# - ANTIFERROMAGNETISM -

## Mean field theory
## &
## Spin wave theory

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# Mean field theory

# Heisenberg model for ferromagnetism

- Coulomb interaction
- Pauli principle

$$H = -\sum_{n,m} J(\vec{R}_m - \vec{R}_n)\, \vec{S}_m \cdot \vec{S}_n$$

$\vec{R}_m, \vec{R}_n$: position of lattice points

$J\left(\vec{R}_m - \vec{R}_n\right)$: exchange interaction ( $J > 0$ )

# Mean field theory

rewrite spin product:

$$\vec{S}_m \cdot \vec{S}_n = \vec{S}_m \langle \vec{S}_n \rangle + \langle \vec{S}_m \rangle \vec{S}_n - \langle \vec{S}_m \rangle \langle \vec{S}_n \rangle + \underbrace{\left( \vec{S}_m - \langle \vec{S}_m \rangle \right)\left( \vec{S}_n - \langle \vec{S}_n \rangle \right)}_{\text{neglect fluctuations}}$$

Mean field Heisenberg Hamiltonian:

$$H = -\sum_{n,m} J(\vec{R}_m - \vec{R}_n)\, \vec{S}_m \langle \vec{S}_n \rangle - \sum_{n,m} J(\vec{R}_m - \vec{R}_n)\, \langle \vec{S}_m \rangle\, \vec{S}_n + C$$

isotropy: $J(\vec{R}_m - \vec{R}_n) = J(\vec{R}_n - \vec{R}_m)$

# Mean field theory

$$H = -\sum_n \vec{S}_n \cdot \underbrace{\left[ 2\sum_m J(\vec{R}_m - \vec{R}_n)\langle \vec{S}_m \rangle \right]}_{\text{effective external field}}$$

- 2-particle interaction $\xrightarrow{\textbf{MFT}}$ interaction with $\vec{B}_{eff}$

$$\boxed{H = -g\mu_B \sum_n \vec{S}_n \cdot \vec{B}_{eff}}$$

homogeneity : $\langle \vec{S}_m \rangle = \langle \vec{S} \rangle$; $\quad \vec{B}_{eff} = \frac{2}{g\mu_B}\lambda \langle \vec{S} \rangle$

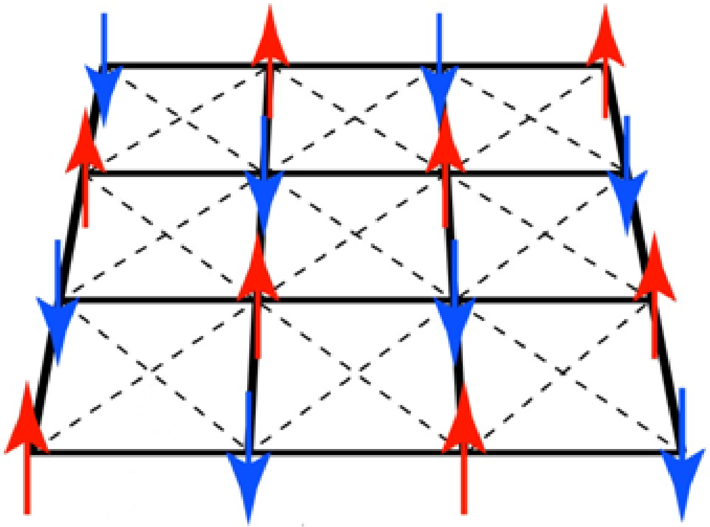
# Mean field theory: antiferromagnetism

- $J(\vec{R}_m - \vec{R}_n) < 0 \longrightarrow$

  antiparallel arrangement

- divison into 2 sublattices:

  plus and minus



MFT:

$$H = -g\mu_B \sum_{n^+} \vec{S}_n \cdot \vec{B}_+ - g\mu_B \sum_{n^-} \vec{S}_n \cdot \vec{B}_-$$

# Mean field theory: antiferromagnetism

effective external fields:

$$g\mu_B \vec{B}_+ = 2\sum_{m^+} J(\vec{R}_m - \vec{R}_n)\langle\vec{S}\rangle_+ + 2\sum_{m^-} J(\vec{R}_m - \vec{R}_n)\langle\vec{S}\rangle_-$$

$$g\mu_B \vec{B}_- = 2\sum_{m^-} J(\vec{R}_m - \vec{R}_n)\langle\vec{S}\rangle_- + 2\sum_{m^+} J(\vec{R}_m - \vec{R}_n)\langle\vec{S}\rangle_+$$

$$g\mu_B \vec{B}_+ = 2\lambda_{++}\langle\vec{S}\rangle_+ + 2\lambda_{+-}\langle\vec{S}\rangle_-$$

$$g\mu_B \vec{B}_- = 2\lambda_{--}\langle\vec{S}\rangle_- + 2\lambda_{-+}\langle\vec{S}\rangle_+$$

symmetry: $\lambda_{++} = \lambda_{--}$ and $\lambda_{+-} = \lambda_{-+}$

# Mean field theory: antiferromagnetism

sublattice magnetization:

$$\vec{M}_+ = \frac{N}{2} g \mu_B \langle \vec{S} \rangle_+ \qquad \vec{M}_- = \frac{N}{2} g \mu_B \langle \vec{S} \rangle_-$$

$$\vec{B}_+ = -\mu_0 \Gamma \vec{M}_+ - \mu_0 A \vec{M}_-$$

$$\vec{B}_- = -\mu_0 \Gamma \vec{M}_- - \mu_0 A \vec{M}_+$$

with $\Gamma = -\frac{4}{g^2 \mu_B^2 N \mu_0} \lambda_{++}$ and $A = -\frac{4}{g^2 \mu_B^2 N \mu_0} \lambda_{+-}$

# Mean field theory: antiferromagnetism

calculation of magnetization of sublattice plus:

$H_+ = -g\mu_B \sum_{n^+} \vec{S}_n \cdot \vec{B}_+;$ $\vec{B}_+ = -\mu_0 \Gamma \vec{M}_+ - \mu_0 A \vec{M}_-$

quantization of spin orientation:

$\vec{S}_n \cdot \vec{B}_+ = \hbar s_n B_+;$ $s_n = -S, -S+1, \ldots, S-1, S$

$\Rightarrow$ $H_+ = -b \sum_{n^+} s_n;$ $b = g\mu_B \hbar B_+$

magnetization:

$$M_+ = \frac{1}{\beta} \frac{\partial \ln Z_+}{\partial B^+}$$

# Mean field theory: antiferromagnetism

partition function:

$$Z_+ = Tr\, e^{-\beta H_+} = \sum_{s_1=-S}^{S} \dots \sum_{s_{\frac{N}{2}}=-S}^{S} \exp\left(\beta b \sum_{n=1}^{\frac{N}{2}} s_n\right)$$

$$= \prod_{n=1}^{\frac{N}{2}} \sum_{s_{\frac{N}{2}}=-S}^{S} e^{\beta b s_n} = \left[e^{\beta b S}\left(1 + e^{-\beta b} + e^{-2\beta b} + \cdots e^{-2\beta b S}\right)\right]^{\frac{N}{2}}$$

$$= \left[e^{\beta b S} \sum_{x=0}^{2S} \left(e^{-\beta b}\right)^x\right]^{\frac{N}{2}} = \left[e^{\beta b S} \frac{1 - e^{-\beta b(2S+1)}}{1 - e^{-\beta b}}\right]^{\frac{N}{2}}$$

$$= \left[\frac{e^{\beta b(S+\frac{1}{2})} - e^{-\beta b(S+\frac{1}{2})}}{e^{\frac{1}{2}\beta b} - e^{-\frac{1}{2}\beta b}}\right]^{\frac{N}{2}} = \left[\frac{\sinh(\beta b(S+\frac{1}{2}))}{\sinh(\frac{1}{2}\beta b)}\right]^{\frac{N}{2}}$$

# Mean field theory: antiferromagnetism

magnetization:

$$M_\pm = \frac{1}{\beta}\frac{\partial \ln Z_\pm}{\partial B_\pm} = \frac{N}{2} g \mu_B S \cdot \mathcal{B}_S(\beta g \mu_B S B_\pm)$$

$$= \frac{N}{2} g \mu_B S \cdot \mathcal{B}_S\left(\beta g \mu_B S \left| -\mu_0 \Gamma \vec{M}_\pm - \mu_0 A \vec{M}_\mp \right|\right)$$

⟶ self consistent equation for $M_\pm$

Brillouin function:

$$\mathcal{B}_S(x) = \frac{2S+1}{2S}\coth\left(\frac{2S+1}{2S}x\right) - \frac{1}{2S}\coth\left(\frac{1}{2S}x\right)$$

for $S = \frac{1}{2}$ particles:

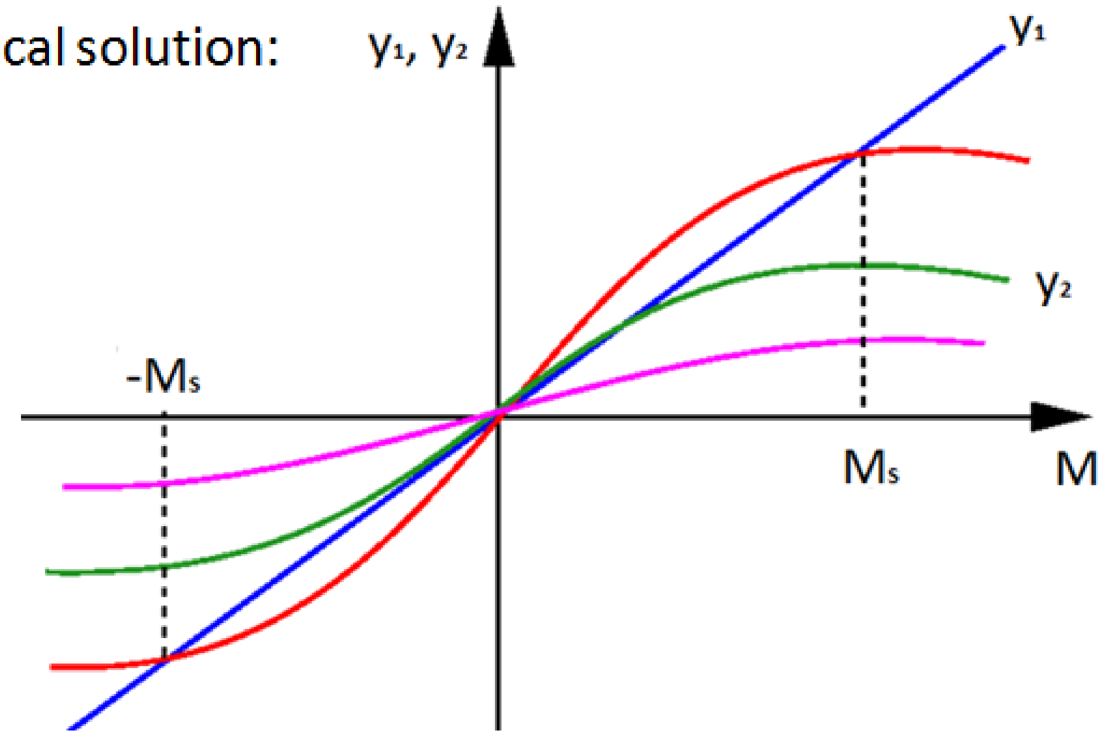
$$\mathcal{B}_{\frac{1}{2}}(x) = \tanh x$$

# Mean field theory: antiferromagnetism

for no external field: $\vec{M}_+ = -\vec{M}_- = \vec{M}$

$$\underbrace{M}_{y_1} = \underbrace{\frac{N}{2} g\mu_B S \cdot \mathcal{B}_S(\beta g \mu_B S \mu_0 (A - \Gamma) M)}_{y_2}$$

graphical solution:



# Mean field theory: antiferromagnetism

beyond $M_S = 0$ there exist 2 other solutions (intersections) for

$$\left(\frac{dy_2}{dM}\right)_{M_S=0} > 1$$

Taylor expansion of $\mathcal{B}_S(x)$ for $x \ll 1$:

$$\mathcal{B}_S(x) = \frac{S+1}{3S}x - \frac{S+1}{2S}\frac{2S^2+2S+1}{30S^2}x^3 + \cdots$$

$$\left(\frac{dy_2}{dM}\right)_{M_S=0} = \frac{N}{2}g\mu_B S\frac{S+1}{3S}\beta g\mu_B S\mu_0(A-\Gamma) = \frac{C}{2}\frac{A-\Gamma}{T}$$

$$\frac{C}{2}\frac{A-\Gamma}{T_N} = 1 \quad \longrightarrow \quad \boxed{T_N = \frac{C}{2}(A-\Gamma)}$$

Néel temperature

# Mean field theory: antiferromagnetism

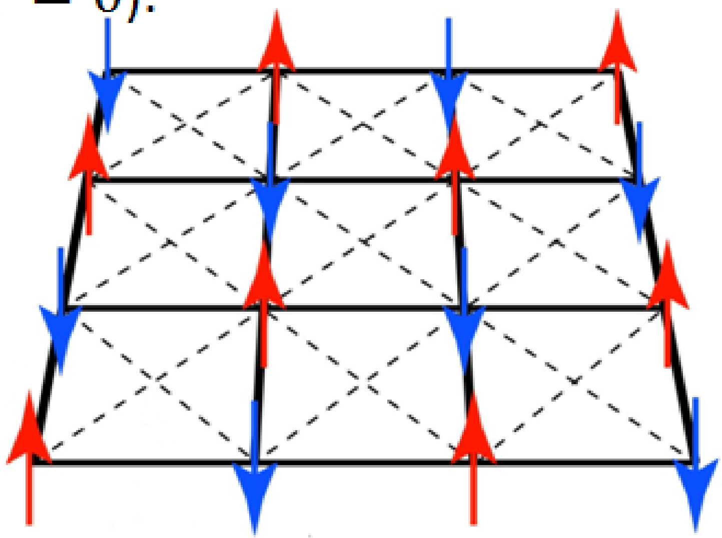
antiferromagnetic phase transition:

- $T > T_N$: sublattice magnetization vanishes $\longrightarrow$ $M_\pm = 0$
- $T < T_N$: antiparallel orientation of spins $\longrightarrow$ $M_\pm \neq 0$
  (spontaneous sublattice magnetization)

classical Néel state (ground state $T = 0$):

$M_\pm(T=0) = M_\pm^{max}$

$\longrightarrow$ perfect antiparallel orientation



# Mean field theory: disadvantages

- Existence of phase transition independent of d

  ⟶ no magnetic ordering in 1d, 2d
- critical exponents are not correct
- low temperature behavior of $M$ is not correct

⟶ Mean field theory only gives a qualitative behavior of an antiferromagnet

# Spin wave theory

# Spin wave theory: introduction

Heisenberg Hamiltonian (antiferromagnet $J > 0$):

$$H = J\sum_{j\delta} \vec{S}_j \cdot \vec{S}_{j+\delta} = J\sum_{j\delta}\left(S_j^x S_{j+\delta}^x + S_j^y S_{j+\delta}^y + S_j^z S_{j+\delta}^z\right)$$

with $S^{\pm} = S^x \pm iS^y$

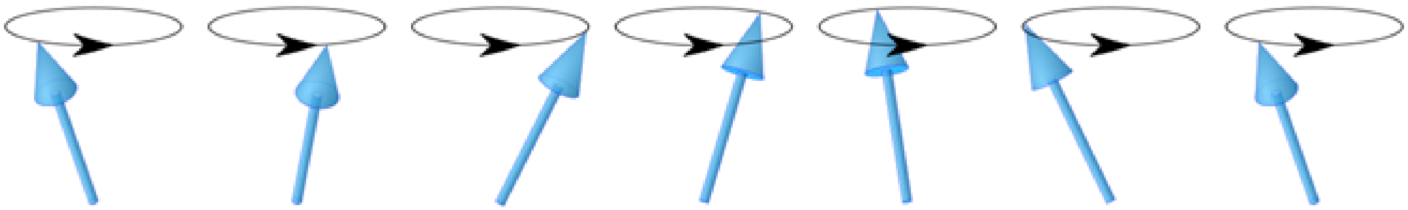
$$\longrightarrow \quad H = J\sum_{j\delta}\Big\{\underbrace{S_j^z S_{j+\delta}^z}_{(1)} + \underbrace{\frac{1}{2}\left(S_j^+ S_{j+\delta}^- + S_j^- S_{j+\delta}^+\right)}_{(2)}\Big\}$$

(1) energy gain through antiparallel orientation (classical Néel state)

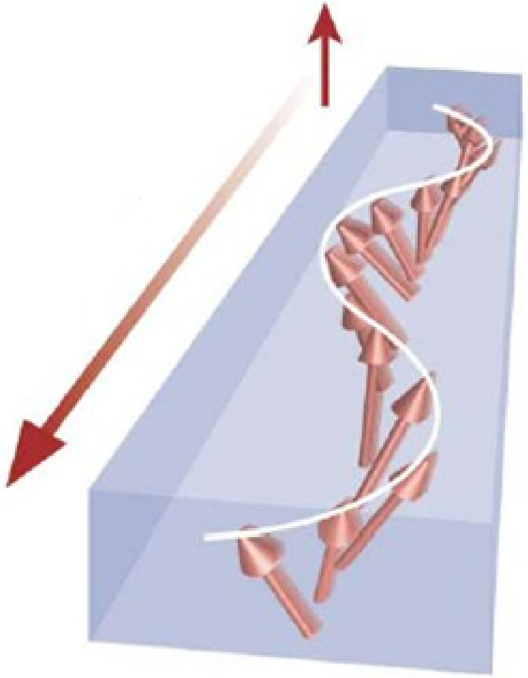
(2) $S^{\pm}$-operators produce a propagating spin flip $\longrightarrow$ propagation of a spin wave

# Spin wave theory: introduction

oscillations about Néel state:



normal modes are spin waves:



- creation of a quasi-particle: magnon (boson)
- aim: rewrite spin operators in terms of boson creation and annihilation operators

# Spin wave theory: antiferromagnetism

## Holstein Primakoff transformation:

division into 2 sublattices A and B:

| A (spin up) | B (spin down) |
|---|---|
| $S_{Aj}^{+} = \sqrt{2S}\left(1 - \frac{a_j^\dagger a_j}{2S}\right)^{\frac{1}{2}} a_j$ | $S_{Bl}^{+} = \sqrt{2S}\, b_l^\dagger \left(1 - \frac{b_l^\dagger b_l}{2S}\right)^{\frac{1}{2}}$ |
| $S_{Aj}^{-} = \sqrt{2S}\, a_j^\dagger \left(1 - \frac{a_j^\dagger a_j}{2S}\right)^{\frac{1}{2}}$ | $S_{Bl}^{-} = \sqrt{2S}\left(1 - \frac{b_l^\dagger b_l}{2S}\right)^{\frac{1}{2}} b_l$ |
| $S_{Aj}^{z} = S - a_j^\dagger a_j$ | $S_{Bl}^{z} = S - b_l^\dagger b_l$ |

Fulfills $[S_j^+, S_j^-] = 2S_j^z$

# Spin wave theory: antiferromagnetism

assumption: low temperatures ⟶ only few excited magnons

$$\frac{\langle a_j^\dagger a_j \rangle}{S} = \frac{\langle n_j \rangle}{S} \ll 1$$

$$S_{Aj}^+ = \sqrt{2S}\left(1 - \frac{a_j^\dagger a_j}{2S}\right)^{\frac{1}{2}} a_j \approx \sqrt{2S}\left(a_j - \frac{a_j^\dagger a_j a_j}{4S}\right)$$

| A (spin up) | B (spin down) |
|---|---|
| $S_{Aj}^+ = \sqrt{2S}\left(a_j - \frac{a_j^\dagger a_j a_j}{4S}\right)$ | $S_{Bl}^+ = \sqrt{2S}\left(b_l^\dagger - \frac{b_l^\dagger b_l^\dagger b_l}{4S}\right)$ |
| $S_{Aj}^- = \sqrt{2S}\left(a_j^\dagger - \frac{a_j^\dagger a_j^\dagger a_j}{4S}\right)$ | $S_{Bl}^- = \sqrt{2S}\left(b_l - \frac{b_l^\dagger b_l b_l}{4S}\right)$ |
| $S_{Aj}^z = S - a_j^\dagger a_j$ | $S_{Bl}^z = S - b_l^\dagger b_l$ |

# Spin wave theory: antiferromagnetism

propagating excitations $\longrightarrow$ $\vec{k}$-space operators:

| A (spin up) | B (spin down) |
|---|---|
| $a_j^\dagger = \sqrt{\frac{2}{N}} \sum_{\vec{k}} e^{i\vec{k}\vec{j}} c_{\vec{k}}^\dagger$ | $b_l^\dagger = \sqrt{\frac{2}{N}} \sum_{\vec{k}} e^{i\vec{k}\vec{l}} d_{\vec{k}}^\dagger$ |
| $a_j = \sqrt{\frac{2}{N}} \sum_{\vec{k}} e^{-i\vec{k}\vec{j}} c_{\vec{k}}$ | $b_l = \sqrt{\frac{2}{N}} \sum_{\vec{k}} e^{-i\vec{k}\vec{l}} d_{\vec{k}}$ |

satisfy boson commutation relations:

$\left[c_{\vec{k}}, c_{\vec{k'}}^\dagger\right]_- = \delta_{\vec{k}\vec{k'}}$ and $\left[c_{\vec{k}}, c_{\vec{k'}}\right]_- = \left[c_{\vec{k}}^\dagger, c_{\vec{k'}}^\dagger\right]_- = 0$

# Spin wave theory: antiferromagnetism

$$H = J\sum_{j\delta}\left\{S^z_{Aj}S^z_{Aj+\delta} + \frac{1}{2}\left(S^+_{Aj}S^-_{Aj+\delta} + S^-_{Aj}S^+_{Aj+\delta}\right)\right\} + J\sum_{j\delta}\left\{S^z_{Bj}S^z_{Bj+\delta} + \frac{1}{2}\left(S^+_{Bj}S^-_{Bj+\delta} + S^-_{Bj}S^+_{Bj+\delta}\right)\right\}$$

| A (spin up) | B (spin down) |
| --- | --- |
| $S^+_{Aj} = \sqrt{2S}\left(a_j - \frac{a_j^\dagger a_j a_j}{4S}\right)$ <br> $S^-_{Aj} = \sqrt{2S}\left(a_j^\dagger - \frac{a_j^\dagger a_j^\dagger a_j}{4S}\right)$ <br> $S^z_{Aj} = S - a_j^\dagger a_j$ | $S^+_{Bl} = \sqrt{2S}\left(b_l^\dagger - \frac{b_l^\dagger b_l^\dagger b_l}{4S}\right)$ <br> $S^-_{Bl} = \sqrt{2S}\left(b_l - \frac{b_l^\dagger b_l b_l}{4S}\right)$ <br> $S^z_{Bl} = S - b_l^\dagger b_l$ |
| $a_j^\dagger = \sqrt{\frac{2}{N}}\sum_{\vec{k}} e^{i\vec{k}\vec{j}} c_{\vec{k}}^\dagger$ <br> $a_j = \sqrt{\frac{2}{N}}\sum_{\vec{k}} e^{-i\vec{k}\vec{j}} c_{\vec{k}}$ | $b_l^\dagger = \sqrt{\frac{2}{N}}\sum_{\vec{k}} e^{i\vec{k}\vec{l}} d_{\vec{k}}^\dagger$ <br> $b_l = \sqrt{\frac{2}{N}}\sum_{\vec{k}} e^{-i\vec{k}\vec{l}} d_{\vec{k}}$ |

# Spin wave theory: antiferromagnetism

Spin wave theory:

$$H = -\frac{Nz}{2}JS^2 + H_0 + H_1$$

classical Néel energy

bilinear terms of c-,d-operators

higher order terms (magnon-magnon interaction)

$$H_0 = JzS\sum_{\vec{k}}\left[\gamma_{\vec{k}}\left(c_{\vec{k}}d_{-\vec{k}} + d^\dagger_{\vec{k}}c^\dagger_{-\vec{k}}\right) + \left(c^\dagger_{\vec{k}}c_{\vec{k}} + d^\dagger_{\vec{k}}d_{\vec{k}}\right)\right]$$

with $\gamma_{\vec{k}} = \frac{1}{z}\sum_\delta e^{i\vec{k}\vec{\delta}}$

➡ aim: diagonalization of $H_0$

# Spin wave theory: antiferromagnetism

rewrite $H_0$:

$$\frac{1}{zJS}H_0 = \frac{1}{2}\left(\sum_{\vec{k}} H_{\vec{k}}^{(1)} + \sum_{\vec{k}} H_{\vec{k}}^{(2)}\right)$$

$$H_{\vec{k}}^{(1)} = \gamma_{\vec{k}}\left(c_{\vec{k}}^{\dagger} d^{\dagger}_{-\vec{k}} + d_{-\vec{k}} c_{\vec{k}}\right) + \left(c_{\vec{k}}^{\dagger} c_{\vec{k}} + d^{\dagger}_{-\vec{k}} d_{-\vec{k}}\right)$$
$$H_{\vec{k}}^{(2)} = \gamma_{\vec{k}}\left(d_{\vec{k}}^{\dagger} c^{\dagger}_{-\vec{k}} + c_{-\vec{k}} d_{\vec{k}}\right) + \left(c^{\dagger}_{-\vec{k}} c_{-\vec{k}} + d_{\vec{k}}^{\dagger} d_{\vec{k}}\right)$$

Diagonalization of $H_{\vec{k}}^{(1)} \longrightarrow H_{\vec{k}}^{(1)} = \lambda_{\vec{k}} \alpha_{\vec{k}}^{\dagger} \alpha_{\vec{k}}$

Bogolyubov transformation:

$$\alpha_{\vec{k}} = u_{\vec{k}} c_{\vec{k}} - v_{\vec{k}} d^{\dagger}_{-\vec{k}}$$
$$\alpha_{\vec{k}}^{\dagger} = u_{\vec{k}} c_{\vec{k}}^{\dagger} - v_{\vec{k}} d_{-\vec{k}}$$

$u_{\vec{k}}, v_{\vec{k}} \in \mathbb{R}$

$u_{\vec{k}}^2 - v_{\vec{k}}^2 = 1 \longrightarrow \left[\alpha_{\vec{k}}, \alpha_{\vec{k}}^{\dagger}\right]_- = 1$

# Spin wave theory: antiferromagnetism

$$H^{(1)}_{\vec{k}} = \lambda_{\vec{k}} \alpha^{\dagger}_{\vec{k}} \alpha_{\vec{k}} \longrightarrow \left[\alpha_{\vec{k}}, H^{(1)}_{\vec{k}}\right]_{-} = \lambda_{\vec{k}} \alpha_{\vec{k}} \quad \text{and} \quad \left[\alpha^{\dagger}_{\vec{k}}, H^{(1)}_{\vec{k}}\right]_{-} = -\lambda_{\vec{k}} \alpha^{\dagger}_{\vec{k}}$$

$$\alpha_{\vec{k}} = u_{\vec{k}} c_{\vec{k}} - v_{\vec{k}} d^{\dagger}_{-\vec{k}}$$

$$\alpha^{\dagger}_{\vec{k}} = u_{\vec{k}} c^{\dagger}_{\vec{k}} - v_{\vec{k}} d_{-\vec{k}}$$

$$H^{(1)}_{\vec{k}} = \gamma_{\vec{k}} \left( c^{\dagger}_{\vec{k}} d^{\dagger}_{-\vec{k}} + d_{-\vec{k}} c_{\vec{k}} \right) + \left( c^{\dagger}_{\vec{k}} c_{\vec{k}} + d^{\dagger}_{-\vec{k}} d_{-\vec{k}} \right)$$

$$\underbrace{\begin{pmatrix} 1 - \lambda_{\vec{k}} & \gamma_{\vec{k}} \\ \gamma_{\vec{k}} & 1 - \lambda_{\vec{k}} \end{pmatrix}}_{det = 0} \begin{pmatrix} u_{\vec{k}} \\ v_{\vec{k}} \end{pmatrix} = 0$$

$$\lambda^2_{\vec{k}} = 1 - \gamma^2_{\vec{k}} \longrightarrow \text{determination of } u_{\vec{k}}, v_{\vec{k}}$$

# Spin wave theory: antiferromagnetism

$$H_0 = -\frac{JzN}{2}S(S+1) + JzS\sum_{\vec{k}}\sqrt{1-\gamma_{\vec{k}}^2}\left\{\left(\alpha_{\vec{k}}^{\dagger}\alpha_{\vec{k}}+\frac{1}{2}\right)+\left(\beta_{\vec{k}}^{\dagger}\beta_{\vec{k}}+\frac{1}{2}\right)\right\}$$

$$\hbar\omega_{\vec{k}} = JzS\sqrt{1-\gamma_{\vec{k}}^2} \qquad \left(\gamma_{\vec{k}} = \frac{1}{z}\sum_{\delta} e^{i\vec{k}\vec{\delta}}\right)$$

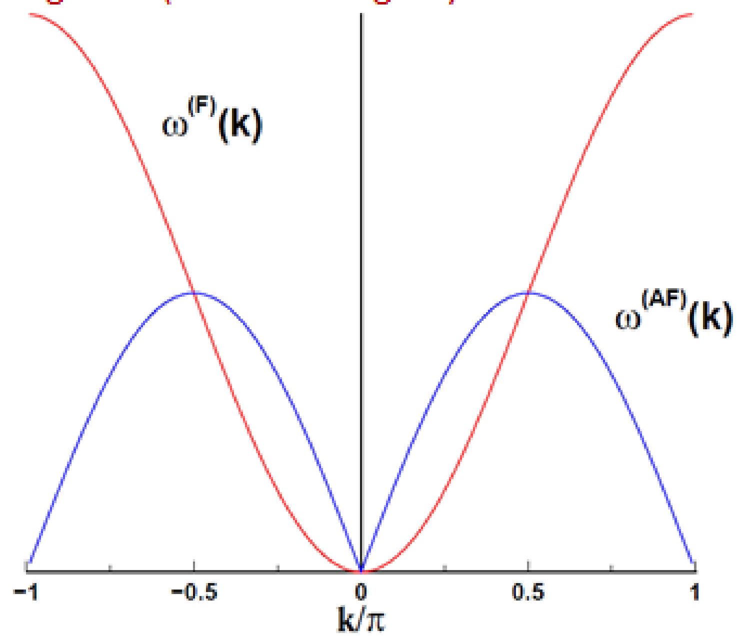
dispersion relation for magnons (antiferromagnet)

long wavelength-limit $a|\vec{k}| \ll 1$ :

$$\sqrt{1-\gamma_{\vec{k}}^2} \approx a|\vec{k}|$$

$$\rightarrow \quad \hbar\omega_{\vec{k}} \approx JzSa|\vec{k}|$$

linear dispersion



# Spin wave theory: antiferromagnetism

$$H = E_0 + \sum_{\vec{k}} \hbar\omega_{\vec{k}} \left(\alpha_{\vec{k}}^{\dagger}\alpha_{\vec{k}} + \beta_{\vec{k}}^{\dagger}\beta_{\vec{k}}\right)$$

ground state energy:

$$E_0 = \underbrace{-\frac{1}{2}NzJS^2}_{\text{classical Néel energy}} + \underbrace{JzS\sum_{\vec{k}}\left(\sqrt{1-\gamma_{\vec{k}}^2}-1\right)}_{\text{excitation due to quantum fluctuation}}$$

magnetization per spin (in ground state):

$$m = S \underbrace{- \frac{1}{N}\sum_{\vec{k}}\left(\frac{1}{\sqrt{1-\gamma_{\vec{k}}^2}}-1\right)}_{\text{spin reduction}}$$

# Summary: Mean field theory

Heisenberg model:

$$H = -\sum_{n,m} J(\vec{R}_m - \vec{R}_n)\vec{S}_m \cdot \vec{S}_n$$

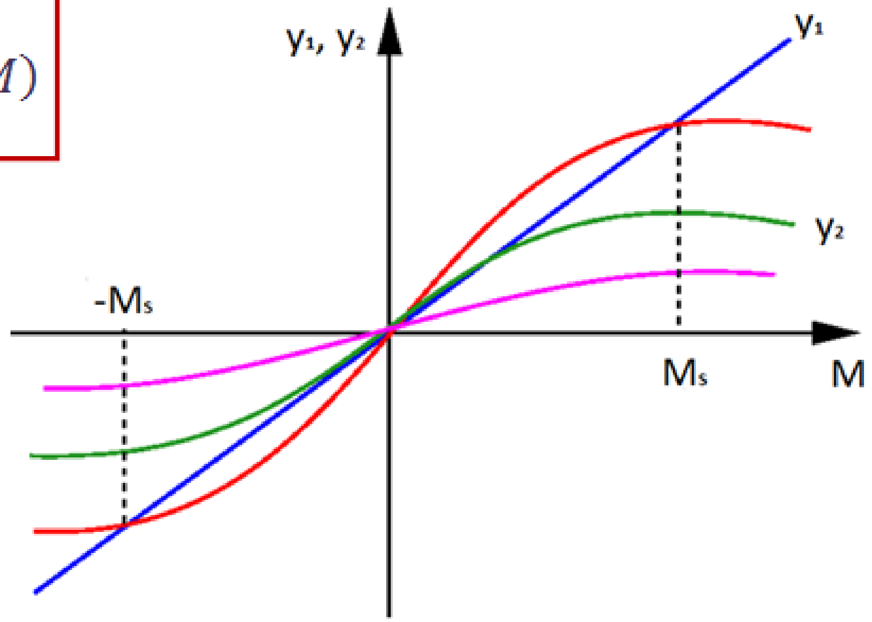
MFT:

$$H = -g\mu_B \sum_{n^+} \vec{S}_n \cdot \vec{B}_+ - g\mu_B \sum_{n^-} \vec{S}_n \cdot \vec{B}_- \quad \text{with} \quad \vec{B}_\pm = -\mu_0 \Gamma \vec{M}_\pm - \mu_0 A \vec{M}_\mp$$

$$M = \frac{N}{2} g\mu_B S \cdot \mathcal{B}_S(\beta g \mu_B S \mu_0 (A - \Gamma) M)$$

$$T_N = \frac{C}{2}(A - \Gamma)$$

$$M_\pm(T = 0) = M_\pm^{max}$$



# Summary: Spin wave theory

Heisenberg model:

$$H = J\sum_{j\delta}\left\{S^z_{Aj}S^z_{Aj+\delta} + \frac{1}{2}\left(S^+_{Aj}S^-_{Aj+\delta} + S^-_{Aj}S^+_{Aj+\delta}\right)\right\} + J\sum_{j\delta}\left\{S^z_{Bj}S^z_{Bj+\delta} + \frac{1}{2}\left(S^+_{Bj}S^-_{Bj+\delta} + S^-_{Bj}S^+_{Bj+\delta}\right)\right\}$$

- Holstein-Primakoff transformation
- $\vec{k}$-space operators
- Bogolyubov transformation

$$H = E_0 + \sum_{\vec{k}} \hbar\omega_{\vec{k}}\left(\alpha^\dagger_{\vec{k}}\alpha_{\vec{k}} + \beta^\dagger_{\vec{k}}\beta_{\vec{k}}\right)$$

$$E_0 = -\frac{1}{2}NzJS^2 + JzS\sum_{\vec{k}}\left(\sqrt{1-\gamma^2_{\vec{k}}} - 1\right)$$

$$\hbar\omega_{\vec{k}} \approx JzSa|\vec{k}|$$

$$m = S - \frac{1}{N}\sum_{\vec{k}}\left(\frac{1}{\sqrt{1-\gamma^2_{\vec{k}}}} - 1\right)$$