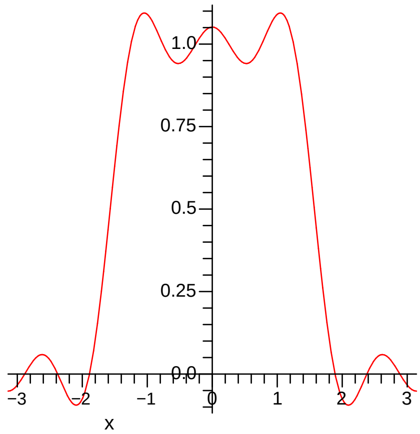
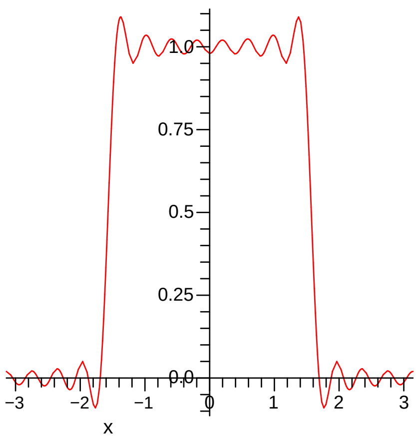
```
> restart:

> # #####################
  # Uebung 8, Aufgabe 2 b)
  # #####################

> N := 5:

  g_expansion := 0:

  for n from 0 to N do

    if (n = 0) then
      g_expansion := g_expansion + (sqrt(Pi)*V0/sqrt(2)) * (1/(sqrt
  (2*Pi))):
    fi:

    if ((n mod 4) = 1) then
      g_expansion := g_expansion + 2*V0/(sqrt(Pi)*n) * cos(n*x)/sqrt
  (Pi):
    fi:

    if ((n mod 4) = 3) then
      g_expansion := g_expansion - 2*V0/(sqrt(Pi)*n) * cos(n*x)/sqrt
  (Pi):
    fi:

  od:

  # g_expansion;

  plot(subs(V0=1, g_expansion), x=-Pi..+Pi);

  # Fehlerabschaetzung.

  Delta := simplify((int(g_expansion^2, x=-Pi..-Pi/2) + int(
  (g_expansion-V0)^2, x=-Pi/2..+Pi/2) +
    int(g_expansion^2, x=+Pi/2..+Pi)) / (2*Pi));

  evalf(subs(V0=1.0, Delta));
```



$$\Delta := \frac{1}{900} \frac{V0^2 \left(225\pi^2 - 2072\right)}{\pi^2}$$

$$0.01673611952 \qquad (1)$$

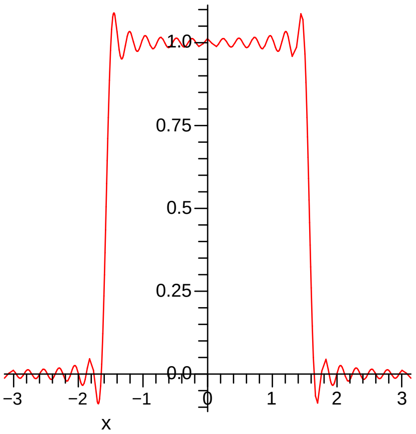
```
> N := 15:

  g_expansion := 0:

  for n from 0 to N do

    if (n = 0) then
      g_expansion := g_expansion + (sqrt(Pi)*V0/sqrt(2)) * (1/(sqrt
  (2*Pi))):
    fi:

    if ((n mod 4) = 1) then
      g_expansion := g_expansion + 2*V0/(sqrt(Pi)*n) * cos(n*x)/sqrt
  (Pi):
    fi:

    if ((n mod 4) = 3) then
      g_expansion := g_expansion - 2*V0/(sqrt(Pi)*n) * cos(n*x)/sqrt
  (Pi):
    fi:

  od:

  # g_expansion;

  plot(subs(V0=1, g_expansion), x=-Pi..+Pi);

  # Fehlerabschaetzung.

  Delta := simplify((int(g_expansion^2, x=-Pi..-Pi/2) + int(
  (g_expansion-V0)^2, x=-Pi/2..+Pi/2) +
    int(g_expansion^2, x=+Pi/2..+Pi)) / (2*Pi));

  evalf(subs(V0=1.0, Delta));
```



$$\Delta := \frac{1}{1623241620} \frac{V0^2 \left(-3903866944 + 405810405\pi^2\right)}{\pi^2}$$

$$0.006324373034 \qquad (2)$$

```
> N := 25:

  g_expansion := 0:

  for n from 0 to N do

    if (n = 0) then
      g_expansion := g_expansion + (sqrt(Pi)*V0/sqrt(2)) * (1/(sqrt
  (2*Pi))):
    fi:

    if ((n mod 4) = 1) then
      g_expansion := g_expansion + 2*V0/(sqrt(Pi)*n) * cos(n*x)/sqrt
  (Pi):
    fi:

    if ((n mod 4) = 3) then
      g_expansion := g_expansion - 2*V0/(sqrt(Pi)*n) * cos(n*x)/sqrt
  (Pi):
    fi:

  od:

  # g_expansion;

  plot(subs(V0=1, g_expansion), x=-Pi..+Pi);

  # Fehlerabschaetzung.

  Delta := simplify((int(g_expansion^2, x=-Pi..-Pi/2) + int(
  (g_expansion-V0)^2, x=-Pi/2..+Pi/2) +
    int(g_expansion^2, x=+Pi/2..+Pi)) / (2*Pi));

  evalf(subs(V0=1.0, Delta));
```



$$\Delta := \frac{1}{11198346445088302500}\,\frac{VO^2\left(-27200318649043270568+27995866112720756 25\,\pi^2\right)}{\pi^2}$$

$$0.003895051061 \qquad (3)$$

```
> # #####################
  # Uebung 8, Aufgabe 2 c)
  # #####################

> # Monombasis.

  N := 11:

  g := array(0..N):

  for i1 from 0 to N do
    g[i1] := x^i1:
  od:

> # Schmidtsches Orthonormalisierungsverfahren.

  h := array(0..N):

  for i1 from 0 to N do

    h[i1] := g[i1]:

    for i2 from 0 to i1-1 do
      h[i1] := h[i1] - h[i2] * int(h[i2]*h[i1], x=-Pi..+Pi):
    od:

    norm_ := sqrt(int(h[i1]*h[i1], x=-Pi..+Pi)):
    h[i1] := h[i1] / norm_:

    print(simplify(h[i1])):

  od:

  plot([h[0], h[1], h[2], h[3]], x=-Pi..+Pi);
```

$$\frac{1}{2}\,\frac{\sqrt{2}}{\sqrt{\pi}}$$

$$\frac{1}{2}\,\frac{x\sqrt{6}}{\pi^{3/2}}$$

$$\frac{1}{4}\,\frac{\left(3x^2-\pi^2\right)\sqrt{10}}{\pi^{5/2}}$$

$$\frac{1}{4}\,\frac{x\left(5x^2-3\pi^2\right)\sqrt{14}}{\pi^{7/2}}$$

$$\frac{3}{16}\,\frac{\left(35x^4+3\pi^4-30\pi^2x^2\right)\sqrt{2}}{\pi^{9/2}}$$

$$\frac{1}{16}\,\frac{x\left(63x^4+15\pi^4-70\pi^2x^2\right)\sqrt{22}}{\pi^{11/2}}$$

$$\frac{1}{32}\,\frac{\left(231x^6-5\pi^6+105\pi^4x^2-315\pi^2x^4\right)\sqrt{26}}{\pi^{13/2}}$$

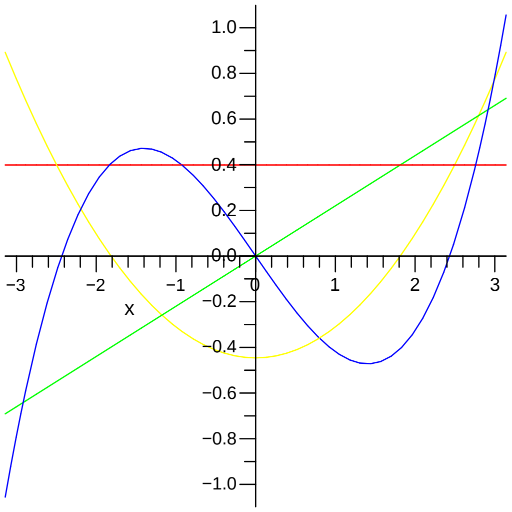
$$\frac{1}{32}\,\frac{x\left(429x^6-35\pi^6+315\pi^4x^2-693\pi^2x^4\right)\sqrt{30}}{\pi^{15/2}}$$

$$\frac{1}{256}\,\frac{\left(6435x^8+35\pi^8-1260\pi^6x^2+6930\pi^4x^4-12012\pi^2x^6\right)\sqrt{34}}{\pi^{17/2}}$$

$$\frac{1}{256}\,\frac{x\left(12155x^8+315\pi^8-4620\pi^6x^2+18018\pi^4x^4-25740\pi^2x^6\right)\sqrt{38}}{\pi^{19/2}}$$

$$\frac{1}{512}\,\frac{1}{\pi^{21/2}}\left(\left(46189x^{10}-63\pi^{10}+3465\pi^8x^2-30030\pi^6x^4+90090\pi^4x^6-109395\pi^2x^8\right)\sqrt{42}\right)$$

$$\frac{1}{512}\,\frac{1}{\pi^{23/2}}\left(x\left(88179x^{10}-693\pi^{10}+15015\pi^8x^2-90090\pi^6x^4+218790\pi^4x^6-230945\pi^2x^8\right)\sqrt{46}\right)$$




```
> # Entwickeln der "Potentialstufe" in der Polynombasis.

  c := array(0..N):

  g_expansion := 0:

  for i1 from 0 to N do

    c := int(h[i1]*V0, x=-Pi/2..+Pi/2):
    print(simplify(c)):

    g_expansion := g_expansion + c * h[i1]:

  od:

  plot(subs(V0=1, g_expansion), x=-Pi..+Pi);

  # Fehlerabschaetzung.

  Delta := simplify((int(g_expansion^2, x=-Pi..-Pi/2) + int(
  (g_expansion-V0)^2, x=-Pi/2..+Pi/2) +
    int(g_expansion^2, x=+Pi/2..+Pi)) / (2*Pi));

  evalf(subs(V0=1.0, Delta));

  # N =  3  -->  Fehler 0.0742
  # N =  7  -->  Fehler 0.0420
  # N = 11  -->  Fehler 0.0225
```

$$\frac{1}{2}\sqrt{2}\,\sqrt{\pi}\,VO$$

$$0$$

$$-\frac{3}{16}\sqrt{10}\,\sqrt{\pi}\,VO$$

$$0$$

$$\frac{45}{256}\sqrt{2}\,\sqrt{\pi}\,VO$$

$$0$$

$$\frac{21}{2048}\sqrt{26}\,\sqrt{\pi}\,VO$$

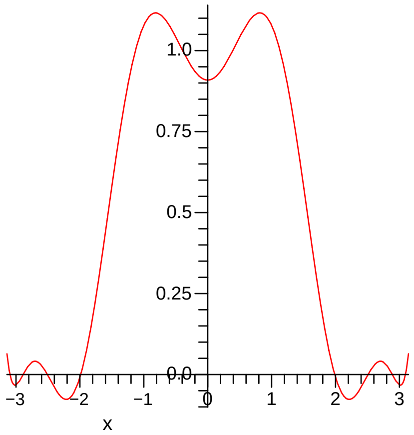
$$0$$

$$\frac{1893}{65536}\sqrt{34}\,\sqrt{\pi}\,VO$$

$$0$$

$$\frac{8283}{524288}\sqrt{42}\,\sqrt{\pi}\,VO$$

0



$$\Delta := \frac{6192348787}{274877906944} V0^2$$

$$0.02252763365 \quad (4)$$

>