Übungsblatt 5

[due: July 4]

Problem 1: **Latent heat from Landau theory**

Consider the Landau free energy

$$\mathcal{L}(\eta, T) = d(T) + \frac{a}{2}\eta^2 + \frac{b}{4}\eta^4 + \frac{c}{6}\eta^6$$

and assume that $a > 0, b < 0$, so that a first-order transition takes place. Derive an expression for the latent heat of the first-order transition. Now let a, b be variable, e.g. $a(T,P), b(T,P)$, and discuss what happens with the latent heat as the tricritical point is approached along the first-order line.

Problem 2: **Correlation functions in Landau theory**

a) The usual expression for the Landau function used in calculating correlation functions is given by

$$L = \int d^3x \left[\mathcal{L}(\eta(\mathbf{x}) + \frac{\gamma}{2}(\nabla\eta(\mathbf{x}))^2\right]. \tag{1}$$

Another possible term consistent with the same symmetries would have been $\eta(\mathbf{x})\nabla^2\eta(\mathbf{x})$. Show that $\int_V d^3x\, \eta\nabla^2\eta$ is proportional to $\int_V d^3x (\nabla\eta)^2$ up to a surface term. The surface term is usually neglected in the thermodynamic limit. Explain why that is justified.

b) In the lecture we have solved the equation for the Green function

$$(-\nabla^2 + \xi^{-2})G(\mathbf{x} - \mathbf{x}') = \frac{1}{\beta\gamma}\delta^3(\mathbf{x} - \mathbf{x}') \tag{2}$$

in terms of Bessel functions. Now solve it in Fourier space and transform the solution back to real space. Discuss your solution also for $T \to T_c$, where $\xi \to \infty$.

Useful integral:

$$\int_0^\infty dx\, \frac{x \sin x}{x^2 + a^2} = \frac{\pi}{2}e^{-a}$$