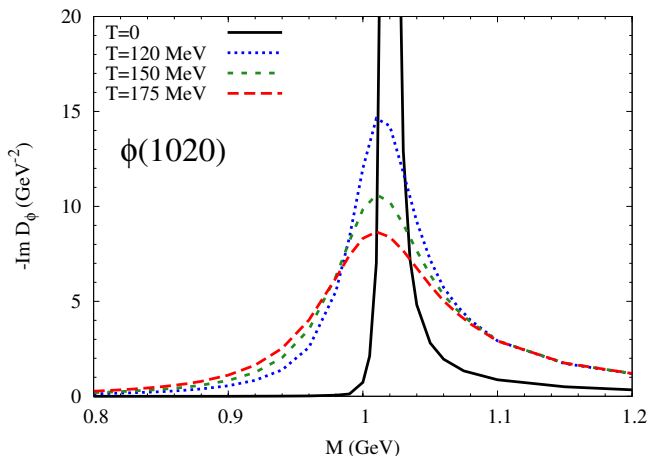
# In-medium spectral functions and baryon effects



[R. Rapp, J. Wambach 99]

- **baryon effects** important
  - large contribution to broadening of the peak
  - responsible for most of the strength at small  $M$

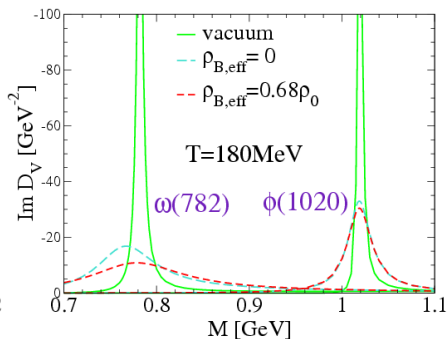
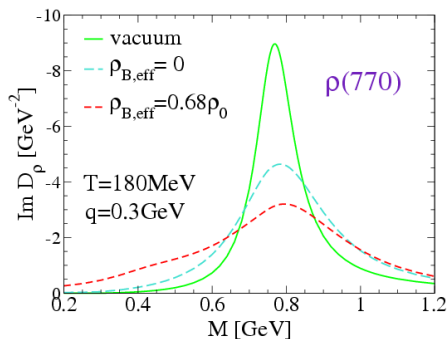
# In-medium spectral functions and baryon effects



[R. Rapp, J. Wambach 99]

- **baryon effects** important
  - large contribution to broadening of the peak
  - responsible for most of the strength at small  $M$

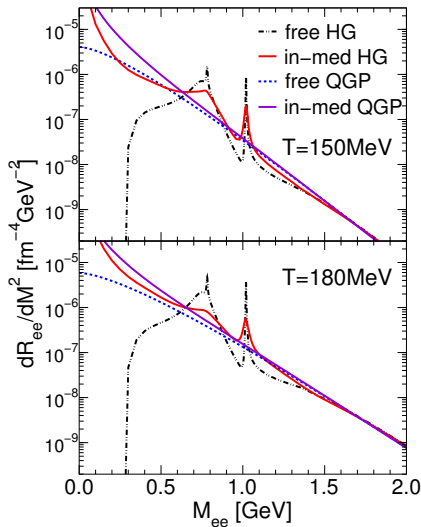
# In-medium spectral functions and baryon effects



[R. Rapp, J. Wambach 99]

- **baryon effects** important
  - large contribution to broadening of the peak
  - responsible for most of the strength at small  $M$

# Dilepton rates: Hadron gas $\leftrightarrow$ QGP



- in-medium **hadron gas** matches with **QGP**
- similar results also for  $\gamma$  rates
- “quark-hadron duality”?
- **hidden local symm.+baryons?**

[BKU<sup>+</sup>85, HY03, HS06, HSW08]



# Bibliography I

- [A<sup>+</sup>86] S. Amendolia, et al., A Measurement of the Space-Like Pion Electromagnetic Form-Factor, Nucl. Phys. B **277** (1986) 168.  
[http://dx.doi.org/10.1016/0550-3213\(86\)90437-2](http://dx.doi.org/10.1016/0550-3213(86)90437-2)
- [BCE<sup>+</sup>85] L. Barkov, et al., Electromagnetic Pion Form-Factor in the Timelike Region, Nucl. Phys. B **256** (1985) 365.  
[http://dx.doi.org/10.1016/0550-3213\(85\)90399-2](http://dx.doi.org/10.1016/0550-3213(85)90399-2)
- [BKU<sup>+</sup>85] M. Bando, T. Kugo, S. Uehara, K. Yamawaki, T. Yanagida, Is the  $\rho$  Meson a Dynamical Gauge Boson of Hidden Local Symmetry?, Phys. Rev. Lett. **54** (1985) 1215.  
<http://dx.doi.org/10.1103/PhysRevLett.54.1215>
- [DGH92] J. F. Donoghue, E. Golowich, B. R. Holstein, Dynamics of the Standard Model, Cambridge University press (1992).
- [FP77] C. D. Frogatt, J. L. Petersen, Phase-Shift Analysis of  $\pi^+\pi^-$  Scattering between 1.0 and 1.8 GeV Based on Fixed Transfer Analyticity (II), Nucl. Phys. B **129** (1977) 89.

# Bibliography II

- [GK91] C. Gale, J. I. Kapusta, Vector dominance model at finite temperature, Nucl. Phys. B **357** (1991) 65.  
[http://dx.doi.org/10.1016/0550-3213\(91\)90459-B](http://dx.doi.org/10.1016/0550-3213(91)90459-B)
- [GS68] G. J. Gounaris, J. J. Sakurai, Finite-Width Corrections to the Vector-Meson-Dominance Prediction for  $\rho \rightarrow e^+e^-$ , Phys. Rev. Lett. **21** (1968) 244.  
<http://link.aps.org/doi/10.1103/PhysRevLett.21.244>
- [Hee00] H. v. Hees, Renormierung selbstkonsistenter Naherungen in der Quantenfeldtheorie bei endlichen Temperaturen, Ph.D. thesis, TU Darmstadt (2000).  
<http://th.physik.uni-frankfurt.de/~hees/publ/doc.pdf>
- [Her92] M. Herrmann, Eigenschaften des  $\rho$ -Mesons in dichter Kernmaterie, Dissertation, Technische Hochschule Darmstadt, Darmstadt (1992).  
[http://www-lib.kek.jp/cgi-bin/img\\_index?200038480](http://www-lib.kek.jp/cgi-bin/img_index?200038480)

# Bibliography III

- [HG08] W. A. Horowitz, M. Gyulassy, Testing AdS/CFT Drag and pQCD Heavy Quark Energy Loss, *J. Phys. G* **35** (2008) 104152.  
<http://dx.doi.org/10.1088/0954-3899/35/10/104152>
- [HS06] M. Harada, C. Sasaki, Dropping  $\rho$  and  $A_1$  meson masses at chiral phase transition in the generalized hidden local symmetry, *Phys. Rev. D* **73** (2006) 036001.  
<http://dx.doi.org/10.1103/PhysRevD.73.036001>
- [HSW08] M. Harada, C. Sasaki, W. Weise, Vector-axialvector mixing from a chiral effective field theory at finite temperature, *Phys. Rev. D* **78** (2008) 114003.  
<http://dx.doi.org/10.1103/PhysRevD.78.114003>
- [HY03] M. Harada, K. Yamawaki, Hidden local symmetry at loop: A new perspective of composite gauge boson and chiral phase transition, *Phys. Rept.* **381** (2003) 1.  
[http://dx.doi.org/10.1016/S0370-1573\(03\)00139-X](http://dx.doi.org/10.1016/S0370-1573(03)00139-X)

- [KLZ67] N. M. Kroll, T. D. Lee, B. Zumino, Neutral Vector Mesons and the Hadronic Electromagnetic Current, *Phys. Rev.* **157** (1967) 1376.  
<http://link.aps.org/abstract/PR/v157/i5/p1376>
- [Krz11] M. Krzewicki, Elliptic and triangular flow of identified particles at ALICE, *J. Phys. G* **38** (2011) 124047.  
<http://dx.doi.org/10.1088/0954-3899/38/12/124047>
- [LAA<sup>+</sup>07] R. A. Lacey, et al., Has the QCD Critical Point been Signaled by Observations at RHIC?, *Phys. Rev. Lett.* **98** (2007) 092301.  
<http://dx.doi.org/10.1103/PhysRevLett.98.092301>
- [LPM98] S. Leupold, W. Peters, U. Mosel, What QCD sum rules tell about the  $\rho$  meson, *Nucl. Phys. A* **628** (1998) 311.  
[http://dx.doi.org/10.1016/S0375-9474\(97\)00634-9](http://dx.doi.org/10.1016/S0375-9474(97)00634-9)

- [LSY95] S. H. Lee, C. Song, H. Yabu, Photon - vector meson coupling and vector meson properties at low temperature pion gas, Phys. Lett. B **341** (1995) 407.  
[http://www.sciencedirect.com/science?\\_ob=GatewayURL&\\_origin=SPIRES&\\_method=citationSearch&\\_volkey=03702693%23341%23407&\\_version=1&md5=05053a52e85b02fde34213175c490b2a](http://www.sciencedirect.com/science?_ob=GatewayURL&_origin=SPIRES&_method=citationSearch&_volkey=03702693%23341%23407&_version=1&md5=05053a52e85b02fde34213175c490b2a)
- [Mei88] U. G. Meissner, Low-Energy Hadron Physics from Effective Chiral Lagrangians with Vector Mesons, Phys. Rept. **161** (1988) 213.  
[http://dx.doi.org/10.1016/0370-1573\(88\)90090-7](http://dx.doi.org/10.1016/0370-1573(88)90090-7)
- [MT85] L. D. McLerran, T. Toimela, Photon and Dilepton Emission from the Quark-Gluon Plasma: Some General Considerations, Phys. Rev. D **31** (1985) 545.  
<http://dx.doi.org/10.1103/PhysRevD.31.545>

# Bibliography VI

- [Pis95] R. D. Pisarski, Where does the  $\rho$  go? Chirally symmetric vector mesons in the quark - gluon plasma, Phys. Rev. D **52** (1995) 3773.  
<http://dx.doi.org/10.1103/PhysRevD.52.R3773>
- [Rap03] R. Rapp, Dileptons in high-energy heavy-ion collisions, Pramana **60** (2003) 675.  
<http://dx.doi.org/10.1007/BF02705167>
- [Rap05] R. Rapp, The vector probe in heavy-ion reactions, J. Phys. G **31** (2005) S217.  
<http://arxiv.org/abs/nucl-th/0409054>
- [RW99] R. Rapp, J. Wambach, Low mass dileptons at the CERN-SPS: Evidence for chiral restoration?, Eur. Phys. J. A **6** (1999) 415.  
<http://dx.doi.org/10.1007/s100500050364>
- [Sak60] J. J. Sakurai, Theory of strong interactions, Ann. Phys. (NY) **11** (1960) 1.  
[http://dx.doi.org/10.1016/0003-4916\(60\)90126-3](http://dx.doi.org/10.1016/0003-4916(60)90126-3)

- [Sas05] C. Sasaki, Chiral phase transition in QCD and vector manifestation (2005).  
<http://arxiv.org/abs/hep-ph/0504073>
- [SJG11] B. Schenke, S. Jeon, C. Gale, Anisotropic flow in  $\sqrt{s} = 2.76$  TeV Pb+Pb collisions at the LHC, Phys. Lett. B **702** (2011) 59.  
<http://dx.doi.org/10.1016/j.physletb.2011.06.065>
- [UBW02] M. Urban, M. Buballa, J. Wambach, Temperature dependence of  $\rho$  and  $a_1$  meson masses and mixing of vector and axial-vector correlators, Phys. Rev. Lett. **88** (2002) 042002.  
<http://dx.doi.org/10.1103/PhysRevLett.88.042002>

# Quiz



- 1 which important “theoretical quantity” can be measured by observing electromagnetic probes in HICs (and elementary reactions)?
- 2 what is chiral-symmetry restoration and in which ways could it be realized in nature?
- 3 what can we learn from QCD sum rules about  $\chi$ SR?
- 4 what tell effective hadronic models about the medium modification of light vector mesons and the related  $\chi$ SR? dilepton data in HICs?
- 5 why are baryon-vector-meson interactions important even at high collision energies, where  $\mu_B \simeq 0$  (nearly 0 net-baryon density)?