

Exercise Sheet #8

Problem 1 (*Time Series with Noise*)

Consider the time series generated by a logical OR (\vee) operation,

$$\sigma_{t+2} = \sigma_{t+1} \vee \sigma_t$$

where $\sigma \in [0, 1]$.

- (a) Evaluate the probability of 0, 1 to occur given random initial conditions.
- (b) Now add noise to the series, assuming that each bit may flip with probability w . With $w^2 \rightarrow 0$, find the frequency of 0, 1 by writing down an equation for the time dynamics and finding a steady solution (write an equation for the probabilities of each steady state of the noiseless case).
- (c) Validate your results by calculating the frequencies numerically. You will need to set $w < 0.0001$.

Problem 2 (*Information Content*)

Two labs want to communicate the results of an experiment between them. In the experimental lab, two fair coins with sides 0, 1 are tossed each time, and their results are summed together. After a long day of running these experiments, the results are sent to the theoretical lab for analysis.

- (a) Calculate the binary entropy of a single coin toss, and of the sum of two coin tosses.
- (b) If the string of results is compressed optimally, how long would the string of toss sums be compared to a string containing the information of all individual tosses?
- (c) Calculate the same for the sum of three coin tosses, and for the sum of two six-sided dice.