
Contents

1 Network Theory	1
1.1 Properties of Real-World Networks	1
1.1.1 The Small World Effect	1
1.1.2 Basic Graph-Theoretical Concepts	4
1.1.3 Network Degree Distribution	9
1.2 Spectral Properties	11
1.2.1 Graph Laplacian	14
1.3 Percolation in Generalized Random Graphs	17
1.3.1 Graphs with Arbitrary Degree Distributions	17
1.3.2 Probability Generating Function Formalism	23
1.3.3 Distribution of Component Sizes	25
1.4 Robustness of Random Networks	29
1.5 Small-World Models	34
1.6 Scale-Free Graphs	36
Exercises	41
Further Reading	43
References	43
2 Bifurcations and Chaos in Dynamical Systems	45
2.1 Basic Concepts of Dynamical Systems Theory	45
2.2 Fixpoints, Bifurcations and Stability	52
2.2.1 Fixpoints Classification and Jacobian	53
2.2.2 Bifurcations and Normal Forms	56
2.2.3 Hopf Bifurcations and Limit Cycles	59
2.3 Global Bifurcations	62
2.3.1 Infinite Period Bifurcation	64
2.3.2 Catastrophe Theory	65
2.3.3 Rate Induced Tipping	69
2.4 Logistic Map and Deterministic Chaos	71
2.4.1 Colliding Attractors	77
2.5 Dynamical Systems with Time Delays	78
2.5.1 Distributed Time Delays	81

Exercises	82
Further Reading	84
References	84
3 Dissipation, Noise and Adaptive Systems	87
3.1 Chaos in Dissipative Systems	87
3.1.1 Phase Space Contraction and Expansion	87
3.1.2 Strange Attractors and Dissipative Chaos	91
3.1.3 Partially Predictable Chaos	94
3.2 Adaptive Systems	97
3.2.1 Conserving Adaptive Systems	102
3.3 Diffusion and Transport	106
3.3.1 Random Walks, Diffusion and Lévy Flights	106
3.3.2 Markov Chains	110
3.4 Stochastic Systems	113
3.4.1 Langevin Equation	113
3.4.2 Stochastic Calculus	116
3.5 Noise-Controlled Dynamics	117
3.5.1 Fokker–Planck Equation	117
3.5.2 Stochastic Escape	119
3.5.3 Stochastic Resonance	121
Exercises	125
Further Reading	126
References	126
4 Self Organization	129
4.1 Interplay Between Diffusion and Reaction	129
4.1.1 Travelling Wavefronts in the Fisher Equation	131
4.1.2 Sum Rule for the Shape of the Wavefront	135
4.1.3 Self-Stabilization of Travelling Wavefronts	136
4.2 Interplay Between Activation and Inhibition	138
4.2.1 Turing Instability	139
4.2.2 Pattern Formation	140
4.2.3 Gray–Scott Reaction Diffusion System	142
4.3 Collective Phenomena and Swarm Intelligence	147
4.3.1 Phase Transitions in Social Systems	147
4.3.2 Collective Decision Making and Stigmergy	149
4.3.3 Collective Behavior and Swarms	152
4.3.4 Opinion Dynamics	154
4.4 Car Following Models	156
4.4.1 Linear Flow and Carrying Capacity	157
4.4.2 Self-Organized Traffic Congestions	158
Exercises	160
Further Reading	162
References	162

5	Information Theory of Complex Systems	163
5.1	Probability Distribution Functions	163
5.1.1	Law of Large Numbers	167
5.1.2	Bayesian Statistics	169
5.1.3	Statistical Binning	171
5.1.4	Time Series Characterization	173
5.2	Entropy and Information	177
5.2.1	Maximal Entropy Distributions	182
5.2.2	Minimal Entropy Principle	183
5.2.3	Mutual Information	185
5.2.4	Kullback-Leibler Divergence	190
5.3	Complexity Measures	193
5.3.1	Complexity and Predictability	195
5.3.2	Algorithmic and Generative Complexity	198
	Exercises	200
	Further Reading	202
	References	202
6	Self-Organized Criticality	203
6.1	Landau Theory of Phase Transitions	203
6.2	Criticality in Dynamical Systems	209
6.2.1	1/f Noise	213
6.3	Cellular Automata	214
6.3.1	Conway's Game of Life	215
6.3.2	Forest Fire Model	216
6.4	Sandpile Model and Self-Organized Criticality	218
6.4.1	Absorbing Phase Transitions	221
6.5	Random Branching Theory	222
6.5.1	Branching Theory of Self-Organized Criticality	222
6.5.2	Galton–Watson Processes	228
6.6	Application to Long-Term Evolution	230
	Exercises	237
	Further Reading	238
	References	239
7	Random Boolean Networks	241
7.1	Introduction	241
7.2	Random Variables and Networks	243
7.2.1	Boolean Variables and Graph Topologies	243
7.2.2	Coupling Functions	246
7.2.3	Dynamics	248
7.3	Dynamics of Boolean Networks	249
7.3.1	Flow of Information Through a Network	250
7.3.2	Mean-Field Phase Diagram	252
7.3.3	Bifurcation Phase Diagram	254
7.3.4	Scale-Free Boolean Networks	258

7.4	Cycles and Attractors	260
7.4.1	Quenched Boolean Dynamics	261
7.4.2	K = 1 Kauffman Network	264
7.4.3	K = 2 Kauffman Network	266
7.4.4	K = N Kauffman Network	267
7.5	Applications	270
7.5.1	Living at the Edge of Chaos	270
7.5.2	Yeast Cell Cycle	271
7.5.3	Application to Neural Networks	274
	Exercises	276
	Further Reading	277
	References	278
8	Darwinian Evolution, Hypercycles and Game Theory	279
8.1	Introduction	279
8.2	Mutations and Fitness in a Static Environment	282
8.3	Deterministic Evolution	286
8.3.1	Evolution Equations	287
8.3.2	Beanbag Genetics: Evolution Without Epistasis	290
8.3.3	Epistatic Interactions and the Error Catastrophe	292
8.4	Finite Populations and Stochastic Escape	297
8.4.1	Adaptive Climbing Under Strong Selective Pressure	298
8.4.2	Adaptive Climbing vs. Stochastic Escape	302
8.5	Prebiotic Evolution	304
8.5.1	Quasispecies Theory	304
8.5.2	Hypercycles and Autocatalytic Networks	306
8.6	Macroecology and Species Competition	310
8.7	Coevolution and Game Theory	313
8.7.1	Tragedy of the Commons	320
	Exercises	322
	Further Reading	324
	References	324
9	Synchronization Phenomena	327
9.1	Frequency Locking	327
9.2	Coupled Oscillators and the Kuramoto Model	329
9.3	Synchronization in the Presence of Time Delays	337
9.4	Synchronization Mechanisms	339
9.4.1	Aggregate Averaging	340
9.4.2	Causal Signaling	344
9.5	Piecewise Linear Dynamical Systems	348
9.6	Synchronization Phenomena in Epidemics	351
9.6.1	Continuous Time SIRS Model	356
	Exercises	357
	Further Reading	359
	References	359

Contents	xv
<hr/>	
10 Complexity of Machine Learning	361
10.1 Computation Units	361
10.1.1 Structured Units	364
10.1.2 The XOR Problem	365
10.2 Recurrent Neural Networks	366
10.2.1 Random Matrix Theory	367
10.2.2 Criticality in Recurrent Networks	369
10.3 Neural Differential Equations	374
10.3.1 Residual Nets	374
10.4 Gaussian Processes	376
10.4.1 Multivariate Gaussians	376
10.4.2 Correlated Stochastic Functions	379
10.4.3 Machine Learning with Neural Tangent Kernels	382
10.5 Attention Induced Information Routing	386
10.5.1 Transformer.....	387
Exercises	390
Further Reading	391
References.....	391
11 Solutions	393
11.1 Solutions to the Exercises of Chap. 1	394
11.2 Solutions to the Exercises of Chap. 2	401
11.3 Solutions to the Exercises of Chap. 3	407
11.4 Solutions to the Exercises of Chap. 4	413
11.5 Solutions to the Exercises of Chap. 5	416
11.6 Solutions to the Exercises of Chap. 6	424
11.7 Solutions to the Exercises of Chap. 7	430
11.8 Solutions to the Exercises of Chap. 8	433
11.9 Solutions to the Exercises of Chap. 9	440
11.10 Solutions to the Exercises of Chap. 10	446
References.....	450
Index	451