Übungsblatt 6

[due: July 18]

Problem 1: XY **model partition function**

The XY model consists of spins attached to lattice sites with nearest neighbour interactions. The lattice can be in any number of spatial dimensions, but the spins all rotate in the same two-dimensional plane. Thus

$$s = (\vec{s}_1, \ldots, \vec{s}_N), \quad |\vec{s}_i|^2 = 1, \quad J_{ij} = \begin{cases} J & \text{if } i,j \text{ nearest neighbours} \\ 0 & \text{otherwise} \end{cases}$$

and $(s, Js) = \sum_{i,j} \vec{s}_i \cdot J_{ij} \vec{s}_j$.

Put the partition function of the XY model in the Landau form, by methods similar to that used for the Ising model in the lecture.

Problem 2: **Correlation functions in the Gaussian model**

First, show that the Lagrangian of the Gaussian model in Fourier space is given by

$$L = \frac{1}{V} \sum_{\mathbf{k}} \frac{1}{2} |\eta(\mathbf{k})|^2 (2at + \gamma \mathbf{k}^2) + a_0 V$$

where a_0 is a constant. Show that the expectation value of the two point correlation functions $\langle \eta(\mathbf{k}) \eta(\mathbf{k}') \rangle$ is non-zero only when $\mathbf{k} = -\mathbf{k}$, and that in this case it has the value

$$\langle \eta(\mathbf{k}) \eta(\mathbf{k}') \rangle = \frac{TV}{2at + \gamma \mathbf{k}^2}.$$

You have to evaluate the integral

$$\langle \eta(\mathbf{k}) \eta(\mathbf{k}') \rangle = \frac{\int D\eta \; \eta(\mathbf{k}) \eta(\mathbf{k}') e^{-\beta L}}{\int D\eta \; e^{-\beta L}}.$$

Note that $\eta(\mathbf{k}) = \eta^*(-\mathbf{k})$. Consider the three cases $\mathbf{k} \neq \pm \mathbf{k}', \mathbf{k} = \mathbf{k}'$ and $\mathbf{k} = -\mathbf{k}$.