Übungsblatt 2

[due: May 23]

Problem 1: Lee-Yang zeroes

Consider a system with grand partition function

$$Z(z,V) = (1+z)^V (1+z^{\alpha V}),$$

where α is a positive constant.

a) Find the roots of $Z(z,V) = 0$ in the complex z-plane at fixed V. Show that as $V \to \infty$ the roots converge toward the real axis at $z = 1$.

b) Consider the equation of state in parametric form,

$$\frac{P}{T} = V^{-1} \ln Z(z,V); \quad \frac{1}{v} = V^{-1} z \frac{\partial}{\partial z} \ln Z(z,V),$$

and show that there is a first-order phase transition. Find the specific volumes of the two phases (i.e. when the entire system is in one or the other phase).
Hint: Examine the behaviour of $1/v$ in the limit $V \to \infty$.

c) Find the equation of state in the gas phase. Show that the continuation of this equation beyond the phase transition density fails to show any sign of the transition. This demonstrates that the order of the operations $z(\partial/\partial z)$ and $V \to \infty$ can be interchanged *only* within a single-phase region!

Problem 2: 1d Ising model

Consider the 1d Ising model for $H = 0$ with open boundary conditions (i.e.. the boundary spins are unconstrained). Calculate the partition function exactly and compare with the case of periodic boundary conditions. How does the difference influence the thermodynamical properties?