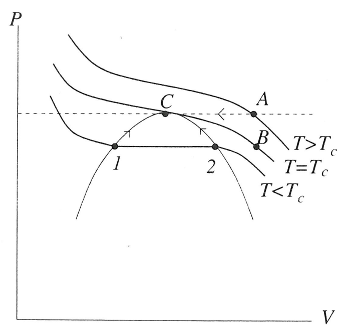
Übungsblatt 4

[due: June 20]

Problem 1: Scaling hypothesis



Consider a fluid near its critical point, with isotherms illustrated qualitatively in the figure. For $T \geq T_c$ assume that the Gibbs free energy of the fluid has a singular part satisfying the scaling form

$$G \sim t^{2-\alpha}\mathcal{G}_+(p/t^\Delta), \quad \text{with} \quad p = \frac{P - P_c}{P_c}, \quad t = \frac{T - T_c}{T_c}, \tag{1}$$

where $\mathcal{G}_+$ is a homogeneous function of p/t^Δ.
Approaching the critical point along $p = 0$ (path AC in the figure), calculate the exponents $x(q)$ that describe the singular parts of the following quantities q, such that $q \sim t^{x(q)}$: Entropy S, heat capacity at constant pressure C_P, isothermal compressibility κ_T, thermal expansion coefficient α. Compare with the exponents given in the lecture.

Problem 2: Specific heat from Landay theory
For an Ising model with $H = 0$, the Landau free energy can be written as

$$\mathcal{L} = at\eta^2 + \frac{b}{2}\eta^4, \quad t = \frac{T - T_c}{T_c}. \tag{2}$$

By eliminating η, write down $\mathcal{L}$ in terms of a, b, t for $T < T_c$ as well as for $T > T_c$. The thermodynamics can be obtained by treating $\mathcal{L}$ like the Gibbs free energy. Thus, define the specific heat as $C_V = -T(\partial^2\mathcal{L}/\partial T^2)$ and compute it for both temperature regimes. Show that there is a discontinuous jump in C_V at $T = T_c$. What is the value of the critical index α? Discuss the meaning of your result.